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Probable Momentum Spectra in the Atmosphere of Long-Lived Heavy Triplets Produced by Primary Cosmic Rays

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A model of high-energy collision processes is constructed on the basis that collisions can be described in terms of a four-momentum transfer q, and that the spectrum of four-momentum transfers varies as $\exp(-q/q_0)$ where q_0 is a parameter of the order of magnitude of 500 MeV/c. The model is used for Monte Carlo calculations of the diffusion through the atmosphere of heavy triplets produced by the nucleons of the primary cosmic radiation. The model parameters are established by fitting the attenuation length of the primary protons in their course through the atmosphere.

I. INTRODUCTION

THE hypothesis that the elementary particles can be associated with elements of irreducible representations of the group SU_3, and that the larger part of the strong interaction is invariant with respect to the transformations of the group, has met with striking success.[1] Only those representations containing elements which occupy the center of the element space ($Y=0$, $T_3=0$) correspond to sets of observed particles. The spinor representations, which do not occupy the center, are not filled. This is analogous to a conceivable, though nonexistent, situation in SU_2 structures, such as spin, where observable particles might all belong to those representations which include an element such that m, the third component of the spin, would be zero; i.e., only integral spins would exist.

All representations of SU_2 can be constructed through the multiplication operation from the fundamental spinor representation of dimension 2, i.e., spin $\frac{1}{2}$. In a similar fashion, all representations of SU_3 can be constructed from spinor representations of dimension 3. It is then attractive to consider that there are one or more sets of three basic spinor particles which are equivalent with respect to the main part of the strong interaction, and that the various meson and baryon states are "compounds" of these fundamental particles. The existence of such particles, which have not yet been observed, appears to be a sufficient, though not necessary, condition in the understanding of much of the strong-interaction phenomena observed at this time.

These particles, which we call here, generically, triplets, are probably quite heavy. The mass-splitting perturbation to the symmetric part of the interaction is small inasmuch as the mass splitting is given, quite precisely, by first-order perturbation theory.[2] We then expect that the symmetric interaction is very strong and that the triplets are correspondingly massive.

According to various views these triplets may have third-integral charges[3]—in which case we call them quarks; or they may have integral charges[4]—in which case we retain the nomenclature "triplet."

The existence of such heavy triplets would be suggested by the establishment of the existence of a heavy, nearly stable, particle of unit charge. The mass of nonrelativistic particles can be measured by simultaneously determining the momentum and velocity of the particle. Heavy triplets, produced by the interaction of high-energy nucleons of the primary cosmic radiation, will likely be produced with velocities quite high in the laboratory system—so near to c that only extremely precise measurements of the velocity are informative. If the particles are moderated in energy by their interactions with nucleons in the atmosphere, it may be possible to combine simple time-of-flight velocity measurements with momentum measurements to determine the mass. Appreciable momentum acceptances and large angular acceptances can be achieved with such techniques.

Lacking a complete theory of dynamics, there is no possibility of calculating an exact production cross section and detailed momentum distribution of the heavy triplets produced by high-energy nucleon-nucleon interactions in the upper atmosphere. Definitive calculations concerning the history of these particles passing through the atmosphere are likewise not possible at this time. It is, however, possible to devise plausible and well-defined models, both for the production of triplets and for their moderation by interactions with the nucleons in the atmosphere. These models can be used to predict momentum spectra at various depths and at various resultant angles in the atmosphere. Hopefully, important qualitative features of the spectra will be largely model-independent. While such features must depend on the broad and plausible foundations of the models, they may be largely independent of choice of detail.

[1] M. Gell-Mann, Phys. Rev. **125**, 1067 (1962); Y. Ne'eman, Nucl. Phys. **26**, 222 (1961).
[2] M. Gell-Mann, Phys. Rev. **125**, 1067 (1962); S. Okubo, Progr. Theoret. Phys. (Kyoto) **27**, 949 (1962).
[3] M. Gell-Mann, Phys. Letters **8**, 214 (1964); G. Zweig, CERN Report (unpublished).
[4] F. Gürsey, T. D. Lee, and M. Nauenburg, Phys. Rev. **135**, B467 (1964).

We have constructed such a model of production and interaction of heavy charged particles, and we have investigated the variation of the predicted spectra with the values of model parameters and with model assumptions. It appears that useful quantitative predictions concerning such spectra have been obtained.

II. PRODUCTION CROSS SECTIONS

The cross section for any particular production channel (i) tends to rise immediately above threshold even as the phase space available for that particular channel increases; the cross section reaches a maximum and then declines as new channels open and the total phase space available to all channels increases faster than that for channel (i). The probability of the production of a particular particle does not necessarily decrease as the result of this radiative damping, however, as some of the new channels which are opened are channels in which the particular particle is produced along with other particles. Even as the cross section for the reaction $p+p\rightarrow p+p+p+\bar{p}$, producing antiprotons, rises from threshold and then decreases, cross sections for reactions such as $p+p\rightarrow p+p+\bar{p}+n\pi$, where n is an integer, increase, and the total probability of production of antiprotons through all channels continues to increase slowly with energy.[5]

