

Depolarization of Positive Muons in Solids*

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When polarized positive muons stop in a solid, they may be depolarized prior to their decay. The presence of a magnetic field along the direction of initial polarization will inhibit, or quench, some of this depolarization. We have studied depolarization and quenching in a variety of solids, including semiconductors, at temperatures as low as 4.2°K. No evidence has been found that any muon depolarization is associated with the formation of a muonium atom which is bound for $>10^{-10}$ sec, even in samples at 4.2°K. For many samples and temperatures, the quenching of the observed depolarization is consistent with a model in which the muon captures and loses electrons repeatedly, forming a succession of briefly bound muonium atoms. In some of the semiconductors at low temperatures, as well as in other samples, a large fraction of the depolarization is quenched by a field of 100 G. This is seen as evidence for another depolarization mechanism. We also find that the time during which depolarization occurs in boron carbide decreases with decreasing temperature.

I. INTRODUCTION

A. Muon Depolarization in Solids

WHEN a beam of polarized positive muons is stopped by ionization loss in a target, the angular distribution of decay positrons will be anisotropic unless the muons are depolarized prior to their decay. For a wide variety of solid targets¹⁻⁵ it has been shown that: (1) muons are not depolarized in conductors, (2) muons are depolarized partially or completely in other substances, (3) depolarization occurs (except in boron carbide) in a time $\leq 0.1 \mu\text{sec}$, and (4) in a sufficiently large magnetic field parallel to the polarization axis the depolarization will be prevented or "quenched." For most samples some depolarization still occurs in longitudinal external fields of several kilogauss, indicating that the depolarization is associated with large internal fields.

The quenching of the depolarization of muons in nuclear emulsion and fused quartz is consistent with a mechanism by which muons capture and lose electrons in rapid succession.^{5,6} This model assumes the repeated formation of ground-state muonium.⁷ During each such formation, 50% depolarization would occur in $\sim 10^{-10}$ sec and be characterized by a field of 1580 G. (The presence of an external field of this size along the polarization axis would quench half of the depolariza-

tion which would occur in the absence of such a field.)

In an attempt to explore this process and to understand the part played therein by conduction electrons, we have studied depolarization and quenching in semiconductors.⁸ In this way we obtain targets with a wide range of electron concentrations. Several other substances were studied for comparison. It was originally expected that if conduction electrons played a major role in the depolarization process in semiconductors, cooling the sample might substantially reduce the number of muonium formations. However, no evidence has been found for the occurrence of long-lived ($\gg 10^{-10}$ sec) formations of ground-state muonium in any sample.

This experiment suggests that during its history in a solid, a muon may be exposed in turn to two depolarizing mechanisms. The earlier mechanism is a succession of ground-state muonium formations which occur toward the end of the range of the slowing muon. Subsequently, if the muon still retains some polarization, it may be depolarized by an interaction which is characterized by a magnetic field ~ 50 G, and which depolarizes the muon within $0.2 \mu\text{sec}$ (the lower limit set by the experimental time resolution).

An exception to this behavior is found in boron carbide, in which depolarization occurs over an interval of several microseconds. At low temperatures, the muon polarization disappears in a shorter time than at room temperature.

B. Measured Asymmetry

In the decay $\mu^+ \rightarrow e^+ + \nu + \bar{\nu}$, the angular distribution of decay positrons relative to the direction of muon spin has the form $1 + \alpha \cos\theta$, where α is approximately one-third.⁹ For a beam of polarized muons which has

⁸ Previous work at this laboratory has been reported in G. Feher, R. Prepost, and A. M. Sachs, *Phys. Rev. Letters* **5**, 515 (1960).

⁹ A comprehensive review of the weak interactions of muons appears in G. Feinberg and L. Lederman, *Ann. Rev. Nucl. Sci.* **13**, 431 (1963).

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¹ R. A. Swanson, *Phys. Rev.* **112**, 580 (1958).

² M. Weinrich, Nevis Report No. 56, Ph.D. thesis, 1958 (unpublished).

³ A. O. Vaisenberg, *Usp. Fiz. Nauk* **70**, 429 (1960) [English transl.: *Soviet Phys.—Uspekhi* **3**, 195 (1960)].

⁴ J. M. Cassels, T. W. O'Keeffe, M. Rigby, A. M. Wetherell, and J. R. Wormald, *Proc. Phys. Soc. (London)* **A70**, 543 (1957).

⁵ J. C. Sens, R. A. Swanson, V. L. Telegdi, and D. D. Yovanovitch, *Phys. Rev.* **107**, 1465 (1957).

⁶ R. Ferrell and F. Chaos, *Phys. Rev.* **107**, 1322 (1957); R. A. Ferrell, Y. C. Lee, and M. K. Pal, *ibid.* **118**, 317 (1960).

⁷ J. Friedman and V. L. Telegdi, *Phys. Rev.* **106**, 1290 (1957).

TABLE I. Fraction of polarization remaining at the time of decay for positive muons stopped in samples of silicon with different dopings and in other targets. The results, at three temperatures, are taken from precession measurements, with no applied longitudinal field. All measurements, except those indicated by an asterisk, were taken with the samples in a cryostat, and corrections have been made only for muons stopping in the walls of the empty cryostat.

Sample	Resistivity (Ω cm)	Phosphorus donors per cm^3	Boron acceptors per cm^3	Residual polarization		
				300°K	77°K	4.2–10°K
Silicon:						
No. 4	0.05		4×10^{18}	0.90 ± 0.01	0.67 ± 0.08	0.45 ± 0.01
No. 14	3000		3×10^{12}	0.52 ± 0.05	0.24 ± 0.03	0.24 ± 0.08
No. 10	350	4×10^{12}		0.45 ± 0.08	0.15 ± 0.04	0.19 ± 0.05
No. 11	50	4×10^{13}		0.07 ± 0.03	0.10 ± 0.03	0.15 ± 0.03
No. 13	0.3	2×10^{16}		0.09 ± 0.03	0.12 ± 0.03	0.16 ± 0.04
No. 15	0.03	3×10^{17}		0.10 ± 0.05		
No. 16	0.01	1.5×10^{18}		1.00 ± 0.15		
No. 5	0.003	1.5×10^{19}		1.07 ± 0.10	0.92 ± 0.09	0.89 ± 0.10
Germanium		10^{15}		0.92 ± 0.08	0.23 ± 0.07	0.18 ± 0.07
Alumina				0.24 ± 0.16	0.24 ± 0.16	0.12 ± 0.10
Sulfur				0.14 ± 0.01	0.22 ± 0.01	0.20 ± 0.02
LiF				$0.00 \pm 0.02^*$		
MgO				$0.52 \pm 0.12^*$		
Red P				$0.44 \pm 0.12^*$		
Black P				$0.07 \pm 0.14^*$		
				$0.72 \pm 0.16^*$		

* Indicates measurement outside cryostat, other measurements made with sample in cryostat.

stopped in a target, the measured angular distribution will have the form $1 + A \cos\theta$, where A may be smaller than α because: (1) The muons have undergone some depolarization and retain only a fraction R of the original polarization. (The object of this experiment is to measure R for various samples.) (2) The muon beam has a polarization p_0 , which is less than 100%. (3) Counter telescopes have finite angular resolutions which reduce the observed asymmetry by a factor G . The effective asymmetry parameter is then given by $A = \alpha R p_0 G$.

