(see Fig. 5) are unresolved, and He^3 particles could increase the yield of apparent α_2 events. At backward angles curves B and E are sufficiently close to allow an overlap of the long tail with α_2, and this may be expected to contribute in an increasing amount to apparent α_2 events. It is not clear whether the reaction C^{12}-$(Li^6,\alpha_{3,4})N^{14}$ (curve C of Fig. 5) would have contaminated the 1.7-MeV α_2 data.

CONCLUSION

The reaction $C^{12}(Li^6,\alpha)N^{14}$ now shows a consistent behavior from 2 to 4 MeV. The cross section grows rapidly with energy, corresponding to the increased penetration of the Coulomb barrier. Behavior of the angular distributions between 40° and 120° changes very little with beam energy, which is consistent with a direct reaction with a high Q value (since the momentum transfer changes slowly). Some fluctuations are seen at the back angles.

ACKNOWLEDGMENTS

We are grateful to Professor J. M. Blair for several helpful discussions and to D. Bauman and the entire staff of the Van de Graaff laboratory for their assistance.

PHYSICAL REVIEW VOLUME 142, NUMBER 3 FEBRUARY 1966

Gamma-Ray Cascades from Nuclei with High Spin*

Daniel Sperber

Physics Division, Illinois Institute of Technology, Research Institute, Chicago, Illinois

(Received 22 July 1965; revised manuscript received 20 October 1965)

The spectra and angular distribution for a cascade of gamma rays from nuclear states with high spin have been calculated. Calculations based on the single-particle and liquid-drop model are presented. It is shown how the angular distribution can be used to determine the appropriate model for the states under consideration.

I. INTRODUCTION

In this paper a calculation of the spectrum and angular distribution for a cascade of gamma rays from nuclear states with high spin is presented.

Most of the deductions about nuclear structure are made by studying nuclear energy levels and nuclear decays. For each nucleus the main rigorously good quantum numbers are the energy E, the total angular momentum J, and the parity π. In the low-energy region the relation between E, J, and π is extremely helpful in understanding nuclear structure. A similar relation for the higher energy region will undoubtedly augment our understanding of this structure. In particular one has to be aware that the well-defined low-lying states constitute only a small fraction of nuclear states. Until recently only a fraction of the higher lying states had been investigated in neutron bombardment. Those states have limited values of angular momenta. A more complete and systematic study includes a variation of energy and angular momentum over a wide range. In the higher energy region parity loses its importance since half the states have positive and half have negative parity. Therefore, in this region it is sufficient to investigate the energies and angular momenta of those states and the relation between the two. Since the states under consideration are dense and have short lifetimes, it is possible only to measure transition rates between such states. The transition rates have to be compared with calculated average values of off-diagonal matrix elements.

The best suited decay for such a study is the electromagnetic one, since for this type of decay the form of the interaction between the electromagnetic field and the nucleus is well known. The off-diagonal elements can be calculated using explicitly model-dependent formulas for decay rates. From comparison between theory and experiment, the appropriate nuclear model can be inferred. Since successive steps in a gamma-ray cascade cannot be distinguished experimentally, the spectrum and angular distribution for a cascade of gamma rays must be calculated.

Nuclear states with high spin may be prepared in heavy-ion bombardment. The direction of the spin of the compound system is close to a plane perpendicular to the direction of the heavy-ion beam. Therefore the anisotropy in the angular distribution of the emitted radiation from such states is expected. Both the spectrum and the angular distribution of the emitted gamma rays give valuable information.

The present calculation is based on the statistical model. The transition probability is expressed as a product of a square of the appropriate nuclear matrix element and the density of final states.

The angular distribution for gamma rays for a single transition from nuclei with high spin was calculated by

* Work supported by the U. S. Atomic Energy Commission.

Strutinskii,[1] Babikov,[2] and Sperber.[3,4] However, the emission of a few successive gamma rays is not infrequent.[5–12] Therefore distributions of cascades of gamma rays have to be studied. Previously Strutinskii[13,14] and Troubetzkoy[15] discussed cascades of gamma rays. In those two calculations no mechanism for the decay is assumed. The nuclear matrix element is introduced as a constant parameter. Also, spin-dependent parameters are neglected, and consequently angular distributions are not calculated.