The cross section for the production of antiprotons by proton-nucleon interactions rises from threshold to a value near a millibarn at laboratory proton energies near 25 BeV, approximately four times the threshold energy of about 6 BeV. This cross section is approximately equal to πa^2, where a is the Compton wavelength of the proton: $a=\hbar/Mc$. Furthermore, the relative production of π, K, and $\bar{p}$ is roughly proportional to the inverse square of the masses. While this might suggest that the cross section for production of triplets may be of the order of πa_t^2, where $a_t=\hbar/M_tc$, this is a long extrapolation, in concept as well as energy, if the triplet mass M_t is of the order of or greater than 10 BeV$/c^2$. At the very high energies necessary for the production of massive triplets, very many competitive channels are open. Since the intensity of the primary cosmic-ray spectrum varies as $dN/dE\approx E^{-2.67}$, where E is the energy,[6] we expect that production of triplets will result primarily from the interactions of protons with energies which are not large compared with the threshold energy for production. In this case, the energy in the center-of-mass system will not likely be large compared with the mass of a triplet pair; a statistical equilibrium will not occur; the triplet production, which can occur only through a few channels, may be very strongly damped by the competition from the very many other open channels; and the cross section may be reduced beyond that estimated from the measured antiproton cross section and dimensional considerations. On the other hand, the coupling between triplets is certainly very strong, so that some elements of the basic matrix element may be much larger than that for antiproton production.

The basic reactions which produce triplets will be

$$p+N\rightarrow N+N+T+\bar{T}+nB, \tag{1a}$$

and possibly

$$p+N\rightarrow N+T_a+T_b+T_c+nB, \tag{1b}$$

or

$$p+N\rightarrow N+T_\alpha+T_\beta+nB, \tag{1c}$$

where N represents nucleon, T and $\bar{T}$ represent a triplet and antitriplet, and T_a, T_b, and T_c are three members of a triplet representation. The T_α and T_β would be members of different triplet representations. The symbol B represents bosons, mainly pions but also K mesons, and n is an integer: $n=0, 1, 2, \cdots$. At any particular bombarding energy, reactions (1a) will have more phase space available than reactions (1b), though a little less than (1c). It is conceivable that the matrix elements for (1b) may be larger, however. The threshold energy for (1a) (with $n=0$) will be about half of the threshold energy for (1b) for heavy triplets with $M_t\gg M_p$, where M_t is the triplet mass and M_p is the proton mass.

While the preceding discussion hardly leads to a definite suggestion for a formula for the production cross section, we choose, as a basis for calculation, a plausible model hypothesis which does not violate the considerations raised, and we use the formula

$$\begin{aligned} \sigma_t&=\sigma_0(E/3E_{\mathrm{th}}-1/3)^2, \quad E_{\mathrm{th}}<E<4E_{\mathrm{th}},\\ \sigma_t&=\sigma_0, \quad E>4E_{\mathrm{th}}; \quad \sigma_0=\pi a^2, \end{aligned} \tag{2}$$

where a is the Compton wavelength of the triplet, E_{th} is the threshold energy for the production of triplet pairs, and E is the energy of the incident proton. The cross section so described rises slowly from threshold to a constant at an energy equal to or greater than four times the threshold energy in the laboratory system, or about twice the threshold energy in the center-of-mass system. Our final results do not seem to be very sensitive to the energy dependence beyond a scale factor. Results using a simple alternative model; $\sigma_t=\pi a^2$, $E>E_{\mathrm{th}}$, present about the same momentum spectra of triplets at sea level, but with about three times the intensity. We note here that our results are finally presented using a constant cross section, $\sigma_t=10^{-30}$ cm^2, in order to facilitate comparison with other estimates. Also, the actual cross section probably increases slowly with energy, perhaps like $E^{1/4}$; such an increase has little effect on the calculations.

Besides the total cross section, it is necessary to make assumptions concerning the angular dependence and momentum dependence of the triplet production. It seems that it is quite adequate to assume that the

[5] W. F. Baker, *et al.*, Phys. Rev. Letters **7**, 101 (1961); other references are presented in this article.

[6] Y. Pal (private communications).

production will be in the forward direction in the laboratory system. In all reactions which have been studied, the probability of particles being produced with transverse momentum greater than ≈ 1 BeV/c is small.[7] Since the incident flux is isotropic, small deviations from production in the forward direction are irrelevant. Again, for heavy triplets, our extrapolations from the behavior of light particles may not be reliable, and we cannot exclude the possibility that larger transverse momenta are important. However, even for the extreme assumption that the production angular distribution were isotropic in the center-of-mass system, the Lorentz transformation to the laboratory system is such that the laboratory production angles are still near 0° for most of the production and our approximation of forward production still leads to no important error.

At threshold the momentum of a triplet in the center-of-mass system will be zero; in the lab system $P_t = M_t P_i/(2M_t+2M_p) \approx \frac{1}{2}P_i$ for $M_t \gg M_p$, where P_i is the momentum in the laboratory system of the incident proton. To the extent that the incident proton and the target nucleon can be considered identical, the average momentum of a triplet in the center-of-mass system P_t must be zero for all proton energies. Then in the lab system, $P_t = P_i(M_t+T_t)/(2M_p{}^2+2E_iM_p)^{1/2}$, where T_t is the mean kinetic energy of the triplet in the center-of-mass system. For $T_t \ll M_t$ and $M_p \ll M_t$, $P_t \approx M_t (P_i/2M_p)^{1/2}$. This remains a good approximation in a statistical model since, for energies important for production by the cosmic rays, the center-of-mass kinetic energies of the triplets will be small.

