Kinetic Supermultiplets of $\tilde{U}(12)$. II

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Possible baryon "kinetic supermultiplets" are discussed as candidates for classifying higher baryonic resonances of negative parity. The "kinetic supermultiplets" had been proposed, in a previous work, as an extension of the supermultiplet scheme of broken $\tilde{U}(12)$. The classification of higher boson resonances in terms of a "kinetic supermultiplet" has proved remarkably successful and led to correct predictions of quantum numbers and accurately verified mass relations. The baryon "kinetic supermultiplet" discussed here employs the hitherto unused representation **220** in $\tilde{U}(12)$, which corresponds to the **20** of SU_6, and is tentatively chosen on the basis of its economy. It groups together a baryon nonet with $J^P=\frac{1}{2}^-$, another nonet with $J^P=\frac{3}{2}^-$, and a singlet with $J^P=\frac{5}{2}^-$. The mass formulas are derived up to first order in SU_3 breaking. Based on the quantum-number predictions and the mass formulas, we attempt a classification of higher baryon resonances, and present a tentative scheme of assignments and consequent predictions. An alternative scheme employing the representation **364** in $\tilde{U}(12)$, corresponding to the **56** of SU_6, is also examined.

INTRODUCTION

In a preceding paper[1] (hereafter referred to as I) we have proposed an extension of the multiplet structure of broken $\tilde{U}(12)$[2] by introducing the notion of "kinetic supermultiplets." The "kinetic supermultiplets" are represented by reducible $\tilde{U}(12)$ tensors and contain separate nondegenerate multiplets. In I the example of the lowest boson "kinetic supermultiplet" was discussed in detail and it was shown that it leads to the scheme proposed by Borchi and Gatto for a classification of higher boson resonances.[3] The mass relations derived in I for the proposed boson kinetic supermultiplet were found to be remarkably accurate. An equidistance relation predicted in I between the squared masses of the resonances A_1, A_2, and B, namely $\frac{1}{2}[m^2(A_1)+m^2(A_2)]=m^2(B)$, appears to be exactly verified. A number of predictions of resonant boson masses was obtained on the basis of the derived mass formulas and of a preliminary classification of existing resonances. Since then evidence has been reported for new resonances which seem to fit the proposed scheme quite well. A predicted $T=0$ meson with $J^{PC}=2^{++}$ at (1560 ± 50) decaying into $K\bar{K}$ may be identified with the $f'(1520)$.[4] The D meson at 1280 MeV[5] could be identified, according to the suggested quantum numbers, with the $J^{PC}=1^{++}$ $T=0$ meson predicted at 1180 ± 190 MeV. Furthermore there seems to be evidence[6] about the existence of a $J^{PC}=0^{++}$, $T=1$ meson, predicted at 970 ± 70 MeV.

In this note we shall discuss a possible baryon kinetic supermultiplet described by a tensor product of the representation **220** of $\tilde{U}(12)$ with the "kinetic" tensor. The irreducible **220** representation is chosen essentially on the basis that it allows for the most economical scheme. Such a baryon kinetic supermultiplet contains a nonet with $J^P=\frac{1}{2}^-$, and nonet with $J^P=\frac{3}{2}^-$ and a singlet with $J^P=\frac{5}{2}^-$. If, instead of a tensor of **220** one chooses a tensor of **364**, each of the above three singlets is substituted by a decuplet, giving a much more cumbersome scheme. The classification of the higher baryon resonances in terms of kinetic supermultiplets, rather than in terms of ordinary $\tilde{U}(12)$ supermultiplets,[7] seems to be required from the evidence for negative parities of a number of higher baryon resonances, at least as long as one likes to avoid representations with large numbers of components. The imposition of the

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equations of motion and the extraction of the irreducible parts allows us to derive the decomposition of the kinetic supermultiplet into its component multiplets and the subsequent derivation of the wave functions. Before comparing with experiment we derive the mass formulas up to first order in the SU_3 symmetry breaking. We obtain a set of three linear mass relations and two bilinear ones, accounting for the possible mixing effects between the two $T=0$ states in each of the two nonets. We carry out a preliminary comparison with the existing data on the basis of the derived mass formulas. We find that a crucial point, for the validity of the scheme, is the existence of a nonet of baryon resonances with $J^P=\frac{1}{2}^-$, all lying in an energy region definitely below 2 GeV. Of the established particles, only $Y_0^*(1405)$, appears to be a possible candidate so far. In spite of the preliminary status of the classification, we feel that an experimental exploration for possible resonant behaviors around the predicted masses may be of interest. We also present a parallel discussion for the alternative choice of **364**, instead of **220**, as a basic representation. The mass relations and the preliminary assignments do not seem to substantiate such an alternative choice. However, the preliminary state of the assignments and the *a priori* impossibility of excluding further symmetry breakings in the derivations of the mass relations again do not allow for a definite conclusion.

THE 220 SUPERMULTIPLET

A kinetic supermultiplet is described in terms of a reducible tensor which is obtained as a product of a basic $\tilde{U}(12)$ irreducible tensor and the kinetic tensor belonging to the regular representation. The imposition of the equations of motion on the reducible tensor and the extraction of its irreducible components leads to the decomposition of the kinetic supermultiplet into its component multiplets. As for the basic $\tilde{U}(12)$ baryonic irreducible tensor, one is immediately led to a choice between the **364** and **220** representations. We shall start here with the irreducible **220** tensor. The particle content of **220** is $(8,\frac{1}{2})+(1,\frac{3}{2})$, where the first number in the parentheses denotes the SU_3 multiplicity and the second number denotes the spin. The baryon kinetic supermultiplet described by the product of the kinetic tensor and the irreducible tensor of **220** will have the content

$$(8,\tfrac{1}{2})+(8,\tfrac{3}{2})+(1,\tfrac{1}{2})+(1,\tfrac{3}{2})+(1,\tfrac{5}{2}). \tag{1}$$

If in place of the irreducible tensor of **220** one starts from the irreducible tensor of **364** the resulting baryon kinetic supermultiplet will have the much larger content

$$(8,\tfrac{1}{2})+(8,\tfrac{3}{2})+(10,\tfrac{1}{2})+(10,\tfrac{3}{2})+(10,\tfrac{5}{2}). \tag{2}$$

It thus appears that the choice of **220** for the representation of the basic irreducible tensor is much more economical in particle content. We shall show in the following that it leads to a baryon kinetic supermultiplet with appropriate quantum numbers for a classification of higher baryon resonances of negative parity.