In this paper a generalization of the work mentioned in Refs. 3 and 4 for a cascade of gamma rays is presented. Nuclear matrix elements are calculated using the single-particle model and the liquid-drop model. The nuclear matrix element and density of levels used in this work are spin-dependent. The angular distribution is calculated with respect to the heavy-ion beam which produces the states with high spin.

II. THEORY

The spectrum and angular distribution of the nth member of a gamma-ray cascade depends on the level occupation after $(n-1)$ emissions and on the spectrum and angular distribution for one transition. The occupation of levels is energy- and spin-dependent. Let $y_n(E,J,M;t)$ be the level occupation after the emission of n gamma rays, and $t=0$ be the time at which gamma emission started. The following equations are satisfied by the functions $y_n(E,J,M;t)$:

$$\frac{\partial y_n(E,J,M)}{\partial t}=\delta_{n,0}\sum_{J'M'}\int_E^{E_0} y_{n-1}(E',J',M';t)\times S(E',J',M';E,J,M)\,dE'-y_n(E,J,M)\times\sum_{J'M'}\int_0^E S(E,J,M;E',J',M')\,dE',\quad n=0,1,2,\cdots. \tag{1}$$

In Eq. (1), E_0 is the maximum energy of the gamma-ray-emitting nucleus, and $S(E,J,M;E',J',M')$ are the transition probabilities between the states characterized by E, J, M and E', J', M'. The transition probability $S(E,J,M;E',J',M')$ can be considered as a sum of transition probabilities of a definite multipolarity. Let $S_L(E,J,M;E',J',M')$ be the transition probability for electric radiation of order L. Then

$$S_L(E,J,M;E',J',M')=\frac{8\pi c(L+1)}{L[(2L+1)!!]^2}\left(\frac{E-E'}{hc}\right)^{2L+1}\times e^2/hc\,|\langle JM|Q_{M-M'}{}^L|J'M'\rangle|^2\times\rho(E',J',M'). \tag{2}$$

In Eq. (2) $\langle JM|Q_{M-M'}{}^L|J'M'\rangle$ are the matrix elements of the components of a multipole tensor of order L and $\rho(E',J',M')$ is the density of final states. A similar equation exists for magnetic radiations. Before a choice for the form of nuclear matrix elements and the density of levels is made, the general method of determining the level occupation regardless of the specific form is discussed.

The total level occupation $y(E,J,M;t)$ is the level occupation with the appropriate quantum numbers after the emission of any number of gamma rays and can be expressed as a sum of terms in which each term represents the level occupation after the emission of a specified number of photons, so that

$$y(E,J,M;t)=\sum_{k=0}^{\infty} y_k(E,J,M;t). \tag{3}$$

Using Eqs. (1) and (3) an integrodifferential equation for the total level occupation is derived.

$$\frac{\partial y(E,J,M;t)}{\partial t}=\sum_{J'M'}\int_E^{E_0} y(E',J',M';t)S(E',J',M';E,J,M)\,dE'-y(E,J,M;t)\sum_{J'M'}\int_0^E S(E,J,M;E',J',M')\,dE'. \tag{4}$$

Let $n_L(E,\Omega,t)$ be the total number of the emitted gamma rays of multipolarity L and energy E which have been emitted into the solid angle $d\Omega$ characterized by Ω up to the time t. Then

$$\frac{\partial n_L}{\partial t}=\sum_{J'J''M'}\int_E^{E_0} y(E',J',M';t)\times S_L(E',J',M';E'-E,J'',M'-M)\,dE'\times|\mathbf{Y}_{M-M'}{}^{L(L1)}|^2. \tag{5}$$