All measurements of A in this experiment are expressed relative to the value of A measured for aluminum under the same conditions. Our measurements and others¹⁻³ are consistent with the assumption of no depolarization in aluminum and other conductors. From this assumption, using the measured value of $A_{Al} = 0.27 \pm 0.01$ and the calculated value of G , the polarization of the incident muon beam was determined to be $p_0 = 90 \pm 9\%$. Furthermore, the ratio A/A_{Al} is just R for the substance in question.

When a sample is held in the cryostat, it is difficult to estimate the fraction of detected positrons which originate from muons stopping in material other than the sample itself. In each case the background detected when the sample was removed from the cryostat has been subtracted. This correction, however, is not accurate, since the presence of the sample changes the number of muons stopping in the cryostat walls, (which are mostly metallic and thus nondepolarizing). For sulfur, for example, the measured value of R is 0.14 ± 0.01 when the sample is at room temperature in the cryostat but is 0.00 ± 0.02 for the same sample outside the cryostat. (See Table I.) No similar measurements of R for targets outside the cryostat were made for

other samples. Since the effect of the cryostat may vary from one sample to another, no attempt was made to correct the measured values of R . We should, therefore, note that a measured value of $R = 0.15$ is consistent with complete depolarization of those muons which actually decay in the sample.

II. DESCRIPTION OF EXPERIMENT

The measurements made in this experiment were of three kinds: (1) A precession measurement, similar to the experiment of Garwin, Lederman, and Weinrich.¹⁰ Polarized positive muons which stopped in various samples were precessed at a frequency of 0.5 Mc/sec by a pair of Helmholtz coils which provided a transverse field of 35 G. The angular distribution of the positrons decaying in a given interval of time after the stopping of the muon was sampled by two counter telescopes used independently as a check. A comparison of the positron rates in one telescope with and without the precessing field allows a determination of the degree of muon polarization at the instant of decay. (2) A "quenching" experiment, in which muons were stopped in a magnetic field parallel to direction of beam polarization. The field of maximum value 4.2 kG, was supplied by a large air-core magnet which once housed a 16-in. cloud chamber. The magnet also served as a mounting for all the experimental equipment. The positron rate in either the forward or the backward telescope in a given field was compared to the rate for no field. From this, the muon polarization at the magnetic field, for which the measurement was made, was determined relative to the no-field case. The no-

¹⁰ R. L. Garwin, L. M. Lederman, and M. Weinrich, Phys. Rev. **105**, 1415 (1957).

field depolarization was assumed to be that measured in the precession experiment. (3) A time distribution measurement, in which the time interval between the stopping of a muon in the precessing field and the detection of a positron in either telescope was measured by a digital time analyzer.

A low-energy, long duty cycle beam from the Nevis synchrocyclotron was stopped in various samples as shown in Fig. 1. The beam was defined by counter No. 3, which was $\frac{1}{16}$ in. thick, 2 in. in diameter, and placed as close as possible to the target. Typically, 200 muons/sec were stopped in a 6-g/cm² target. Positrons from the decay of muons in the sample were detected by counter telescopes placed at 0° and 180° with respect to the beam.

During most of the precession runs, no attempt was made to cancel the stray field present at the sample. This was measured to be 0.6 G transverse to (vertical), and 0.9 G along the axis of polarization of the muon beam. The third component of stray field was less than 0.1 G.

A cryostat, shown in Fig. 2, was built to allow samples to be cooled by liquefied gases. It was designed so that the central reservoir could be easily removed to allow samples to be changed during a run. In order to minimize background, both the vacuum jacket and target sample container were provided with thin beam windows.

In an early part of the experiment the samples were held in vacuum, clamped in a copper frame to the bottom of the central reservoir, and cooled by thermal conduction. When the liquid helium was in the central reservoir, the sample temperatures were estimated to be $10 \pm 5^\circ\text{K}$; the large uncertainty is due to thermal gradients between carbon resistor thermometers and the samples on which they were mounted.

In order to achieve a lower and better known temperature, the samples for the latter part of the experiment were immersed in liquid helium in $\frac{1}{8}$ -in. wall stainless steel boxes with 0.002-in.-thick beam windows. To facilitate changing the samples, the boxes were connected by copper tubes to sweat-soldered unions on the bottom of the central reservoir.

For the sulfur sample and some of the semiconductors, it was of interest to obtain stable temperatures in the range 100–300°K. This was accomplished by sealing a sample in a cavity between two sheets of polyurethane foam and circulating cold nitrogen gas around the sample. The gas was cooled by passing it through copper tubing in liquid nitrogen, and the sample temperature was measured by a copper-constantin thermocouple. The temperature was easily varied by adjusting the rate of flow of the gas, and the foam target holder contributed little background.

In order to reduce background, target samples were selected (wherever possible) which were at least 6 g/cm² thick, in order to stop all the muons. Most of

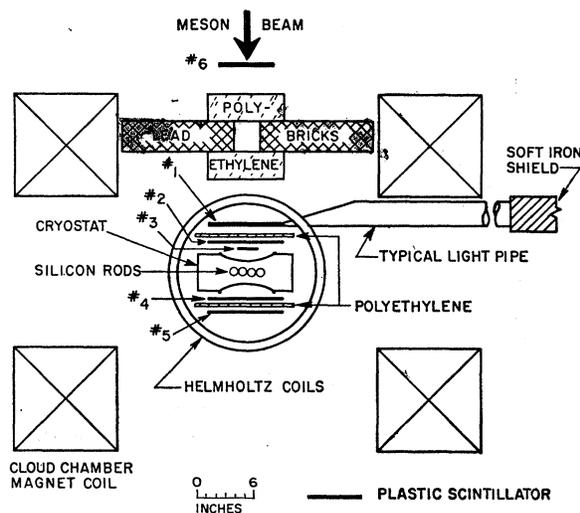


Fig. 1. Experimental layout as seen from above. Polyethylene absorber is chosen so that positive muons in the meson beam stop in the target of silicon rods. Helmholtz coils provide a transverse field of 35 G for experiments in which muon spins are precessed. The cloud chamber magnet provides a longitudinal field of up to 4.2 kG for experiments on quenching of the depolarization.

the semiconductors¹¹ were in the form of rods about 6 in. long and 1 in. in diameter. Samples of similar resistivity and type of doping were used together as targets. When the rods were arranged side-by-side, "thin spots" occurred which allowed some muons to stop in the rear wall of the cryostat.

