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Weak Interactions and G_2†

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A speculative model of the weak interactions constructed in the framework of the G_2 symmetry scheme is discussed. The model leads to the existence of $\Delta Q=-\Delta S$ matrix elements in the vector interaction and their absence in the axial-vector contribution. In particular, it leads in a natural manner to the existence of $\Sigma^+\to n+e^++\nu$ at a rate compatible with the present upper limit, to the existence of $\Delta Q=-\Delta S$ $(K^0)_{e3}$ decays, and to a strong inhibition of the reaction $K^+\to\pi^++\pi^++e^-+\bar{\nu}$. It also gives rise to rigorous predictions involving the $(K^+)_{e3}$ and $(K^0)_{e3}$ decays. The scheme allows for the possible violation of CP invariance in the $(K^0)_{e3}$ and is qualitatively compatible with Sach's explanation of the order of magnitude of the CP violation in $K_2^0\to\pi^++\pi^-$. Several qualitative and quantitative predictions, some of which hold to all orders in the symmetry-breaking interactions, are discussed and compared, whenever possible, with experimental information.

I. INTRODUCTION

IN this note we discuss a speculative model for the weak interactions constructed in the framework of the G_2 symmetry scheme.[1] The interest of the model is closely connected with the possible existence of $\Delta Q=-\Delta S$ processes. In fact, the model leads in a natural manner to the existence of $\Delta Q=-\Delta S$ matrix elements in the vector interaction and their absence in the axial vector contribution. In particular, it leads to the existence of the process $\Sigma^+\to n+e^++\nu$ at a rate compatible with the present experimental upper limit and to a strong inhibition of the reaction $K^+\to\pi^++\pi^++e^-+\bar{\nu}$.

The model also allows for the possible violation of CP invariance in the $(K^0)_{e3}$ decays without abandoning the close connection between vector currents and the generators of the infinitesimal group or the CP invariance of the strong interactions. Moreover, it is qualitatively compatible with Sachs's explanation of the order of magnitude of the CP violation in the $K_2^0\to\pi^++\pi^-$ process.[2]

Specifically, we will consider the following assumptions:

(a) The space integrals of the fourth components of the vector currents are linear combinations of the generators of the G_2 algebra.[3] Therefore, the vector currents belong to the regular or 14-dimensional representation of this group.

(b) The axial-vector currents are linear combinations of currents which transform as components of seven dimensional representations.

We will also incorporate some assumptions usually made in the theories of weak interactions:

(c) The $\Delta S=0$ leptonic Lagrangian obeys the $|\Delta I|=1$ rule.

(d) The Lagrangian describing the nonleptonic decays is a bilinear expression of the $\Delta S=0$ and $\Delta S=1$ currents. One may further construct the nonleptonic Lagrangian in such a manner that it satisfies the $|\Delta S|\leq 1$ and the $|\Delta I|=\frac{1}{2}$ rule for nonleptonic decays but we will not emphasize this point here.[4]

(e) The strong and medium-strong interactions are invariant under isospin rotations and under CP.

Assumption (a) is a natural generalization of the "conserved vector hypothesis" in the framework of G_2 and has been discussed in the past.[1] Assumption (b) is motivated essentially by simplicity: the seven-dimensional representations are the multicomponent representations of lowest dimensionality (the 7 and 14 are the two fundamental representations of G_2). As it is pointed out later, by taking bilinear combinations of the eight baryonic fields N, Ξ, Σ, and Λ one can construct in G_2 two different septets of currents satisfying (b) and (c). For this reason, it is natural to assume in the present model that the axial-vector currents are linear combinations of currents transforming as components of two different septets (Sec. IV).

The assumptions described above lead to a number of qualitative and quantitative predictions, some of which are rather striking. The aim of this note is to point out these predictions and, whenever possible, compare them with the available experimental information.

Some predictions are obtained from general properties such as the isotopic spin content of various representations and isospin transformation properties. Such predictions hold to all orders in the medium-strong

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[1] R. E. Behrends and A. Sirlin, Phys. Rev. **121**, 324 (1961); Phys. Rev. Letters **5**, 476 (1960); *ibid.* **8**, 221 (1962).

[2] R. G. Sachs, Phys. Rev. Letters **13**, 286 (1964).

[3] See e.g. R. E. Behrends, J. Dreitlein, C. Fronsdal, and W. Lee, Rev. Mod. Phys. **34**, 1 (1962).

