πN Charge-Exchange Scattering and the ρ Trajectory*

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πN charge-exchange scattering is considered within the Regge framework. Among meson exchanges only ρ exchange is possible in this reaction. On the basis of the nucleon electromagnetic-structure data it is found that at the momentum-transfer variable $t=m_\rho^2$ the ratio of the spin-flip to non-spin-flip residues is quite large, ~ 4.5, and arguments are given in favor of an even larger ratio, ~ 20, at $t=0$. This latter result helps explain the hump near the forward direction in the charge-exchange differential cross section. With the ρ trajectory and residues consistent with the $\pi^\pm p$ total and differential cross-section data, good fits for $-t \leqslant 0.5$ (BeV/c)^2 are obtained to the recent $\pi^- + p \to \pi^0 + n$ data between 9 and 18 BeV/c.

IN the πN charge-exchange scattering the quantum numbers of the crossed channel are such that among meson exchanges only ρ exchange is the possible candidate. Because of this, it provides an important tool to investigate the validity of various high-energy models. Most reactions, in particular the elastic scattering, are not so fortunate in this respect, since they involve exchanges of more than one type of meson. Among the models that have been considered in the past, one is the so called absorptive model[1] where the competition due to the many open channels is approximately taken into account, but the exchanged system is considered to be an elementary particle. The other is the Regge-pole model where the exchanged system is considered to lie on a Regge trajectory. It has been shown recently that the absorptive model fails very badly in the charge-exchange reaction.[2] For the differential cross section near forward directions it predicts a value roughly an order of magnitude larger than the experimental value in the region between 5 and 10 BeV/c, and as a function of energy it predicts an increasing differential cross section while experimentally the cross section decreases.[2] This failure is all the more striking in view of the rather ideal conditions that are present to satisfy the requirements of the model; there is only a single pole with known coupling constants and the absorption coefficients of both the initial and final states are known. The forward angle appears to follow a power law $\sim \mathbf{E}^{\alpha-1}$, where $\mathbf{E}$ is the lab energy and α lies between 0.5 and 0.6, while in the nonforward directions there appears to be some evidence of shrinkage.[3,4] Such a behavior is typical of a single Regge pole. Thus on the basis of the charge-exchange reaction, it appears that at high energies ρ should be considered as a Regge pole rather than an elementary pole.

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[1] For details on this model see K. Gottfried and J. D. Jackson, Nuovo Cimento 34, 735 (1964). References to the earlier theoretical work are given there.

[2] V. Barger and M. Ebel, Phys. Rev. 138, B1148 (1965). It is pointed out here that if the spin-flip amplitude is taken into account, then the angular distribution becomes even worse.

[3] I. Mannelli, A. Bigi, R. Garrara, M. Wahlig, and L. Sodickson, Phys. Rev. Letters 14, 408 (1965).

[4] A. V. Stirling, P. Sonderegger, J. Kirz, P. Falk-Vairant, O. Guisan, C. Bruneton, P. Borgeaud, M. Yvert, J. P. Guillaud, C. Caverzasio, and B. Amblard, Phys. Rev. Letters 14, 763 (1965).

We shall consider the $\pi^- + p \to \pi^0 + n$ reaction within the Regge framework. The experimental data on nucleon charge and magnetic moment form factors provides useful information on the behavior of the ρ residues in the neighborhood of the ρ resonance. One of our chief results, based on this information, is that the spin-flip residue is quite large near the ρ mass in the crossed channel and becomes larger at zero momentum transfers. This latter fact helps explain the hump in the differential cross section at small momentum transfers. The fits to the experimental data are quite good.