The alumina (Al_2O_3) and boron carbide (B_4C) were in the form of tiles sintered from high-purity powder.

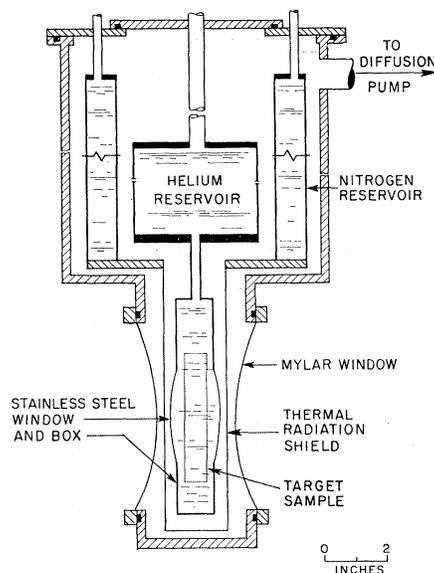


Fig. 2. Cryostat for holding target samples in liquid helium.

¹¹ We would like to thank T. Benedict of Merck and Company, Inc., and several members of the Bell Telephone Laboratories for supplying the semiconductor samples used in these experiments.

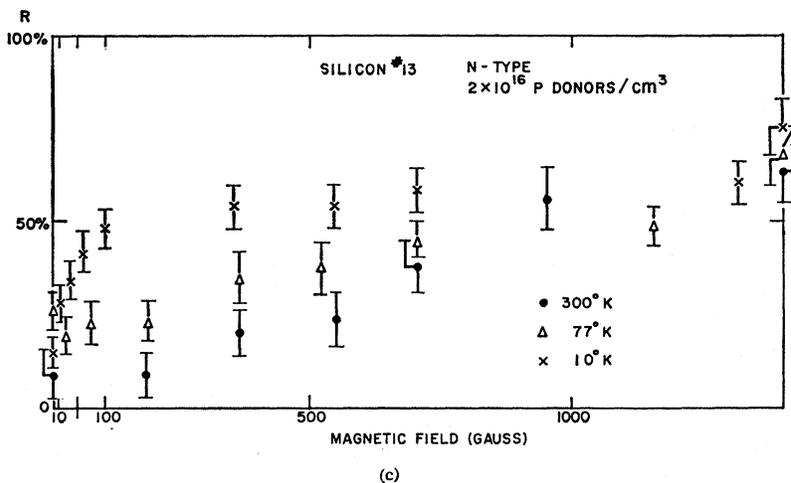
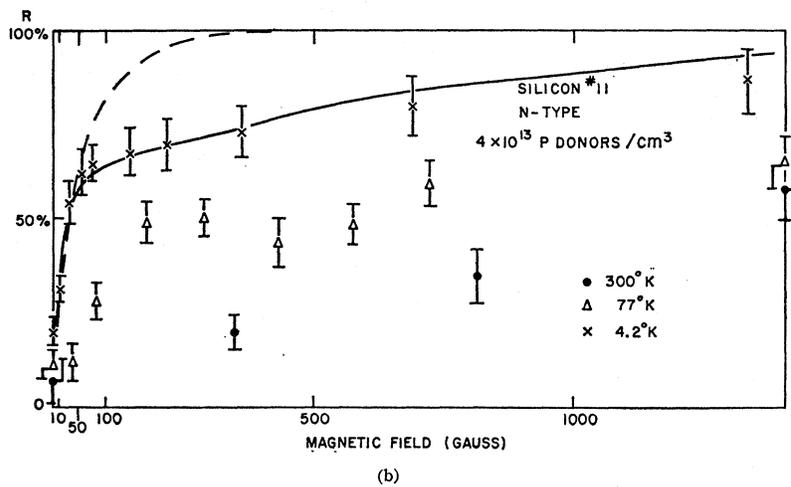
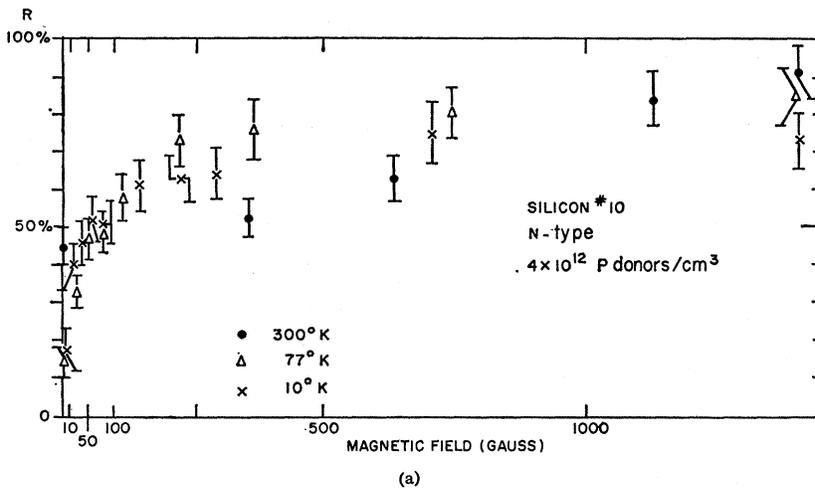


FIG. 3. Quenching of the depolarization in five samples of silicon with different doping at three temperatures. The fraction R of the muon polarization at the time of decay is measured for various values of the applied longitudinal magnetic field. (b) The dashed curve is a fit of Eq. (1) (many short-lived formations of $n=1$ muonium) to the low-field points for silicon No. 11 at 4.2°K, using $N=5$, $\tau=1.2(\times 3.6 \times 10^{-11}$ sec), and $H_0=30$ G. The solid curve shows a fit to the data assuming, for example, 10 formations of $n=1$ muonium with $\tau=0.3(\times 3.6 \times 10^{-11}$ sec) and $H_0=160$ G followed by 2 formations of $n=2$ muonium with $\tau=3.2(\times 8 \times 3.6 \times 10^{-11}$ sec) and $H_0=160/8$ G=20 G.

