(vi) Our results come closest to solution I of Phillips and Rarita.[8] Our slope for the ρ trajectory is slightly larger. This may be due to the pure exponential behavior for the residues assumed in PR while we have used (7). The number of parameters needed in our fits is less than in PR. This is due to the fact that we have used extra information from the nucleon form factor data and also because our parametrization of the residues involve a lesser number of parameters. We have confined our fits to $-t\leq 0.5$ $(\mathrm{BeV}/c)^2$ partly because beyond this range the parametric representations of α and r are expected to be unreliable and partly because at large momentum transfers there is no reason to ignore N and N^* exchanges.

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Sum Rules for Magnetic Quadrupole and Electric Dipole Moments: An Application of the Algebra of Current Components*

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Assuming simple commutation relations between the various components of the vector current density, we derive three commutation relations between electric dipole and magnetic quadrupole operators. Taking these commutators between proton states at rest, we get three sum rules. Considering only one intermediate state, namely the $I=\frac{1}{2}$, $J=\frac{3}{2}^-$ pion-nucleon resonance $N^{**}(1518)$, we have a consistency relation between them. The sum rules are then used for deriving the ratio between the E_{2-} and M_{2-} amplitudes in single-pion photoproduction, and one obtains a result in good agreement with experiment.

I. INTRODUCTION

RECENTLY, various applications of the commutation relations between components of current densities, the so-called algebra of current components,[1,2] have been given. One of the most important results is certainly the sum rule for the magnetic moment of the proton, derived by Dashen and Gell-Mann,[1] and Lee.[3] These authors, assuming a commutation relation between the space components of the vector current density J_i^α, where α is an $SU(3)$ index, derived a commutation relation for the space integrals of the magnetic moment operators $\frac{1}{2}\epsilon_{ikl}r_kJ_l^\alpha$. Then, taking the expectation value of this commutator between proton states at rest, and inserting a suitable set of intermediate states, namely the nucleon and the 33-resonance $N^*(1238)$, they obtain a relation between the magnetic moment and the charge radius of the proton[4]:

$$(\mu_p/2M)^2=\tfrac{1}{6}\langle r^2\rangle_p\,, \tag{1}$$

which is fairly well satisfied by the experimenta numbers.

Moreover, the commutation relations also give the vanishing of the electric quadrupole transition between the proton and the N^*.

In the present paper, we want to apply the same method to other moments of the electromagnetic current, and particularly to the electric dipole and magnetic quadrupole moments. The starting point will be, as in the case of the Dashen-Lee sum rules, the commutation relations between the components of the vector current density:

$$[J_4^\alpha(\mathbf{x}),J_4^\beta(\mathbf{x}')]=if_{\alpha\beta\gamma}\delta(\mathbf{x}-\mathbf{x}')J_4^\gamma(\mathbf{x})\,, \tag{2}$$

$$[J_4^\alpha(\mathbf{x}),J_i^\beta(\mathbf{x}')]=if_{\alpha\beta\gamma}\delta(\mathbf{x}-\mathbf{x}')J_i^\gamma(\mathbf{x})+(\text{terms symmetric in }\alpha,\beta)\,, \tag{2'}$$

$$[J_i^\alpha(\mathbf{x}),J_j^\beta(\mathbf{x})]=i\delta_{ij}f_{\alpha\beta\gamma}\delta(\mathbf{x}-\mathbf{x}')J_4^\gamma(\mathbf{x})+(\text{terms symmetric in }\alpha,\beta)\,. \tag{2''}$$

It is rather reasonable to believe that these commutators are correct. Adler and Callan[5] have proved Eqs. (2) and (2′) for some simple models: the quark model and two models which involve baryons and mesons, like the Gell-Mann and Lévy[6] σ model. It must be noted that in the quark model the last term in Eq. (2″)

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[1] R. F. Dashen and M. Gell-Mann, Phys. Letters 17, 142, 145 (1965).

[2] R. P. Feynman, M. Gell-Mann, and G. Zweig, Phys. Rev. Letters 13, 678 (1964).

[3] B. W. Lee, Phys. Rev. Letters 14, 676 (1965).