[4] Phenomenological models for the weak interactions satisfying these rules and allowing for $\Delta Q=-\Delta S$ reactions have been discussed, for example, by R. E. Behrends and A. Sirlin, Phys. Rev. **121**, 324 (1961); *Lecture Notes on Weak Interactions*, edited by C. Fronsdal (W. A. Benjamin and Company, Inc., New York), p. 110; and International Conference on Fundamental Aspects of Weak Interactions, Brookhaven National Laboratory Report No. BNL 837(C-39), p. 266, 1963 (unpublished). The last paper contains references to later works on this subject.

symmetry-breaking interactions and are, therefore, quite reliable consequences of our assumptions. Other predictions involve the evaluation of vector matrix elements in the symmetric limit. In this case one expects second-order symmetry-breaking effects.[5] Finally other consequences depend on the evaluation of axial-vector matrix elements in which one expects first-order symmetry-breaking effects. Because of the different nature of the approximations involved, we consider in Sec. II some general qualitative properties not affected by symmetry-breaking effects while in Secs. III and IV we point out quantitative predictions obtained from the detailed consideration of vector and axial-vector matrix elements, respectively. Finally in Sec. V we discuss predictions involving the decay amplitudes of $\Lambda^0 \to N+\pi$ and $\Xi \to \Lambda+\pi$.

II. PREDICTIONS HOLDING TO ALL ORDERS IN THE SYMMETRY-BREAKING INTERACTION

(1) Assumptions (a) and (b) lead immediately to the possible existence of $\Delta Q=-\Delta S$ matrix elements in the vector interaction and their absence in the axial-vector contribution. This follows from the isotopic content of the 14- and 7-dimensional representations. The 14-dimensional representation contains isotopic spin-$\frac{3}{2}$ currents while these are absent in the 7-dimensional representation. Therefore, the model leads in a natural manner to the existence of the processes $\Sigma^+ \to n+l^+ +\nu$, $K^0 \to \pi^+ + l^- +\bar{\nu}^-$ and to the fact that the $\Delta Q=-\Delta S$ process $K^+ \to \pi^+ +\pi^+ +e^- +\bar{\nu}$ should be greatly inhibited with respect to $K^+ \to \pi^+ +\pi^- +e^+ +\nu$. A detailed discussion of the first three processes is given in Sec. III. With regard to the $(K^+)_{e4}$ decays, we note that in the present model $K^+ \to \pi^+ +\pi^+ +e^- +\bar{\nu}$ can only occur via the $\Delta Q=-\Delta S$ vector current and that this contribution is inhibited by several orders of magnitude because of centrifugal barriers.[6] Experimentally,

$$R(K^+ \to \pi^+ +\pi^+ +e^- +\bar{\nu})/R(K^+ \to \pi^+ +\pi^- +e^+ +\nu) \lesssim 1/40.$$

(2) The $\Delta S=1$ vector currents are components of an $I=\frac{3}{2}$ multiplet and the $I=\frac{1}{2}$ vector currents are absent. This statement follows again from the isotopic spin content of the 14-dimensional representation. An immediate consequence is that such processes as $\Sigma^- \to \Lambda^0 + l^- +\nu$, $\Lambda^0 \to p+l^- +\bar{\nu}$ and $\Xi^- \to \Lambda^0 + l^- +\bar{\nu}$ should occur only via the axial-vector interaction. Experimentally, the ratio G_A/G_V has been studied in the $\Lambda^0 \to p +e^- +\bar{\nu}$ decay, although the results do not appear to be conclusive: Baglin *et al.*[7] rule out pure V but do not decide between pure A and $|G_V|=|G_A|$; Lind *et al.* and Barlow *et al.*[8] find that $G_A/G_V \approx -1$ is considerably more likely than pure A, and Ely *et al.*[9] favor pure A although they do not rule out $|G_V|=|G_A|$.

Block[10] has suggested an experiment to determine $(G_V)_{\Lambda p}$ from the zero-to-zero transition ${}_{\Lambda}\mathrm{H}^4 \to H_e{}^4 +e^- +\bar{\nu}$. We note that this process is forbidden in the present model as an allowed nuclear transition and hence such an experiment would provide an excellent test of the predictions $(G_V)_{\Lambda p}=0$.

The absence of $I=\frac{1}{2}$ vector currents leads also to a rigorous prediction in the K_{e3} decays which we discuss in Sec. III.

Let us further point out that if the existence of both $I=\frac{1}{2}$ and $I=\frac{3}{2}$ vector currents is established and the connection between the vector currents and the group generators is not abandoned, then it is necessary to consider groups of rank higher than two (see, e.g., the third paper in Ref. 1).