We must first summarize the content of **220** of broken $\tilde{U}(12)$. In the $U_3\otimes\tilde{U}(4)$ decomposition the wave functions are the following: For the SU_3 octet

$$\begin{aligned}\psi^k{}_{[\alpha\beta]\gamma}[\epsilon_{cas}(T_k)_b{}^s+\epsilon_{cbs}(T_k)_a{}^s]&\\ +\psi^k{}_{[\beta\gamma]\alpha}[\epsilon_{abs}(T_k)_c{}^s+\epsilon_{acs}(T_k)_b{}^s]&\\ +\psi^k{}_{[\gamma\alpha]\beta}[\epsilon_{bcs}(T_k)_a{}^s+\epsilon_{bas}(T_k)_c{}^s]&,\end{aligned} \tag{3}$$

where Greek indices refer to $\tilde{U}(4)$ and Latin indices to $U(3)$, T_k are the unitary matrices, the brackets $[\cdots]$ indicate antisymmetrization, and

$$\psi_{[\alpha\beta]\gamma}=[(1+p\!\!\!/ /m)\gamma_5C]_{\alpha\beta}\psi_\gamma(p)\,, \tag{4}$$

$$(p\!\!\!/-m)\psi(p)=0\,. \tag{5}$$

Equations (4) and (5) follow from the imposition of the Bargmann-Wigner equations. In Eq. (4) C is the charge-conjugation matrix.[2] For the SU_3 singlet one has the wave function

$$\psi_{\{\alpha\beta\gamma\}}\epsilon_{abc}\,, \tag{6}$$

where the brackets $\{\cdots\}$ indicate symmetrization. Imposition of the Bargmann-Wigner equations gives

$$\psi_{\{\alpha\beta\gamma\}}=(\psi^\mu)_\alpha[(1+p\!\!\!/ /m)\gamma_\mu C]_{\beta\gamma}\,, \tag{7}$$

with the conditions

$$p_\mu\psi^\mu(p)=0\,,\quad\text{and}\quad (p\!\!\!/-m)\psi_\mu=0\,. \tag{8}$$

So much for the treatment of **220** in broken $\tilde{U}(12)$. We now turn to the reducible tensor describing our kinetic supermultiplet.

DECOMPOSITION AND WAVE FUNCTIONS

Let us fix our attention on the $\tilde{U}(4)$ indices. Multiplication by the kinetic tensor gives for the octet terms, Eqs. (3), (4), and (5),

$$\psi_{[\alpha\beta]\gamma,\delta}{}^\epsilon=[(\gamma_5C)_{\alpha\beta}(\psi_{,\lambda})_\gamma+i(\gamma_\mu\gamma_5C)_{\alpha\beta}(\psi^\mu{}_{,\lambda})_\gamma](\gamma^\lambda)_\delta{}^\epsilon. \tag{9}$$

We note that mixed symmetry in α, β, and γ requires

$$\psi_{,\lambda}=i\gamma_\mu\psi^\mu{}_{,\lambda}\,. \tag{10}$$

The imposition of the Bargmann-Wigner equations gives

$$p_\mu\psi_{\nu,\lambda}-p_\nu\psi_{\mu,\lambda}=0\,, \tag{11}$$

$$p\!\!\!/\psi_{,\lambda}=im\psi_{\mu,\lambda}\,, \tag{12}$$

$$p^\mu\psi_{\mu,\lambda}=-im\psi_{,\lambda}\,. \tag{13}$$

We must also impose a transversality condition

$$p^\lambda\psi_{,\lambda}=0\,,\quad p^\lambda\psi_{\mu,\lambda}=0\,. \tag{14}$$

Furthermore each ψ will satisfy Dirac equations. The functions $\psi_{,\lambda}$ and $\psi_{\mu,\lambda}$ will be decomposed into terms of definite spin. We note first that the function

$$\varphi_\lambda=\psi_{,\lambda}-\tfrac{1}{3}(\gamma_\lambda+p_\lambda/m)\gamma^\mu\psi_{,\mu} \tag{15}$$

is a Rarita-Schwinger spinor. In fact it satisfies the Dirac equation together with

$$p^\lambda\varphi_\lambda=0\,,\quad \gamma^\lambda\varphi_\lambda=0\,. \tag{16}$$

Similarly

$$\chi_\lambda=\tfrac{1}{3}(\gamma_\lambda+p_\lambda/m)\gamma^\mu\psi_{,\mu}=\tfrac{1}{3}(\gamma_\lambda+p_\lambda/m)\gamma_5\psi \tag{17}$$

describes a spin-$\frac{1}{2}$ particle. In Eq. (17) we have introduced ψ such that $\gamma^\mu\psi_{,\mu}=\gamma_5\psi$. It is apparent that ψ satisfies the Dirac equation. The required decomposition is then

$$\psi_{,\lambda}=\varphi_\lambda+\chi_\lambda \tag{18}$$

with φ_λ and χ_λ given by Eqs. (15) and (17), respectively. We have thus concluded the discussion of the SU_3 octets [see Eq. (1)], and we have obtained the following wave functions

$$(\mathbf{8},\tfrac{1}{2}):\quad \psi_{[\alpha\beta]\gamma,\delta}{}^\epsilon=[(\gamma_\lambda+p_\lambda/m)\gamma_5\psi]_\gamma \times[(1+\not{p}/m)\gamma_5C]_{\alpha\beta}(\gamma^\lambda)_\delta{}^\epsilon, \tag{19}$$

$$(\mathbf{8},\tfrac{3}{2}):\quad \psi_{[\alpha\beta]\gamma,\delta}{}^\epsilon=[(1+\not{p}/m)\gamma_5C]_{\alpha\beta}(\varphi_\lambda)_\gamma(\gamma^\lambda)_\delta{}^\epsilon \tag{20}$$

[together with the Rarita-Schwinger conditions, Eq. (16)].

