Spectroscopic Factors from Deuteron Stripping Reactions Leading to Ground and First Excited States of Even Spherical Nuclei*

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(Received 5 August 1965; revised manuscript received 28 October 1965)

Deuteron stripping reactions on odd-neutron targets from Cr to Sm leading to the ground and first excited states of the even system are analyzed with distorted-wave Born approximation calculations. Theoretical spectroscopic factors are computed for these reactions using wave functions derived from the pairing-plus-$P^{(2)}$ model. One such factor is calculated and measured for the ground-state transition, while for the first excited state the analysis is in terms of several spectroscopic factors corresponding to several l values for the transferred neutron. For the ground state, the agreement between experiment and theory is excellent. For the excited state the agreement is not so good, but most of the spectroscopic factors are in agreement to about a factor of 2.

INTRODUCTION

DURING the last few years deuteron stripping reactions analyzed by means of the distorted-wave Born Approximation (DWBA) have become a rather precise quantitative tool for determining properties of the nuclear wave functions involved in the reactions. A careful study of the reaction $Ca^{40}(d,p)Ca^{41}$ indicated that spectroscopic factors can be extracted with an accuracy of 20% or better.[1]

The most extensive systematic analyses have been for the case of even-even targets leading to various states in the adjacent odd-neutron nuclei. For these cases, since the target has spin and parity 0^+, the DWBA parity and angular-momentum selection rules require a unique value, l, for the orbital angular momentum of the stripped neutron for transitions to a final state of angular momentum and pairty J, π. The proton angular distribution determines the l value and thus helps determine J, π of the final state. Furthermore the magnitude of the cross section determines the spectroscopic factor, the amount of even 0^+ state plus neutron in the final-state wave function. Such analysis has served to determine the sequence and separation of the neutron single-particle states and their rate of filling with neutrons throughout the periodic table.[2] The experimental occupation of the single-particle levels is well represented by the pairing model of nuclear structure if the level energies and pairing strength are chosen appropriately.[3] There has been less success, however, in finding a theoretical description of the distribution of the single-particle strength among the various odd-nuclear levels. Systematic agreement between experiment and theory is qualitative at best.[4]

The model of Kisslinger and Sorensen, hereafter referred to as KS, in which odd nuclei are described in terms of a linear combination of quasiparticles coupled to various numbers of quadrupole phonons[5] has had considerable success in describing the lowest few states of odd nuclei, particularly in regard to their energies and $B(E2)$ transition rates.[6] Thus, while higher states are only qualitatively described, the lowest few states may be quantitatively described by the model. This model is also successful in giving good agreement with experimental energies and ground-state electromagnetic transition rates for the 2^+ vibrational state of even spherical nuclei. The purpose of this paper is to report on deuteron stripping experiments on odd-N nuclei leading to the 0^+ ground state and 2^+ first excited states of spherical nuclei from chromium to the rare earths. Spectroscopic factors are extracted by the use of the DWBA, and used as a further check on the validity of the pairing plus $P^{(2)}$ wave functions for odd-neutron ground states and for the ground state and one-phonon states of even nuclei.

THEORETICAL SPECTROSCOPIC FACTORS

Expressions for the theoretical spectroscopic factors for stripping to the ground state and one-phonon vibrational states of even nuclei for wave functions of the type used by KS have been derived by Yoshida,[3] and we will not repeat these derivations. Since the KS wave functions will be used we simply rewrite the

* This work was supported by the National Science Foundation and by the U. S. Office of Naval Research. The experimental work was performed at the Sarah Mellon Scaife Radiation Laboratory.
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[1] L. L. Lee, J. P. Schiffer, B. Zeidman, G. R. Satchler, R. M. Drisko, and R. H. Bassel, Phys. Rev. 136, B971 (1964).
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[4] R. A. Sorensen, Nucl. Phys. 25, 674 (1961); A. Lande and G. E. Brown, Princeton University Report No. PUC-1965-159 (unpublished).
[5] L. S. Kisslinger and R. A. Sorensen, Rev. Mod. Phys. 35, 853 (1963), referred to hereafter as KS.
[6] R. A. Sorensen, Phys. Rev. 133, B281 (1964).