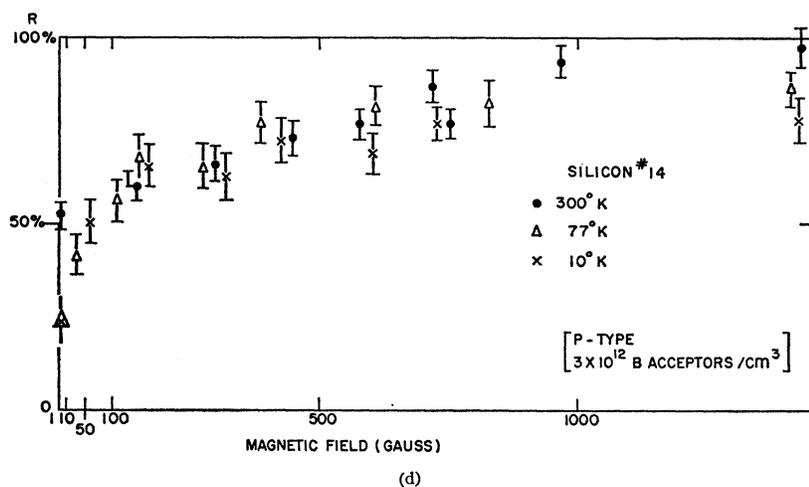
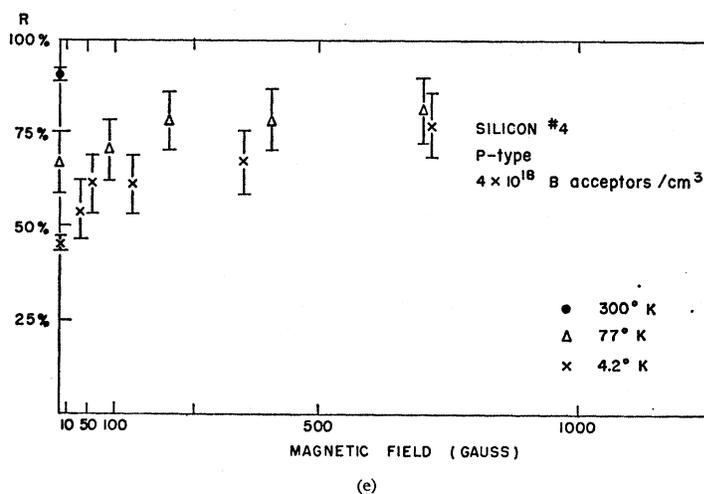


FIG. 3. (continued).



The red phosphorus, magnesium oxide, and lithium fluoride were held in polyethylene bags. The black phosphorus was pressed in the form of a block, and the sulfur target was cast in the same shape from melted "brimstone."

The six scintillation counters employed RCA 6810A photomultipliers and had light pipes 44 in. long to remove the tubes from the region of high magnetic field. In addition, each photomultiplier was surrounded by two thin cylindrical shields of high permeability material and an outer shell of soft iron $\frac{1}{2}$ in. thick. As the central field was raised to 4.2 kG, no appreciable change was observed in the height of counter pulses.

In a quenching measurement performed on aluminum, A_{Al} was constant up to a central field of 1 kG, but deviated from constant for a field of >1 kG. The deviations were of opposite sign in the two positron telescopes, and reached a maximum of 10% at 4.2 kG. This effect is attributed to focusing of the decay positrons by the central field. Since R for any sample is expressed relative to a similar measurement in aluminum, this effect is removed from the final results.

A stopping muon was identified by a coincidence among pulses in counters No. 6, 1, 2, and 3, together with no pulse in counter No. 4 (anticoincidence). The stopping muon coincidence pulse opened a gate of variable width after a delay whose length could also be varied. A "forward positron" was identified by a coincidence between pulses in counters No. 4 and 5, together with no pulse in counters No. 6 or 2. A "backward positron" was similarly identified by a coincidence between counters No. 1 and 2, and no pulse in No. 6 or 4. The requirement that no pulse occur in No. 6 rejected events which were in prompt coincidence with an incident beam particle.

Positron coincidences were scaled if they occurred during the muon gate. At the same time, electronically delayed pulses of positrons from earlier muons were also counted if they fell during the gate. These counts, called "accidental positrons," form a rate-dependent background, and were subtracted from the real rate. The accidentals were never found to comprise more than 5% of the real rate.

The resolving time of the coincidence circuits was 15

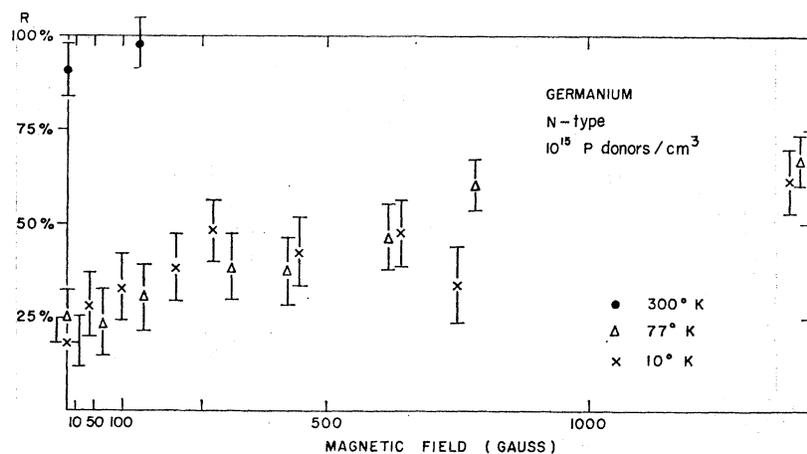


FIG. 4. Quenching of the depolarization in *N*-type germanium at three temperatures.

nsec, the width of the input pulses. Pulses used in anticoincidence were stretched to increase their efficiency for rejecting "bad" events. However, since a stopping muon produces pulses in counters No. 6 and 2, these pulses will then persist in the anticoincidence circuits and create a dead time during which no positrons can be detected. This dead time was 100 nsec during most of the experiment, but was reduced to 30 nsec for later measurements of time dependence.

For precession measurements, the muon gate extended from 0.5 to 1.5 μ sec after the muon stopped. A correction to the data was made to account for variation of the precession of the muon about the average angle of 180° during the gate. For quenching measurements, the gate extended from 0.2 to 4.2 μ sec after the muon stopped.

The time distribution measurements were made with a digital time analyzer designed at this laboratory and described elsewhere.¹² Briefly, the pulses from the stopping muon and decay positron formed a gate during which the analyzer scaled pulses from a 100-

Mc/sec oscillator. Time dependent background was eliminated by "interference remover" logic, which passed muon or positron pulses only when no other pulse appeared during 15 μ sec before or 8 μ sec after the pulse being processed. The measured time intervals for backward and forward positrons were stored in two halves of a random-access addressed pulse height analyzer.

III. EXPERIMENTAL RESULTS

The samples of original interest in this experiment were doped silicon and germanium semiconductors, which have a large static dielectric constant ($\epsilon_{Si}=12$; $\epsilon_{Ge}=16$).