Let A' and B be the usual πN amplitudes.[5] Under the assumption of the dominance of the ρ trajectory, we have as $s \to \infty$,

$$A'(s,t) \to \frac{2^\alpha(\pi)^{1/2}(2\alpha+1)\Gamma(\frac{1}{2}+\alpha)}{\Gamma(1+\alpha)} \frac{8\pi m r_+(t)}{4m^2-t} \times\left(\frac{1-e^{-i\pi\alpha}}{\sin\pi\alpha}\right)\left(\frac{s}{2m^2}\right)^\alpha,$$

$$B(s,t) \to \frac{2^\alpha(\pi)^{1/2}\alpha\Gamma(\frac{1}{2}+\alpha)}{\Gamma(1+\alpha)} \frac{4\pi r_-(t)}{m^2}\left(\frac{1-e^{-i\pi\alpha}}{\sin\pi\alpha}\right)\left(\frac{s}{2m^2}\right)^{\alpha-1},$$

where s and t are the usual Mandelstam variables, m is the nucleon mass, and $\alpha(t)$ is the position of the ρ pole; $r_+(t)$ and $r_-(t)$ are dimensionless quantities given by

$$r_+(t) = m^{2\alpha-1}\gamma_+(t)\,, \tag{1}$$

$$r_-(t) = m^{2\alpha}\gamma_-(t)\,, \tag{2}$$

where γ_+ and γ_- are the residues of the non-spin flip amplitude $f^{(-)}{}_{+,J}(t)$ and the spin flip amplitude $f^{(-)}{}_{-,J}(t)$, respectively. The difference, $\Delta\sigma$, between $\pi^- p$ and $\pi^+ p$ total cross sections, and the differential cross section $d\sigma/dt$ are given by

$$\Delta\sigma = [2/(\omega^2-1)^{1/2}]\,\mathrm{Im}A'(s,0)\,, \tag{3}$$

$$\frac{d\sigma}{dt} = \frac{m^2}{2\pi s^2}\left\{\left(1-\frac{t}{4m^2}\right)|A'|^2 + \frac{t}{4m^2}\left(s-\frac{(m+\omega)^2}{1-t/4m^2}\right)|B|^2\right\}, \tag{4}$$

where

$$\omega = (s-m^2-m_\pi^2)/2m\,.$$

[5] We use the notations of V. Singh, Phys. Rev. 129, 1889 (1963).

We take the following parametric representation for α and $r_\pm$:

$$\alpha(t)=a+[b/(1-ct)] \tag{5}$$

and

$$r_\pm(t)=\frac{r_\pm(0)}{\alpha'(0)}\left(\frac{m^2}{m_{0\pm}{}^2}\right)^{\alpha(t)-\alpha(0)}\frac{d\alpha}{dt}\left(1+\frac{t}{t_{0\pm}}\right). \tag{6}$$

The expression for α is essentially a pole approximation and is useful for $t<1/c$. The expression for $r(t)$ follows from potential theory where it is known that for small t the reduced residue $\gamma(t)$ is given by[6]

$$\gamma(t)\approx(m^2R_t^2)^{\alpha(t)}\frac{1}{R_t}\frac{d\alpha}{dt}, \tag{7}$$

where R_t is the effective radius of interaction in the t channel. In our case $m_0\sim 1/R_t$. If zeros are present in $\gamma(t)$ then one must multiply the right-hand side of (7) by appropriate terms. In our case we use the term $(1+t_0/t)$. Generally, the zeros lie in the negative t region.

The knowledge of the experimental value of $\Delta\sigma$ determines $\alpha(0)$ and $r_+(0)$. The phenomenological fits to the total cross-section data have already been made.[7,8] The best value seems to be[9]