pertinent equations in the notation of KS and discuss them. These wave functions are derived from a shell-model Hamiltonian in which the interaction term is the sum of a pairing force and a quadrupole force. The pairing force is treated by the BCS method[7] with the use of the quasiparticle transformation of Bogoliubov and Valatin,[8] while the quadrupole force is treated by the quasiparticle random-phase approximation.[9]

In the absence of ground-state correlations which are unimportant for this process and ignored here, the ground state of the even nucleus is the quasiparticle vacuum:

$$\psi_{g.s.}=|0\rangle. \tag{1}$$

The phonon state is obtained by operating with the phonon creation operator

$$\psi_{2+}=B_{2\mu}^{\dagger}|0\rangle, \tag{2}$$

where

$$B_{2\mu}^{\dagger}=\tfrac{1}{2}\sum_{ij}(r(ij)[\beta_i^{\dagger}\beta_j^{\dagger}]_{2\mu}-(-1)^{\mu}s(ij)[\beta_j\beta_i]_{2-\mu}). \tag{3}$$

The sum on ij goes over neutron pairs and proton pairs which can couple to $J=2+$. The $\beta^{\dagger}$, β are the quasiparticle creation and destruction operators of the transformation

$$b_{ljm}^{\dagger}=U_{lj}\beta_{ljm}^{\dagger}+(-1)^{l+j-m}V_{lj}\beta_{lj-m}, \tag{4}$$

where $b^{\dagger}$ is the ordinary particle creator.[10] For the odd-neutron nuclei, the wave functions of KS are obtained by diagonalizing the Hamiltonian in the space of wave functions made up of one neutron quasiparticle in the ljm states of a major shell and zero, one, and two phonons. For the ground state the two-phonon component is always small (for the few cases treated here, but not in KS, we have not included the two-phonon part at all). These wave functions are of the form

$$\psi_{ljm}=C_{lj00}{}^{lj}\beta_{ljm}^{\dagger}|0\rangle+\sum_{l'j'}C_{l'j'12}{}^{lj}[\beta_{l'j'}{}^{\dagger}B_2^{\dagger}]_{jm}|0\rangle. \tag{5}$$

In KS, expressions are given for the r and s coefficients and the C coefficients are numerically tabulated.

For these wave functions the spectroscopic factor S for stripping on an odd-N target of spin j to the 0^+ state of the resulting even nucleus is[3]

$$S_l{}^{0+}=(2j+1)(V_{lj}C_{lj00}{}^{lj})^2. \tag{6}$$

The l value, and thus the angular distribution is uniquely predicted by the angular momentum and parity of the odd ground state, and the spectroscopic factor measures the amount of pure quasiparticle in this state together with the fullness of the level in the final state. The spectroscopic factor for stripping to the 2^+ one-phonon state is more complicated. In this case the transferred neutron need not have the $j\pi$ of the target, but may have any $j'\pi'$ which can couple to that of the target to produce the final 2^+ state. The contributions from the different $j'\pi'$ for the neutron contribute incoherently to the angular distribution which is thus a sum of cross sections corresponding to different l'. The spectroscopic factor for each l', which may be determined separately, receives two contributions corresponding to $j'=l'\pm\frac{1}{2}$. Finally for each j' the corresponding spectroscopic factor is[3]

$$S_{l'j'}{}^{2+}=\Big|(-1)^{l'}U_{l'j'}C_{lj00}{}^{lj}r(jj')+\left(\frac{2j+1}{5}\right)^{1/2}V_{l'j'}\times C_{l'j'12}{}^{lj}+5^{1/2}(2j+1)^{1/2}V_{l'j'}\sum_{ab}C_{b12}{}^{lj}\times r(aj')r(ab)\begin{Bmatrix}j' & a & 2\\ b & j & 2\end{Bmatrix}\Big|^2, \tag{7}$$

where the phase factor is such that the first two terms add or subtract corresponding to whether $(U_{lj}U_{l'j'}-V_{lj}V_{l'j'})$ is plus or minus, respectively. The third term, not included by Yoshida, is of minor importance. Finally, we have:

$$S_{l'}{}^{2+}=S_{l'j'=l'-\frac{1}{2}}{}^{2+}+S_{l'j'=l'+\frac{1}{2}}{}^{2+}. \tag{8}$$

Thus the spectroscopic factors for the 2+ state measure a rather complicated relation involving the wave functions of both the odd-nucleus ground state and the phonon itself.