[4] Actually, it should be $\langle r^2\rangle_V$, the charge radius of the isovector form factor which differs from $\langle r^2\rangle_P$ by about 20%.

[5] S. L. Adler and C. G. Callan, CERN Report 65/1227/5-Th. 587 (unpublished).

[6] M. Gell-Mann and M. Lévy, Nuovo Cimento 16, 705 (1960).

has the simple form

$$id_{\alpha\beta\gamma'}\delta(\mathbf{x}-\mathbf{x}')\epsilon_{ijk}A_k^{\gamma'}(\mathbf{x}),$$

where $\mathbf{A}^{\gamma'}$ is the space part of the axial-vector current density. There is some indication that such an expression can be valid, without any additional term involving gradients of δ functions, at least for the magnetic moment commutation relations.[1,7]

According to Schwinger,[8] the last term in Eq. (2″) contains the factor $d_{\alpha\beta\gamma}\nabla_x\delta(\mathbf{x}-\mathbf{x}')$. In this paper, however, we shall not need to worry about the terms symmetric in the $SU(3)$ indexes in Eqs. (2′) and (2″), because the sum rules that will be discussed involve only the $f_{\alpha\beta\gamma}$ part of the commutators.

Using Eqs. (2)–(2″), we will derive in Sec. II three commutation relations between isovector electric dipole and magnetic quadrupole operators: one between two electric dipoles, another between two magnetic quadrupoles, and a last one between an electric dipole and a magnetic quadrupole.

Taking then, as usual, the expectation value of the three commutators between proton states at rest, we obtain three sum rules. If we restrict the intermediate states to only one, the $I=\frac{1}{2}$, $J=\frac{3}{2}^-$ pion-nucleon resonance $N^{**}(1518)$, we get a consistency relation between the sum rules.

In Sec. III we will express the matrix elements of the multipole moments in terms of the CGLN[9] amplitudes in photoproduction.

Actually, the matrix elements that we get from the commutation relations involve an off-mass-shell photon, so that some assumption has to be made on the form-factor dependence of the matrix element. Assuming the same form-factor dependence for the electric dipole and the magnetic quadrupole transitions, we can compare the ratio of these two matrix elements with the results we get from photoproduction experiments, and we find a remarkable agreement. Furthermore, we discuss the simple model in which the form factor is dominated by the ρ meson.

II. THE SUM RULES

We introduce now the electric dipole operator

$$\mathbf{E}^{\alpha}=\int(\mathbf{r}J_4^{\alpha})d^3r \tag{3}$$

and the magnetic quadrupole

$$M_{ij}^{\alpha}=\int[r_i(\mathbf{r}\times\mathbf{J}^{\alpha})_j]d^3r. \tag{4}$$

Using Eqs. (2)–(2″), we obtain the following commutation relations between the operators (3) and (4):

$$[E_i^{\alpha},E_j^{\beta}]=if_{\alpha\beta\gamma}\int r_ir_jJ_4^{\gamma}d^3r, \tag{5}$$

$$[M_{ij}^{\alpha},M_{i'j'}^{\beta}]=if_{\alpha\beta\gamma}\int r_ir_{i'}(\delta_{jj'}r^2-r_jr_j')J_4^{\gamma}d^3r + (\text{symmetric terms in } \alpha,\beta), \tag{5′}$$

$$[E_i^{\alpha},M_{jk}^{\beta}]=if_{\alpha\beta\gamma}\int r_ir_j(\mathbf{r}\times\mathbf{J}^{\gamma})_kd^3r + (\text{symmetric terms in } \alpha,\beta). \tag{5″}$$

The last term in Eqs. (5′) and (5″) obviously comes from the analogous term in Eq. (2″). If we take now, in Eqs. (5)–(5″), $\alpha=1$, $\beta=2$, $i=j=i'=j'=k=3$, we obtain

$$[E_3^1,E_3^2]=i\int z^2J_4^3d^3r, \tag{6}$$

$$[M_{33}^1,M_{33}^2]=i\int z^2(x^2+y^2)J_4^3d^3r, \tag{6′}$$

$$[M_{33}^1,E_3^2]=i\int z^2(\mathbf{r}\times\mathbf{J}^3)_zd^3r. \tag{6″}$$

Let us take now the expectation value of Eqs. (6)–(6″) between proton states at rest. The right-hand sides of the equations are then easily interpreted in terms of the derivatives with respect to momentum transfer squared of the isovector electric and magnetic form factors of the nucleon.