It was previously found at this laboratory⁸ that R at room temperature is a function of the type and amount of impurity concentration. Measurements were made then for many samples, and in the current work representative samples were chosen for further study. In general, *P*-type silicon at room temperature partially depolarizes muons. For a range of *N*-type impurity concentrations between about 10^{14} and 3.5×10^{17} phosphorus donors/cm³, the results are consistent with complete depolarization, while in a sample with 1.5×10^{18} /cm³, no depolarization occurs.

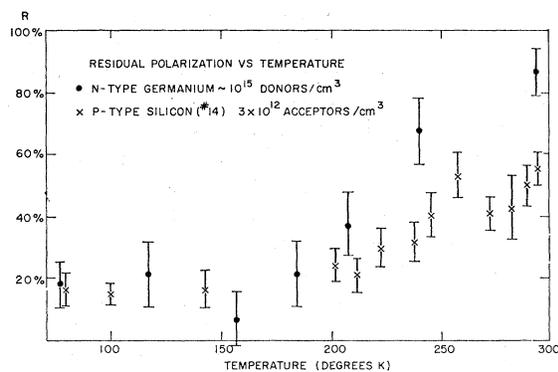


FIG. 5. Variation of R with temperature in silicon No. 14 and germanium in the absence of a longitudinal magnetic field.

¹² S. L. Meyer, E. W. Anderson, E. Bleser, L. M. Lederman, J. L. Rosen, J. E. Rothberg, and I-T. Wang, Phys. Rev. **132**, 2693 (1963).

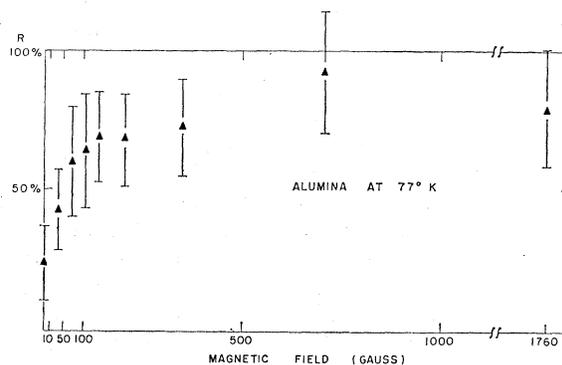


FIG. 6. Quenching of the depolarization in alumina at 77°K.

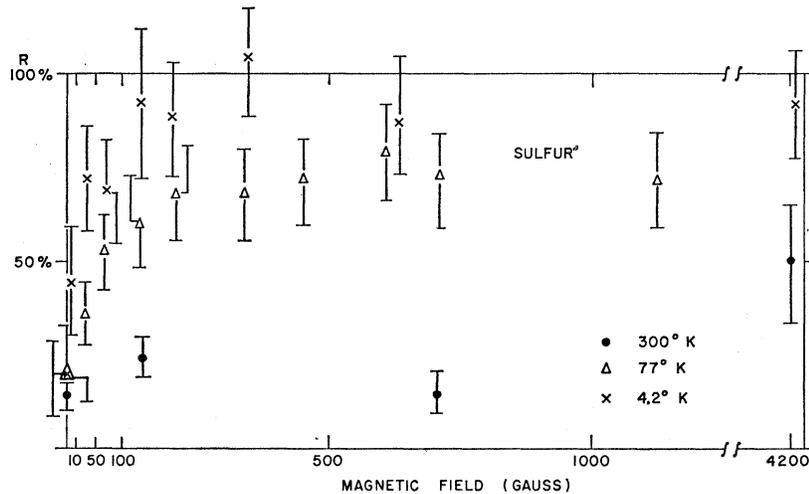


FIG. 7. Quenching of the depolarization in sulfur at three temperatures.

The values of R measured in this experiment are tabulated in Table I. It is of interest that at low temperatures only No. 5 (with a high concentration of donors) produces no depolarization (No. 16 was not measured at low temperatures, but would presumably be similar to No. 5). The reduction in R for No. 4 at 4.2°K suggests that at somewhat lower temperatures there would be almost complete depolarization, as in other samples. (The residual polarization in the other samples is consistent with $R=0$ if allowance is made for background.)

It is found that at the maximum field used, 4.2 kG, most of the depolarization in any sample is quenched. The effect of a small field varies with the sample and temperature. In particular, although a field of 100 G has little effect at 300°K in any semiconductor sample, at low temperatures this field is sufficient to quench ~50% of the depolarization in the samples that completely depolarize.

Quenching curves of R versus external field are shown in Figs. 3 and 4. They are plotted to only 1500 G because in general all samples exhibit quenching in the region of higher fields which can be fitted by a straight line of small slope, approaching ~90% at 4 kG.

For purposes of discussion it is convenient to define a "transition" temperature T^* , such that when $T < T^*$, approximately half the depolarization is quenched by a field of less than 100 G. (Since we cannot predict T^* , its physical significance is not stressed.)

From Figs. 3(a) through 3(e), we see that at the lowest temperature the quenching curves for No. 10, 11, 13, and 14 are indistinguishable. However, for samples No. 10 and 14, $77^\circ\text{K} < T^* < 300^\circ\text{K}$, while for No. 11 and 13, $4.2^\circ\text{K} < T^* < 77^\circ\text{K}$, and for No. 4 and germanium we infer that $T^* < 4.2^\circ\text{K}$. The temperature variation of R (precession measurement) for No. 14 and germanium is shown in Fig. 5.

In order to determine whether the preceding behavior is specific to a semiconductor, an insulator with a

large static dielectric constant was also studied. This was alumina, Al_2O_3 , for which $\epsilon=10$. The sample depolarized "completely" and had identical quenching curves at all temperatures. A large effect was always seen at 100 G, indicating that $T^* > 300^\circ\text{K}$. The depolarization was almost completely quenched at 4.2 kG, and a typical quenching curve at lower fields is shown in Fig. 6. It is seen to be similar to silicon for $T < T^*$.

An insulator with a low static dielectric constant ($\epsilon=4$), sulfur, exhibited a more complex behavior¹³ (Fig. 7). This sample depolarized "completely" at all temperatures. At 4.2°K all of the depolarization was quenched by a field of ~150 G. At 77°K, there was also a large effect at 150 G, but the polarization reached a maximum of ~70% at 500 G and no more quenching occurred even at 4 kG. At room temperature the quenching was very gradual and can be fit by a straight line which reaches $R \sim 50\%$ at 4 kG. R was also measured as a function of temperature in a constant longitudinal field of 240 G (Fig. 8).