Instead of trying to interpret the experimental results as implying values for the wave-function coefficients we simply compare the experimental results with the wave functions given by KS. The theoretical spectroscopic factors obtained from Eqs. 6, 7, and 8 for these wave functions with the exceptions noted below are shown in Table I along with the experimental values. For Kr, Fe, Nd, and Sm, new wave functions are used. For Kr and Fe no wave functions are given in KS and so new wave functions are generated from the pairing-plus-$P^{(2)}$ model for this region of the periodic table. The single-particle neutron levels used are: $\epsilon_{f_{7/2}}=-4.0$, $\epsilon_{p_{3/2}}=0.0$, $\epsilon_{f_{5/2}}=1.5$, $\epsilon_{p_{1/2}}=3.0$, $\epsilon_{g_{9/2}}=4.0$, and the other parameters are extrapolated from KS. For Nd and Sm the single-particle energies listed in KS Table XI are clearly inadequate, and for these nuclei a single-particle spectrum is used which differs from KS by having the neutron $h_{9/2}$ level raised by several MeV so that it lies between the $p_{1/2}$ and $f_{5/2}$ levels at the top of the well in agreement with Cohen's results.[2]

EXPERIMENTAL PROCEDURE

Studies were made of (d,p) reactions on odd-neutron nuclei induced by 15-MeV deuterons from the Univer-

[7] J. Bardeen, L. N. Cooper, and J. R. Schrieffer, Phys. Rev. **108**, 1175 (1957).
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[9] R. Arview and M. Veneroni, Compt. Rend. **250**, 992, 2155 (1960); T. Marumori, Progr. Theoret. Phys. (Kyoto) **24**, 331 (1960); M. Baranger, Phys. Rev. **120**, 957 (1960).
[10] Here and in subsequent equations, we use Condon and Shortley phases as suggested in unpublished notes of Ben Bayman.

TABLE I. Comparison of experimental and theoretical spectroscopic factors. E and I_f are the energy and spin of the final state, while S' and S are the spectroscopic factors defined in the text for the transfer of an "l" neutron. Each reaction is preceded by the target spin and parity.