For obtaining sum rules from Eqs. (6)–(6″), we now have to make some choice on the intermediate states in the commutators.

Since both $\mathbf{E}$ and M_{ij} are negative-parity operators, and the intermediate states are at rest [because of the integration over space in Eqs. (3) and (4)], we cannot have the nucleon as an intermediate state. If we now look among the states which are present in photoproduction, we find that the $J=\frac{3}{2}^+$, $N^*(1238)$ state does not contribute for the same reason. We see, therefore, that in the commutator (6) it is possible to have only intermediate states with $J=\frac{1}{2}^-$ and $J=\frac{3}{2}^-$, while in (6′) only states with $J=\frac{3}{2}^-$ and $J=\frac{5}{2}^-$ can contribute, and in (6″), only states with $J=\frac{3}{2}^-$.

Since from the photoproduction experiments there is clear evidence of an $I=\frac{1}{2}$, $J=\frac{3}{2}^-$ pion-nucleon resonance, the $N^{**}(1518)$, we assume that this is the only one intermediate state to consider in the three commutators (6), (6′), and (6″). With this hypothesis, we get the three sum rules

$$|\langle E_{13}\rangle|^2=-\tfrac{1}{2}(dG_E{}^V(k^2)/dk^2)_{k=0}=\tfrac{1}{12}\langle r^2\rangle_V, \tag{7}$$

$$|\langle M_{23}\rangle|^2=2[d^2G_E{}^V(k^2)/(dk^2)^2]_{k=0}, \tag{8}$$

$$\langle E_{13}\rangle\langle M_{23}\rangle^*=(i/2M)(dG_M{}^V(k^2)/dk^2)_{k=0}, \tag{9}$$

[7] A. Bietti, Phys. Rev. **140**, B908 (1965).
[8] J. Schwinger, Phys. Rev. Letters **3**, 296 (1959).
[9] G. F. Chew, M. L. Goldberger, F. E. Low, and Y. Nambu, Phys. Rev. **106**, 1345 (1957).

where

$$\langle E_{13}\rangle = \langle N^{**}|E_3{}^3|p\rangle, \quad (10)$$

$$\langle M_{23}\rangle = \langle N^{**}|M_{33}{}^3|p\rangle, \quad (11)$$

and

$$G_{E,M}{}^V = G_{E,M}{}^P - G_{E,M}{}^N.$$

The $G_{E,M}{}^{P,N}(k^2)$ are the electric and magnetic form factors of the proton and of the neutron as defined by Sachs[10] and discussed by Hand, Miller, and Wilson.[11]

It is now evident that from Eqs. (7), (8), and (9) we get a consistency relation, namely

$$\frac{\langle r^2\rangle_V}{6}\left[\frac{d^2G_E{}^V(k^2)}{(dk^2)^2}\right]_{k=0} = \frac{1}{4M^2}\left(\frac{dG_M{}^V(k^2)}{dk^2}\right)^2_{k=0}. \quad (12)$$

In some sense, Eq. (12) is similar to Eq. (1) only "shifted" by a derivative. If we take the experimental results on the proton and neutron form factors,[11,12] Eq. (12) gives

$$2.7\times10^{-3}\ \mathrm{F}^6 = 2.8\times10^{-3}\ \mathrm{F}^6.$$

We can therefore say that this result, to some extent, supports our assumption of the dominance of the N^{**} intermediate state in the commutators (6)–(6″).

III. COMPARISON WITH PHOTOPRODUCTION

Equation (12) is a most encouraging result, and we would like now to compare our sum rules (7), (8), and (9) with some experimental data on the electromagnetic transition between proton and N^{**}.