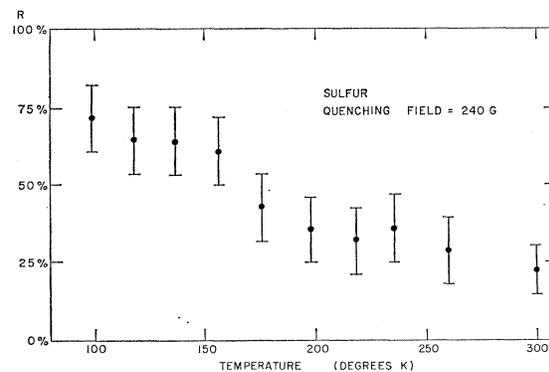


FIG. 8. Variation of R with temperature in sulfur in a longitudinal field of 240 G.

¹³ S. Gorodetzky, Th. Muller, M. Port, and A. Zichichi, Phys. Letters 2, 133 (1962).

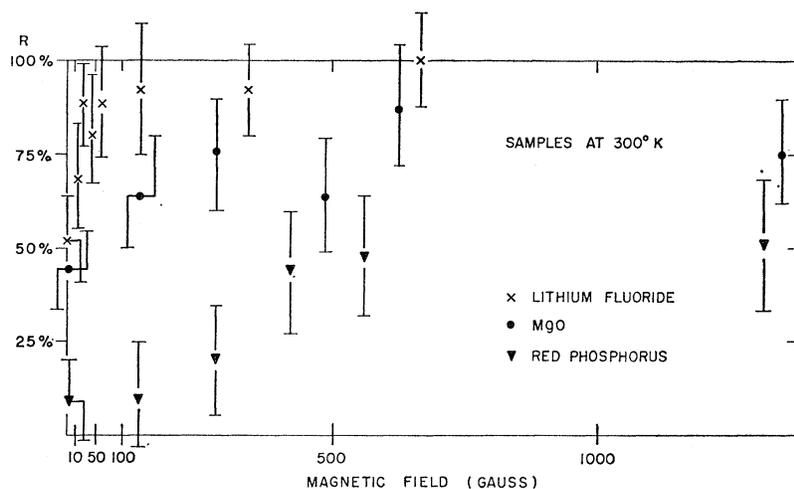


FIG. 9. Quenching of the depolarization in LiF, MgO, and red P at room temperature.

Several other available samples were studied at room temperature. Black phosphorus, a semiconductor, depolarized only slightly. Quenching curves for the other samples, all insulators, are shown in Fig. 9. For all, 4 kG is sufficient to quench most of the depolarization.

The digital time analyzer was used for several representative samples. In the absence of a precessing field, a longitudinal field of 7 G was used to eliminate the effect of the stray field. Except for boron carbide, no sample exhibited any change in effective asymmetry parameter over the time of observation, several microseconds. For these samples, all of the depolarization must have occurred within 0.2 μ sec after the muon stopped.

The data for boron carbide were fitted to $P(t) = Ke^{-t/\tau}[1 \pm Ae^{-t/\tau_d}] + B$, where $P(t)$ is the positron rate, B the time-independent background, τ the muon lifetime, τ_d the relaxation time for the polarization, and K the stopping muon rate. A was assumed to be the same as for aluminum, and the plus and minus signs refer to the backward and forward telescopes, respectively. At room temperature $\tau_d = 5.1 \pm 1.5 \mu$ sec, in agreement with the results of Swanson.¹ However, at 77°K, $\tau_d = 2.8 \pm 0.5 \mu$ sec, and at 4.2°K, $\tau_d = 2.5 \pm 0.4 \mu$ sec. This decrease of τ_d with temperature suggests that the relaxation effect is not one of static fields introduced by impurities in the sample. A quenching field of 70 G at 4.2°K was sufficient to inhibit the depolarization completely.

IV. INTERPRETATION

In considering a possible depolarization mechanism, we may eliminate depolarization by collision. It has been calculated that the incident muons will suffer less than 0.1% depolarization in being slowed to "thermal" energies.¹⁴ In the 0.2 μ sec after slowing down, the muon will undergo many collisions as it moves through

the solid. The cumulative effect of many random collisions can cause depolarization, but calculations based on a simple model³ show that negligible depolarization will occur in 0.2 μ sec. Furthermore, the field which is observed to quench the depolarization is much smaller than it would have to be if the depolarization occurred in collisions.

Finally, if one assumed that the depolarization in boron carbide were due to collisions made by a thermal muon, one would expect that as the temperature (and thus muon thermal velocity) were reduced more time would be required for the depolarization process, i.e., τ_d would increase.

An alternative to this picture of a "free" muon is one in which the muon is bound to, or interacts coherently with, the source of a local field in the solid. Since the largest fields are due to electrons, it is natural to consider the effect of ground-state muonium formation on the polarization of the muon.

Previous attempts to detect muonium in solids have been unsuccessful.¹ No evidence has been found in this experiment, even at 4.2°K, that any depolarization results from a formation of muonium which remains bound for at least a hyperfine period (2.24×10^{-10} sec). (In that case, as we shall see, ~50% of the observed depolarization would have been quenched by a small field.)

It is difficult to predict whether a muon stopping in a solid will form muonium. The muon slows down from the MeV energy range by ionization loss. When its velocity is comparable to electron velocities, capture of electrons from the atoms in the material through which the muon passes is most probable. The capture of an electron into the ground state ($n=1$) of muonium is most probable at a muon velocity of $v_0 = ac$, the K -shell electron velocity. (Capture in the n th state is most probable at v_0/n , where n is the principal quantum number.) However, the probability for ionization in a collision with an atom is also a maximum near this

¹⁴G. W. Ford and C. J. Mullin, Phys. Rev. **108**, 477 (1957).

velocity. The muonium, if formed, would remain bound for a time $\ll 10^{-11}$ sec.

In order that the muonium remain bound for many collision times, the kinetic energy of the muonium formed must be insufficient for ionization on impact. (Of course the probability of electron capture at such low muon velocities is considerably reduced from the probability at v_0 .)

Since radiative lifetimes of the first few excited states of muonium are on the order of 10^{-8} to 10^{-9} sec, such states might also contribute to depolarization. Excited states, having smaller binding energies, could survive enough collisions to produce depolarization only if formed at lower muon velocities than required in the case of ground-state formation.

The time required for the muon to reach "thermal" energies in our target is difficult to calculate. It has been estimated as $\sim 10^{-11}$ sec in a gas,¹⁵ but there is no conclusive reason for assuming that it is less than 10^{-10} sec in a solid. Thus there may be sufficient time available for the muon to lose some or all of its polarization while it is slowing down.

Once slowed to "thermal" energies, a muon might still capture conduction or free electrons, or it might be bound in a molecular formation with the lattice or an impurity ion. In a molecule, muon depolarization would occur as a result of interaction with much weaker fields than in the ground state of muonium.