At very high energies peripheral production may be important—it is plausible that radiation damping may not be so important for such processes. A limiting case will be obtained if we consider that the incident proton is sometimes the virtual state: $p \rightarrow p+T+\bar{T}$. At high energies only a small four-momentum transfer is required to "push" the virtual state onto the energy shell. The momentum, in the laboratory system, of the triplet will then be $P_t \approx P_i M_t/(2M_t+M_p)$; for $M_t \gg M_p$, $P_t = \frac{1}{2}P_i$. This is the largest plausible important laboratory momentum. The smallest laboratory momentum we can expect with any large probability will result from production by the same mechanism where the incident proton disassociates the target proton with a small four-momentum transfer. In the high-energy limit the four-momentum transfer will be zero and $P_t \approx M_t(M_t/M_p+\frac{1}{2})$ will be independent of P_i. For $M_t \gg M_p$, $P_t \approx M_t^2/M_p$. Both of these calculations assume implicitly that the triplet is at rest in the system of the excited nucleon. If the momenta of the triplet in that system are not not large, $P_t \lesssim 1$ BeV$/c^2 \ll M_t$, the laboratory momenta will be spread out by a factor $(1 \pm P_t/M_t)$ which we can safely neglect. In this model the mean center-of-mass kinetic energy is larger, leading to a somewhat larger mean laboratory momentum and a somewhat larger spread of laboratory momenta. In our calculations we use the center-of-mass model for $E_i < 4E_{\text{th}}$, and the peripheral model for $E_i > 4E_{\text{th}}$. The character of the triplet spectra at sea level is almost independent of the choice of model used.

With such a model of production the main contribution to the total production of triplets comes from primary cosmic-ray protons with energies between 300 and 3000 BeV. The integral flux in this region is approximated rather well by the expression[6]

$$N(E) = \int_E^\infty \frac{dN}{d\epsilon} d\epsilon = 1.4E^{-1.67} \text{ cm}^{-2} \text{ sr}^{-1}, \tag{3}$$

where E is measured in BeV. For definiteness we further consider that the total interaction cross section per nucleon[8] is 30 mb, which is the value measured at 30 BeV.

The area of the nucleus was taken as $A\sigma$, where σ is the nucleon-nucleon cross section and A is the number of nucleons in an "air" nucleus, taken as 14. The probability of the proton undergoing N collisions in the nucleus is then $(1/e)(1/N!)$. The mean free path for proton *interactions* with nuclei is then 88 g; the average number of nucleon-nucleon interactions for each nuclear interaction is $e/(e-1) = 1.57$. This mean free path is in excellent agreement with those measured at accelerators[8] and with other measurements of the size of nuclei.[9]

By interaction cross section we mean the total cross section excluding diffraction scattering. Triplets are then produced at the rate of $2\sigma_t/(30 \text{ mb})$ per proton interaction, where σ_t is the cross section for the production of triplet pairs. The results of the triplet spectra calculations are almost independent of the mean free path assigned to the proton. The absorption cross section enters only as a scale factor.

We assume that the interacting proton characteristically keeps its identity as a nucleon and find that it retains about 58% of its energy on the average. This corresponds to a mean four-momentum transfer of about 750 MeV/c. Protons corresponding to the high-energy tail of the cosmic-ray spectrum then have several chances to produce triplets. This is a rather important factor, about doubling the number of triplets produced. We approximate the situation by assuming that the probability of a four-momentum transfer q is proportional to $\exp(-|t|^{1/2}/0.75)$, where t is the square of the four-momentum transfer[10] measured in BeV/c. [Since we are primarily interested in energy transfer in this paper, it is convenient to consider the first and

[7] Such measurements have been made by many experimenters, e.g., L. F. Hansen and W. B. Fretter, Phys. Rev. **118**, 812 (1960).

[8] A. Ashmore, G. Cocconi, A. N. Diddens, and A. M. Wetherall, Phys. Rev. Letters **5**, 576 (1960).

[9] See Ref. 8; and R. W. Williams, Rev. Mod. Phys. **36**, 815 (1964).

[10] The experimental results are not all in agreement; Y. Pal and B. Peters, Kgl. Danske Videnskab. Selskab, Mat. Fys. Medd. **33**, (1964).

fourth components of the four-momentum separately. Throughout this paper, except where explicitly noted, $t=q^2=p_x^2-(E/c)^2$, and the x direction is along the beam.] Such a model is in excellent accord with experimental observations concerning the mean energy retained by high-energy protons,[11] and, as will be demonstrated, the attenuation length in the atmosphere. Occasionally a very high energy pion is produced which carries a large fraction of the incident proton energy and can contribute to the production of triplets. This probability is sufficiently small that we can neglect it.