E (MeV)	I_f	l	σ_{max} (mb/sr)	S'_{expt}	S_{expt}	S_{theory}
			(3/2⁻) $Cr^{53}(d,p)Cr^{54}$			
0	0	1	1.31	0.23	0.91	0.97
0.86	2	1	3.76	0.57	0.46	0.52
		+ 3		<0.03	<0.02	0.06
			(1/2⁻) $Fe^{57}(d,p)Fe^{58}$			
0	0	1	0.18	0.036	0.072±0.01	0.067
0.81	2	1	0.74	0.11	0.044±0.009	0.06
		+ 3	0.17	0.16	0.064±0.025	0.05
			(3/2⁻) $Ni^{61}(d,p)Ni^{62}$			
0	0	1	2.61	0.50	2.0±0.30	1.7
1.18	2	1	2.09	0.33	0.26±0.04	0.32
		+ 3	0.015	0.02	0.016±0.015	0.04
			(5/2⁻) $Zn^{67}(d,p)Zn^{68}$ [a]			
0	0	3	0.14	0.37	2.2±0.3	2.9
1.08	2	1	0.44	0.09	0.11±0.02	0.21
		+ 3		<0.01	<0.01	0.11
			(9/2⁺) $Ge^{73}(d,p)Ge^{74}$			
	0	4				1.2
	2	4				0.83
			(1/2⁻) $Se^{77}(d,p)Se^{78}$ [b]			
0	0	1	1.21	0.34	0.68±0.010	0.83
0.62	2	1	0.32	0.074	0.029±0.005	0.07
		+ 3		<0.03	<0.01	0.05
			(9/2⁺) $Kr^{83}(d,p)Kr^{84}$			
	0	4				5.58
	2	4				0.03
			(5/2⁺) $Zr^{91}(d,p)Zr^{92}$			
0	0	2	0.47	0.24	1.44±0.20	1.6
0.94	2	2	2.50	1.11	1.33±0.20	1.5
		+ 0		0.06	0.07±0.04	0.09
			(5/2⁺) $Mo^{95}(d,p)Mo^{96}$ [c]			
0	0	2	0.61	0.41	2.48±0.37	2.7
0.81	2	2	0.46	0.25	0.30±0.05	0.68
		+ 0	0.15	0.067	0.08±0.02	0.23
			(5/2⁺) $Ru^{99,101}(d,p)Ru^{100,102}$			
0(100)	0	2	0.54	0.46	2.74±0.4	3.3
0(102)	0	2	{0.68	{0.54	3.0±0.5	2.9
0.54(100)	2	2			···	0.06
		+ 0	0.087	0.073	0.087±0.04	0.37
0.48(102)	2	2	0.037	0.027	0.032±0.008	0.11
		+ 0	0.094	0.073	0.087±0.03	0.23
			(5/2⁺) $Pd^{105}(d,p)Pd^{106}$			
0	0	2	0.39	0.29	1.74±0.25	2.8
0.52	2	2	0.083	0.057	0.068±0.03	0.14
		+ 0	0.076	0.054	0.065±0.03	0.04
			(1/2⁺) $Cd^{111,113}(d,p)Cd^{112,114}$			
(112)	{0	0				0.79
	{2	2				0.11
(114)	{0	0				0.90
	{2	2				0.15
			(1/2⁺) $Sn^{115}(d,p)Sn^{116}$ [d]			
0	0	0	0.79	0.54	1.08±0.16	1.03
1.30	2	2	0.33	0.26	0.10±0.015	0.18
			(1/2⁺) $Sn^{117}(d,p)Sn^{118}$ [d]			
0	0	0	1.19	0.70	1.4±0.2	1.2
1.22	2	2	0.49	0.39	0.16±0.025	0.19
			(1/2⁺) $Sn^{119}(d,p)Sn^{120}$ [d]			
0	0	0	1.13	0.65	1.3±0.2	1.4
1.17	2	2	0.20	0.15	0.06±0.01	0.15
			(1/2⁺) $Te^{125}(d,p)Te^{126}$ [e]			
0	0	0	0.94	0.60	1.2±0.3	0.99
0.69	2	2	0.081	0.068	0.027±0.004	0.06
			(1/2⁺, 3/2⁺) $Xe^{129,131}(d,p)Xe^{130,132}$			
(130)	{0	0				0.68
	{2	2				0.05
(132)	{0	2				1.84
	{2	2				0.00
		+ 0				0.05
			(3/2⁺) $Ba^{135,137}(d,p)Ba^{136,138}$			
0(136)	0	2	0.27	0.60	2.4±0.3	2.6
0.83(136)	2	2	0.21	0.40	0.32±0.04	0.02
		+ 0	0.026	0.019	0.015±0.015	0.03
			(7/2⁻) $Nd^{143,145}(d,p)Nd^{144,146}$			
0(144+146)	0	3	0.16	0.30	2.4±1.2	{1.4(144) {1.4(146)
0.69(144) + 0.46(146)	2	3 + 1	0.50	0.86	1.4±0.7	{0.5(144) {0.7(146)
			0.17	0.067	0.11±0.06	{0.05(144) {0.13(146)
			(7/2⁻) $Sm^{147,149}(d,p)Sm^{148,150}$			
0(148+150)	0	3	0.084	0.17	1.36±0.2	{1.4(148) {2.1(150)
0.55(148) + 0.33(150)	2	3 + 1	0.064	0.11	0.18±0.05	{0.7(148) {1.0(150)
			0.12	0.051	0.08±0.03	{0.11(148) {0.22(150)

[a] From Ref. 15. [b] From Ref. 16. [c] From Ref. 17. [d] From Ref. 18. [e] From Ref. 19.

sity of Pittsburgh cyclotron. Angular distributions for proton groups leading to the ground and first excited states were measured at roughly 5° intervals between 10° and 40°. These were then analyzed with DWBA calculations[11] to obtain spectroscopic factors, S. The optical model parameters used in these calculations are listed in Table II. As previously noted, the ground state (0^+) groups require a single known l value to satisfy conservation of angular momentum and parity, while the transitions to the first excited (2^+) state can have a combination of two (or more) l values. The former were in all cases (except Nd and Sm) well fitted by the DWBA calculations, while the latter were fitted by the sum of two angular distributions, one with the l of the 0^+ transition which was taken from the experimental data for the 0^+, and the other of different l which was taken from DWBA calculations. Some of the fits are shown in the figures.