$$\alpha(0)=0.55, \tag{8}$$

and

$$r_+(0)=0.16. \tag{9}$$

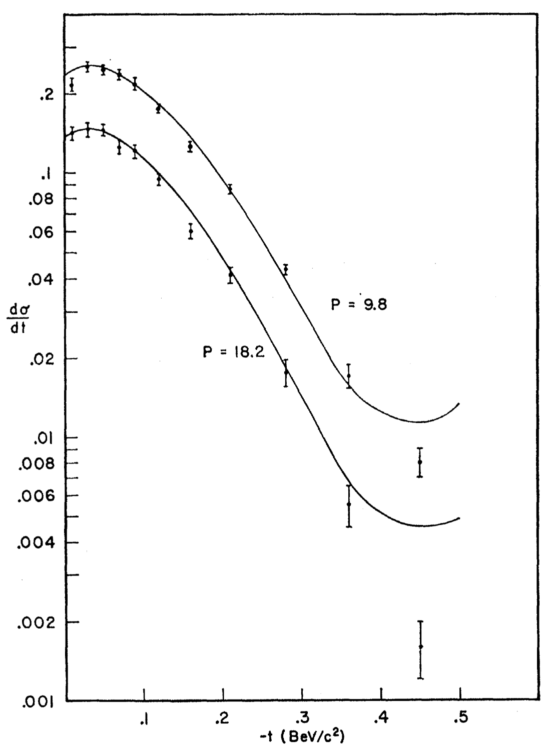

FIG. 1. Fits to the data on charge-exchange scattering, $\pi^-+p\rightarrow\pi^0+n$, at lab momenta P of 9.8 and 18.2 BeV/c. Differential cross section $d\sigma/dt$ in mb/(BeV/c)^2 is plotted versus momentum transfer squared $-t$ up to 0.5 (BeV/c)^2. The data are from Ref. 4. The experimental values of $d\sigma/dt$ beyond $-t=0.5$ (BeV/c)^2, not plotted here, increase as $-t$ increases. Our theoretical curve predicts an increase at a somewhat lower value of $-t$.

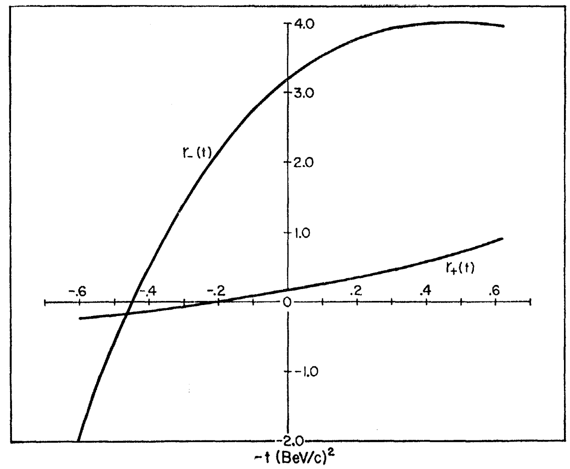

FIG. 2. The dimensionless quantities $r_+(t)$ and $r_-(t)$ versus t (BeV/c)^2.

In order to explain the crossover phenomenon[10] between π^-p and π^+p differential cross sections, it is found that the non-spin-flip residue should change sign.[8] The crossover occurs around $t=-0.2$ (BeV/c)^3 and, therefore, we take

$$t_{0+}=0.2\text{ (BeV/}c)^2. \tag{10}$$

We also have some conditions at the ρ mass. Clearly,

$$\alpha(m_\rho{}^2)=1. \tag{11}$$

Calculations of the $\pi\pi\rightarrow N\bar{N}$ amplitudes in $I=1$, $J=1$ states have been made in Refs. 11 and 12, where the experimental information on the nucleon charge and magnetic moment form factors were used. We use the

[6] B. R. Desai, Phys. Rev. **138**, B1174 (1965).

[7] R. K. Logan, Phys. Rev. Letters **14**, 414 (1965).

[8] R. J. N. Phillips and W. Rarita, Phys. Rev. **140**, B200 (1965); hereafter referred to as PR.

[9] Our value of α is essentially an average of the values in Refs. 7 and 8.