An examination of the quenching curves shows that in most cases some depolarization still occurs in an external field of several kilogauss. The simplest depolarization mechanism in which complete quenching would require larger fields is the formation of ground-state muonium,⁷ and we shall consider that system in some detail. The data of this experiment are in general consistent with the hypothesis that depolarization occurs in a succession of brief muon-electron attachments.

A muonium atom in the ground state will have a z component of total angular momentum M equal to zero or one. An atom with $M=1$ will precess in a transverse magnetic field at ~ 1.4 Mc/sec G. In a measurement of muon polarization in which free muons are precessed at the (lower) muon Larmor frequency, (14 kc/sec/G), muons bound in $M=1$ muonium would appear unpolarized. The stray magnetic field at the target, in the absence of a precessing field, would reorient, and effectively depolarize, $M=1$ muonium within $0.2 \mu\text{sec}$. If, however, $M=1$ muonium is formed in a magnetic field along the direction of muon polarization is preserved. Thus in a quenching field of ~ 10 G, much larger than the stray field at the target, $M=1$ muonium would not be depolarized.

A muonium atom with $M=0$ loses its polarization completely in several hyperfine periods. (The hyper-

fine period is 2.24×10^{-10} sec in vacuum.) If, however, $M=0$ muonium is formed in a magnetic field along the direction of muon polarization, it retains a fraction R of residual polarization given by⁶ $R(x) = x^2/(1+x^2)$, where x is the external field measured in units of the hyperfine field H_0 . In vacuum, H_0 is 1580 G, the field of the muon at the distance of the electron. Clearly, only 50% depolarization will occur when $M=0$ muonium is formed in an external field of 1580 G.

If all the polarized muons which stopped in a target formed muonium, complete depolarization would be observed in a precession experiment. However, in a quenching experiment in a longitudinal field of $\gtrsim 10$ G, the field dependence of the residual polarization of the initially polarized muons would be given by $R(x) = \frac{1}{2} + x^2/2(1+x^2)$. Since no sample exhibits a sharp inhibition of depolarization on the order of 50% in a quenching field of 10 G, we conclude that no muons are depolarized by forming $M=1$ muonium for as long as $\sim 0.2 \mu\text{sec}$. ($M=1$ muonium in a 1 G stray field will precess $\sim 90^\circ$ in $0.2 \mu\text{sec}$, the time during which the depolarization must have occurred.) In addition, the observed $R(x)$ in fields > 10 G does not agree with the theoretical $R(x)$, which one would expect for muonium that was stable for as short a time as several hyperfine periods.

The results are consistent with complete depolarization in several samples in the temperature range $T > T^*$. Although no $M=1$ muonium was detected, the size of the field required to quench the depolarization suggests $M=0$ muonium. This is consistent with the hypothesis that a muon forming muonium will lose its electron after a brief (~ 1 hyperfine period) attachment, but will subsequently capture another electron, the process being repeated many times. Our picture of the behavior of a muon toward the end of its range supports such an assumption.

If we consider the average duration τ of one attachment in units of $(2\pi)^{-1} \times$ (hyperfine period), or 3.58×10^{-11} sec, it can be shown that⁶

$$R(x) = \left\{ 1 - \frac{1}{2(1+x^2+\tau^2)} \right\}^N, \quad (1)$$

where N is the number of muonium formations. Here only formations in the ground state are included. If 10^{-9} sec were available for depolarization, various combinations of τ and N could produce any observed $R(0)$. (See Fig. 10.)

If muonium were formed in a solid of high dielectric constant, the polarization of the medium would reduce the electrostatic attraction between the muon and electron and produce a hyperfine field smaller than 1580 G. This effect is seen in the case of a "shallow" phosphorus donor in silicon,¹⁶ where the field is reduced by a factor of ~ 30 . In such situations the increased

¹⁵ V. W. Hughes, D. W. McColm, K. Ziocck, and R. Prepost, Phys. Rev. Letters 5, 63 (1960); R. Prepost, Nevis Report No. 99, Ph.D. thesis, 1962 (unpublished).

¹⁶ G. Feher, Phys. Rev. 114, 1219 (1959).

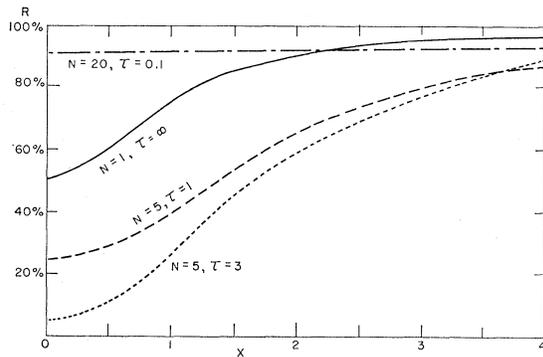


FIG. 10. Theoretical predictions of $R(x)$ given by Eq. (1), where x is the longitudinal magnetic field in units of H_0 , for four combinations of N , the number of muonium formations, and τ , the mean duration of each formation in units of 3.6×10^{-11} sec.

mean radius of the electron orbit cannot be predicted from a knowledge of the static dielectric constant only.¹⁷

It is in fact found that although the quenching curves for muons for $T > T^*$ cannot be fit with Eq. (1) when $H_0 = 1580$ G, fits are possible with smaller values of H_0 . Fits of Eq. (1) to the data with typical parameters are shown in Figs. 11 and 12. The experimental errors permit fits for a wide range of values of H_0 , N , and τ .

No muon depolarization is observed in conductors, such as aluminum. Assuming that a muon slows down in a conductor by interacting with a dense plasma of electrons, we expect a thermalization time of $\sim 10^{-13}$ sec at 300°K .¹⁸ This is too short for any depolarization. After thermalization, the muon cannot interact coherently with any "single" electron since conduction electrons are not localized in the lattice.

Silicon samples No. 5 and 16 share some properties with a conductor. The donor electron concentration for No. 16 is just above that at which the electronic wave functions of different donors overlap (No. 15 is just below).¹⁶ These samples possess an "impurity conduction band" and should behave like conductors at all temperatures. It is found (Table I) that No. 16 and No. 5 do not depolarize muons at room temperature (and No. 5 does not even at 10°K), while No. 15 depolarizes muons completely at room temperature, in agreement with the model.

If we consider the quenching curves for samples at $T < T^*$, where $\sim 50\%$ of the depolarization is inhibited by a field of less than 100 G, we see two distinct slopes. The multiple formation model predicts that if $\sim 50\%$ of the depolarization is quenched in a certain field, then a field only several times larger will quench most of the remaining depolarization. This clearly does not occur in the case of those samples with $T < T^*$.

¹⁷ W. Kohn, in *Solid State Physics*, edited by F. Seitz and D. Turnbull (Academic Press Inc., New York, 1957), Vol. 5, p. 257.