(3) The model allows for a possible CP violation in the $(K^0)_{l3}$ decays and is qualitatively compatible with Sachs' explanation of the smallness of the CP violation in $K_2{}^0 \to \pi^+ +\pi^-$ decay.[2] Assumptions (a) and (e) imply that the vector currents have a definite behavior under CP. However, in the present model there are two independent partially conserved vector currents which contribute to $K_2{}^0 \to \pi^- + l^+ +\nu$, namely the currents with $I=\frac{3}{2}$, $I_3=-\frac{1}{2}$ and $I=\frac{3}{2}$, $I_3=-\frac{3}{2}$. As the parameters which characterize the coupling of these two currents to the lepton current can be relatively complex, we see that the model allows for a possible CP violation in $K_2{}^0 \to \pi^\mp + l^\pm +\nu$ even if we assume the validity of (a) and (e). On the other hand, it is easy to see that the scheme is still compatible with a CP-invariant nonleptonic Lagrangian satisfying (d). Thus, in this model we may envisage a situation in which the $K_0 \to \pi^+ +\pi^-$ and $K_2{}^0 \to \pi^\pm + l^\mp +\nu$ amplitudes are CP-invariant and CP-noninvariant, respectively. As Sachs has pointed out, such a situation can lead to a qualitative understanding of the smallness of the CP violation in the $K_2{}^0 \to \pi^+ +\pi^-$ decay. Some experimental evidence for the existence of a $\Delta Q=-\Delta S$ amplitude and CP violation in the $(K^0)_{e3}$ decays has been recently reported by Baldo-Ceolin *et al.*[11] and by Aubert *et al.*[12] We emphasize the fact that the model is compatible with a possible CP violation in $(K_2{}^0)_{l3}$ decays and, therefore, in $K_2{}^0 \to \pi^+ +\pi^-$, without abandoning the close connection between vector currents and the generators of the infinitesimal group and the CP invariance of the strong interactions [assumptions (a) and (e)].

[5] For the proof of such theorems, see, e.g., S. Fubini and G. Furlan, Physics 1, 229 (1965).
[6] A. Sirlin, Phys. Rev. 129, 1377 (1963).
[7] Baglin *et al.*, Nuovo Cimento 35, 977 (1965). These authors quote $|G_V/G_A|=0.84_{0.8}{}^{+0.9}$.
[8] V. G. Lind *et al.*, Phys. Rev. 135, B1483 (1964) and J. Barlow *et al.*, Phys. Letters 18, 64 (1965).
[9] R. P. Ely *et al.*, Phys. Rev. 137, B1302 (1965).
[10] M. Block, Summer Institute of Theoretical Physics, University of Colorado, 1965 (unpublished).
[11] M. Baldo-Ceolin *et al.*, Nuovo Cimento 38, 684 (1965).
[12] B. Aubert *et al.*, Phys. Letters 17, 59 (1965).

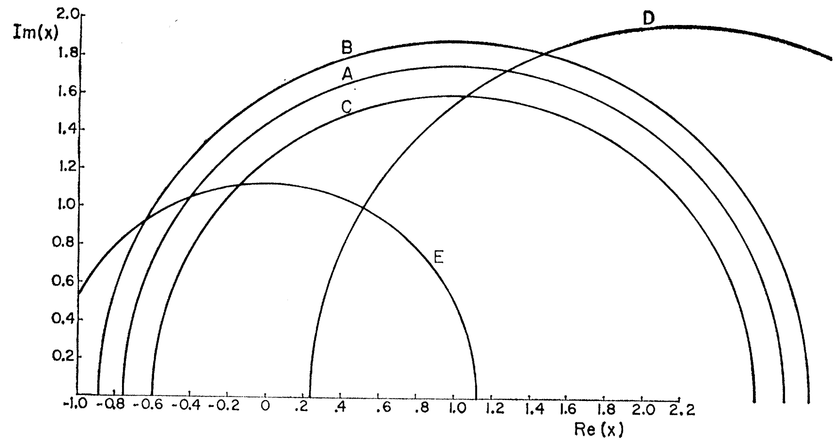

FIG. 1. Curves A, B, and C represent solutions of Eq. (2) corresponding to the values $R_2/R_+ = 0.765$, 0.888, and 0.642, respectively. The interior of the semicircle D is excluded, within the experimental error, by the experiment of Kirsch *et al.* (Ref. 15). Finally, the interior of the semicircle E is the region compatible with $B\,(\Sigma^+ \to n+e^+ +\nu) \leq 10^{-4}$ according to Eq. (4a).