III. DIFFUSION OF PROTONS AND TRIPLETS THROUGH THE ATMOSPHERE

It appears that all strong interactions are improbable if high four-momentum transfers are involved. Typically, for a particular channel, cross sections vary as $\exp(-|t|^{1/2}/q_0)$, where t is the square of the four-momentum transfer and q_0 has a value near the meson mass. Such a statement implies a judicious, though ill-defined, description of the process. For example, in the case of the simple elastic scattering of neutrons and protons, backscattering of the neutrons, with large nominal values of four-momentum transfer, are reinterpreted as charge exchange, with small values of four-momentum transfer. In the case of the interaction of conserved triplets with nucleons, there is probably no important element of triplet exchange, as there would appear to be no single light particle which could carry the triplet symmetry in an exchange. It then seems plausible to describe the effect on the triplet of the interaction with a nucleon as a change in four-momentum q, where $t=q^2$, such that the probability of the change q is proportional to $\exp(-|t|^{1/2}/q_0)$.

This model of momentum transfer, together with the value of the nucleon-interaction cross section used,

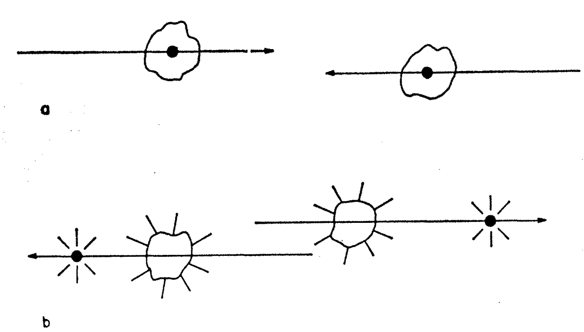

FIG. 1. Schematic model of a nucleon-nucleon collision at high energy in the center-of-mass system. Diagram (a) represents the approach of two nucleons, with their accompanying field, before the collision. Diagram (b) represents the situation immediately after the collision. The fields have been detached from the nucleons and excited to high temperatures by the collision. They will radiate mesons. The "bare" nucleons proceed in their original direction and radiate mesons in the reconstruction of their field.

results in a description of nucleon-nucleon interactions which is in good agreement with observations. Indeed the basic surmise of the model used for triplet-nucleon interactions is that the triplet-nucleon and nucleon-nucleon interactions at high energies are very much the same and that both interactions may be described loosely as shown by the diagrams of Fig. 1. In Figure 1(a) the two particles, together with their accompanying fields, are approaching in the center-of-mass system.[12] The second diagram, 1(b), represents the situation after the interaction. The two heavy particles a and b continue with only a small change, $q\approx 0.5$ BeV/c, in their four-momentum, but with their fields largely stripped: They are bare nucleons. The two meson clouds have interacted, have been excited by their interaction, and are moving with a lower velocity in the center-of-mass system in their original direction. These are the fireballs observed in high-energy interactions and they will radiate mesons and pairs. The amplitude for the bare nucleon states, $|\psi\rangle$, is given by $|\psi\rangle=\sum c_i|\psi_i\rangle$. Most of the clothed nucleon states $|\psi_i\rangle$ are unstable with respect to decay to nucleons, and hence the bare nucleons radiate mesons. If such mechanisms dominate triplet interactions at very high energies, the excitation energy of the firegalls will be about the same for nucleons and triplets of the same velocity. This is essentially the result if we consider that the mean invariant four-momentum transfer q_0 is the same in the two cases. Then the proportion of its energy lost by the triplet in the production of fireballs in a collision is not large, and further, it tends to vary inversely with the mass. For $q\ll M_t$ and $q\ll E_t$, where E_t is the laboratory energy of the incident triplet (as will usually be valid at all but the lowest triplet energies), the proportional energy lost to the production of fireballs in a collision will be $\Delta E_t/E_t\approx q/M_t$. For a triplet of mass 10 BeV/c^2 this would be only $\approx 5\%$ per collision if q_0, the mean value of $|t|^{1/2}$, is 0.5 BeV/c.

The mean laboratory energy E_n of the nucleon remaining after the radiation of the bare nucleon with energy E_b will be about equal to $E_n=E_b(M_n+T_n)/M^*$, where T_n is the final kinetic energy of the clothed nucleon in the center-of-mass system of the bare nucleon, and M^* is the mean mass of the isobar states comprising the bare nucleon. The energy of an interacting nucleon is then lost in two stages: (1) Energy is lost in the formation of fireballs from the interactions of the fields of the incident and target nucleon, and (2) energy is lost in the radiation of mesons which result in the re-creation of the meson field of the nucleon. The total energy loss or inelasticity has been measured variously to be from 30% to 50% of the incident energy.[11] Our calculations discussed in Sec. IV show that a value of 42% for the inelasticity is indicated from measurements of the attenuation length for

[11] Y. Pal and B. Peters, Kgl. Danske Videnskab. Selskab, Mat. Fys. Medd. 33, 51 (1964), use $\alpha=0.45$.

[12] G. Cocconi, in *Proceedings of the 1962 International Conference on High Energy Physics at CERN*, edited by J. Prentki (CERN, Geneva, 1962).

nucleons in the atmosphere. The division of this energy loss between the two processes is less certain. It would appear, from the intensity of high-energy muons observed as products of the cosmic radiation (which appear to come primarily from pions from isobar decays), that not much less than 20% of the incident nucleon energy must go into such pions, setting M^* as no more than 1300 MeV and leaving not more than 25% for the formation of fireballs.[13]

The four-momentum transfer q (neglecting here transverse components) is completely determined by the incident and final energies of the nucleon, and, for values of $q \ll E$, depends only on the value of the inelasticity. A value of 42% corresponds to $q \approx 750$ MeV/c. The mean four-momentum transfer involved in the creation of the fireballs would be of the order of 300 MeV/c.