Next let us discuss the SU_3 singlets [Eqs. (6), (7), and (8)]. By insertion of the kinetic tensor one obtains the wave function

$$\psi_{\{\alpha\beta\gamma\},\delta}{}^\epsilon=(\psi_{\mu,\lambda})_\alpha[(1+\not{p}/m)\gamma^\mu C]_{\beta\gamma}(\gamma^\lambda)_\delta{}^\epsilon. \tag{21}$$

We have to impose the transversality condition

$$p^\lambda\psi_{\mu,\lambda}=0\,. \tag{22}$$

Moreover, the imposition of the Bargmann-Wigner equations gives

$$(\not{p}-m)\psi_{\mu,\lambda}=0\,, \tag{23}$$

$$\gamma^\mu\psi_{\mu,\lambda}=0\,,\quad\text{and}\quad p^\mu\psi_{\mu,\lambda}=0\,. \tag{24}$$

The separation of the components of different spin is uniquely given by the decomposition

$$\psi_{\mu,\lambda}=\mathcal{S}_{\mu\lambda}+\mathcal{A}_{\mu\lambda}+\mathcal{T}_{\mu\lambda}\,, \tag{25}$$

where $\mathcal{S}_{\mu\lambda}$, $\mathcal{A}_{\mu\lambda}$, and $\mathcal{T}_{\mu\lambda}$ will describe the spin $\frac{5}{2}^-$, the spin $\frac{3}{2}^-$, and the spin-$\frac{1}{2}^-$ singlets, respectively [compare with Eq. (1)]. The symmetric tensor $\mathcal{S}_{\mu\lambda}$ satisfies, in addition to $\mathcal{S}_{\mu\lambda}=\mathcal{S}_{\lambda\mu}$, the conditions

$$\gamma^\mu\mathcal{S}_{\mu\lambda}=0\,,\quad p^\mu\mathcal{S}_{\mu\lambda}=0\,,\quad\text{and}\quad(\not{p}-m)\mathcal{S}_{\mu\lambda}=0\,. \tag{26}$$

The tensor $\mathcal{A}_{\mu\lambda}$ is expressed in terms of the antisymmetric tensor $A_{\mu\lambda}$:

$$\mathcal{A}_{\mu\lambda}=A_{\mu\lambda}+\tfrac{1}{5}[(\gamma_\mu+p_\mu/m)\gamma^\rho A_{\lambda\rho} +(\gamma_\lambda+p_\lambda/m)\gamma^\rho A_{\mu\rho}]\,, \tag{27}$$

where $A_{\mu\lambda}$, in addition to the antisymmetry condition $A_{\mu\lambda}=-A_{\lambda\mu}$, satisfies

$$\sigma^{\mu\lambda}A_{\mu\lambda}=0\,,\quad\text{and}\quad(\not{p}-m)A_{\mu\lambda}=0\,. \tag{28}$$

The remaining term in (25), $\mathcal{T}_{\mu\lambda}$, is directly expressed in terms of $\psi^\mu{}_{,\mu}=\psi$:

$$\mathcal{T}_{\mu\lambda}=\{(g_{\mu\lambda}-p_\mu p_\lambda/m^2) +\tfrac{1}{2}[i\sigma_{\mu\lambda}+(1/m)(\gamma_\mu p_\lambda-\gamma_\lambda p_\mu)]\}\psi\,, \tag{29}$$

with

$$(\not{p}-m)\psi=0\,.$$

Explicit counting of the components, in the rest system, shows directly that $\mathcal{S}_{\mu\lambda}$, $\mathcal{A}_{\mu\lambda}$, and $\mathcal{T}_{\mu\lambda}$ describe systems with spin $\frac{5}{2}$, $\frac{3}{2}$, and $\frac{1}{2}$, respectively.

In conclusion, the wave functions of the SU_3 singlets of our baryon kinetic supermultiplet are the following:

$$(\mathbf{1},\tfrac{5}{2}):\quad \psi_{\{\alpha\beta\gamma\},\delta}{}^\epsilon=(\mathcal{S}_{\mu\nu})_\alpha[(1+\not{p}/m)\gamma^\mu C]_{\beta\gamma}(\gamma^\nu)_\delta{}^\epsilon\,,$$

$$(\mathbf{1},\tfrac{3}{2}):\quad \psi_{\{\alpha\beta\gamma\},\delta}{}^\epsilon=(\mathcal{A}_{\mu\nu})_\alpha[(1+\not{p}/m)\gamma^\mu C]_{\beta\gamma}(\gamma^\nu)_\delta{}^\epsilon\,,$$

$$(\mathbf{1},\tfrac{1}{2}):\quad \psi_{\{\alpha\beta\gamma\},\delta}{}^\epsilon=(\mathcal{T}_{\mu\nu})_\alpha[(1+\not{p}/m)\gamma^\mu C]_{\beta\gamma}(\gamma^\nu)_\delta{}^\epsilon\,.$$

The parity of the supermultiplet can be directly verified by direct application of the parity operation to the wave functions. All multiplets are found to have negative parity. Under the the same parity definition the representation **364** of $\tilde{U}(12)$, which corresponds to the **56** of SU_6 and contains the stable baryon octet and the decuplet, has parity $+1$. It has to be noted that the representation **572** of $\tilde{U}(12)$, which corresponds to the **70** of SU_6, has been discarded as a candidate for classifying some of the higher baryon resonances because of its positive parity.[7]