Two experimental techniques were used. The protons were passed through a magnetic spectrometer, and in cases where good energy resolution was not needed (Fe, Ni, Pd), they were detected by the triple scintillator described previously.[12] In cases where there are two isotopes involved so that good energy resolution is

[11] The authors are indebted to R. M. Drisko for many helpful discussions in connection with these calculations.

[12] R. K. Jolly, E. K. Lin, and B. L. Cohen, Phys. Rev. **130**, 2391 (1963).

TABLE II. Optical-model parameters used in DWBA calculations. For Ru and Nd, the Pd and Sm calculations were used, and for Ni and Zr, calculations made in connection with previous papers were used.

	Deuteron optical-potential parameters						Proton optical-potential parameters					
	V (MeV)	r_0 (F)	a (F)	W' (MeV)	r_0' (F)	a' (F)	V (MeV)	r_0 (F)	a (F)	W' (MeV)	r_0' (F)	a' (F)
$Cr^{53}(d,p)$[a]	90.8	1.15	0.81	75.2	1.34	0.68	45.8	1.25	0.65	48.8	1.25	0.47
$Fe^{57}(d,p)$	59.4	1.105	0.884	53.6	1.389	0.712	45	1.25	0.65	46.4	1.25	0.47
$Pd^{105}(d,p)$	76.3	1.032	0.974	59.2	1.353	0.771	49.2	1.25	0.65	53	1.25	0.47
$Ba^{135}(d,p)$	98.3	1.15	0.81	61	1.34	0.68	53	1.25	0.65	42	1.25	0.47
$Sm^{149}(d,p)$	104	1.15	0.81	68	1.34	0.68	53	1.25	0.65	68	1.25	0.47

[a] The calculation (as well as the data) for Cr is for a deuteron energy of 13.5 MeV.

required (Zr, Ru, Ba, Nd, Sm), the detection was with photographic plates.[13] In one case (Cr), data were taken from the literature[14] on experiments done with 13.5-MeV deuterons, and analyzed with DWBA calculations. In five cases (Zn, Se, Mo, Sn, and Te), the results were already available[15–19] from previous work, so no further data were obtained. For all cases except Fe^{57} and Ni^{61}, targets were not isotopically enriched; enrichment was unnecessary since the proton groups being studied were the highest energy groups in the spectrum.

EXPERIMENTAL RESULTS

The results are listed in Table I. $\sigma_{\max}$ denotes the cross section at the maximum in the angular distribution, S' is the ratio of this cross section to that calculated from DWBA, and S is the spectroscopic factor as usually defined, which here is

$$\sigma=[(2I_f+1)/(2I_i+1)]\sigma_{\mathrm{DWBA}}S, \qquad (9)$$

whence it is related to S' by

$$S'=[(2I_f+1)/(2I_i+1)]S. \qquad (10)$$

The error estimates in Table I are subjective estimates of the experimental uncertainties. They do not include uncertainties in the DWBA calculation or in the philosophy of the methods used.

Some of the detailed problems in obtaining the results will now be discussed. Each paragraph is headed by the name of the target nucleus, with its spin and parity in parenthesis.

Cr^{53} $(3/2^-)$

The data from Ref. 14 was analyzed with DWBA calculations. This procedure gave $S=0.8$ for the closed shell to single-particle transition in $Cr^{52}(d,p)$ for which S should be 1.0, so a 10% renormalization was made to give some weight to this information.

Fe^{57} $(1/2^-)$

The fit of the 2^+ data to the sum of $l=1$ and $l=3$ angular distributions is shown in Fig. 1. There is fair agreement with previous work[20] which was done with much poorer statistics and optical-model parameters which are not now considered appropriate.

