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Transport of Heat from Metals to Insulators at Low Temperatures

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The thermal relaxation times of thin indium and lead films evaporated on sapphire and quartz single-crystal substrates have been measured. These times are compared with the predictions of two models for the heat-loss process. The first assumes perfect thermal contact between the two materials and corresponds to perfect black-body phonon radiation from the film into the insulator. The second, which predicts a somewhat reduced rate of heat loss (up to ~40% less), corresponds to an acoustic-mismatch model in which the reflection of phonons at the interface is considered. The possible relevance of these two models to the experimental situation in which a metal film is involved is discussed. Experimentally it is found that the results for the indium films approach the predictions for the perfect-thermal-contact model, while the results for the lead films show a slower rate of heat loss than either model predicts. Comparison to other experimental results is also made.

INTRODUCTION

THE fact that a temperature discontinuity can exist at the interface between two dissimilar materials when a heat flux is crossing this interface was first shown by Kapitza.[1] This thermal barrier at the interface is largest and has been studied most completely for cases in which one of the materials is liquid helium. This effect has not been studied as extensively when both materials are solids. This is in part because the effects are not as large; however, it is also due to the fact that it is more difficult to bring the two surfaces into intimate thermal contact and that in the process of doing so (and during subsequent thermal cycling) large strains may be set up near the boundary. Both these problems are overcome to a large degree by evaporating thin metal films onto an insulating substrate since intimate contact is automatically ensured and this procedure cannot appreciably strain the substrate. The thermal properties of such evaporated structures are also of technical interest in the operation of low-temperature electrical circuits.

In brief, there are two basic reasons for the existence of a thermal barrier at the interface. The more fundamental is simply the law of blackbody radiation, but in this case applied to phonons rather than to photons: The maximum possible heat flow from a surface heated to a given temperature is limited by the finite number of phonons that are excited at this temperature and by their finite velocity. This is most apparent at temperatures sufficiently low that the phonon mean free path is long. This source of barrier resistance can be considered to be localized within a distance of one phonon mean free path on one (or both) sides of the interface (depending on whether or not the heat flux is predominantly carried by the phonons in both materials). The second reason for a barrier resistance is that, *if* the heat flux is predominantly carried by the phonons in both materials, the reflection of phonons caused by the acoustic mismatch will further reduce the phonon transmission and hence the heat flow. It is not at all obvious that this second mechanism should be effective at the boundary between a metal and an insulator since the electrons can also act as carriers of heat.[2]

It is thus the intent of this paper to present measurements of the thermal time constants of thin evaporated metal films on single-crystal insulating substrates and to compare the results obtained to other available experimental results and to the theoretical predictions made on the basis of various models of the thermal-conduction process.

THEORETICAL BACKGROUND

In order to treat the rate of energy loss from the thin metallic film theoretically, it is necessary to set up some idealized models for this process. These models are not complete enough to predict the experimentally determined thermal time constants in detail, but should be sufficient to define limits between which the experimentally observed values might be expected to lie. The following discussion, although not leading to any radically new results, should also give some insight into the physical processes involved and into the physical basis of the assumptions made.

In the following it will be assumed that, owing to the very short electron-phonon relaxation time,[3] the electrons and phonons in the alloyed metallic film come into substantial thermal equilibrium with each other in times of the order of 10^{-9} sec or less. Because of the short electron and phonon mean free paths in the metal and due to the small thicknesses of the films studied, it will also be assumed that the temperature of the film is a well-defined quantity and that it is uniform throughout

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* Partially supported by the U. S. Army Electronics Laboratories under Contract DA 36-039 AMC-02280(E).

[1] P. L. Kapitza, J. Phys. (Moscow) 4, 181 (1941).

[2] If the metal is strongly superconducting ($T\rightarrow 0$), the phonons are the dominant carriers of heat because of the ineffectiveness of the electrons. Our metal films were only slightly superconducting ($T\approx T_c$) and the electrons could thus still be effective.

[3] A. H. Wilson, *The Theory of Metals* (Cambridge University Press, New York, 1953).

the film.[4] The phonon mean free path in the insulator will be assumed to be essentially infinite. This is permissible if the mean time for a phonon-scattering process in the insulator is long compared to the thermal relaxation time of the metal film and if the heating pulse is short, as was generally the case in our experiments. This assumption simplifies the calculations since backscattering of the additional phonons supplied by the heater is then not involved in the calculations.