Considering that the $N^{**}\to N+\pi$ decay mode represents about 80% of the total, the best sources of information are certainly the single-pion photoproduction experiments in the region of the N^{**} resonance. Unfortunately, we can not directly compare the electric and magnetic amplitudes of photoproduction with the amplitudes we get from (7), (8), and (9).

In fact, as we said before, the integration over space in Eqs. (3) and (4) makes the three-momenta of the proton and the N^{**} equal in the amplitudes (10) and (11), so that, with the proton at rest, these amplitudes are induced by a virtual photon with $\mathbf{k}=0$ and "mass" $k_0=W-M$, where W is the c.m. energy of the pion-nucleon system. In our case, $W=W_R=1518$ MeV so that $k_0=580$ MeV. However, we assume that the effect of the virtual photon can be taken into account by means of a form factor which we shall take to be the *same* for electric and magnetic amplitudes. Therefore, we can compare the ratios between the amplitudes obtained from (7), (8), and (9), and those derived from photoproduction experiments.

Since the photoproduction data are given in terms of the CGLN multipoles, we have to express our amplitudes (10) and (11) in terms of these multipole expansions. We have

$$\langle E_{13}\rangle_{ss'} = \chi_s(2\pi)^{-3/2}\left(\frac{M}{2\omega E}\right)^{1/2}\left(i\frac{\partial}{\partial k_z}j_4(\mathbf{k})\right)_{\mathbf{k}=0}\chi_{s'},$$

$$\langle M_{23}\rangle_{ss'} = \chi_s(2\pi)^{-3/2}\left(\frac{M}{2\omega E}\right)^{1/2}\left[-\frac{\partial}{\partial k_z}(\nabla_{\mathbf{k}}\times\mathbf{j})_z\right]_{\mathbf{k}=0}\chi_{s'},$$

where χ_s and $\chi_{s'}$ are two-component Pauli spinors for the initial and final nucleons, and because the photon is virtual, we have to use electroproduction[13] instead of photoproduction amplitudes, so that $\mathbf{j}$ is given by[14]

$$\mathbf{j}(\mathbf{k}) = i\boldsymbol{\sigma}\mathfrak{F}_1 + \frac{(\boldsymbol{\sigma}\cdot\mathbf{q})(\boldsymbol{\sigma}\times\mathbf{k})}{qk}\mathfrak{F}_2 + \frac{(\boldsymbol{\sigma}\cdot\mathbf{k})\mathbf{q}}{kq}\mathfrak{F}_3 + \frac{i(\boldsymbol{\sigma}\cdot\mathbf{q})\mathbf{q}}{q^2}\mathfrak{F}_4 + \frac{i(\boldsymbol{\sigma}\cdot\mathbf{k})\mathbf{k}}{k^2}\mathfrak{F}_5 + \frac{i(\boldsymbol{\sigma}\cdot\mathbf{q})\mathbf{k}}{qk}\mathfrak{F}_6$$

and

$$j_4(\mathbf{k}) = i\mathbf{j}(\mathbf{k})\cdot\mathbf{k}/k_0.$$

$(\mathbf{k},k_0)$ is the four-momentum of the photon in the c.m. system; $(\mathbf{q},\omega)$ is the four-momentum of the pion in the c.m. system, and $E=(q^2+M^2)^{1/2}$. $\mathfrak{F}_{1\cdots6}$ are given in terms of the conventional CGLN[9] multipole expansions:

$$\mathfrak{F}_1 = \sum_{l=0}^{\infty}\{(lM_{l+}+E_{l+})P_{l+1}'(x) + [(l+1)M_{l-}+E_{l-}]P_{l-1}'(x)\},$$

$$\mathfrak{F}_2 = \sum_{l=1}^{\infty}[(l+1)M_{l+}+lM_{l-}]P_l'(x),$$

$$\mathfrak{F}_3 = \sum_{l=1}^{\infty}[(E_{l+}-M_{l+})P_{l+1}''(x)+(E_{l-}+M_{l-})P_{l-1}''(x)],$$