Even as we would expect that the energy lost to fireball formation by triplets would be the same as for nucleons of the same velocity, we would expect that the meson radiation involved in reconstituting the long-range meson field of the triplet would be similar. The proportional energy radiated would, however, be smaller by a factor of about M_p/M_t. Of course, a triplet is not a proton, and these numbers can only be suggestive.

The energy of the triplets will then be moderated by their strong interactions with the nucleons in the atmosphere. While it is not possible at this time to calculate the nucleon-triplet collision cross section it seems probable that the cross section is not very different from the nucleon-nucleon cross section, and the interaction itself, when considered in terms of appropriate variables, is not very different from the nucleon-nucleon interaction.

The effective "size" of the triplet will not likely be larger than the magnitude of the Compton wave length of the lightest particle emitted virtually; this particle will be the pion. From this point of view the triplet-nucleon cross section, at high energies, would not be much different from that for other elementary particles. It is plausible that the coupling of pions to the triplets is extremely large, just as the dynamic theories relating triplets to other elementary particles suggest enormous triplet-triplet binding energies. If this is the case, the triplet-nucleon cross section may be appreciably larger than the nucleon-nucleon cross section of about 30 mb but hardly larger than $\pi(h/m_\pi c)^2$ or about 60 mb. We consistently neglect diffraction scattering, which is not relevant to our interest in energy loss. Since, in the dynamic theories, the three triplets which form a nucleon are together not coupled with excessive strength to the pion, it seems unlikely that one is—though we must allow for the possibility of some neat cancellation. From this view we would ascribe the enormous triplet couplings to very short-range forces which would not affect total cross sections very much and assume that the meson cloud about the triplet is similar to that about the nucleon.

The number of collisions a triplet will make with nucleons in the course of a path of length L will be equal to $NL\sigma$, where N is the number of nucleons per cm^2 in the path and σ is the nucleon-triplet cross section. Since the nucleons are grouped in clusters of 14 or 16 in nitrogen or oxygen nuclei, the intervals between collisions (in terms of grams/cm^2 traversed in the atmosphere) are not distributed as a normal exponential. There are long paths between nuclei and very short paths in nuclear matter. We choose to describe the distribution by the same model we used for the proton interactions. The mean free path for interactions with nuclei was taken as 88 g, and the interactions with the nuclei were taken as interactions with a layer of nuclear matter one mean free path deep. The numerical value of 88 g is taken in accord with the choice of nucleon-triplet interaction cross section of 30 mb. The results are not sensitive to reasonable choices of the distribution of path lengths between internuclear travel and intranuclear travel, and hence the proton-nitrogen cross section.

It is certainly not quite correct to treat the several interactions of the incident nucleon with the nucleons of the nucleus as separate incoherent incidents. In the system of the incident nucleon, the time between collision is very short, of the order of $h/(m_\pi c\gamma)$, and there is not time for the nucleon, with its field disturbed, to recreate this field. The momentum transfer and energy loss are then probably slightly overestimated by our model. However, the error is not important compared with other uncertainties in our treatment of triplets.

There is also some evidence[14] that for some interactions the mean momentum transfer to heavy particles is greater than to light particles. The value of 0.4 BeV, which we use, is dimensionally complementary to the interaction radius defining the cross section $\sigma = 30$ mb. If a much larger mean momentum transfer is relevant, the effective interaction cross section probably should be smaller. Since the energy lost in an interval is proportional to the product of the number of interactions times the mean energy loss, or to σq, the values chosen here are probably reasonable: The number chosen for the product of the momentum transfer and the cross section is in accord with that which fits our experience with the nucleon-nucleon interaction.

It is interesting to consider, as an extreme alternative, the statistical model where the triplet would emerge from a fireball with small momentum in the center-of-mass system. The mean final energy E_f in the laboratory system, after a collision, of a triplet with initial energy E_i, will be $E_f \approx E_i M_p (M_t^2 + 2E_i M_n)^{1/2}$, where M_n is the nucleon mass. The energy loss in such a collision is very large for very high-energy triplets. For

[13] This must be fitted to the high-energy muon spectrum in the atmosphere. See Ref. 11.

[14] A. Bigi, *et al.*, in *Proceedings of the 1962 International Conference on High Energy Physics at CERN*, edited by J. Prentki (CERN, Geneva, 1962).

triplets of moderate energy, the mean energy loss is not great: $E_f = E_i M_t^2/(M_t+M_n)^2$.

It would appear that the statistical model is quite unlikely to be relevant at high energies. It does not fit observations at all well for nucleon-nucleon interactions and there appears to be no reason to believe that it should be better suited for the description of conserved triplet interactions. At low energies, below the threshold for inelastic processes, where the characteristic four-momentum transfer is not extremely small compared with the triplet momentum, the statistical model may adequately represent reality—it is essentially correct for isotropic elastic scattering. As a calculational procedure we represent the triplet-nucleon interaction below the threshold for the production of a pion, by the assumption of isotropic scattering in the center-of-mass system. This may not fit any actual state of affairs too badly. We would expect a larger cross section than 30 mb, but a smaller probability of large momentum transfer per collision than implied by isotropic scattering in the center-of-mass system.

