approximations involved. We also note that the prediction $\mathrm{Sign}\beta_\Xi = \mathrm{Sign}\beta_\Lambda$ is quite consistent with the experimental values $\beta_\Lambda = 0.18 \pm 0.24$ and $\beta_\Xi = 0.13 \pm 0.17$.

VI. A BRIEF SUMMARY OF SU_3 RESULTS

It is perhaps convenient to recall briefly some of the comparable results of the well known theory constructed by Cabibbo on the basis of the SU_3 algebra.[21] This theory predicts the rule $\Delta Q = \Delta S$, [for example $x=0$, $R(\Sigma^+ \to n + l^+ + \nu) = R(\Xi^0 \to \Sigma^- + l^+ + \nu) = 0$] and the relation $R_2 = R_+$. The fact that in this theory $x=0$ implies that no observable violation of CP can be found in the time distribution of the $(K^0)_{e3}$ decays. In addition the Cabibbo theory leads to the following ratios for G_A/G_V in leptonic decays[22]:

$$(G_A/G_V)_{\Lambda^0 p} = -0.68,$$
$$(G_A/G_V)_{\Sigma^- n} = 0.305,$$
$$(G_A/G_V)_{\Xi^-\Lambda^0} = -0.19,$$
$$(G_A/G_V)_{\Xi^-\Sigma^0} = (G_A/G_V)_{\Xi^0\Sigma^+} = -1.18.$$

The Cabibbo theory also predicts the branching ratios

$$B(\Lambda \to p + e^- + \bar\nu) = 0.91 \times 10^{-3},$$
$$B(\Sigma^- \to n + e^- + \bar\nu) = 1.32 \times 10^{-3},$$
$$B(\Sigma^- \to \Lambda + e^- + \bar\nu) = 0.61 \times 10^{-4},$$
$$B(\Xi^- \to \Lambda + e^- + \bar\nu) = 0.65 \times 10^{-3}.$$

[21] N. Cabibbo, Phys. Rev. Letters **10**, 531 (1963).

[22] W. Willis *et al.*, Phys. Rev. Letters **13**, 291 (1964). We quote solution A(i).

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Electromagnetic Interactions and G_2†

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The electromagnetic predictions of the particle symmetry G_2 with symmetry breaking are given. Since none of these predictions are in conflict with present experimental results, it is concluded that arguments against G_2 based on its electromagnetic predictions without symmetry breaking are invalid.

Some of the arguments against G_2 as a particle symmetry have been that it predicts, in the symmetric limit, incorrect electromagnetic properties for the particles.[1] For example, G_2 contains CA (charge conjugation times A parity) as an inner automorphism. From CA, it follows, to all orders in the electromagnetic field and to zeroth order in the moderately strong symmetry-breaking interaction $H_{\mathrm{S.B.}}$, that

$$\begin{array}{llll} m_{\Xi^-} - m_{\Xi^0} = m_p - m_n, & \text{(a)} & \Gamma_{\Sigma^+} = -\Gamma_{\Sigma^-}, & \text{(e)} \\ m_{\Sigma^+} - m_{\Sigma^-} = 0, & \text{(b)} & \Gamma_p = -\Gamma_{\Xi^-}, & \text{(f)} \\ \Gamma_{\Sigma^0} = 0, & \text{(c)} & \Gamma_n = -\Gamma_{\Xi^0}, & \text{(g)} \\ \Gamma_\Lambda = 0, & \text{(d)} & & \end{array} \quad (1)$$

where Γ_p is the electromagnetic vertex operator for the proton. Moreover, there exist other inner automorphisms of G_2 which, in the same approximations, give the additional results

$$\begin{array}{llll} \Gamma_n = 0, & \text{(h)} & m_{\Sigma^+} - m_{\Sigma^0} = m_p - m_n. & \text{(j)} \\ \Gamma_p = \Gamma_{\Sigma^+}, & \text{(i)} & & \end{array} \quad (1')$$