In our experiments, phonons leave the metal film and flow outward into the insulator and we are interested in the rate of energy loss from the heater when the heater is at a temperature $T_1 = T_2 + \Delta T$. Here T_2 is the ambient temperature of the insulator. Although it is not proper to assign a temperature to the phonon disturbance in the insulator since the phonon mean free path there is quite long, there will be an outgoing disturbance in the phonon distribution which travels at the acoustic velocity c_2. In the region near the heater, these additional phonons will pass through a cross-sectional area equal to the area of the heater. The spectral density of these additional phonons will be called $\Delta N(\nu)$, so that in the region of the disturbance

$$N_2(\nu)d\nu = N_2^0(\nu) + \Delta N(\nu)\,, \tag{1}$$

where

$$N_2^0(\nu) = \frac{(4\pi)}{c_2^3}\frac{\nu^2 d\nu}{(e^{h\nu/kT_2}-1)}\,. \tag{2}$$

The net heat flux per unit area is then given by

$$\frac{dQ}{dt} = 2\pi\int_0^{\nu_{\max}}\int_0^{\pi} N_2(\nu)h\nu c_2 \cos\theta_2 \sin\theta_2\, d\theta_2 d\nu\,. \tag{3}$$

Here θ_2 is the angle between the direction of the phonon propagation in the insulator and the normal to the film. It is clear that there is no net contribution caused by $N_2^0(\nu)$ and that, because of the assumptions about the phonon mean free path in the insulator, the contribution from $\Delta N(\nu)$ is zero from $\theta = \frac{1}{2}\pi \rightarrow \pi$. Therefore,

$$dQ/dt = 2\pi\int_0^{\nu_{\max}}\int_0^{\pi/2} \Delta N(\nu)h\nu c_2 \cos\theta_2 \sin\theta_2\, d\theta_2 d\nu\,. \tag{4}$$

Thus, the problem is to determine the form of $\Delta N(\nu)$, which may have a complicated and even unknown dependence on frequency and on angle.

At this point one advantage of the present experimental method of measuring interfacial thermal resistances can be seen. In steady-state thermal-resistance measurements (see, for example, Ref. (8), the temperature is measured at several positions along the two materials and extrapolated to give the temperature at the two sides of the interface. However, the temperature is not a well-defined quantity in such experiments when the temperature is low and phonon-phonon collisions are infrequent. Significantly different temperatures at the interface and hence different interfacial resistances can be obtained depending on the specific assumptions made about the effects on the phonon distribution function caused by wall collisions and phonon-phonon interactions. On the other hand, the interpretation of measurements of the thermal relaxation time of thin films does not require any special assumptions if the film is sufficiently thin and if the phonon mean free path in the substrate is sufficiently long. If these conditions hold, the relaxation-time method measures the excess energy, $\sum_\nu h\nu\Delta N(\nu)$, carried away from the film by the nonequilibrium phonons without reference to the exact form of $\Delta N(\nu)$ or whether or not it can be characterized by an equilibrium temperature.

It is possible to determine the form of $\Delta N(\nu)$ for two simplified cases, that of perfect thermal contact between the film and the insulator (the blackbody phonon-radiation model)[5,6] and the acoustic-mismatch model.[5] The first can be thought of as a special case of the second. However, as will be described later, the relevance of these models to the case of a *metallic* film in contact with an insulator is not too clear. However, it can be argued that these two models may furnish upper and lower limits between which our present experimental results on the thermal relaxation time might be expected to lie.

These two models consider only the phonons. In the metal film the number of phonons of frequency ν striking a unit area of the surface in a unit of time at an angle θ_1 is just $2\pi N_1(\nu)c_1\cos\theta_1\sin\theta_1 d\theta_1 d\nu$. If $\alpha_1(\theta_1)$ is the transmission coefficient of the surface for these phonons, the rate at which they are transferred across the surface in the positive direction is

$$2\pi\alpha_1(\theta_1)N_1(\nu)c_1\cos\theta_1\sin\theta_1 d\theta_1 d\nu\,. \tag{5}$$

In the insulator there is a flow of reflected phonons away from the surface given by

$$2\pi[1-\alpha_2(\theta_2)]N_2(\nu)c_2\cos\theta_2\sin\theta_2 d\theta_2 d\nu\,. \tag{6}$$