Energy loss by ionization is considered in all stages of the calculation on the basis of unit charge. For momenta $\gtrsim M_t/2$ the energy loss by ionization is almost irrelevant as the collision losses dominate the situation, for lower momenta the ionization losses are important. Therefore, the features of the calculations presented here are valid for quarks of charge $\frac{1}{3}e$ or $\frac{2}{3}e$ for high momenta, but the weakly charged particles will have a little longer range before stopping. Also, as a result of charge exchange, some triplets will have spent part of their life in a neutral mode. The history of slow particles $P_t < \frac{1}{2}M_t$ is the least reliable part of these estimates in any case.

IV. CALCULATIONAL PROCEDURES AND RESULTS

The momentum spectra of triplets and of protons in the atmosphere were calculated by means of Monte Carlo techniques using the IBM 7094 at Brookhaven National Laboratory. The history of individual particles in their passage through the atmosphere was calculated using a random-number generator and theoretical probability distributions representing the character of physical processes according to the results of the discussions of Sec. II and III. The triplet and proton spectra were calculated using somewhat different procedures.

Since the parameters used for the triplet calculations depend, at least qualitatively, upon the values determined by the fitting of the proton interaction calculations to the experimental data concerning the mean free path of protons in the atmosphere, we discuss the proton-spectrum calculations first.

Incident protons were selected such that their energy distributions correspond to the cosmic ray proton spectrum of Eq. (3). For each proton a path length in g/cm^2 was chosen at random according to a procedure

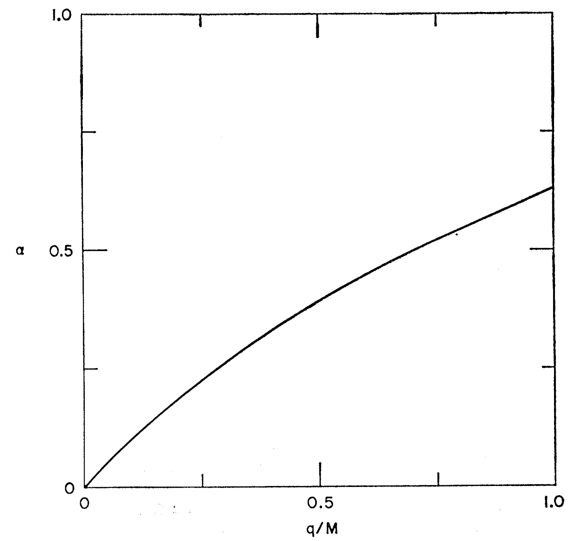

FIG. 2. The inelasticity coefficient α plotted as a function of q/M, where q is the four-momentum transfer to a particle of mass M.

which established probabilities appropriate to the proton mean-free-path choice of Sec. II. At the point of collision, a four-momentum transfer, $q = |t|^{1/2}$, was chosen according to a probability distribution such that $P(q) \approx \exp(-|t|^{1/2}/q_0)$. The proportional energy loss for $E_i \gg q$, where E_i is the proton energy, depends only on q/M. The energy loss or inelasticity versus q/M is plotted in Fig. 2. For the particular value of q chosen in the Monte Carlo calculation, the energy loss of the incident proton is calculated. The momentum and energy of the proton are evaluated after the collision; another path is then selected and the procedure is continued. The proton is thus followed through a history of such collisions as it diffuses through the atmosphere. The energy of the proton is noted and stored as the proton passes each 200 g/cm^2 level. Where the momentum falls below 10 BeV/c, the proton is dropped by the program, another incident proton is chosen from the cosmic-ray spectrum, and the procedure is repeated.

The proton energy spectrum at various levels of the atmosphere was calculated using different values of q_0, the mean four-momentum transfer, and compared with experimental observations. Typically, such observations consist of measurements of counting rates of detectors, which have a specific energy threshold, as a function of altitude or depth of atmosphere. The attenuation of the primary cosmic rays appears to be exponential with the attenuation length[15] of 120 g/cm^2. Figure 3 shows the attenuation length for the intensity of protons over 10 BeV as a function of q_0, as calculated by the Monte-Carlo procedures. The results indicate that the mean momentum transfer is 750 MeV/c and that the mean inelasticity is 0.42 ± 0.04. The error is estimated from the errors in the input

[15] For example, J. H. Tinlot, Phys. Rev. 73, 1476 (1948).

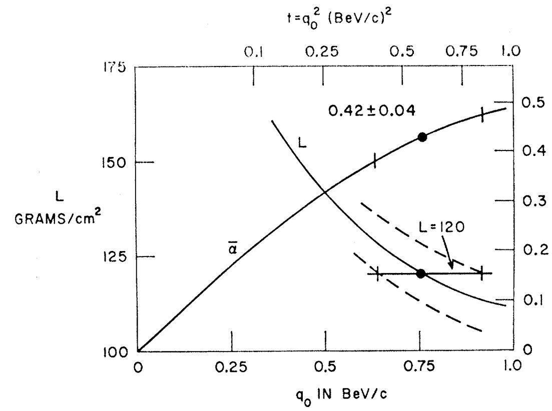

FIG. 3. The relation of interaction parameters and attenuation length for high-energy nucleons passing through the atmosphere. One curve represents the attenuation length as a function of q_0, the mean momentum transfer, the other represents the mean value of the inelasticity, $\bar{\alpha}$, as it depends upon q_0. The solid circles represent the values of $\bar{\alpha}$ and q_0 which fit the measured attenuation length of 120 g/cm^2.

quantities: $\sigma_p=30\pm2$ mb, the mean free path $=88\pm5$ g/cm^2, and γ, the coefficient of the integral cosmic-ray spectrum, is taken as $\gamma=1.67\pm0.1$. The largest uncertainty is due to the error in σ_p.