By using these and other properties of G_2, one can also show, to the same approximation, that the following processes are forbidden

$$\begin{array}{llll} \omega^0 \to \pi^0 + \gamma, & \text{(k)} & \rho^0 \to \gamma, & \text{(n)} \\ \rho^0 \to \pi^0 + \gamma, & \text{(l)} & \Sigma^0 \to \Lambda^0 + \gamma. & \text{(o)} \\ \omega^0 \to \gamma, & \text{(m)} & & \end{array} \quad (1'')$$

It is immediately apparent that all of these predictions on which data exists are in violent disagreement with experiment. Clearly, *to this degree of approximation* in electromagnetic processes, G_2 is a very poor candiate for a particle symmetry.

Now, it would be convenient for physicists if the particle symmetry of nature were such that all its predictions in the symmetric limit were valid to a high degree of accuracy. Unfortunately, however, nature does not always arrange itself for our mathematical convenience. It thus seems entirely reasonable to examine G_2 and its electromagnetic predictions a little more closely before arriving at any conclusion as to its

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[1] See, for example, G. Feinberg and R. E. Behrends, Phys. Rev. **115**, 745 (1959); R. E. Behrends and A. Sirlin, *ibid.* **121**, 324 (1961). Y. Dothan and H. Harari, Nuovo Cimento **32**, 498 (1964). A. J. Macfarlane, N. Mukunda, and E. C. G. Sudarshan, Phys. Rev. **133**, B475 (1964); N. Mukunda, A. J. Macfarlane, and E. C. G. Sudarshan, *ibid.* **138**, B665 (1965); J. B. Bronzan and F. E. Low, Phys. Rev. Letters **12**, 522 (1964); S. Okubo and R. E. Marshak, Nuovo Cimento **28**, 56 (1963).

validity in particle physics. It is the purpose of this paper to establish the electromagnetic predictions of G_2 which hold when the symmetry-breaking effects are included. We denote by β the coupling strength of the symmetry-breaking interaction, $H_{\text{S.B.}}$, which transforms like the hypercharge generator of G_2.[2]

In an accompanying article,[3] we have seen that the *relative* forbiddeness of the processes $(1''k)$-$(1''n)$ is consistent with the present experimental situation; the results were $|g_{\omega\pi^0\gamma}|\sim\beta|g_{\rho^0\pi^0\gamma}|$, $|g_{\rho^0\gamma}|\sim\beta|g_{\omega^0\gamma}|$. Also from the electromagnetic-form-factor analysis, we concluded $|g_{\omega NN}|\sim\beta|g_{\rho NN}|$. (Note that $g_{\omega NN}$ and $g_{\rho NN}$ are not related by symmetry because the ρ and ω belong to different representations.) As one can easily establish, both $g_{\rho^0\pi^0\gamma}$ and $g_{\omega^0\gamma}$ are forbidden in first order (i.e., $g_{\rho^0\pi^0\gamma}\propto\alpha^{1/2}\beta$ and $g_{\omega^0\gamma}\propto\alpha^{1/2}\beta$ where α is the fine-structure constant), and hence can be related to other constants only through a dynamical model. Thus, by including symmetry-breaking effects, the processes $(1''k)$–$(1''n)$ do occur and there is no inconsistency with experiment.

Let us now turn our attention to the form-factor predictions. Up to first order in $H_{\text{S.B.}}$, and first order in the electromagnetic interaction, the form factors will transform like $Q+QH_{\text{S.B.}}$, where Q transforms like the electric charge. For the seven dimensional representation, to which the N, Σ, and Ξ are assigned, the quantity $QH_{\text{S.B.}}$ will transform as a linear combination of the $I_3=0$ and $Y=0$ matrices $D^1(0)$, $D^{27}(1)$, and $D^{27}(0)$ (the superscript is the dimensionality of the representation and the argument is the total isotopic spin). If one notes the further condition that $(E_5,(E_5,QH_{\text{S.B.}}))=0$, then one obtains, to first order in the symmetry-breaking interaction, the following relations[4]:

$$\begin{aligned}\Gamma_n&=\Gamma_{\Xi^0}=\Gamma_{\Sigma^0},\\ \Gamma_p-\Gamma_{\Xi^-}&=\Gamma_{\Sigma^+}-\Gamma_{\Sigma^-},\\ \Gamma_{\Sigma^+}+\Gamma_{\Sigma^-}&=2\Gamma_{\Sigma^0}.\end{aligned}\tag{2}$$

Thus, we see that already in first order all the objectionable form-factor predictions have been removed.