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[4] That the temperature of the film should become uniform in a very short time is most readily shown for the Pb-Bi alloy films since detailed information on the thermal conductivity of such alloys at ~8°K is available. It can be shown from the data of J. L. Olsen [Proc. Phys. Soc. (London) **A65**, 518 (1952)] that in a 1.5% alloy the mean free paths of the electrons and the phonons are both approximately 400 Å. Thus the times for diffusion of these carriers across the thickest films employed (~2000 Å) will be $\sim 10^{-12}$ sec for the electrons and $\sim\frac{1}{2}\times 10^{-9}$ sec for the phonons. Both of these times are sufficiently fast that they will not contribute appreciably to the observed decay times. More recent experiments on Pb-Bi alloys by P. Lindenfeld [Phys. Rev. Letters **6**, 613 (1961)] and by H. Montgomery [Proc. Roy. Soc. (London) **A244**, 85 (1958)] indicate that the electron diffusion time may be slightly slower (~25%) than the value given above and the phonon diffusion time decreased to $\sim\frac{1}{4}\times\frac{1}{4}0^{-9}$ sec. Data on In-Sn films [P. Lindenfeld and H. Rohrer, Phys. Rev. **139**, 206 (1965)] indicate that for 6% Sn in In at 4°K the electron diffusion time is also about 10^{-12} sec and the phonon diffusion time about 10^{-10} sec. Also the fact that our observed thermal decay times for the In-Sn films are substantially as fast as the maximum speeds theoretically allowed shows that there can be no appreciable contribution to the decay time from thermal diffusion within the In-Sn alloy film.

[5] W. A. Little, Can. J. Phys. **37**, 334 (1959).

[6] R. E. Jones and W. B. Pennebaker, Cryogenics **3**, 215 (1963).

The flow toward the surface is

$$2\pi N_2(\nu)c_2\cos\theta_2\sin\theta_2 d\theta_2 d\nu. \tag{7}$$

The net flow found by combining Eqs. (5), (6), and (7) is

$$2\pi[\alpha_1(\theta_1)N_1(\nu)c_1\cos\theta_1\sin\theta_1\, d\theta_1 d\nu \\ -\alpha_2(\theta_2)N_2(\nu)c_2\cos\theta_2\sin\theta_2\, d\theta_2 d\nu]. \tag{8}$$

The angles θ_1 and θ_2 are connected by Snell's law,

$$c_2\sin\theta_1 = c_1\sin\theta_2, \tag{9}$$

and, therefore,

$$\sin\theta_1\cos\theta_1 d\theta_1 = (c_1/c_2)^2\sin\theta_2\cos\theta_2 d\theta_2. \tag{10}$$

Equation (4) thus becomes

$$\frac{dQ}{dt}=2\pi\int_0^{\nu_{\max}}\int_0^{\pi/2}\frac{\nu^2 h\nu}{c_2^2}\left[\frac{\alpha_1(\theta_1)}{e^{h\nu/kT_1}-1}-\frac{\alpha_2(\theta_2)}{e^{h\nu/kT_2}-1}\right] \\ \times\cos\theta_2\sin\theta_2 d\theta_2 d\nu, \tag{11}$$

where Eq. (2) has been used. Obviously $\alpha_1(\theta_1)$ must equal $\alpha_2(\theta_2)$ in order for microscopic balancing to hold when $T_1 = T_2$.

Integration of Eq. (11) over ν then yields, when the temperature is sufficiently low that $\nu_{\max}$ can be replaced by ∞,

$$\frac{dQ}{dt}=\frac{8\pi^5}{15}\frac{k^4}{c_2^2h^3}T_1^3\Delta T\int_0^{\pi/2}\alpha_2(\theta_2)\cos\theta_2\sin\theta_2 d\theta_2. \tag{12}$$

This result was obtained by Little.