Of course the model itself is subject to criticism. We point out that the lumping of the two sources of energy loss—the creation of fireballs, and the radiation involved in reconstituting the stripped field—in one parameter no doubt results in an incorrect spectrum of energy loss in a collision. This may not be serious, however, as the various moments of this distribution should be much less important than the mean.

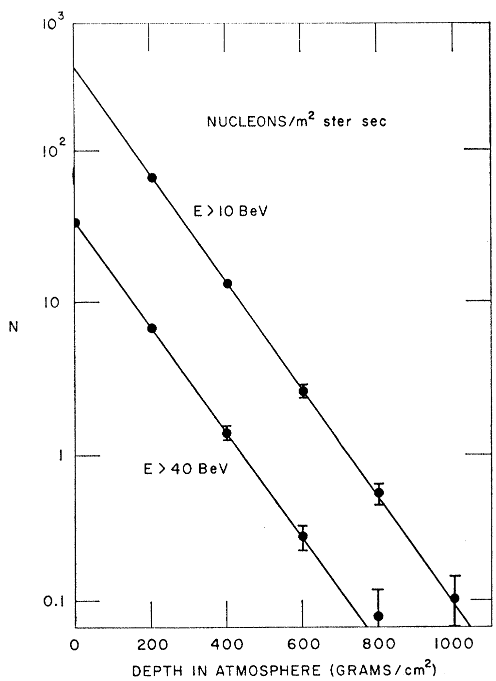

FIG. 4. The intensity of nucleons as a function of depth in the atmosphere for nucleons with $p>10$ and with $p>40$ BeV/c where p is the triplet momentum.

Figure 4 shows the intensity of protons with energies above 10 BeV as a function of depth in the atmosphere for a mean momentum transfer of 750 MeV/c, exhibiting the exponential character of the variation of intensity with depth.

For the calculation of the triplet spectra, incident protons were again selected such that their energy distributions corresponded to the cosmic-ray proton spectrum, and again, for each proton a path length was chosen at random to fit the proton mean free path. At the point of collision the probability of producing a triplet was taken as 1 for $E_p>4E_{\text{th}}$, and as $(3\,E_{\text{th}})^2/(4\,E_{\text{th}}-E_p)^2$ for $E_{\text{th}}<E_p<4\,E_{\text{th}}$. The over-all normalization to the ratio σ_t/σ, where σ_t is the asymptotic cross section for triplet production and σ is the total nucleon-nucleon absorption cross section, is separate

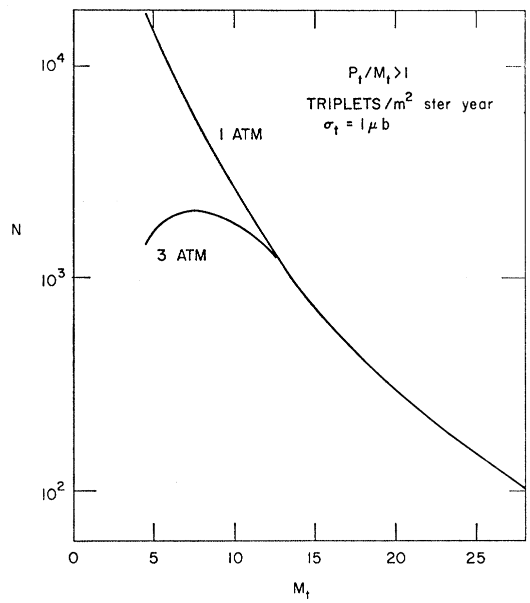

FIG. 5. The intensity of triplets such that $p/M>1.0$, as a function of triplet mass M. Here p is the triplet momentum. The asymptotic production cross section is taken as $\sigma_t=10^{-30}$ cm^2.

from the Monte Carlo calculation. The threshold energy E_{th}, as well as the production spectra and collision energy losses, depends upon the triplet mass M_t. The calculation was conducted separately for various values of M_t.

The momentum of the triplet after production was calculated according to the recipes of Sec. II. Then the triplet was followed through a history of random collisions using a mean four-momentum transfer of 0.4 BeV/c, through an ideal infinite atmosphere for a total path of 6 kg/cm^2, which is 6 atm. The momentum of the triplet was noted and stored as the triplet passed each kg/cm^2 level. If the momenta fell below the limit $P_t<0.1M_tc$, the triplet was assumed to travel beyond the next kilogram mark and then stop before the next kilogram division.

After each triplet either stops or travels through 6 kg/cm^2, the program drops the triplet and takes up again the history of the incident proton. The proton momentum transfer during the collision was calculated, the momentum of the proton was reevaluated, a path length was determined, and the probability of making a triplet in the course of the next collision was evaluated. If a triplet was produced, the triplet was followed through its history before returning to the proton. The proton was followed until its energy dropped below the threshold energy for triplet production. Another incident proton was then chosen and the whole procedure repeated.