But we can do better. Proceeding in exactly the same manner, only now to second order in the symmetry-breaking interaction and first order in the electromagnetic interaction, we find the relations[5]

$$\Gamma_n+\Gamma_{\Xi^0}=\Gamma_{\Sigma^+}+\Gamma_{\Sigma^-}=2\Gamma_{\Sigma^0}.\tag{3}$$

It should be noted that these relations among form factors hold for all four-momentum transfers.

If one confines his attention to the low four-momentum transfer part of the form factors (say, to the region where the neutron and proton form factors are now experimentally known) one can say a little more. Recent form-factor analyses[6] of n and p seem to indicate that Γ_n and Γ_p are dominated by the ρ, ω, φ, and an $I=1$, $J^P=1^-$ resonance above the φ. (Whether it is the B or not is an open question; however, in G_2, the φ is assigned to a 14-dimensional representation and hence must have such a partner in the nearby mass spectrum.) Because of the higher symmetry, then, we expect that all the baryon form factors in the low four-momentum region will be dominated by this same set of resonances. In fact, if we assume that these four resonances (at four different masses) are coupled in a G_2 symmetric way to the baryons, then we may easily derive the relationships

$$\Gamma_n+\Gamma_{\Xi^0}=\Gamma_p+\Gamma_{\Xi^-}=\Gamma_{\Sigma^+}+\Gamma_{\Sigma^-}=2\Gamma_{\Sigma^0},\tag{4}$$

which should hold for low four-momentum transfers, e.g. static magnetic moments.

The form-factor analyses seem to suggest that the contributions from these four resonances are roughly comparable.[6] Since $|g_{\rho^0\gamma}|\sim\beta|g_{\omega\gamma}|$ we were led to $|g_{\omega NN}|\sim\beta|g_{\rho NN}|$. Similarly, since $\varphi\to\gamma$ and $B^0\to\gamma$ are allowed while $\omega\to\gamma$ is first-forbidden, we might guess that the full relation $|g_{\rho^0\gamma}|\sim\beta|g_{\omega^0\gamma}|\sim\beta^2|g_{\varphi\gamma}|\sim\beta^2|g_{B\gamma}|$ roughly holds. We then would be led to the relation $|g_{\varphi NN}|\sim|g_{BNN}|\sim\beta|g_{\omega NN}|\sim\beta^2|g_{\rho NN}|$ in order to obtain comparable contributions to the nucleon form factors.[7]

Since the Λ is assigned to a one-dimensional representation of G_2, its form factor cannot be related to the other baryons. However, it would not be unreasonable to expect its form factor to be dominated in the low four-momentum transfer region by the ω and the φ. If such is the case, and if, through some accident, $|g_{\omega\Lambda\Lambda}|\sim|g_{\omega NN}|$ and $|g_{\varphi\Lambda\Lambda}|\sim|g_{\varphi NN}|$, then the form factor for the Λ would be the same order of magnitude as the form factors for the other baryons, which seems to be the case experimentally. One might also note that just from the symmetry properties, the Λ form factor is nonzero in the same order of symmetry breaking as the neutron form factor is nonzero. It, therefore, seems to us that although no relation can be given between the Λ and other baryon form factors, the experimental results on the Λ do not contradict the results expected from G_2 with symmetry breaking.

With regard to the transition form factor $\Gamma_{\Sigma\Lambda}$ for the process $\Sigma^0\to\Lambda^0+\gamma$, we note that this becomes non-

[2] R. E. Behrends and L. F. Landovitz, Phys. Rev. Letters **11**, 296 (1963).