[10] It is found experimentally that while $d\sigma/dt$ $(\pi^-p)>d\sigma/dt$ (π^+p) near forward directions, at larger momentum transfers the π^-p value goes below the π^+p value. This is what we mean by the crossover phenomenon. A similar situation holds in $K^\pm p$ and pp, $\bar{p}p$ scattering. In a recent paper by T. O. Binford and B. R. Desai, Phys. Rev. **138**, B1167 (1965), this phenomenon was taken into account for $K^\pm p$ and for pp, $\bar{p}p$, but not for $\pi^\pm p$. This was due to the fact that unlike the other cases the difference between π^-p and π^+p cross sections is very small above 10 (BeV/c) and, therefore good fits can be obtained for the $\pi^\pm p$ data even when the crossover phenomenon is ignored (i.e., even when one neglects the contribution of the ρ trajectory). The smallness of π^-p, π^+p cross-section difference is, or course, reflected in the small magnitude of the charge-exchange cross section.

[11] V. Singh and B. M. Udgaonkar, Phys. Rev. **128**, 1820 (1962).

[12] J. S. Ball and D. Y. Wong, Phys. Rev. **130**, 2112 (1963), hereafter referred to as BW. The solution II of BW, corresponding to a two-pole approximation to the $\pi\pi$ amplitude, is used. I would like to thank Professor J. S. Ball for supplying me with the numerical results of this paper.

results of Ball and Wong[12] to determine $r_+(t)$ and $r_-(t)$ at $t=m_\rho^2$. We find

$$r_+(m_\rho^2)=0.87\,, \tag{12}$$

$$r_-(m_\rho^2)=3.98\,. \tag{13}$$

It may be noted that the nonspin-flip and spin-flip residues get major contributions from the charge and magnetic moment form factors, respectively. It is interesting to note that r_-/r_+ at the ρ mass is quite large, $\sim$4.5. To determine the behavior of the residues in the neighborhood of the ρ mass, we need another quantity which we define to be

$$R(m_\rho^2)=\frac{d}{dt}\ln\left(\frac{r_+(t)}{r_-(t)}\right)\bigg|_{t=m_\rho^2}. \tag{14}$$

The quantity $R(m_\rho^2)$ can be qualitatively determined by making the plausible assumption

$$R(m_\rho^2)\approx\frac{d}{dt}\ln\left(\frac{f^{(-)}{}_{+,J}(t)}{f^{(-)}{}_{-,J}(t)}\right)\bigg|_{t=m_\rho^2} \tag{15}$$

at $J=\alpha(t)$. However, we choose to keep $R(m_\rho^2)$ as a parameter. Eventually we will compare our value with the value on the right hand side of (15) given by BW. There are two more parameters left before we can determine the charge-exchange differential cross section. We take them to be $\alpha'(0)$ and t_{0-}. It may be noted that the zero in the spin-flip residue would be responsible for the second dip in the differential cross section. A glance at the charge-exchange data indicates that t_{0-} should be around 0.5 $(\mathrm{BeV}/c)^2$. We may fix t_{0-} at this value. However, we choose to keep it as a parameter to be determined from experiments.

We thus have three parameters $\alpha'(0)$, t_{0-}, and $R(m_\rho^2)$. We have fitted our results with the data of Ref. 4 for lab momenta above 9 BeV/c and for $-t\leqslant 0.5$ $(\mathrm{BeV}/c)^2$. In Fig. 1 the fits to the highest and the lowest energies in this interval are given. The fits are quite good. With the input values given in (8)–(13) we obtain the following values for the three parameters

$$\begin{aligned}\alpha'(0)&=0.72\ (\mathrm{BeV}/c)^{-2},\\ t_{0-}&=0.45\ (\mathrm{BeV}/c)^{2},\\ R(m_\rho^2)&=1.9\ (\mathrm{BeV}/c)^{-2}.\end{aligned} \tag{16}$$

We also determine the following quantities:

$$\begin{aligned}m^2/m_{0+}^2&=1.75\,,\\ m^2/m_{0-}^2&=0.21\,.\end{aligned} \tag{17}$$

The curves for $r_\pm$ and α are given in Figs. 2 and 3, respectively. The following points must be noted:

(i) The spin-flip residue is quite large. At the ρ mass r_-/r_+ is about 4.5. Because $R(m_\rho^2)$ is positive this ratio at $t=0$ is even larger, $\sim$20. In fact if we write

$$d\sigma/dt=a-bt$$

for extremely small $-t$ then $b/a\sim 25$.