¹⁸ B. M. Smirnov, *Zh. Eksperim. i Teor. Fiz.* 44, 192 (1963) [English transl.: *Soviet Phys.—JETP* 17, 133 (1963)].

As an example, Eq. (1) has been plotted [dashed curve on Fig. 3(b)] for silicon sample No. 11 at 4.2°K , using the parameters $H_0 = 30$ G, $N = 5$, and $\tau = 1.2$. These have been chosen so that Eq. (1) is in good agreement with the data in the region of low magnetic field. It is clear that for fields of several hundred gauss, the observed depolarization cannot be explained by this theory.

The depolarization in those samples could be caused by two depolarizing mechanisms, the earlier being multiple formation of ground-state muonium. If the muon is thereby completely depolarized, the second process, if it occurs, is unobservable. If, however, the muon is only partially depolarized in ground-state muonium formations, it may experience a second interaction which is characterized by a much smaller field (~ 50 G). We must assume that the occurrence of this second interaction depends on the temperature of the sample, since it occurs in No. 14 at 77°K but not at room temperature.

This second interaction cannot be the precession of a free muon in a field of 50 G, since in $0.2 \mu\text{sec}$ a muon would precess only 20° . The interaction can involve the formation of a bound state in which the muon-electron separation is greater than that in the ground state of muonium. In the case where the static dielectric constant $\epsilon > 1$ and $H_0 \sim 240$ G in the ground state, an increase in separation by a factor of 2 will reduce H_0 by a factor of 2^{-3} to ~ 30 G.

Such a separation could be encountered in an excited ($n > 1$) state of muonium or in a "molecular formation" in which the muon is bound locally in an atom. These states would be bound much more weakly than the ground state of muonium, and would be stable during their lifetimes against ionization by collision at lower muon kinetic energies than would be the case for

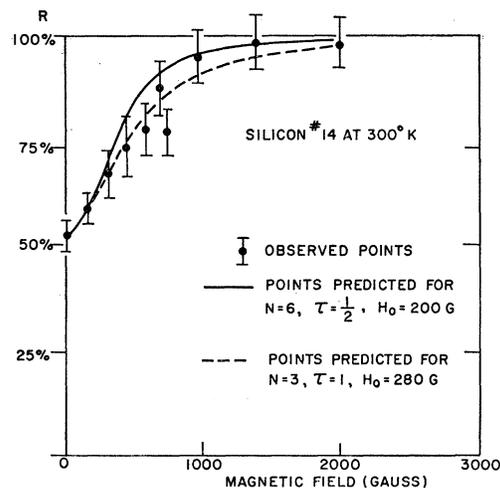


FIG. 11. Experimental points for quenching of the depolarization in silicon No. 14 at room temperature, fit by Eq. (1) for two of the many possible combinations of N , τ , and H_0 .

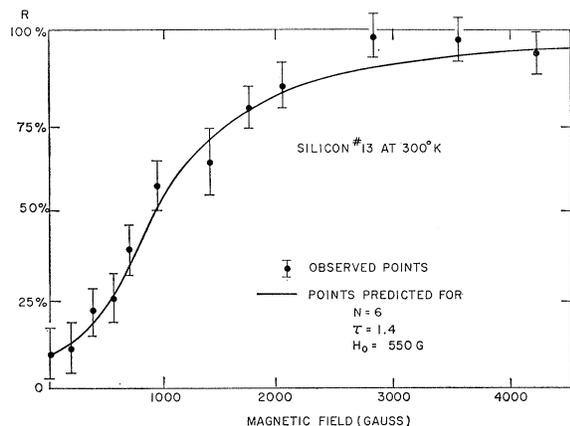


FIG. 12. Experimental points for quenching of the depolarization in silicon No. 13 at room temperature, fit by Eq. (1) for one of the many possible combinations of N , τ , and H_0 .

ground-state muonium. Thus they would be formed later in the slowing down of the muon. (Excited states of muonium could also be briefly formed earlier, together with ground-state formations, but would not then significantly contribute to the depolarization.)

As an example, the solid line in Fig. 3(b) shows a fit to the quenching in silicon No. 11 at 4.2°K. The assumed model is 10 formations of ground state muonium of mean duration 0.3 (in units of 3.6×10^{-11} sec) and hyperfine field 160 G, followed by 2 formations of $n=2$ muonium of mean duration 3.2 [in units of $(8) \times (3.6) \times 10^{-11}$ sec] and hyperfine field $160/8$ G = 20 G. Although the fit is good, this only indicates consistency with the model, since with 5 free parameters a wide range of curves can be approximated.

V. CONCLUSIONS

We have seen that no single mechanism for muon depolarization is adequate to describe the process in all materials and at all temperatures encountered in this experiment. The following summary outlines a relatively simple model which we believe may be valid.

The depolarization of positive muons in semiconductors agrees qualitatively with the following picture:

Depolarization occurs while the muon is slowing down, as a result of consecutive short-lived formations of muonium. Only formations in the ground state will be important in the early stages of slowing down. In

a given sample, a reduction in temperature decreases the depolarization by these ground state formations. In samples at room temperature, the depolarization associated with this mechanism decreases with decreasing donor impurity concentration. (This does not include the very heavily doped samples.)

At low temperatures ($T < T^*$) the muon may later in the slowing down process form more weakly bound states in which the muon-electron separation is greater than in the ground state of muonium, and in which the "depolarizing field" is much weaker.

The depolarization in alumina is also consistent with this model. Here, however, the weakly bound state is formed even at room temperature.

The depolarization in MgO and red phosphorus at room temperature is consistent with the first mechanism, multiple ground-state muonium formation.

The depolarization in boron carbide cannot be explained in terms of a bound state with an electron. This is because a polarization relaxation time of ~ 1 μ sec corresponds to an H_0 of ~ 1 G, and the 7-G external field would have quenched this depolarization. A field of 70 G does, however, completely quench the low-temperature depolarization in boron carbide.

The depolarization in lithium fluoride at room temperature and in sulfur at 4.2°K is completely quenched in a field of ~ 100 G. Although the lithium fluoride was not investigated for a time dependence, there was none observed in sulfur. (In terms of the two mechanism model, this behavior would correspond to there being no depolarization from formations of ground state muonium. This seems unlikely; more probably the depolarization is due to some other mechanism.)

The room-temperature quenching of the depolarization in sulfur is not clearly related to the low-temperature behavior. The transition between the two types of behavior occurs in the neighborhood of 200°K. It is possible that at room temperature so many formations of ground-state muonium occur that there is little quenching in a field of 4 kG. However, the commercial sulfur sample probably contained a high concentration of impurities which may play an important role in the depolarization process.

ACKNOWLEDGMENT

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