The blackbody model, and hence perfect thermal contact, corresponds to $\alpha_2(\theta_2)=1$ and the value of the integral, to be called Γ, is then $\frac{1}{2}$. In general, phonon reflection does occur since $\alpha_2(\theta_2)<1$ and Γ is found to have a value less than $\frac{1}{2}$. In general, if $\alpha_2(\theta_2)$ is not a constant, evaluation of Eq. (12) must be made numerically. One special case for which this can be done is the acoustic-mismatch model. According to Rayleigh, the acoustic reflection coefficient between two acoustic media is given by

$$\alpha_2(\theta_2)=\alpha_1(\theta_1)=\frac{(4\rho_2c_2/\rho_1c_1)(\cos\theta_2/\cos\theta_1)}{(\rho_2c_2/\rho_1c_1+\cos\theta_2/\cos\theta_1)^2}, \tag{13}$$

where ρ_1 and ρ_2 are the densities of the two media. Using Eq. (13), Little has given numerical values of Eq. (12) over certain ranges of ρ_2/ρ_1 and of c_2/c_1. He has also considered the fact that both longitudinal and transverse phonons are involved and has derived numerical results for Γ for several values of the ratio of the longitudinal and transverse velocities (that is, the Poisson ratio). In this case, the quantity Γ/c_2^2 in Eq. (12) should be replaced by $(\Gamma_{2,l}/c_{2,l}^2+2\Gamma_{2,t}/c_{2,t}^2)$, where Γ_l and Γ_t are the integrated reflection coefficients for longitudinal and transverse phonons.

The question now is, in what way is $\alpha_2(\theta_2)$ changed when we consider not just the phonons, but also the effects of the electrons. Since the electrons rapidly reach equilibrium with the phonons in the metal film, their presence should not interfere with the number of phonons which at a given temperature strike and pass through the surface from either direction. Thus values of Γ calculated with Eq. (13) should give a lower limit for the rate of energy loss (as long as the surfaces are clean and smooth and in intimate contact). However, the electrons may furnish an additional mechanism for transferring heat from the metal to the insulator via a direct interaction between the electrons in medium 1 and the phonons in medium 2. It is difficult to estimate theoretically how effective this mechanism might be, but, from the experimental data on the thermal time constants, it will be seen that in some cases (the In-Sn films) the energy transfer is significantly more rapid than would have been predicted from Eqs. (12) and (13) with the known values of the velocities and densities. Thus, the electrons may indeed contribute to the heat flow.

The rate of energy transfer, even if the electrons play an important role, can never exceed that given by the black-body model (i.e., $\Gamma=\frac{1}{2}$) as long as the effects of surface modes are small as expected. The electrons, because they are in equilibrium with the phonons in the film, simply cannot transfer more energy to the insulator than the phonons would have transferred if they had suffered no reflection at the interface. It will be seen that the experimental thermal time constants in some cases (the In-Sn films) approach this blackbody limit. In other cases (the Pb-Bi films), these time constants are longer even than those predicted from Eq. (12) including acoustic reflection. Presumably this latter can only be caused by some kind of nonideal effect at the interface, such as poor contact between the two materials.

Since Eq. (12) is of the form $dQ/dt=K\Delta T$ and since $dQ/dt=-Cd(\Delta T)/dt$, where C is the heat capacity of the film, it is easily seen that the temperature of the film is given by $\Delta T=\Delta T_0e^{-t/\tau}$. The thermal relaxation time τ is thus given by $\tau=C/K$.

EXPERIMENTAL RESULTS

The experimental data presented here are measurements of the thermal relaxation time of superconducting bolometers similar to the ones previously described in a report[7] on the propagation of heat pulses in insulators. However, the data to be presented are for the most part for films especially prepared to measure the relaxation times. Data is not included for those samples on which heat pulse data was previously obtained and for which the relaxation time was measured only very approximately.

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[7] R. J. von Gutfeld and A. H. Nethercot, Jr., Phys. Rev. Letters **12**, 641 (1964).

The relaxation time measurements were made by heating the evaporated alloy films slightly above the ambient temperature by the use of short pulses of light from either a suitably attenuated giant pulse ruby laser or from a GaAs diode. The ambient temperature for the In-Sn films was usually around 3.8°K, and around 8°K for the Pb-Bi films. These temperatures correspond to resistance values of the films intermediate between those of the normal and totally superconducting states ($T \approx T_c$). The laser light was directed onto the surface of the film by means of a quartz light pipe extending into the Dewar. The decay time of the ruby laser light was $\sim$20 nsec so that this source could only be used to measure thermal decay times substantially greater than 20 nsec. However, the decay times ($<$4 nsec, including the time constants of the photo tube and the electrical circuit) of the GaAs diodes were considerably faster than any of the experimental thermal decay times observed and therefore these diodes could be used for all the relaxation-time measurements.