$$\mathfrak{F}_4 = \sum_{l=1}^{\infty}[M_{l+}-E_{l+}-M_{l-}-E_{l-}]P_l''(x),$$

$$\mathfrak{F}_5 = -\mathfrak{F}_1 - x\mathfrak{F}_3 + \sum_{l=0}^{\infty}[(l+1)Y_{l+}P_{l+1}'(x) - lY_{l-}P_{l-1}'(x)],$$

$$\mathfrak{F}_6 = -x\mathfrak{F}_4 + \sum_{l=1}^{\infty}[lY_{l-}-(l+1)Y_{l+}]P_l'(x),$$

with $x=\mathbf{q}\cdot\mathbf{k}/qk$, and the $M_{l\pm}$, $E_{l\pm}$, $Y_{l\pm}$ refer, respectively, to the magnetic, electric, and longitudinal multipoles with orbital angular momentum l and $J=l\pm\frac{1}{2}$.

[10] F. J. Ernst, R. G. Sachs, and K. C. Wali, Phys. Rev. **119**, 1105 (1960); R. G. Sachs, Phys. Rev. **126**, 2256 (1962).

[11] L. N. Hand, D. G. Miller, and R. Wilson, Rev. Mod. Phys. **35**, 335 (1963).

[12] The value of $[d^2G_E{}^V(k^2)/(dk^2)^2]_{k=0}$ is actually taken from the pole curves in Ref. 11, which fit the experimental data rather well.

[13] In our case, we must remember that the photon is time-like, that is, $k_\mu{}^2=-k_0{}^2<0$.

[14] See, for instance, I. M. Barbour, Nuovo Cimento **27**, 1382 (1963). Our CGLN multipole amplitudes are bigger than these by a factor of $\sqrt{137}$.

For $\mathbf{k}=0$, the following relations can be derived[15]:

$$\lim_{\mathbf{k}\to 0}[E_{l+}-Y_{l+}]=0, \qquad (13)$$

$$\lim_{\mathbf{k}\to 0}[(l-1)E_{l-}+lY_{l-}]=0. \qquad (14)$$

Taking into account now only states with $l=2$ and $J=\frac{3}{2}$, and using the threshold behavior in k of the multipoles,

$$M_{2-}=m_{2-}k^2,\; E_{2-}=E_{2-}(0)+e_{2-}k^2,\; Y_{2-}=Y_{2-}(0)+y_{2-}k^2,$$

a straightforward calculation gives

$$\langle E_{13}\rangle_{ss'}=i(2\pi)^{-3/2}\left(\frac{M}{2\omega E}\right)^{1/2}\frac{E_{2-}(0)}{k_0}\times\chi_s\left[\frac{3(\boldsymbol{\sigma}\cdot\mathbf{q})q_z}{q^2}-\sigma_z\right]\chi_{s'} \qquad (15)$$

and

$$\langle M_{23}\rangle_{ss'}=-6(2\pi)^{-3/2}\left(\frac{M}{2\omega E}\right)^{1/2}m_{2-}\chi_s\times\left[\frac{3(\boldsymbol{\sigma}\cdot\mathbf{q})\sigma_z q_z}{q^2}-1\right]\chi_{s'}. \qquad (16)$$

Using (15) and (16), we can write the sum rules (7), (8), and (9), summing over the intermediate spin states,

$$\frac{1}{2\pi^2}\int\frac{Mq\,dW}{W}\frac{|E_{2-}(0)|^2}{k_0^2}=\frac{\langle r^2\rangle_V}{12}, \qquad (7')$$

$$\frac{18}{\pi^2}\int\frac{Mq\,dW}{W}|m_{2-}|^2=2\left[\frac{d^2G_E{}^V(k^2)}{(dk^2)^2}\right]_{k=0}, \qquad (8')$$

$$\frac{-3i}{\pi^2}\int\frac{Mq\,dW}{W}\frac{E_{2-}(0)}{k_0}m_{2-}{}^*=\frac{i}{2M}\left(\frac{dG_M{}^V(k^2)}{dk^2}\right)_{k=0}. \qquad (9')$$

The main contribution to the three integrals comes when W is near $W_R=1518$ MeV.