Ni^{61} $(3/2^-)$

The angular distribution for the 2^+ appears to be a pure $l=1$. However, if the ratio of the intensities of the 0^+ to 2^+ peaks is plotted versus angle as in Fig. 2, there is a dip at the angle where the $l=3$ angular distribution has its peak. The result is calculated assuming that this dip is due to an $l=3$ mixture in the angular distribution for the 2^+ state. For the $l=1$ components, there is good agreement with Ref. 20 (but note the preceding paragraph).

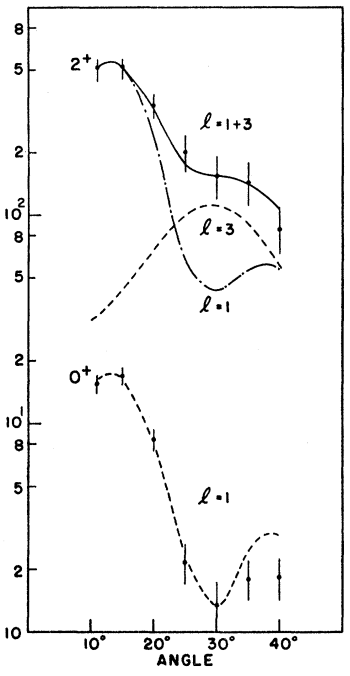

FIG. 1. The data points are angular distributions of proton groups leading to the lowest 0^+ and 2^+ states of Fe^{58} from the reaction $Fe^{57}(d,p)Fe^{58}$. The dashed curves are the results of DWBA calculations, and the dot-dash curve is obtained from a curve through the data for the 0^+ group. The upper part of the figure shows the components of $l=1$ and $l=3$ angular distributions used to fit the data for the 2^+ group; the solid curve shows the sum of these two components.

[13] B. L. Cohen, R. H. Fulmer, and A. L. McCarthy, Phys. Rev. **126**, 698 (1962), and other references given therein.
[14] M. V. Pasechnik and P. G. Ivanitski, Zh. Eksperim. i Teor. Fiz. **44**, 1129 (1963) [English transl.: Soviet Phys.—JETP **17**, 761 (1963)].
[15] E. K. Lin and B. L. Cohen, Phys. Rev. **132**, 2632 (1963).
[16] E. K. Lin, Phys. Rev. **139**, B340 (1965).
[17] S. A. Hjorth and B. L. Cohen, Phys. Rev. **135**, B920 (1964).
[18] E. J. Schneid, A. Prakash, and B. L. Cohen, Phys. Rev. (to be published).
[19] R. K. Jolly, Phys. Rev. **136**, B683 (1964).
[20] R. H. Fulmer and A. L. McCarthy, Phys. Rev. **131**, 2133 (1963).

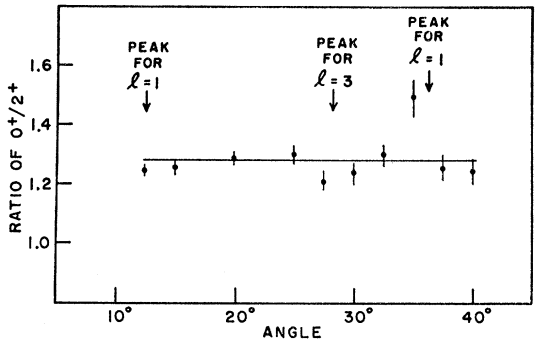

FIG. 2. Ratio of intensity of $0^+/2^+$ transitions in $Ni^{61}(d,p)Ni^{62}$.

Zr^{91} $(5/2^+)$

The angular distribution for the 2^+ state is almost pure $l=2$, but there is some deviation at 8° where $l=0$ is very large and $l=2$ is small; this was used for the analysis. Some weight was also given to an analysis at 30° where $l=0$ is at its second maximum while $l=2$ is at a minimum.