The change in the film temperature caused by the laser light produced a change in the resistance of the film proportional to this temperature change. Measurements were made with a 1 mA bias current through the detector to provide a voltage proportional to the temperature change. Typically, the rise in temperature of the film due to the laser pulse was $\sim$0.001°K, producing a voltage change of $\sim 10^{-3}$ V. The range over which the superconducting bolometer showed good linearity between output voltage and temperature was generally 0.020°K or more. This voltage was amplified and displayed on a fast oscilloscope. The laser beam was monitored by an appropriate phototube in order that the laser pulse and the response of the thin film could be compared. This provided information for making corrections to the observed thermal decay times for circuit and laser response times when necessary.

Several tracings of typical oscilloscope records of some of the thermal response curves are shown in Fig. 1. Measurements from the *original* oscilloscope records were plotted on log paper and the resultant straight lines demonstrated that the temperature did indeed decay approximately exponentially with time after the light pulse from the laser was completely turned off. The time constants were found from the slope of these straight lines.

Table I gives the data on some of the samples measured. The table is divided into two sections comprising (1) In-Sn alloy films evaporated on sapphire and quartz single-crystal substrates and (2) Pb-Bi films, also on sapphire and quartz single-crystal substrates. These substrates were cylindrical and approximately 1 cm long and 1 cm in diameter. The experimental relaxation times appearing in the table have been normalized to a 1000-Å film thickness. The theoretical values expected from the ideal blackbody-radiation model and from the acoustic-mismatch model are given in the last two columns. The film thicknesses were measured interferometrically directly from the samples themselves.

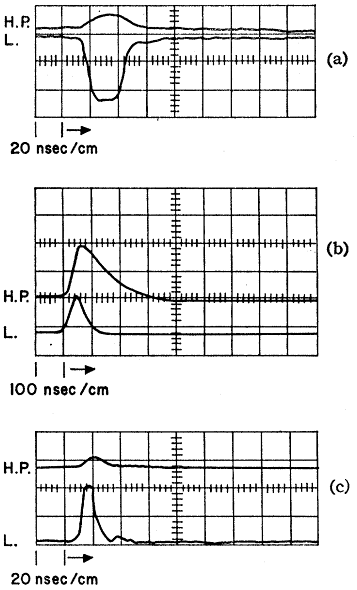

FIG. 1. (a) Thermal response of an indium-tin alloy film on z-cut sapphire. The heat source is a gallium arsenide diode. The upper trace (H.P.) is the thermal response of the film; the lower trace (L.) is the response of the photomultiplier tube to the light. (b) Response of Pb-Bi film on x-cut sapphire (upper trace), and monitor pulse (lower trace). The light source is a giant pulse ruby laser. (c) In-Sn on quartz showing the thermal response of the film (upper trace) and the monitor (lower trace). The heat source is a gallium arsenide diode.

In order to determine the value of Γ for our particular velocity and density ratios so that the theoretical relaxation times predicted by the mismatch theory could be calculated, it was necessary to extrapolate beyond the range of c_2/c_1 given by Little. The same method of extrapolation was employed as that used by Neeper and Dillinger.[8] This corresponds to the approximation that Γ depends only on the ratio $\rho_1 c_1/\rho_2 c_2$ when $c_2/c_1 \ll 1$. We have used those curves in Little's paper for which the Poisson ratio is closest to the ratio appropriate for our films and substrates. Since the values of c_2/c_1 show considerable anisotropy, this produces a range of possible values of Γ to use in the calculation of the relaxation

TABLE I. The measured and calculated relaxation times.