We can now make some comparison with the results of photoproduction experiments. In photoproduction, in the neighborhood of the resonance,[16] we can write for the multipoles E_{2-} and M_{2-}

$$M_{2-}{}^r=\mu_{2-}{}^r k^2/(W^2-W_R{}^2-i\Gamma_R W_R), \qquad (17)$$
$$E_{2-}{}^r=(\epsilon_{2-}{}^r(0)-\mu_{2-}{}^r k^2)/(W^2-W_R{}^2-i\Gamma_R W_R),$$

where the index r stands for real photon, and Γ_R is the width of the N^{**} ($\simeq$120 MeV).

[15] A proof of these relations has been given by R. P. Feynman (private communication). It follows from the assumption that for $\mathbf{k}=0$, $\mathbf{j}(\mathbf{k})$ should go like the components k_x, k_y, k_z of $\mathbf{k}$.

[16] Ph. Salin, Nuovo Cimento **28**, 1294 (1963).

From the behavior of the forward differential cross section in π^+ and π^0 photoproduction, one estimates[17]

$$E_{2-}{}^r/M_{2-}{}^r=3 \quad\text{for}\quad W=W_R; \qquad (18)$$

that is,

$$\epsilon_{2-}{}^r(0)/\mu_{2-}{}^r k^2=4. \qquad (18')$$

Remembering now our assumption about the form factor dependence of $E_{2-}(0)$ and m_{2-}, and taking for them a Breit and Wigner form analogous to (17), namely

$$m_{2-}=\mu_{2-}/(W^2-W_R{}^2-i\Gamma_R W_R),$$
$$E_{2-}(0)=\epsilon_{2-}(0)/(W^2-W_R{}^2-i\Gamma_R W_R),$$

we have

$$\epsilon_{2-}(0)/\mu_{2-}=\epsilon_{2-}{}^r(0)/\mu_{2-}{}^r \quad\text{for}\quad W=W_R,$$

so that, taking the ratio between Eqs. (7′) and (9′), we obtain

$$\frac{\epsilon_{2-}{}^r(0)}{\mu_{2-}{}^r k^2}=\frac{\epsilon_{2-}(0)}{\mu_{2-}k^2}=-6\frac{k_0}{k^2}\frac{\langle r^2\rangle_V}{6}\Big/\left[\frac{1}{M}\left(\frac{dG_M{}^V(k^2)}{dk^2}\right)_{k=0}\right]=3.7, \qquad (19)$$

where we have used the value $k=480$ MeV, given in photoproduction and the experimental values[11] for $\langle r^2\rangle_V/6$ and $[dG_M{}^V(k^2)/dk^2]_{k=0}$ (the last quantity is negative).

We have thus obtained a rather good agreement with the value (18′) in magnitude and in sign. It is worthwhile to note that Eqs. (18) and (18′), holding for π^+ and π^0 photoproduction, are valid for a combination of isovector and isoscalar amplitudes, while Eq. (19) holds only for isovector amplitudes. Equation (19) therefore suggests a dominance of the isovector amplitudes in the transition between proton and N^{**}.

This result seems to agree with the experiments on π^+ and π^- photoproduction, at least for the electric dipole amplitude.[18]

There is another way to obtain isovector dominance more directly. We can take, in fact, in Eq. (5), $\alpha=3$ and $\beta=8$, so that we obtain the commutator between the isovector and isoscalar electric dipole moments. The right-hand side of Eq. (5) now vanishes, because $f_{38\gamma}=0$ for every γ, so if we take $i=1$ and $j=2$, we obtain, using Eq. (16),

$$\int\frac{q\,dW}{W}\frac{E_{2-}{}^V(0)E_{2-}{}^{s*}(0)}{k_0^2}=0.$$

Since $E_{2-}{}^V(0)$ from Eq. (7′) is different from zero, in our approximation $E_{2-}{}^s(0)$ must vanish. We conclude

[17] D. S. Beder, Nuovo Cimento **33**, 94 (1964).

[18] G. Neugebauer, W. Wales, and R. L. Walker, Phys. Rev. **119**, 1726 (1960); and R. L. Walker (private communication).

again from this argument that the biggest part of E_{2-}, namely $E_{2-}(0)$, is isovector.