MASS FORMULAS

We shall now derive the mass relations. In compact notation the reducible tensor describing our baryon kinetic supermultiplet can be written as $\psi_{ABC,D}{}^E$ where $A, B, \cdots, E$ are $\tilde{U}(12)$ indices. Without any further breaking of the symmetry, we can form the following mass terms

$$[1]=\bar{\psi}^{ABC,D}{}_E\psi_{ABC,D}{}^E$$
$$[2]=\bar{\psi}^{ABC,D}{}_A\psi_{EBC,D}{}^E$$
$$[3]=\bar{\psi}^{ABC,D}{}_A\psi_{EBD,C}{}^E$$
$$[4]=\bar{\psi}^{ABC,D}{}_E\psi_{ABD,C}{}^E$$

The "central" mass term [1] defines the relative normalizations of the wave functions. We introduce next first-order breaking of SU_3. At such order, the following mass term must be included:

$$[\alpha_1]=\bar{\psi}^{ABC,D}{}_E(\lambda_8)_A{}^F\psi_{FBC,D}{}^E, \qquad [\beta_1]=\bar{\psi}^{ABC,D}{}_E(\lambda_8)_D{}^F\psi_{ABC,F}{}^E,$$
$$[\gamma_1]=\bar{\psi}^{ABC,D}{}_E(\lambda_8)_F{}^E\psi_{ABC,D}{}^F,$$
$$[\alpha_2]=\bar{\psi}^{ABC,D}{}_F(\lambda_8)_A{}^F\psi_{EBC,D}{}^E, \qquad [\beta_2]=\bar{\psi}^{ABC,D}{}_A(\lambda_8)_F{}^E\psi_{EBC,D}{}^F,$$
$$[\gamma_2]=\bar{\psi}^{ABC,D}{}_A(\lambda_8)_B{}^F\psi_{EFC,D}{}^E, \qquad [\delta_2]=\bar{\psi}^{ABC,D}{}_A(\lambda_8)_B{}^F\psi_{EBC,F}{}^E,$$
$$[\alpha_3]=\bar{\psi}^{ABC,D}{}_F(\lambda_8)_A{}^F\psi_{EBD,C}{}^E, \qquad [\beta_3]=\bar{\psi}^{ABC,D}{}_A(\lambda_8)_B{}^F\psi_{EFD,C}{}^E,$$
$$[\gamma_3]=\bar{\psi}^{ABC,D}{}_A(\lambda_8)_C{}^F\psi_{EBD,F}{}^E, \qquad [\delta_3]=\bar{\psi}^{ABC,D}{}_A(\lambda_8)_D{}^F\psi_{EBF,C}{}^E,$$
$$[\epsilon_3]=\bar{\psi}^{ABC,D}{}_A(\lambda_8)_F{}^E\psi_{EBD,C}{}^F,$$
$$[\alpha_4]=\bar{\psi}^{ABC,D}{}_E(\lambda_8)_A{}^F\psi_{FBD,C}{}^E, \qquad [\beta_4]=\bar{\psi}^{ABC,D}{}_E(\lambda_8)_C{}^F\psi_{ABD,F}{}^E,$$
$$[\gamma_4]=\bar{\psi}^{ABC,D}{}_E(\lambda_8)_D{}^F\psi_{ABF,C}{}^E, \qquad [\delta_4]=\bar{\psi}^{ABC,D}{}_E(\lambda_8)_F{}^E\psi_{ABD,C}{}^F.$$

We note that under charge conjugation $[\alpha_1]$ goes into itself, whereas $[\beta_1]\leftrightarrow[\gamma_1]$. Also $[\alpha_2]\leftrightarrow[\beta_2]$, $[\gamma_2]\leftrightarrow[\gamma_2]$, $[\delta_2]\leftrightarrow[\delta_2]$. Similarly $[\alpha_3]\leftrightarrow[\epsilon_3]$, $[\beta_3]\leftrightarrow[\beta_3]$, $[\gamma_3]\leftrightarrow[\delta_3]$. Finally $[\alpha_4]\leftrightarrow[\alpha_4]$, $[\beta_4]\leftrightarrow[\gamma_4]$, and $[\delta_4]\leftrightarrow[\delta_4]$. Before SU_3 breaking, with the mass term [1], [2], [3], and [4], one finds that the multiplets $(\mathbf{8},\frac{1}{2})$ and $(\mathbf{1},\frac{3}{2})$ are degenerate:

$$m(\mathbf{8},\tfrac{1}{2})=m(\mathbf{1},\tfrac{3}{2}),$$

and the masses of the remaining multiplets $(\mathbf{1},\frac{5}{2})$, $(\mathbf{1},\frac{3}{2})$ and $(\mathbf{8},\frac{3}{2})$ satisfy

$$3m(\mathbf{1},\tfrac{5}{2})+2m(\mathbf{1},\tfrac{3}{2})=5m(\mathbf{8},\tfrac{3}{2}).$$

From the complete set of mass terms, including first order SU_3 breaking, one can derive the following expressions for the elements of the mass matrix in terms of six conveniently chosen parameters, m_1, m_3, m_5, a_3, b_1, and b_3:

$$\langle\mathbf{8},\tfrac{3}{2}|\mathfrak{M}|\mathbf{8},\tfrac{3}{2}\rangle=\tfrac{1}{5}(3m_5+2m_3)+a_3\gamma+b_3Y, \quad (30a)$$
$$\langle\mathbf{8},\tfrac{1}{2}|\mathfrak{M}|\mathbf{8},\tfrac{1}{2}\rangle=m_3-2a_3\gamma+b_1Y, \quad (30b)$$
$$\langle\mathbf{1},\tfrac{1}{2}|\mathfrak{M}|\mathbf{1},\tfrac{1}{2}\rangle=m_1, \quad (30c)$$
$$\langle\mathbf{1},\tfrac{3}{2}|\mathfrak{M}|\mathbf{1},\tfrac{3}{2}\rangle=m_3, \quad (30d)$$
$$\langle\mathbf{1},\tfrac{5}{2}|\mathfrak{M}|\mathbf{1},\tfrac{5}{2}\rangle=m_5, \quad (30e)$$
$$\langle\mathbf{8},\tfrac{3}{2}|\mathfrak{M}|\mathbf{1},\tfrac{3}{2}\rangle=(\sqrt{5})a_3, \quad (30f)$$
$$\langle\mathbf{8},\tfrac{1}{2}|\mathfrak{M}|\mathbf{1},\tfrac{1}{2}\rangle=-\sqrt{2}a_3. \quad (30g)$$