[3] R. E. Behrends, L. F. Landovitz, and B. Tunkelang, Phys. Rev. **142**, 1092 (1966) (first paper of this series).

[4] N. Mukunda, A. J. Macfarlane, and E. C. G. Sudarshan, Phys. Rev. **138**, B665 (1965). By considering representation mixing of the states, these authors obtain the relations $\Gamma_n=\Gamma_{\Xi^0}$, $\Gamma_p-\Gamma_{\Sigma^+}=\Gamma_{\Xi^-}-\Gamma_{\Sigma^-}=3(\Gamma_n-\Gamma_{\Sigma^0})$.

[5] In fact, by using $(E_5,\ (E_5,Q^{2n+1}H_{\text{S.B.}}))=0$ where n is any integer and by noting that $CA\,(Q^{2n+1}+Q^{2n+1}H_{\text{S.B.}}{}^2)\,(CA)^{-1}=-(Q^{2n+1}+Q^{2n+1}H_{\text{S.B.}}{}^2)$, one can show that $\Gamma_n+\Gamma_{\Xi^0}=2\Gamma_{\Sigma^0}$ holds to second order in the symmetry-breaking interaction and to all orders in the electromagnetic interaction. The relation $\Gamma_{\Sigma^+}+\Gamma_{\Sigma^-}=2\Gamma_{\Sigma^0}$, on the other hand, is the well-known isotopic-spin relation which holds to all orders in the symmetry-breaking interaction but to only first order in the electromagnetic interaction.

[6] E. B. Hughes *et al.*, Phys. Rev. **139**, B458 (1965).

[7] It appears that such a relation is not completely absurd since there seems to be some evidence for $|g_{\varphi NN}|\ll|g_{\omega NN}|$ when one compares $K^-p\to\Lambda\omega$ and $K^-p\to\Lambda\varphi$. See P. Schlein, lecture at Summer Institute for Theoretical Physics, University of Colorado, 1965 (unpublished).

zero to order $O(\beta^2)$. In G_2, however, there is no way to relate this transition moment to any of the baryon form factors. Thus, although we know that the process is inhibited to $O(\beta^2)$, there is no way of testing this prediction short of using a dynamical model.

We are now left with the question of the electromagnetic mass differences of the baryons. In a manner similar to that which we used on the form factors, we note that for the seven-dimensional representation, the first-order contribution to the electromagnetic mass differences will transform as a linear combination of the $I_3=0$, $Y=0$ matrices $D^7(1)$, $D^{14}(1)$, and $D^{14}(0)$. It is then easy to determine the relations which hold to first order in symmetry breaking and all orders in the electromagnetic coupling:

$$\delta m_p+\delta m_{\Xi^-}=\delta m_{\Sigma^+}+\delta m_{\Sigma^-},$$
$$\delta m_n+\delta m_{\Xi^0}=2\delta m_{\Sigma^0}.$$

Written in terms of electromagnetic mass differences, these become the one relation

$$m_n-m_p-(m_{\Xi^-}-m_{\Xi^0})=2m_{\Sigma^0}-m_{\Sigma^+}-m_{\Sigma^-}. \qquad (5)$$

Although this relation is still badly violated experimentally, we can see that, in fact, it is a world of improvement over the relations (1a) and (1b).

If one assumes that D^7 is dynamically suppressed (we shall discuss such a model in a moment), then one obtains an additional relation

$$m_n-m_p+m_{\Xi^-}-m_{\Xi^0}=m_{\Sigma^-}-m_{\Sigma^+} \qquad (6)$$

which, with the present numerical values, holds within experimental error.

By proceeding to second order in the symmetry breaking, one finds that there are no restrictions on the electromagnetic mass differences of the baryons. If, however, one again imposes the dynamical assumption that $D^7(1)$ is suppressed, one finds that the relation (6) holds to second order in the symmetry-breaking interaction.