The calculation, as performed in this way, does not imply that the behavior of the proton which actually produces triplets is adequately described by the calculation—it surely is not—but that this proton is representative of the whole class of interacting protons. The specific procedure is convenient inasmuch as it is economical of calculating time and insures a correct and simple normalization.

The results of the set of Monte Carlo calculations are in the form of intensity of triplets, in appropriate units, as a function of M_t, P_t, and the depth of the atmosphere, where M_t is the triplet mass and P_t the triplet momentum. It is inconvenient to present more than a subset of the total information which might be useful in experimental designs. For convenience relating to other estimates, the asymptotic cross section is taken as 10^{-30} cm^2 rather than $\pi(\hbar/M_tc)^2$. Figure 5 presents the intensity as a function of triplet mass, for triplets such that $P_t/M_t \geq 1.0$, for various values of M_t. The curves of Fig. 6 represent the intensity of stopped triplets as a function of depth in the atmosphere for various values of M_t. Figure 7 shows the intensity of triplets such that $0.5 \leq P_t/M_t < 1.1$, as a function of the depth of the atmosphere: $d = 1(\text{kg/cm}^2)/\cos\theta$, where θ is the angle of inclination. The multiple scattering of the triplets through the atmosphere has an effect such that any specific angle represents a spread of total path lengths.

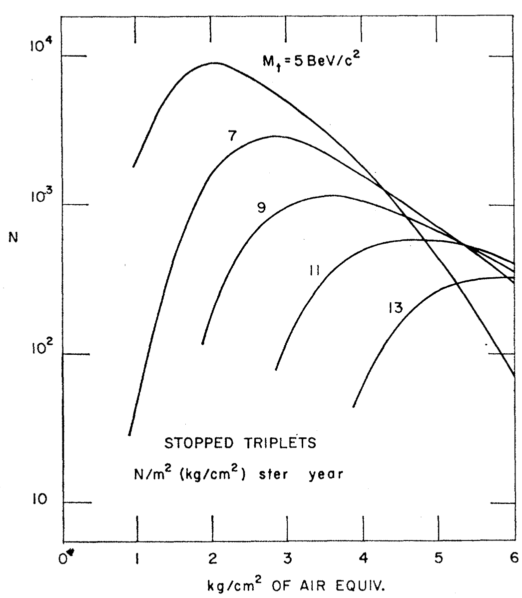

FIG. 6. The intensity of stopped triplets as a function of equivalent atmospheres, for various values of the triplet mass. The asymptotic production cross section is taken as 10^{-30} cm^2.

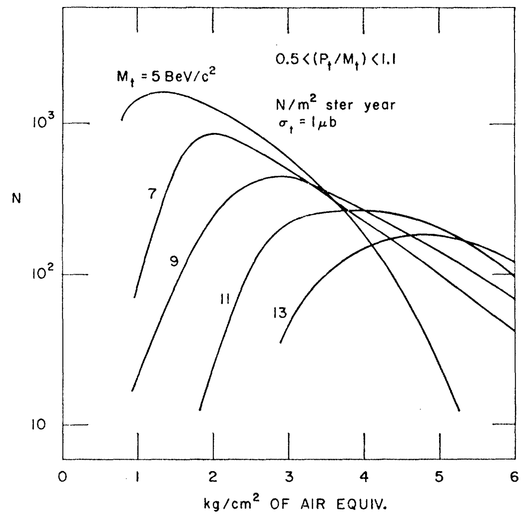

FIG. 7. The intensity of triplets with values of momenta β such that $0.5 < p/M < 1.1$, where p is the triplet momentum and M is the triplet mass. The asymptotic production cross section is taken as 10^{-30} cm^2.

The angular deviation of the triplet from the direction of the primary proton was not calculated specifically. We expect that the mean angular deflection is $\approx q/P$ for each collision, where q is a transverse momentum transfer with an average value of about 0.4 BeV/c, and P is the momentum of the triplet after the collision. The mean square angular deviation $\bar{\theta}^2$ of a triplet of momentum P will be about equal to $(q/P)^2$ from the last collision plus $(q/P)^2\alpha^2$ from the collision before, plus $(q/P)^2\alpha^4$ from the collision before that, etc., where α is the mean change in momentum at a collision. For our model $\alpha \approx 1 - q/M$; then $\bar{\theta}^2 = (q/P)^2/(1-\alpha^2) \approx (q/P)^2(M_t/2q)$.

Typically, for $M_t = 10$ BeV/c^2 and $q = 0.4$ BeV/c, $\bar{\theta}^2 \approx 0.02$ and $\bar{\theta}^2 = 0.15 \approx 8^0$. For a 10-BeV proton the standard deviation in angle is about 2.5^0. The angular deviation is proportional to the momentum transfer, and proportional to the square root of the number of collisions required to moderate the energy, or $(M/q)^{1/2}$. The proton is moderated in fewer collisions; therefore, its mean angular deviation is less.

ACKNOWLEDGMENTS

We are indebted to Dr. Henry Kasha for many discussions of these problems. We thank Professor Y. Pal[16] and Professor B. Peters, who suggested useful modifications and corrections to the first draft of this paper.

[16] We have received an unpublished report by Y. Pal and S. Tandon, who have used a model similar to that discussed in this paper and have reached nearly equivalent conclusions.