In Eqs. (30), Y is the hypercharge and $\gamma=[T(T+1)-\frac{1}{4}Y^2-1]$. The multiplet structure of our kinetic supermultiplet has been shown in Eq. (1). The $T=0$ component of $(\mathbf{8},\frac{1}{2})$ can mix with the singlet $(\mathbf{1},\frac{1}{2})$; similarly the $T=0$ component of $(\mathbf{8},\frac{3}{2})$ can mix with $(\mathbf{1},\frac{3}{2})$. We are thus actually dealing with a nonet (octet+singlet) of spin $\frac{1}{2}$ and negative parity, which we indicate as $\frac{1}{2}^-$, with a nonet $\frac{3}{2}^-$, and a singlet $\frac{5}{2}^-$. We call $\Lambda(\frac{1}{2}^-)$, $\Lambda'(\frac{1}{2}^-)$, $\Sigma(\frac{1}{2}^-)$, $N(\frac{1}{2}^-)$ and $\Xi(\frac{1}{2}^-)$ the physical particles of the $\frac{1}{2}^-$ nonet; similarly $\Lambda(\frac{3}{2}^-)$, $\Lambda'(\frac{3}{2}^-)$, $\Sigma(\frac{3}{2}^-)$, $N(\frac{3}{2}^-)$ and $\Xi(\frac{3}{2}^-)$ the physical particles of the $\frac{3}{2}^-$ nonet; and, finally, we indicate with $\Lambda(\frac{5}{2})$ the $\frac{5}{2}^-$ singlet. From the mass matrix parametrization, Eqs. (30), we obtain, by elimination of the parameters, three linear and two bilinear mass relations for the physical particle masses. We denote each mass by the symbol for the particle. The relations are

$$\tfrac{1}{2}[N(\tfrac{1}{2}^-)+\Xi(\tfrac{1}{2}^-)]+N(\tfrac{3}{2}^-)+\Xi(\tfrac{3}{2}^-)=\Sigma(\tfrac{1}{2}^-)+2\Sigma(\tfrac{3}{2}^-), \quad (31)$$

$$\Lambda(\tfrac{3}{2}^-)+\Lambda'(\tfrac{3}{2}^-)=\tfrac{1}{3}[\Sigma(\tfrac{1}{2}^-)+N(\tfrac{1}{2}^-)+\Xi(\tfrac{1}{2}^-)+2N(\tfrac{3}{2}^-)+2\Xi(\tfrac{3}{2}^-)-\Sigma(\tfrac{3}{2}^-)], \quad (32)$$

$$\tfrac{1}{3}[\Sigma(\tfrac{3}{2}^-)+N(\tfrac{3}{2}^-)+\Xi(\tfrac{3}{2}^-)]=\tfrac{1}{5}\{3\Lambda(\tfrac{5}{2}^-)+\tfrac{2}{3}[\Sigma(\tfrac{1}{2}^-)+N(\tfrac{1}{2}^-)+\Xi(\tfrac{1}{2}^-)]\}, \quad (33)$$

$$\begin{aligned}\Lambda(\tfrac{3}{2}^-)\Lambda'(\tfrac{3}{2}^-)=&\tfrac{1}{9}[\Sigma(\tfrac{1}{2}^-)+N(\tfrac{1}{2}^-)+\Xi(\tfrac{1}{2}^-)]\\&\times[2N(\tfrac{3}{2}^-)+2\Xi(\tfrac{3}{2}^-)-\Sigma(\tfrac{3}{2}^-)]\\&+(10/3)[\tfrac{1}{2}(N(\tfrac{3}{2}^-)+\Xi(\tfrac{3}{2}^-))-\Sigma(\tfrac{3}{2}^-)], \quad (34)\end{aligned}$$

$$\begin{aligned}\Lambda(\tfrac{1}{2}^-)\Lambda'(\tfrac{1}{2}^-)=&\tfrac{1}{9}[3\Lambda(\tfrac{1}{2}^-)+3\Lambda'(\tfrac{1}{2}^-)-\Sigma(\tfrac{1}{2}^-)-N(\tfrac{1}{2}^-)\\&-\Xi(\tfrac{1}{2}^-)+2N(\tfrac{3}{2}^-)+2\Xi(\tfrac{3}{2}^-)-4\Sigma(\tfrac{3}{2}^-)]\\&\times[2N(\tfrac{1}{2}^-)+2\Xi(\tfrac{1}{2}^-)-\Sigma(\tfrac{1}{2}^-)]\\&+\tfrac{4}{3}[\tfrac{1}{2}(N(\tfrac{3}{2}^-)+\Xi(\tfrac{3}{2}^-))-\Sigma(\tfrac{3}{2}^-)]^2. \quad (35)\end{aligned}$$

It may be worthwhile to examine more closely the above relations. First of all, they are all identically satisfied—as expected—if all masses of the supermultiplet are equal. Second, the bilinear relations, Eqs. (34) and (35), become identical to Eqs. (33) and (31), respectively, at first order in the deviation from the common central mass value. Starting from the mass formulas, Eqs. (31)–(35), and on the basis of some preliminary assignments, we shall exhibit a complete set of predictions for the masses of the baryons of the supermultiplet. In spite of the fact that our speculations should still be regarded as tentative, it appears that more experimental effort should be devoted to examine the possible realizations of the supermultiplet, especially in view of the fact that, as we have seen, it offers the most economical picture for higher baryon resonances of negative parity.