In Eq. (5) $\mathbf{Y}_{M-M'}{}^{L(L1)}$ are the vector spherical harmonics

[1] V. M. Strutinskii, Zh. Eksperim. i Teor. Fiz. **37**, 861 (1953) [English transl.: Soviet Phys.—JETP **15**, 1143 (1962)].
[2] V. V. Babikov, Zh. Eksperim. i Teor. Fiz. **42**, 1647 (1962) [English transl.: Soviet Phys.—JEPT **15**, 1143 (1962)].
[3] D. Sperber, Nuovo Cimento **36**, 1164 (1965).
[4] D. Sperber, Phys. Rev. **138**, B1024 (1965).
[5] J. Alexander and G. N. Simonoff, Phys. Rev. **131**, B93 (1964).
[6] J. Alexander and G. N. Simonoff, Phys. Rev. **131**, B104 (1964).
[7] J. Alexander and G. N. Simonoff, in *Proceedings of the Third Conference on Reactions Between Complex Nuclei*, edited by A. Ghiorso, R. M. Diamond, and H. E. Conzett (University of California Press, Berkeley, California, 1963), p. 345.
[8] J. R. Grover, Phys. Rev. **123**, 267 (1961).
[9] J. R. Grover, Phys. Rev. **127**, 2142 (1962).
[10] T. D. Thomas, Phys. Rev. **116**, 703 (1959).
[11] J. F. Mollenauer, Phys. Rev. **126**, 867 (1962).
[12] Yuits Oganesyau, Yu V. Lobanov, B. H. Markov, and O. N. Flerov, Zh. Eksperim. i Teor. Fiz. **44**, 1171 (1963) [English transl.: Soviet Phys.—JETP **17**, 791 (1963)].
[13] V. M. Strutinskii, L. V. Groshev, and M. K. Akimova, Nucl. Phys. **16**, 657 (1959).
[14] A. M. Demidov, L. V. Groshev, and V. A. Pelekhev, Nucl. Phys. **16**, 645 (1960).
[15] E. S. Troubetzkoy, Phys. Rev. **122**, 212 (1961).

of order L. Since at the time $t=0$ there are no gamma rays the solution of this equation becomes

$$n_L(E,\Omega,t)=\sum_{J'J''M'}\int_0^t\int_E^{E_0} y(E',J',M';t)\times S_L(E',J'M';E'-E,J'',M'-M)dE'\,dt\times|\mathbf{Y}_{M-M'}{}^{L(L1)}|^2. \quad (6)$$

For all practical purposes one is interested in the function n_L at a time which is infinitely large as compared with the lifetime for electric dipole or electric quadrupole transitions. One therefore is interested in $n_L(E,\Omega,t=\infty)$.

It is convenient now to introduce the radiation width Γ_γ such that

$$\Gamma_\gamma(E,J,M)=\hbar\sum_{J'M'}\int_0^E S(E,J,M;E',J',M')\,dE', \quad (7)$$

and a branching ratio $T(E,J,M;E',J',M')$ such that

$$S(E,J,M;E',J',M')=\frac{\Gamma_\gamma(E,J,M)T(E,J,M;E',J',M')}{\hbar}, \quad (8)$$

and also a function $Z(E,J,M)$ such that

$$Z(E,J,M)=\int_0^\infty y(E,J,M;t)\Gamma_\gamma(E,J,M)\,dt. \quad (9)$$

Using Eqs. (6)–(8) and (9), $n_L(E,\Omega,t=\infty)$ may be rewritten as

$$n_L(E,\Omega,t=\infty)=\sum_{J''J'M'}\int_E^{E_0} Z(E',J',M')\times T(E',J',M';E'-E,J'',M'-M)\,dE'\times|\mathbf{Y}_{M-M'}{}^{L(L1)}|^2. \quad (10)$$

Therefore, if $Z(E',J',M')$ is known, $n_L(E,\Omega,\infty)$ can be calculated.

Now an equation for $Z(E,J,M)$ is derived. Integrating Eq. (4) over t from $t=0$ to $t=\infty$, one obtains

$$y(E,J,M;\infty)-y(E,J,M;0)=\sum_{J'M'}\int_E^{E_0} Z(E',J',M')\times T(E',J',M';E,J,M)\,dE'-Z(E,J,M). \quad (11)$$

Since for $E>0$ the first term on the left-hand side of Eq. (11) vanishes, the following equation for $Z(E,J,M)$ is obtained.

$$Z(E,J,M)=\sum_{J'M'}\int_E^{E_0} Z(E',J',M')T(E',J',M';E,J,M)\,dE'+y(E,J,M;0). \quad (12)$$

In Eq. (12), $y(E,J,M;0)$ depends on the level occupation at $t=0$ which has to be supplied as an initial condition.