Film	Thickness (Å)	Substrate	τ (Exptl., normalized to 1000 Å) (nsec)	τ_{bb} (nsec)	τ_{am} (nsec)
In(94%)-Sn(6%)	910	z-quartz	9±2	7.5	8–10
In(94%)-Sn(6%)	1,000	x-sapphire	16±2	18	22–29
In(94%)-Sn(6%)	1,200	z-sapphire	17±2	18	22–29
In(94%)-Sn(6%)	1,800	z-sapphire	14±3		
Pb(98.5%)-Bi(1.5%)	1,400	z-quartz	33±4	12	12–14
Pb(98.5%)-Bi(1.5%)	2,000	x-sapphire	50±4	29	33–39

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[8] D. A. Neeper and J. R. Dillinger, Phys. Rev. **135**, A1028 (1964).

times. The range of values given in Table I in the column labeled τ_{am} corresponds to the range of velocities observed for indium in the [100], [001], [110] directions and in the [001], [101], and [111] directions for lead. The values of velocity used for quartz and sapphire are the phase velocities given in the literature. For the substrates we have assumed velocity isotropy. This approximation is quite good for sapphire though less valid for quartz. The values of the specific heat for indium were taken from Bryant and Keesom[9] and those for lead from Horowitz *et al.*[10] We have assumed that the specific heats of the dilute alloys are identical to those of the pure metals. The values of the relaxation times predicted by the ideal-blackbody-radiation model, τ_{bb}, were obtained by setting Γ equal to 0.5. It is clear from Table I that the relaxation times calculated from the ideal-blackbody-radiation model and from the acoustic-mismatch model differ at most by $\sim 40\%$. Since this maximum difference is relatively small and since various assumptions about isotropy, the Poisson ratio, and extrapolations have been made, it is not clear that this difference is readily observable unless the acoustic mismatch is rather large, as would be the case for liquid helium.

The observed relaxation time in closest agreement with the mismatch theory was found to be for In-Sn on z-cut quartz. However, the difference between τ_{bb} and τ_{am} is not large for this case. In-Sn on single-crystal sapphire seems to be in better agreement with ideal-blackbody theory than with the acoustic-mismatch model. The fact that this relaxation time is even slightly faster than predicted by the theory previously stated as setting the shorter limit to the relaxation time requires some explanation. One factor tending to shorten the observed relaxation times as compared to the calculated values is the fact that the film density is assumed in the calculation to be equal to the bulk density. It is likely that this density will be less than the bulk density, in which case the effective heat capacity of the film would be proportionately decreased. Therefore, use of the bulk density may cause the calculated ideal-blackbody relaxation time to appear longer than the observed value. Evidence for such a density decrease of as much as 20% has been experimentally observed for a 6000-Å evaporated indium film.[11]

Although the thermal relaxation time has not been previously measured for indium-quartz and indium-sapphire interfaces, experimental values of the thermal conductances for these interfacial boundaries have been reported. Jones and Pennebaker[6] have measured the thermal conductance of indium films surrounded on both sides by silicon oxide layers. The calculated thermal time constants using their experimental conductance value normalized to a 1000-Å film (with a substrate assumed to be present on only one side) are a factor of 3 slower than our observed relaxation times for the indium-quartz sample. Possibly their assumption that their evaporated SiO substrate layers are similar to single-crystal quartz may not be justified.

Neeper and Dillinger[8] have published experimental values of the conductance of indium-sapphire interfaces that would give a relaxation time approximately a factor of 2 slower than we have experimentally observed. This discrepancy could possibly be explained by strains near the interface originating from their ultrasonic soldering operation. The intimacy of the contact between the indium and the sapphire might also have been different for their ultrasonic soldering technique and our vacuum evaporation of the indium. Since the observed rate of heat flow is a factor of 2 larger in our measurements, it appears that our results may more closely approximate those expected for an ideal interface.[12]

The Pb-Bi films on both sapphire and quartz gave experimental values of the relaxation time considerably slower than those predicted by either theory. However, our normalized relaxation time for Pb on sapphire is $\sim 30\%$ faster than the value obtained by Broom and Simpson[13] for a 600-Å lead film on a sapphire substrate. In their experiment, Broom and Simpson measured the critical current required to switch a superconducting lead film to the normal state. The current was pulsed and the decrease in the value of the current necessary for switching in each successive pulse gave an indirect measure of the temperature of the film.

Neither the blackbody nor the acoustic-mismatch model can explain the relatively slow relaxation times observed for the lead-bismuth films on the quartz and sapphire single-crystal substrates. This might be explained by less fundamental causes. Since the same substrates were used for both In-Sn and Pb-Bi films, it appears that the physical properties of the lead film itself contribute to the slow relaxation time in some way other than through its specific heat. Our lead films were less uniform and more grainy in appearance than our indium films. Such graininess of the film may cause a poor contact to the substrate and thus reduce the surface area in contact with the substrate, especially if the grain structure consists of small spherical islands with a relatively small contact area compared to the

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[9] C. A. Bryant and P. H. Keesom, Phys. Rev. **123**, 491 (1961).