(ii) The value of $R(m_\rho^2)$ given by (15) on the basis of the results of BW is 1.1 $(\mathrm{BeV}/c)^{-2}$. This value is reasonably close to the one we have obtained here. What is more interesting is the fact that both have the same (positive) sign. This gives strong support to the argument that as t decreases from m_ρ^2 towards the negative region, the ratio of the spin flip to the non-spin flip becomes larger.

(iii) The second dip in $d\sigma/dt$ is not well reproduced by our fits. However, it is clear that there is a close connection between the second dip and the zero in the spin-flip residue. We tried to fit the data without any zeros in the spin-flip residue but arranged $\alpha(t)$ and, therefore, the amplitude B to vanish around $-t=0.5$. This can be done if a straight line trajectory is assumed. The fits, however, were not as good.

(iv) Contrary to the assertion in Ref. 13, the Regge fits to the charge-exchange data are not in contradiction with the elastic scattering data. The crossover phenomenon[10] between elastic π^-p and π^+p differential cross section is explained in terms of a zero in the non-spin flip residue. At the crossover point the spin-flip residue is non-negligible and therefore the charge-exchange differential cross section does not vanish at that point.

(v) The zeros in the residues may come about if there is a short-range repulsion in addition to a long-range attraction.[6,14] In the ρ case, the $\pi\pi\rightarrow N\bar{N}$ amplitude gets repulsion from the N^* exchange, while the N exchange is attractive. It may be because of this change of sign in the potential that the residues also change sign.

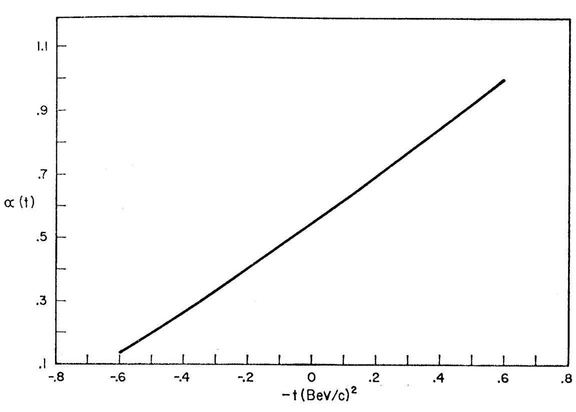

FIG. 3. The ρ trajectory $\alpha(t)$ versus t $(\mathrm{BeV}/c)^2$.

[13] R. K. Logan, Phys. Rev. Letters **14**, 921 (1965).

[14] If the Mandelstam symmetry holds [S. Mandelstam, Ann. Phys. (N. Y.) **19**, 254 (1962)], then, unless very special conditions are satisfied, the residues in the presence of short-range repulsive (Yukawa-type) potentials leave the real axis at large $-t$ values and go into the complex plane. I am grateful to Professor P. E. Kaus for pointing this out to me. In the relativistic case, of course, the Mandelstam symmetry need not hold.

(vi) Our results come closest to solution I of Phillips and Rarita.[8] Our slope for the ρ trajectory is slightly larger. This may be due to the pure exponential behavior for the residues assumed in PR while we have used (7). The number of parameters needed in our fits is less than in PR. This is due to the fact that we have used extra information from the nucleon form factor data and also because our parametrization of the residues involve a lesser number of parameters. We have confined our fits to $-t\leq 0.5$ (BeV/c)^2 partly because beyond this range the parametric representations of α and r are expected to be unreliable and partly because at large momentum transfers there is no reason to ignore N and N^* exchanges.