Two different forms were used for $y(E,J,M;0)$. First a δ function was considered.

$$y(E,J,M;0)=C\times\delta(E_0-E)\delta(J-J_0)\delta(M). \quad (13)$$

In Eq. (13), C is a proportionality constant. The above form for the function $y(E,J,M;0)$ is not too removed from physical reality as the following argument shows. In heavy-ion bombardment the spin of the compound system is in a direction perpendicular to the heavy-ion beam. Therefore the projection of the spin in the direction of the beam vanishes. The neutrons which precede gamma emission carry away only a small fraction of the spin; therefore when gamma emission sets in the z component of the spin in the direction of the heavy-ion beam is still very small, hence the factor $\delta(M)$. The spread of the spin of the compound nucleus in heavy-ion bombardment is not considerable and the change due to evaporated neutrons is insignificant, hence the factor $\delta(J-J_0)$. Finally, energy considerations limit the spread of the energy range at which gamma emission sets in, to an energy around E_0, hence the factor $\delta(E-E_0)$ in Eq. (13). To determine the effect of a more realistic form for $y(E,J,M;0)$ a Gaussian form was used

$$y(E,J,M;0)=C\exp\left\{-\left[\left(\frac{E-E_0}{\Delta E}\right)^2+\left(\frac{J-J_0}{\Delta J}\right)^2+\left(\frac{J-J_0}{\Delta M}\right)^2\right]\right\}. \quad (14)$$

In Eq. (14), ΔE, ΔJ, and ΔM are the respective spreads in energy, spin, and the z component of the spin. The linearity of Eq. (12) suggests that if a solution for a y used in Eq. (13) is known, then the solution for a y used in Eq. (14) may easily be generated. In particular if the solution of Eq. (12) using a δ-type y is $Z_1(E,J,M)$, then the solution of Eq. (12) using a Gaussian y is

$$Z_2(E,J,M)=\sum_{J'M'}\int Z_1(E',J',M')\exp\left\{-\left[\left(\frac{E'-E}{\Delta E}\right)^2+\left(\frac{J'-J}{\Delta J}\right)^2+\left(\frac{M'}{\Delta M}\right)^2\right]\right\}dE'. \quad (15)$$

Equation (12) can be solved by successive approximations such that

$$Z(E,J,M)=\sum_{k=0}^{\infty} Z_k(E,J,M) \quad (16)$$

and

$$Z_k(E,J,M)=\sum_{J'M'}\int Z_{k-1}(E',J',M)\times T(E',J',M';E,J,M)\,dE',\quad k=1,2,3,\cdots,\tag{17}$$

$$Z_0(E,J,M)=y(E,J,M;0).\tag{18}$$

Now the choice of the form of the nuclear matrix element and the forms of the density of states are discussed.

Two extreme models for the decay were used, (a) the single-particle model and (b) the liquid-drop model. According to the shell model the radiation is mainly electric dipole, for which the average value of the appropriate matrix element for a transition from a high-spin state have been given previously.[3]

$$|\langle J_iM_i|Q_M{}^1|J_fM_f\rangle|^2=(e^2R_0{}^2/576)(2J_i+1)(2J_f+1)\times\begin{pmatrix}J_f & 1 & J_i\\ -M_f & M & M_i\end{pmatrix}^2.\tag{19}$$

In Eq. (19), R_0 is the radius of the nucleus and

$$\begin{pmatrix}J_f & 1 & J_i\\ -M_f & M & M_i\end{pmatrix}$$

is a Wigner $3j$ coefficient. According to the liquid-drop model the radiation is predominantly electric quadrupole and the nuclear matrix elements are as given previously.[4] These nuclear matrix elements were calculated for the case where the shapes of equilibrium are oblate spheroids and the matrix elements are

$$\langle Q_0{}^2\rangle=\tfrac{1}{4}(5/\pi)^{1/2}\tfrac{1}{5}ZeR_0{}^2(2/\eta^{2/3})(\eta^2-1),\tag{20a}$$

$$\langle Q_{\pm1}{}^2\rangle=\langle Q_{\pm2}{}^2\rangle=0.\tag{20b}$$

Here Z is the nuclear charge, R_0 the radius of the spherical nucleus, and η is the ratio of the minor to the major axis. The dependence of η on spin can be found in Ref. 16. For spins higher than a critical value Beringer and Knox[16] find that the shapes of equilibrium are prolate spheroids rotating around one of their minor axes. For such spheroids the components of the quadrupole tensor calculated previously[4] become