$Ru^{99,101}$ $(5/2^+)$

The 0^+ state for Ru^{102} and the 2^+ state for Ru^{100} could not be resolved. The combined angular distribution was analyzed as shown in Fig. 3a, into $l=0$ and $l=2$ components using the $l=2$ angular distribution from the 0^+ state for Ru^{100} and the $l=0$ from the DWBA calculation. Since one expects the cross section for the Ru^{102} (0^+) transition to be by far the larger of the two, its S value is estimated by assuming the S values for the $l=2$ components of the 2^+ transitions in the two isotopes to be about the same; if they differ by a factor of 2, it would not materially change the result. The 2^+ state in Ru^{102} is analyzed in routine fashion into $l=0$ and $l=2$ components as shown in Fig. 3b. No account was taken of possible $l=4$ components for either isotope, since there was no indication of excessive intensity at the larger angles.

Pd^{105} $(5/2^+)$

The data for the 2^+ state are shown in Fig. 4. The peak in this angular distribution is between the peaks for $l=0$ and $l=2$; this places very stringent requirements on the ratio of the two, and gives little leeway in fitting the rest of the angular distribution. The over-all fit is not especially good; it would not be improved by the inclusion of an $l=4$ component. The fit could be greatly improved by adjustments in the $l=0$ angular distribution which are within the uncertainties inherent in the DWBA calculations. These adjustments would not greatly affect the results.

$Ba^{135,137}$ $(3/2^+)$

The transition to the 0^+ from Ba^{137} is single-hole to closed-shell, so that S must be $(2j+1),=4$. This was used to normalize the experimental cross section, since

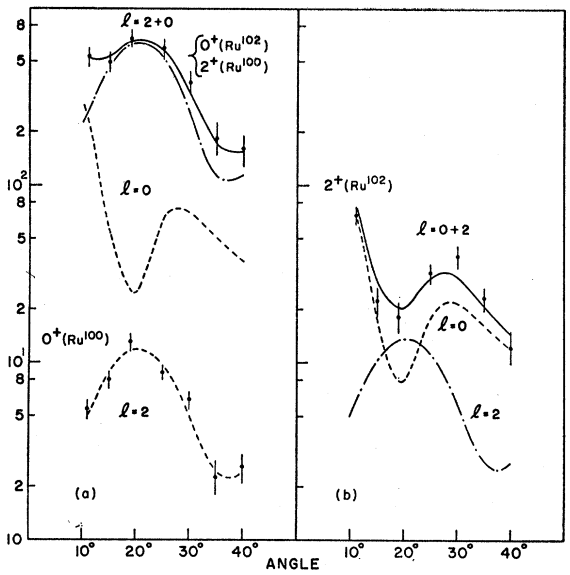

FIG. 3. Data for $Ru(d,p)$; see caption for Fig. 1. The upper part of Fig. 3a shows the sum of the intensity from the 0^+ state of Ru^{102} and the 2^+ state of Ru^{100}, which were not resolved; and the fit of this sum to the sum of $l=0$ and $l=2$ angular distributions. The $l=2$ components in this fit and in the fit of Fig. 3b were obtained from the Ru^{100} 0^+ data, and the $l=0$ component is from DWBA calculations.

the target thickness was very poorly known. Data were usable at only six angles, and only up to 27°. The fit to the sum of $l=0$ and $l=2$ angular distributions is shown in Fig. 5.

$Nd^{143,145}$ $(7/2^-)$

In Nd, the 0^+ states from the two isotopes, and likewise the 2^+ states, were not clearly resolved, so

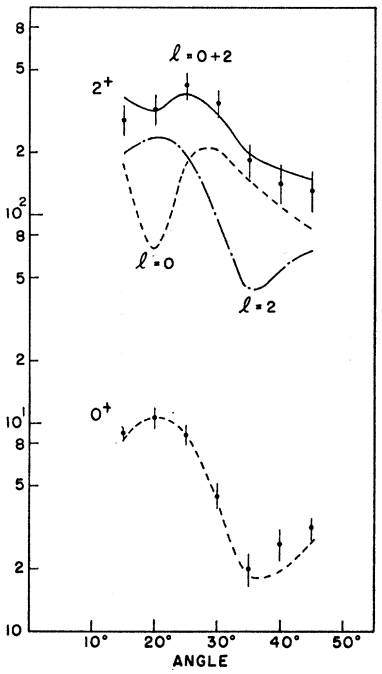

FIG. 4. Data of $Pd^{105}(d,p)Pd^{106}$. See caption for Fig. 1.

that the data for the two isotopes were combined. The data were generally of very poor statistical accuracy, and the angular distributions did not agree well with others found in this region.[21] The results are therefore quoted with large errors, and should be considered qualitative rather than quantitative.