[10] M. Horowitz, A. A. Silvidi, S. F. Malaker, and J. G. Daunt, Phys. Rev. **88**, 1182 (1952).

[11] G. P. Motulevich and A. A. Shubin, Zh. Eksperim. i Teor. Fiz. **44**, 48 (1963) [English transl.: Soviet Phys.—JETP **17**, 33 (1963)].

[12] Our calculated values for the thermal conductance of the indium-sapphire interface, estimated from the acoustic-mismatch theory, differ from those given in the paper of Neeper and Dillinger. Dr. Neeper agrees that the presently derived values of $(0.04 \rightarrow 0.05)T^3$ W/cm^2 deg are more appropriate. Dr. Neeper also informs us that a different temperature extrapolation of his data to give the temperatures at the interface would increase his experimental conductance values by approximately 50% and would thus improve the agreement with our values. However, it is not clear which extrapolation is preferable.

[13] R. F. Broom and M. A. Simpson, Brit. J. Electronics **11**, 78 (1960).

film surface area. Formation of a lead oxide layer at the interface could also greatly reduce the effective heat flow from the film. These factors might help to explain the relatively slow relaxation times observed for the Pb-Bi films. Broom and Simpson also mention film nonuniformities as a possible significant factor in their results.

The observed thermal relaxation times of the In-Sn films on sapphire single crystals provide strong evidence that the blackbody-radiation model is valid for certain films when the substrate has a long phonon mean free path so that phonon back-scattering can be neglected. Also, this data indicates that phonon reflection at the boundary as predicted by the acoustic-mismatch theory does not play a significant role in decreasing the rate of thermal flow across the boundary. It would thus appear that the acoustic-mismatch model may overestimate the size of the interfacial surface resistance in some cases of contact between a metal and an insulator. The data for indium-tin films on quartz indicates a rate of heat flow somewhat less than that predicted by the blackbody model. However, in this case, the predictions of the two models do not differ greatly. Also, since the phonon mean free path in quartz is not as long as that in sapphire,[7] phonon back-scattering may in part influence this result. Finally, since our measured thermal relaxation times are in every instance faster than those which are indicated by previous measurements, it appears that our experimental results may more closely approximate those expected for an ideal interface.

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Fermi Surface of Arsenic*

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The Fermi surface of arsenic has been determined by means of a pseudopotential band-structure calculation. Recent experiments on the de Haas–van Alphen effect permit a more accurate determination of the pseudopotential parameters. The surface consists of three electron pieces located at the point L in the Brillouin zone and the holes are distributed in a multiply connected surface around T. The hole surface can be thought of as being formed by six pockets joined by thin cylinders. Good agreement with experiment is found throughout.

1. INTRODUCTION

THE pseudopotential approach for calculating the band structure of solids has been proved particularly successful in the case of the group-V semimetals.[1–3] In addition, several new experiments concerning the electronic properties of As, Sb, and Bi have been published recently[4,5] or are now under way.[6,7] These two facts considered together have, for the first time, allowed a quantitative determination of the electronic structure of the semimetals, in particular Sb[3] and, as reported here, As.

In this paper we describe a calculation of the Fermi surface of As, which has been obtained by determining the pseudopotential parameters as was done previously for Sb[3] and readjusting slightly the Fermi energy so as to satisfy a few experimental observations. Complete analysis of the surface has been carried out and good agreement with all available experimental data has been found throughout.

Until recently the only experimental data reported in the literature were those of Berlincourt.[8] The information there included was accurately and exhaustively analyzed, but it was not complete since no compensation of electrons and holes was found, i.e.,

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[7] M. G. Priestley, L. R. Windmiller, J. Ketterson, and Y. Eckstein (private communication and to be published).

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[1] L. M. Falicov and S. Golin, Phys. Rev. **137**, A871 (1965); (referred to as I).

[2] S. Golin, Phys. Rev. **140**, A993 (1965).

[3] L. M. Falicov and P. J. Lin, Phys. Rev. (to be published) (referred to as II).

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