$$\langle Q_0{}^2\rangle=\tfrac{1}{4}(5/\pi)^{1/2}\tfrac{1}{5}ZeR_0{}^2(1/\eta^{4/3})(\eta^2-1),\tag{21a}$$

$$\langle Q_{\pm1}{}^2\rangle=0,\tag{21b}$$

$$\langle Q_{\pm2}{}^2\rangle=\tfrac{1}{4}(15/2\pi)^{1/2}\tfrac{1}{5}ZeR_0{}^2(1/\eta^{4/3})(\eta^2-1).\tag{21c}$$

Various spin-dependent and spin-independent forms for the density of levels have been suggested. For the present treatment spin-dependent factors are of paramount importance and are therefore included. Thomas,[17,18] in the analysis of nuclear evaporation, concluded that the most accurate form for the densities of the levels is

$$\rho(E,J)=\frac{(2J+1)}{\pi^{1/2}(2c\tau_J)^{3/2}}\exp\left[\frac{-(J+\frac{1}{2})^2}{2c\tau_J}\right]\rho(E),\tag{22}$$

where

$$\rho(E)=B(E+t)^{-5/4}\exp[2(aE)^{1/2}].\tag{23}$$

Here the thermodynamic temperature t is determined by

$$E=at^2-t\tag{24}$$

and the spin-dependent nuclear temperature τ_J is defined by

$$\frac{1}{\tau}=\frac{d\ln\rho(E,J)}{dE}=\left(\frac{a}{E}\right)^{1/2}-\frac{5}{2}\frac{1}{(2E+t)},\qquad \frac{1}{\tau_J}=\frac{1}{\tau}+\left(\frac{(J+\frac{1}{2})^2}{2c\tau}-\frac{3}{2}\right)\frac{d\ln\tau}{dE}.\tag{25}$$

The moment of inertia of the nucleus is $c\hbar^2$. Equations (22) and (23) were first derived by Lang and LeCouteur.[19]

III. RESULTS AND CONCLUSION

Equation (12) was solved numerically using the method of successive approximation of Eqs. (17) and (18). For single-particle transitions, Eq. (19) was used for the nuclear matrix element while Eqs. (20) and (21) were used for transitions according to the liquid-drop model. The forms (22), (23), and (24) were used for the densities of levels for both models.

A sample calculation was performed for $A=200$, and excitation of 10 MeV and a spin of 30 units of $\hbar$, with the parameter a as 20 MeV^{-1}.

The repeated integration in Eq. (17) was performed numerically using Simpson's $\frac{3}{8}$ rule and coded for a computer. The energy range was divided into intervals of 0.25 MeV. First a calculation with the initial condition [Eq. (13)] was performed.

To check the accuracy of the numerical integration a comparison was made between the numerical and analytically calculated results. For this comparison a less realistic form for the density of levels was chosen such that

$$\rho(E,J)=\exp[(E/t)-J^2/2ct],\tag{26}$$

for which $Z_1(E,J,M)$ can be calculated analytically. The largest deviation was 0.5% of the analytically calculated value. This deviation was obtained for energies below 1.0 MeV. For such low energies the density of levels becomes meaningless so that the results are of no

[16] R. Beringer and W. J. Knox, Phys. Rev. **121**, 1095 (1961).

[17] T. Darrah Thomas, Nucl. Phys. **53**, 558 (1964).

[18] T. Darrah Thomas, Nucl. Phys. **53**, 577 (1964).

[19] J. M. Lang and K. J. LeCouteur, Proc. Phys. Soc. (London) **A67**, 586 (1954).