$Sm^{147,149}$ $(7/2^-)$

As in the case of Nd, the data from the two isotopes were not well resolved, and were therefore combined. The $l=3$ angular distribution was not well fit by the DWBA calculations, so that the experimental curve for the 0^+ was used in analyzing the data for the 2^+. The DWBA calculations did agree well with $l=1$ angular distributions from Ref. 21, so that they could be used with some confidence for $l=1$ here. The fitting of the 2^+ to the sum of the $l=1$ and $l=3$ is shown in Fig. 6. It is reasonably satisfactory.

DISCUSSION

The quality of agreement between the experimental and theoretical spectroscopic factors can be seen from Table I. For the ground-state transitions the agreement is usually within better than 20% showing that the pairing plus $P^{(2)}$ model with the parameters of KS does give the correct fraction of pure quasiparticle in the odd ground state together with the fullness of the level. Even for Fe^{57} the agreement is excellent.

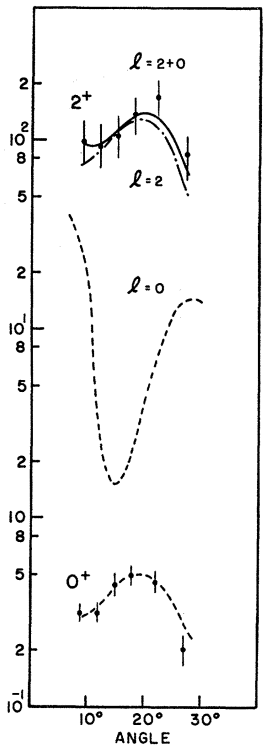

FIG. 5. Data for $Ba^{135}(d,p)Ba^{136}$. See caption for Fig. 1.

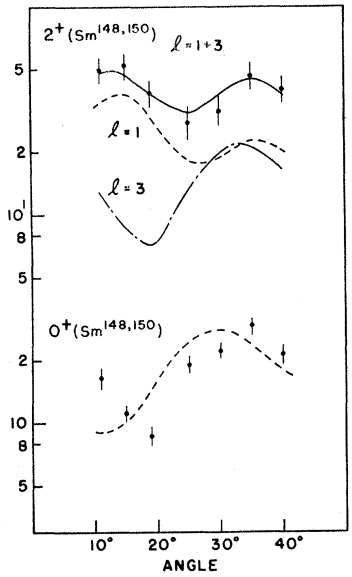

FIG. 6. Data for $Sm^{147,149}(d,p)Sm^{148,150}$. Data for the two isotopes are combined. See caption for Fig. 1.

The agreement between experiment and theory for the 2^+ transitions is not nearly as good. This is not surprising for several reasons. First, the experimental spectroscopic factors are less reliable particularly when several l values are involved. Second, the theoretical value has contributions coming from several parts of the odd and even wave functions and is thus often quite sensitive to the parameters of the theory particularly for small spectroscopic factors. Nevertheless, even where spectroscopic factors for two l's are each compared with theory, most of them agree within about a factor of 2. This is not too bad since the quantities being compared range over two orders of magnitude in value. In addition, in the cases with $l=0$ and $l=2$, the theoretical $l=4$ spectroscopic factor is always so small as to be undetectable. Likewise the predicted $l=5$ is always small in the $l=1$, $l=3$ cases. Thus in most cases, agreement between theory and experiment is about as expected, but there are a few cases in which the disagreement is so substantial that it must be assumed either that the experimental analysis in terms of spectroscopic factors failed or that the wave functions used were inadequate. These unsatisfactory cases need not be taken as evidence for the existence of a new reaction mechanism such as stripping with 2+ excitation by the proton or deuteron.[22–24] Instead, in view of the generally satisfactory agreement, it would probably be more useful to try to improve our quantitative understanding of these reactions in terms of the conventional stripping theory until more clear-cut discrepancies arise.

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