COMPARISON WITH THE EXPERIMENTAL DATA

We note that in Eqs. (31)–(35), $N(\frac{1}{2}^-)$ and $\Xi(\frac{1}{2}^-)$ only occur in the combination $N(\frac{1}{2}^-)+\Xi(\frac{1}{2}^-)$, and the same

TABLE I. A possible scheme of higher baryonic resonances. (Masses are in MeV.)

	$T=0,\ Y=0$	$T=1,\ Y=0$	$T=\frac{1}{2},\ Y=1$	$T=\frac{1}{2},\ Y=-1$
$\frac{1}{2}^-$	$\left\{\begin{array}{l} Y_0^*(1405)^{a} \\ Y_0^*(1660)^{b}\end{array}\right.$	predicted: $Y_1^*(1530\pm40)$	$N^*(1510)^{e}$	predicted: $\Xi^*(1520\pm60)$
$\frac{3}{2}^-$	$\left\{\begin{array}{l} Y_0^*(1519)^{c} \\ \text{predicted: } Y_0^*(1670\pm30)\end{array}\right.$	$Y_1^*(1660)^{d}$	$N^*(1518)$	$\Xi^*(1816)^{f}$
$\frac{5}{2}$	predicted: $Y_0^*(1760\pm25)$			

[a] Reference 12.
[b] Reference 13.
[c] Reference 11.
[d] Reference 10.
[e] Reference 14.
[f] Reference 9.

happens for $N(\frac{3}{2}^-)$ and $\Xi(\frac{3}{2}^-)$. The identification of $N(\frac{3}{2}^-)$ with $N^*(1518)$ is strongly suggested from the quantum assignment, $J^P=\frac{3}{2}^-$, to this resonance.[8] Unfortunately to apply the mass formulas one needs as an input the value of $N(\frac{3}{2}^-)+\Xi(\frac{3}{2}^-)$, and the identification of $\Xi(\frac{3}{2}^-)$ is less secure. It has been suggested that $\Xi^*(1816)$ has $J^P=\frac{3}{2}^-$.[9] We shall here tentatively identify $\Xi(\frac{3}{2}^-)$ with $\Xi^*(1816)$. It has been proposed that $Y_1^*(1660)$ also has $J^P=\frac{3}{2}^-$,[10] suggesting the identification of $\Sigma(\frac{3}{2}^-)$ with such a resonance. A further identification that is strongly consistent with the quantum-number assignments[11] is that of $\Lambda(\frac{3}{2}^-)$ with $Y_0^*(1519)$, which most likely has $J^P=\frac{3}{2}^-$. For the other isotopic singlet of the $\frac{3}{2}^-$ nonet we do not have any assignment to propose and we shall derive a prediction for its mass from our mass formulas. Turning now to the $\frac{1}{2}^-$ nonet we note that for $\Lambda'(\frac{1}{2}^-)$ the most likely candidate would be $Y_0^*(1405)$, consistently with its attributed quantum numbers.[12] For the other isosinglet member of the $\frac{1}{2}^-$ nonet we do not have a well-established candidate. Evidence however has been repeatedly reported for a $Y_0^*(1660)$, the so-called Λ-η^0 resonance, with the quantum numbers required to fit its identification with $\Lambda'(\frac{1}{2}^-)$.[13] We shall thus identify $\Lambda'(\frac{1}{2}^-)$ with the suggested $Y_0^*(1660)$. A last identification that we make here is that of $N(\frac{1}{2}^-)$ with recently discussed $N^*(1510)$ with $J^P=\frac{1}{2}^-$.[14] For the members $\Sigma(\frac{1}{2}^-)$ and $\Xi(\frac{1}{2}^-)$ of the $\frac{1}{2}^-$ nonet we do not have any identification to propose with any of the reported resonances. Similarly we do not have any established candidate for the SU_3 singlet member $\Lambda(\frac{5}{2}^-)$. We can however, predict the masses of $\Sigma(\frac{1}{2}^-)$, $\Xi(\frac{1}{2}^-)$, $\Lambda(\frac{5}{2}^-)$, together with that of the other missing particle $\Lambda'(\frac{3}{2}^-)$ from our mass formulas, Eqs. (31)-(35). From Eqs. (33) and (34) we obtain $\Lambda'(\frac{3}{2}^-)=1670\pm30$ MeV (the errors come from the reported experimental errors on the input masses), and, together with Eq. (31), we also obtain $\Sigma(\frac{1}{2}^-)=1530\pm40$ MeV and $\Xi(\frac{1}{2}^-)=1520\pm60$ MeV. Finally, from Eq. (32) we derive $\Lambda(\frac{5}{2}^-)=1760\pm25$ MeV. The scheme is now complete with all its predicted masses; however, we still have to verify the consistency of the remaining quadratic equation, Eq. (35), with the proposed masses. Inserting all the relevant mass values into Eq. (35), one finds the equality

$$(2.33\pm0.02)\ \mathrm{GeV}^2=(2.35\pm0.12)\ \mathrm{GeV}^2,$$

which is apparently well consistent. It may be worthwhile to note that the quantity $\Delta=\frac{1}{2}[N(\frac{3}{2}^-)+\Xi(\frac{3}{2}^-)]-\Sigma(\frac{3}{2}^-)$ which appears twice in the right-hand side of Eq. (35), is, with our assignments, $\Delta=7\pm17$ MeV. Neglecting Δ in Eq. (35), one obtains in its place the simpler equation [the asymmetry between $\Lambda(\frac{3}{2}^-)$ and $\Lambda'(\frac{3}{2})$ is only apparent]:

$$\Lambda(\tfrac{1}{2}^-)\Lambda'(\tfrac{1}{2}^-)\simeq[\Lambda(\tfrac{1}{2}^-)+\Lambda'(\tfrac{1}{2}^-)-\Lambda(\tfrac{3}{2}^-)]\Lambda(\tfrac{3}{2}^-).$$

This equation connects the isosinglet members of the nonets, which only involve input masses and is well verified in our proposed assignment. The complete scheme, summarizing both the adopted identifications and the predicted masses, is reported in Table I.