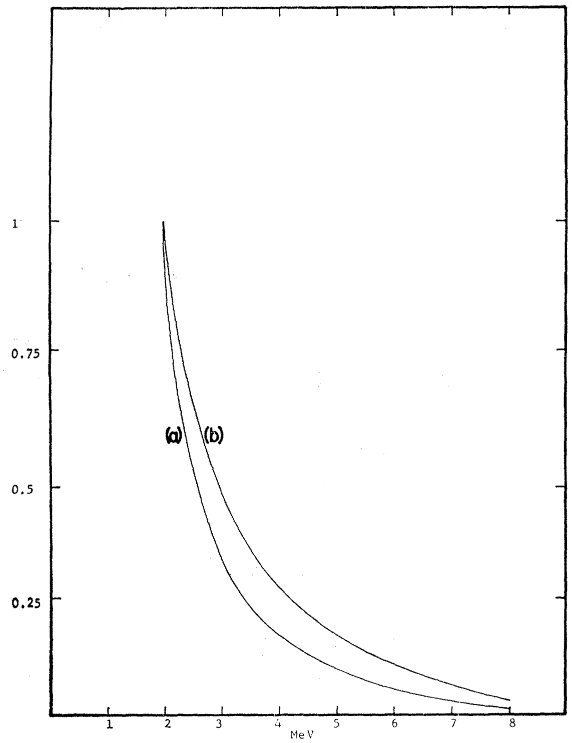

FIG. 1. The spectrum for a cascade of gamma rays in the energy region $2<E<8$ MeV. Curve (a) represents the spectrum according to the single-particle model and the curve (b) represents the spectrum according to the liquid-drop model. The gamma-ray intensity is expressed in arbitrary units; for comparison both intensities are normalized to 1 at 2 MeV.

interest; also the contribution to radiation from such low-lying states is not appreciable. If energies below 2 MeV are disregarded the deviation does not exceed 10^{-5} of the analytically calculated result.

The series [Eq. (16)] converges very quickly. For a test of convergence, the functions $Z(E)$, $Z_k(E)$, and θ_k are introdoced such that

$$Z_k(E)=\sum_{JM} Z_k(E,J,M)\,, \tag{27a}$$

$$Z(E)=\sum Z_k(E)=Z_1(E)\sum \theta_k(E)\,, \tag{27b}$$

where

$$\theta_k(E)=Z_k(E)/Z_1(E)\,. \tag{28}$$

The function $Z_k(E)$ is closely related to the total population of states at an energy E after k gamma emissions. This population after k emissions is compared to the population after one emission by means of the function $\theta_k(E)$. Therefore the function $\theta_k(E)$ measures the relative contribution to the population of states from the kth step of the cascade when the contribution from the first step is normalized to unity. For low energies the function $\theta_k(E)$ first increases and then decreases with k. For higher energies this function always decreases with k (see Table I). The requirement that

$$\theta_k(E)>\theta_{k+1}(E) \quad \text{for} \quad 0\leq E\leq E_0\,, \tag{29a}$$

which may be replaced by the stronger requirement that

$$\theta_k(0)>\theta_{k+1}(0)\,, \tag{29b}$$

is used as criterion for convergence. The satisfaction of this criterion guarantees that for all energies the contribution from the $(k+1)$st member of the cascade is smaller than the contribution from the kth member of this cascade. This occurs (see Table I) for $k=6$. A glance at Table I also shows that for higher energies the sum of the contribution due to the first terms of the cascade is much larger than to all the others.

The shape of the spectrum is shown graphically in Fig. 1. Curves (a) and (b) illustrate the spectra calculated on the basis of the single-particle model and of the liquid-drop model, respectively. In the first case the radiation is mainly electric dipole and in the second case, mainly electric quadrupole. The gamma-ray intensity is expressed in arbitrary units and for both cases is normalized to unity at 2 MeV. The spectra in Fig. 1 correspond to a δ-type of initial population of states. The shape of the spectrum changes only slightly if, instead, a Gaussian population of states with $\Delta E=2$ MeV is used.

The shape of the spectrum is sensitive to the parameter a appearing in Eq. (25), the equation for the density of states. Therefore the shape of the spectrum by itself cannot be used as a criterion for the appropriate nuclear model.

On the other hand, the angular distribution is very sensitive to the model and less sensitive to the initial type of population of states. This fact is demonstrated in the following equations for the angular distribution.

$$I_1=1+0.030\sin^2\theta\,, \tag{30a}$$

$$I_2=1+0.028\sin^2\theta\,, \tag{30b}$$

$$I_3=1+0.04\sin^2\theta\,, \tag{30c}$$

$$I_4=1+2.34\sin^2\theta-2.46\sin^4\theta\,, \tag{31a}$$

$$I_5=1+2.30\sin^2\theta-2.40\sin^4\theta\,, \tag{31b}$$

$$I_6=1+2.52\sin^2\theta-2.61\sin^4\theta\,. \tag{31c}$$

For each model the angular distribution for a cascade of gamma rays is compared with the angular distribu-

TABLE I. The functions $\theta_k(E)$, tabulated for the energy region $0<E<8$ MeV. For higher energy regions the value of these functions becomes so small they are not tabulated. Here the $\theta_k(E)$ are calculated according to the single-particle model. Similar results according to the liquid-drop model predict even faster convergence.