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Sum Rules for Magnetic Quadrupole and Electric Dipole Moments: An Application of the Algebra of Current Components*

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Assuming simple commutation relations between the various components of the vector current density, we derive three commutation relations between electric dipole and magnetic quadrupole operators. Taking these commutators between proton states at rest, we get three sum rules. Considering only one intermediate state, namely the $I=\frac{1}{2}$, $J=\frac{3}{2}^-$ pion-nucleon resonance $N^{**}(1518)$, we have a consistency relation between them. The sum rules are then used for deriving the ratio between the E_{2-} and M_{2-} amplitudes in single-pion photoproduction, and one obtains a result in good agreement with experiment.

I. INTRODUCTION

RECENTLY, various applications of the commutation relations between components of current densities, the so-called algebra of current components,[1,2] have been given. One of the most important results is certainly the sum rule for the magnetic moment of the proton, derived by Dashen and Gell-Mann,[1] and Lee.[3] These authors, assuming a commutation relation between the space components of the vector current density J_i^α, where α is an $SU(3)$ index, derived a commutation relation for the space integrals of the magnetic moment operators $\frac{1}{2}\epsilon_{ikl}r_kJ_l^\alpha$. Then, taking the expectation value of this commutator between proton states at rest, and inserting a suitable set of intermediate states, namely the nucleon and the 33-resonance $N^*(1238)$, they obtain a relation between the magnetic moment and the charge radius of the proton[4]:

$$(\mu_p/2M)^2=\tfrac{1}{6}\langle r^2\rangle_p\,, \tag{1}$$

which is fairly well satisfied by the experimenta numbers.

Moreover, the commutation relations also give the vanishing of the electric quadrupole transition between the proton and the N^*.

In the present paper, we want to apply the same method to other moments of the electromagnetic current, and particularly to the electric dipole and magnetic quadrupole moments. The starting point will be, as in the case of the Dashen-Lee sum rules, the commutation relations between the components of the vector current density:

$$[J_4^\alpha(\mathbf{x}),J_4^\beta(\mathbf{x}')]=if_{\alpha\beta\gamma}\delta(\mathbf{x}-\mathbf{x}')J_4^\gamma(\mathbf{x})\,, \tag{2}$$

$$[J_4^\alpha(\mathbf{x}),J_i^\beta(\mathbf{x}')]=if_{\alpha\beta\gamma}\delta(\mathbf{x}-\mathbf{x}')J_i^\gamma(\mathbf{x})+(\text{terms symmetric in }\alpha,\beta)\,, \tag{2$'$}$$

$$[J_i^\alpha(\mathbf{x}),J_j^\beta(\mathbf{x})]=i\delta_{ij}f_{\alpha\beta\gamma}\delta(\mathbf{x}-\mathbf{x}')J_4^\gamma(\mathbf{x})+(\text{terms symmetric in }\alpha,\beta)\,. \tag{2$''$}$$

It is rather reasonable to believe that these commutators are correct. Adler and Callan[5] have proved Eqs. (2) and (2′) for some simple models: the quark model and two models which involve baryons and mesons, like the Gell-Mann and Lévy[6] σ model. It must be noted that in the quark model the last term in Eq. (2″)

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[1] R. F. Dashen and M. Gell-Mann, Phys. Letters 17, 142, 145 (1965).

[2] R. P. Feynman, M. Gell-Mann, and G. Zweig, Phys. Rev. Letters 13, 678 (1964).

[3] B. W. Lee, Phys. Rev. Letters 14, 676 (1965).

[4] Actually, it should be $\langle r^2\rangle_V$, the charge radius of the isovector form factor which differs from $\langle r^2\rangle_P$ by about 20%.

[5] S. L. Adler and C. G. Callan, CERN Report 65/1227/5-Th. 587 (unpublished).

[6] M. Gell-Mann and M. Lévy, Nuovo Cimento 16, 705 (1960).