Let us now turn to the question of a dynamical suppression of $D^7(1)$. In most of the popular models for calculating the electromagnetic mass differences, the dominant contributions are of order α and involve only the low four-momentum part of two-baryon electromagnetic vertex operators (for an emission and absorption of a photon). We have already discussed the G_2 form factors when they are dominated by the ρ, ω, φ, and B, i.e., the low four-momentum part of the form factors, and have listed certain relationships among the various couplings. If one uses these results in a model in which the form factors appear twice, one finds that the dominant contribution (up to second order in $H_{\text{S.B.}}$) to the electromagnetic mass differences will appear in the representations contained in $14\otimes 14$, $1\otimes 14$, $1\otimes 1$, and in the $N=1$ and 27 representations contained in $7\otimes 14$. Since $N=7$ is not contained in any of these products, it then follows that $D^7(1)$ is dynamically suppressed in such a model [the suppression is of order $O(\beta)$]. Thus, we see that on the basis of a rather general dynamical model, relation (6) is expected to hold in G_2 with symmetry breaking.

Let us recapitulate. The electromagnetic predictions of G_2 with symmetry breaking which we expect to hold with some degree of accuracy are Eq. (3) for all four-momentum transfers, Eq. (4) for low four-momentum transfers and Eq. (6). The objectionable electromagnetic predictions of Eq. (1) which follow from G_2 without symmetry breaking are all changed, by including the symmetry-breaking interaction, into relations which either do not yet contradict experiment or in fact agree with experiment. We thus conclude that the arguments presented, so far, against G_2 as a particle symmetry, based on its electromagnetic predictions, are not valid.

For purposes of comparison, one should note that the symmetry SU_3 predicts Eq. (6) for the electromagnetic mass differences and the relations $\Gamma_{\Sigma^+}=\Gamma_p, \Gamma_{\Xi^0}=\Gamma_n=2\Gamma_\Lambda =(-2/\sqrt{3})\Gamma_T=-2\Gamma_{\Sigma^0}$, $\Gamma_{\Xi^-}=\Gamma_{\Sigma^-}=-(\Gamma_n+\Gamma_p)$ in the limit of no symmetry breaking.[8] To first order in the symmetry-breaking interaction, there is no relationship among the observed masses of the baryons while, for the electromagnetic vertex operators, two relations survive[9]: the usual isotopic-spin result $\Gamma_{\Sigma^-}+\Gamma_{\Sigma^+}=2\Gamma_{\Sigma^0}$ and $(-2\sqrt{3})\Gamma_T=2\Gamma_{\Xi^0}+2\Gamma_n-\Gamma_{\Sigma^0}-3\Gamma_\Lambda$.

An interesting comparison of the SU_3 and G_2 predictions are given by the static magnetic moments of the Ξ^- and the Σ^-. In SU_3 in the symmetry limit, these are $\mu_{\Sigma^-}=\mu_{\Xi^-}=-0.88(e/2m)$. For G_2 with first-order symmetry breaking, we have from Eq. (2) $\mu_{\Xi^-}=\mu_p+\mu_{\Sigma^-}-\mu_{\Sigma^+}=\mu_p+2\mu_{\Sigma^0}-2\mu_{\Sigma^+}=\mu_p+2\mu_n-2\mu_{\Sigma^+}$ and $\mu_{\Sigma^-}=2\mu_{\Sigma^0}-\mu_{\Sigma^+}=2\mu_n-\mu_{\Sigma^+}$ which gives $\mu_{\Xi^-}\sim(-9.6\pm3.0)(e/2m)$ and $\mu_{\Sigma^-}\sim(-8.1\pm1.5)(e/2m)$ for the present experimental value $\mu_{\Sigma^+}=(4.3\pm1.5)(e/2m)$. Although these relations may not hold to better than 20% for both SU_3 and G_2, the predictions for μ_{Σ^-} and μ_{Ξ^-} are sufficiently different that they can easily be distinguished experimentally.

[8] S. Coleman and S. L. Glashow, Phys. Rev. Letters **6**, 423 (1961).
[9] S. Okubo, Phys. Letters **4**, 14 (1963).