	$E=0$	$E=1$	$E=2$	$E=3$	$E=4$	$E=5$	$E=6$	$E=7$
$k=2$	11.30	4.06	2.06	1.14	0.65	0.32	0.19	0.07
$k=3$	51.03	5.08	1.18	0.35	0.11	0.03	0.01	0.00
$k=4$	105.53	2.53	0.25	0.03	0.00	0.00	0.00	0.00
$k=5$	109.20	0.60	0.02	0.00	0.00	0.00	0.00	0.00
$k=6$	60.74	0.08	0.00	0.00	0.00	0.00	0.00	0.00

tion of a single transition. Also, for each model, the angular distribution of the cascade of gamma rays is calculated for two types of initial conditions. First the initial population of states is assumed to be of a δ-type, and second, a Gaussian initial population of states with $\Delta J=5$ and $\Delta M=5$ is used. In Eqs. (30) and (31), the angular distribution according to the single-particle model is given as a function of the polar angle with respect to the direction of the heavy-ion beam. The angular distributions in Eqs. (30a) and (30b) are for a cascade of gamma rays with an initial δ type and with a Gaussian-type population of states, respectively, while the angular distribution in Eq. (30c) is an angular distribution for one emitted gamma ray according to the single-particle model. The equivalent equations for the angular distribution according to the liquid-drop model are found in Eqs. (31). All angular distributions are so normalized that the angle-independent term is one.

By comparing the angular distribution in Eqs. (30) and (31) one immediately notices the big difference in the angular distribution as predicted by the two models. Furthermore, it can be seen that the angular distribution is more isotropic for a cascade of gamma rays than for a single emitted gamma ray, and that the angular distribution is more isotropic for a Gaussian type of initial population of states than for a δ-type initial population of states.

For a cascade of dipole gamma rays when $J\rightarrow J-1\rightarrow J-2\cdots$ a much larger anisotropy is expected than the one predicted by Eq. (30). The reduction in the anisotropy is due to dipole transition from a state with angular momentum J to states with angular momenta J and $J+1$. This point can be best demonstrated by considering a single gamma transition. The angular distributions $A(J\rightarrow J';\theta)$ for transitions from a state with a spin of $30\hbar$ for a nucleus of $A=200$ were calculated as

$$A(30\rightarrow 29;\theta)=1+0.5\sin^2\theta, \tag{32a}$$

$$A(30\rightarrow 30;\theta)=1-0.48\sin^2\theta, \tag{32b}$$

$$A(30\rightarrow 31)=1+0.44\sin^2\theta. \tag{32c}$$

The relative intensities $I(J\rightarrow J')$, which depend on the magnitude of the nuclear matrix element and the spin dependence of density of levels are

$$I(30\rightarrow 29)=1.00, \tag{33a}$$

$$I(30\rightarrow 30)=1.14, \tag{33b}$$

$$I(30\rightarrow 31)=0.35. \tag{33c}$$

The transition probability of $30\rightarrow 30$ is larger than the transition $30\rightarrow 29$ despite the fact that there are more final states with $J=29$ than states with $J=30$. However, the nuclear matrix element for the transition $30\rightarrow 30$ is almost twice as large as the nuclear matrix element for the transition $30\rightarrow 29$. Using Eqs. (32) and (33) the combined angular distribution becomes

$$I_3=1+0.04\sin^2\theta. \tag{30c}$$

The average energy of a gamma ray in this cascade was calculated and found to be 1.65 MeV.

In the present sample calculation the dependence of the spin of the compound nucleus excitation was not incorporated. In more realistic cases the inclusion of this dependence would provide the dependence of the angular distribution on the excitation energy.

In conclusion, the angular distribution is sensitive to the nuclear model, but is less sensitive to the number of emitted gamma rays and to the initial population of states.