THE ALTERNATIVE SCHEME USING 364

The kinetic supermultiplet based on the **364** representation has the content reported in Eq. (2), namely, it consists of an octet+decimet with $J^P=\frac{1}{2}^-$, an octet+decimet with $J^P=\frac{3}{2}^-$, and a decimet with $J^P=\frac{5}{2}^-$. For the SU_3 reducible baryonic octet+decuplet we shall use the name "baryonic octodecimet." The reducible tensor describing our kinetic supermultiplet $\psi_{ABC,D}{}^{E}$ is now completely symmetric in the $\tilde{U}(12)$ indices A, B, C. The decomposition according to $U(3)\otimes\tilde{U}(4)$ can be written as follows: For $(\mathbf{8},\frac{1}{2})$ and $(\mathbf{8},\frac{3}{2})$ we have terms

$$[\psi^k{}_{[\alpha\beta]\gamma,\delta}{}^{\epsilon}\epsilon_{abr}(T_k)_c{}^r+\psi^k{}_{[\beta\gamma]\alpha,\delta}{}^{\epsilon}\epsilon_{bcr}(T_k)_a{}^r+\psi^k{}_{[\gamma\alpha]\beta,\delta}{}^{\epsilon}\epsilon_{car}(T_k)_b{}^r]\delta_d{}^e;$$

for $(\mathbf{10},\frac{1}{2})$, $(\mathbf{10},\frac{3}{2})$, and $(\mathbf{10},\frac{5}{2})$ we have

$$\psi^l{}_{\{\alpha\beta\gamma\},\delta}{}^{\epsilon}(d_l)_{abc}\delta_d{}^e,$$

where $\psi^k{}_{[\alpha\beta]\gamma,\delta}{}^{\epsilon}$ and $\psi^l{}_{\{\alpha\beta\gamma\},\delta}{}^{\epsilon}$ are the same tensors

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appearing in Eqs. (19) and (20). As usual $(T_k)_c{}^r$ are the unitary spin matrices and $(d_l)_{abc}$ $(l=1,\cdots,10)$ are the SU_3 decuplet wave functions.

The mass terms, calculated up to first-order breaking of SU_3, have the same forms of the terms [1], $\cdots$, [4], [α_1], $\cdots$, [δ_4] of the **220** kinetic supermultiplet considered before, and they also have the same charge-conjugation properties. By explicit calculation one obtains the following expressions for the elements of the mass matrix [the notation is the same as in Eqs. (30)]:

$$\langle \mathbf{10},\tfrac{5}{2}|\mathfrak{M}|\mathbf{10},\tfrac{5}{2}\rangle = m_5+A_5Y\,, \tag{36a}$$

$$\langle \mathbf{10},\tfrac{3}{2}|\mathfrak{M}|\mathbf{10},\tfrac{3}{2}\rangle = m_3+A_3Y\,, \tag{36b}$$

$$\langle \mathbf{10},\tfrac{1}{2}|\mathfrak{M}|\mathbf{10},\tfrac{1}{2}\rangle = m_1+A_1Y\,, \tag{36c}$$

$$\langle \mathbf{8},\tfrac{3}{2}|\mathfrak{M}|\mathbf{8},\tfrac{3}{2}\rangle = \tfrac{1}{5}(3m_5+2m_3)-3F_3Y+2D_3\gamma\,, \tag{36d}$$

$$\langle \mathbf{8},\tfrac{1}{2}|\mathfrak{M}|\mathbf{8},\tfrac{1}{2}\rangle = m_3-3F_1Y-4D_3\gamma\,, \tag{36e}$$

$$\langle \mathbf{8},\tfrac{3}{2}|\mathfrak{M}|\mathbf{10},\tfrac{3}{2}\rangle = 2\sqrt{(5)}D_3(\bar{\mathbf{\Sigma}}\cdot\mathbf{Y}^*-\bar{\Xi}\Xi^*)\,, \tag{36f}$$

$$\langle \mathbf{8},\tfrac{1}{2}|\mathfrak{M}|\mathbf{10},\tfrac{1}{2}\rangle = 2\sqrt{2}D_3(\bar{\mathbf{\Sigma}}\cdot\mathbf{Y}^*-\bar{\Xi}\Xi^*)\,, \tag{36g}$$

with the following relations among the parameters $F_1, \cdots, A_5$

$$3D_3-3F_1=A_3\,, \tag{37}$$

$$3F_3+D_3=-\tfrac{1}{5}(3A_5+2A_3)\,. \tag{38}$$

The off-diagonal matrix elements connect octet and decimets of equal spin and are nonzero only between states of the same hypercharge and isotopic spin, i.e., between states

$$(Y=0, I=1):\quad \Sigma_u(\text{octet})\,,\quad Y_u{}^*(\text{decuplet})$$

and

$$(Y=-1, I=\tfrac{1}{2}):\quad \Xi_u(\text{octet})\,,\quad \Xi_u{}^*(\text{decuplet})$$

where the index u stands for "unphysical."

Mixing effects will thus occur in each of the "baryonic octodecimets" between $\Sigma_u(J^P)$ and $Y_u{}^*(J^P)$ and between $\Xi_u(J^P)$ and $\Xi_u{}^*(J^P)$. In the following we call the physical particles of each baryonic octodecimet $\Lambda(J^P)$, $N(J^P)$, $\Sigma(J^P)$ and $\Sigma^*(J^P)$ (resulting from the Σ_u-Y_u mixing), $\Xi(J^P)$ and $\Xi^*(J^P)$ (resulting from the $\Xi_u-\Xi_u{}^*$ mixing), $N^*(J^P)$ and $\Omega(J^P)$. In the nonmixed $\frac{5}{2}^-$ decuplet the particles are $N^*(\frac{5}{2}^-)$, $\Sigma^*(\frac{5}{2}^-)$, $\Xi^*(\frac{5}{2}^-)$, and $\Omega(\frac{5}{2}^-)$. One sees that the mass matrix is given in terms of seven independent parameters, and, by eliminating them, one expects 13 independent sum rules among the 20 isotopic-spin multiplets of the kinetic supermultiplet. Four of the 13 relations are quadratic and are obtained through a diagonalization of the nondiagonal part of the mass operator. We do not report here these quadratic equations, but give instead the explicit form of the remaining nine linear relations:

$$N(\tfrac{1}{2}^-)=N^*(\tfrac{3}{2}^-)\,, \tag{39}$$

$$5N(\tfrac{3}{2}^-)=3N^*(\tfrac{5}{2}^-)+2N(\tfrac{1}{2}^-)\,, \tag{40}$$

$$N^*(\tfrac{5}{2}^-)-\Sigma^*(\tfrac{5}{2}^-)=\Sigma^*(\tfrac{5}{2}^-)-\Xi^*(\tfrac{5}{2}^-)\,, \tag{41}$$

$$N^*(\tfrac{5}{2}^-)-\Sigma^*(\tfrac{5}{2}^-)=\Xi^*(\tfrac{5}{2}^-)-\Omega(\tfrac{5}{2}^-)\,, \tag{42}$$

$$\Sigma(\tfrac{3}{2}^-)+\Sigma^*(\tfrac{3}{2}^-)=\Lambda(\tfrac{1}{2}^-)+\Lambda(\tfrac{3}{2}^-)\,, \tag{43}$$

$$2\Lambda(\tfrac{3}{2}^-)+\Lambda(\tfrac{1}{2}^-)=(8/5)[2\Sigma^*(\tfrac{5}{2}^-)+2N^*(\tfrac{3}{2}^-)+\Omega(\tfrac{3}{2}^-)]\,, \tag{44}$$

$$\Xi(\tfrac{3}{2}^-)+\Xi^*(\tfrac{3}{2}^-)=\tfrac{4}{3}\Lambda(\tfrac{1}{2}^-)+(11/3)\Lambda(\tfrac{3}{2}^-)-\Sigma^*(\tfrac{5}{2}^-)-N(\tfrac{1}{2}^-)-N(\tfrac{3}{2}^-)\,, \tag{45}$$

$$\Xi^*(\tfrac{1}{2}^-)+\Xi(\tfrac{1}{2}^-)=(14/9)\Lambda(\tfrac{1}{2}^-)+(10/9)\Lambda(\tfrac{3}{2}^-)+\tfrac{2}{3}\Omega(\tfrac{1}{2}^-)+\tfrac{1}{3}N^*(\tfrac{1}{2}^-)-\tfrac{2}{3}\Sigma^*(\tfrac{5}{2}^-)-N(\tfrac{1}{2}^-)\,, \tag{46}$$

$$\Sigma(\tfrac{1}{2}^-)+\Sigma^*(\tfrac{1}{2}^-)=\tfrac{1}{9}\Lambda(\tfrac{1}{2}^-)+(20/9)\Lambda(\tfrac{3}{2}^-)+\tfrac{1}{3}N(\tfrac{1}{2}^-)+\tfrac{2}{3}\Omega(\tfrac{1}{2}^-)-\tfrac{4}{3}\Sigma^*(\tfrac{5}{2}^-)\,. \tag{47}$$

We have again denoted the mass value by the particle symbol. Equations (41) and (42) express the equispacing law of the nonmixed $\frac{5}{2}^-$ decuplet. We note that all masses are fixed from the above linear relations once the following masses are known: $N(\frac{1}{2}^-)$, $\Lambda(\frac{1}{2}^-)$, $N(\frac{3}{2}^-)$, $\Lambda(\frac{3}{2}^-)$, $N^*(\frac{1}{2}^-)$, $\Omega(\frac{1}{2}^-)$, $\Sigma^*(\frac{5}{2}^-)$. As for a possible particle assignment we note that, in addition to the negative-parity baryonic resonances considered before, in discussing the assignments to the **220** kinetic supermultiplet, the following negative parity states can now be included among the candidates: $N^*(1688)$, a recently suggested $T=\frac{3}{2}$, $J^P=\frac{1}{2}^-$ resonant state (S_{31} resonance)[15]; $\Sigma^*(1765)$ with possible $J^P=\frac{5}{2}^-$ [16]; and the suggested $\Xi^*(1933)$ with possible $J=\frac{5}{2}$ and undetermined parity.[17] A possible scheme would then assume the following identifications: $\Lambda(\frac{1}{2}^-)=Y_0{}^*(1405)$, $N(\frac{1}{2}^-)=N^*(1510)$, $N^*(\frac{1}{2}^-)=N^*(1688)$, $\Lambda(\frac{3}{2}^-)=Y_0{}^*(1519)$, $N(\frac{3}{2}^-)=N^*(1518)$, $\Sigma(\frac{3}{2}^-)=Y_1{}^*(1660)$, $\Xi(\frac{3}{2}^-)=\Xi^*(1816)$, $\Sigma(\frac{5}{2}^-)=Y_1{}^*(1765)$, and $\Xi^*(\frac{3}{2}^-)=\Xi^*(1933)$. However, when one applies the mass equations one obtains unpleasant results. In fact a large number of unobserved low-mass states is predicted and it seems difficult to find reasons for their absence in the reported mass spectra. In particular one is led to predict $\Omega(\frac{3}{2}^-)$ and $\Xi^*(\frac{3}{2}^-)$ at masses below 1 GeV, which seems to us an unacceptable result. We recall, however, that the predictions follow from our adopted mechanism of mass breaking and we cannot exclude that by including additional mass-breaking terms the scheme based on **364** might become consistent. Alternatively one may consider the exclusion of some of the accepted resonances from the assignments, but we shall not dwell here on a discussion of such possible alternatives. We merely note that, in any case, of the two proposed resonances with $T=0$ $J^P=\frac{1}{2}^-$, $Y_0{}^*(1405)$ and $Y_0{}^*(1660)$, only one can be included in a scheme based on **364** (for the whole set of l-values), whereas they both fit in the scheme based on **220**.

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