An estimate of the isoscalar contribution to m_{2-} is difficult to obtain, because we have to use the part of Eqs. (5) or (5″) symmetric in α, β, which as we said in the Introduction, depends very much on the model Lagrangian.[5] Moreover, the photoproduction experiments are not complete enough to determine the isoscalar part in $M_{2-}{}^{r}$.[19]

We want now to make a rough consistency check on our form-factor hypothesis, assuming as a simple approximation that the form factor is dominated by the ρ meson, namely

$$E_{2-}(0)=m_\rho^2/\left((W-M)^2-m_\rho^2-i\Gamma_\rho m_\rho\right)\quad E_{2-}{}^{r}(0).\tag{20}$$

We can use the experimental value[20] $M/W(1/\sqrt{137})E_{2-}{}^{r}\simeq(6\pi)^{1/2}(1.2)\times10^{-15}$ cm for $W=W_R$, and Eq. (18) to obtain $E_{2-}{}^{r}(0)$.

With Eq. (20), we can then evaluate the integral in Eq. (7′) over the peaks of the N^{**} resonance and of the form factor. We obtain in this manner a value of the order of 7×10^{-28} cm^2, to be compared with $\simeq6.5\times10^{-28}$ cm^2 which is the value of $\langle r^2\rangle_V/12$ given in Ref. 11.

Of course, if the form factor is not present, we get a value ten times smaller for the integral, in strong disagreement with the right-hand side of Eq. (7′). This very rough calculation indicates therefore that, as a first approximation, one can to a reasonable extent assume the ρ-meson dominance of the form factor in the E_{2-} and M_{2-} transitions. This fact enables us to understand, to some extent, the success of the consistency relation (12).

In fact, for the N^{**}, W_R-M is of the order of the ρ-meson mass, so that the matrix elements of the operators (3) and (4) between proton and N^{**}, are enhanced both by the resonance in the pion-nucleon system and the pole in the form factor. This does not happen to matrix elements connecting the proton to other intermediate states.[21]

Analogous considerations can be applied to the analysis of the transitions between proton and the $N^{***}(1688)$ resonance. This being a $J=\frac{5}{2}^+$ state, we need commutation relations between higher moments of the electromagnetic current, namely electric quadrupole and magnetic octupole. Work in this direction is in progress.

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[19] There is some indication [R. L. Walker (private communication)] that the ratio (18) may be slightly bigger. According to Eq. (19), this could suggest the existence of an isoscalar contribution to $M_{2-}{}^{r}$, because at $W=W_R$, $M_{2-}=-(\frac{2}{3})(M_{2-}{}^{V}-M_{2-}{}^{S})$.

[20] R. L. Walker (private communication).

[21] Of course, the situation becomes more complicated if there are other resonant states with $J=\frac{1}{2}^-$ or $\frac{5}{2}^-$ in the same mass region. There is recently some indication, not yet definite, of the existence of such states from the phase shifts analysis in π-N scattering. However, up to now, these states do not seem to show themselves in photoproduction.

Errata

Error and Convergence Bounds for the Born Expansion, Irwin Manning [Phys. Rev. **139**, B495 (1965)]. In Eq. (4.9) insert a factor $\exp i\delta$.

Mechanisms for the $T=\frac{1}{2}$ and $T=\frac{3}{2}$ Recurrences, P. R. Auvil and J. J. Brehm [Phys. Rev. **140**, B135 (1965)]. Equation (21) should read

$$\Gamma_f=\tfrac{1}{10}(k^5/\Delta^2)(f^2/4\pi),$$

and should be followed by the sentence "For a width of 100 MeV we obtain $f^2/4\pi=0.354\,\mu^{-2}$." This leads to the following determination of the $N_{3/2}{}^*N_{3/2}{}^*\pi$ coupling constant [replacing part of Eq. (58)]: $g_{33}{}^2/4\pi=55$. The line above Eq. (61) should read: . . . we get $(5/27)g_{33}{}^2/g^2$, and hence we conclude that $g_{33}{}^2/4\pi=(27/5)g^2/4\pi=81$ . . . . The result of the model is in very reasonable agreement with this.