(4) On the other hand, the amplitude for $K^+ \to \pi^0 + l^+ + \nu$ must be CP-invariant so that, for example, the μ^+ should exhibit no longitudinal polarization along a direction perpendicular to the plane defined by $\mathbf{p}_{\pi^0}$ and $\mathbf{p}_{\mu^+}$. The reason for this is that only one current, namely the vector current with $I=\frac{3}{2}$, $I_3=-\frac{1}{2}$ can contribute to this process. There exists some experimental evidence supporting this prediction.[13]

We also note that the processes $\Sigma^+ \to n+l^+ +\nu$, $\Xi^0 \to \Sigma^- + l^+ + \nu$, must be CP invariant. In addition the model is compatible with CP invariance in $\Sigma^- \to n+l^- +\bar{\nu}$, $\Xi^- \to \Sigma^0 + l^- +\bar{\nu}$, $\Xi^0 \to \Sigma^+ + l^- + \bar{\nu}$, $K^+ \to \pi^\pm + \pi^\mp + l^\mp + \nu$, $\Lambda \to p + l^- + \bar{\nu}$, $\Xi^- \to \Lambda + l^- + \bar{\nu}$ and the nonleptonic decays.

III. VECTOR AMPLITUDES

Calling R_1, R_2 and R_+ the rates for $K_1^0 \to \pi^- + e^+ + \nu$, $K_2^0 \to \pi^- + l^+ + \nu$ and $K^+ \to \pi^0 + l^+ + \nu$, respectively, one readily obtains the relations[1]

$$R_1/R_2 = |(1+x)/(1-x)|^2, \tag{1}$$

$$R_2/R_+ = |1-x|^2/4, \tag{2}$$

where

$$x \equiv M(\bar{K}^0 \to \pi^- + l^+ + \nu)/M(K^0 \to \pi^- + l^+ + \nu)$$

stands for the ratio of the $\Delta Q = -\Delta S$ and $\Delta Q = \Delta S$ amplitudes in the $(K^0)_{e3}$ decays. In Eq. (1) we have used the fact that K_1^0 and K_2^0 are very approximately eigenstates of CP. Equation (2) follows from the absence of the $I=\frac{1}{2}$ vector currents implied by the isospin content of the 14-dimensional representation. It is a rigorous prediction of this model and is not affected by the medium-strong symmetry-breaking interaction.

The structure of the 14-dimensional currents and the absence of the $\Delta Q = -\Delta S$ axial-vector currents lead also to the relations

$$R(\Sigma^+ \to n + l^+ + \nu) = 0.264|x|^2 R_+, \tag{3a}$$

$$R(\Xi^0 \to \Sigma^- + l^+ + \nu) = 0.727 \times 10^{-2} |x|^2 R_+, \tag{3b}$$

where the R's stand for the corresponding rates. Equations (3a) and (3b) follow in the present model in the symmetric limit. In this case the effects of the symmetry-breaking interactions are of second order[5] and one expects these predictions to be fairly reliable. A more detailed discussion of the expected errors on the vector and axial-vector matrix elements is given in Sec. IV.

Inserting the experimental values for R_+ and the Σ^+ and Ξ^0 lifetimes,[14] Eqs. (3a) and (3b) lead to the following expressions for the predicted branching ratios:

$$B(\Sigma^+ \to n + l^+ + \nu) = 0.81|x|^2 \times 10^{-4}, \tag{4a}$$

$$B(\Xi^0 \to \Sigma^- + l^+ \nu) = 0.87|x|^2 \times 10^{-5}. \tag{4b}$$

In general the parameter x is complex. In the complex plane the possible solutions of Eq. (2) define a circle of radius $2(R_2/R_+)^{1/2}$ and center on the positive real axis at a distance 1 from the origin. In Fig. 1 we have represented these solutions (curves A, B, and C) using the experimental value $R_2/R_+ = 0.765 \pm 0.123$ (for simplicity we have drawn only the upper half semicircles; curves A, B, and C correspond to the values $R_2/R_+ = 0.765$, and 0.888 and 0.642, respectively). Some information on the value of x is given by the experiment of Kirsch *et al.*[15] who obtain $|1+x|^2/|1-x|^2 = 0.85_{-0.85}^{+1.8}$. Within the quoted errors this result rules out the region of the complex x plane inside a circle of radius ~ 1.97 and center at $\mathrm{Re}(x) = 2.21$, $\mathrm{Im}(x) = 0$. In Fig. 1 we have represented a part of this circle (curve D). It is clear from the figure that Eq.

[13] U. Camerini *et al.*, Phys. Rev. Letters **14**, 989 (1965).

[14] Unless otherwise noted, we use the experimental data as compiled in A. H. Rosenfeld *et al.*, Rev. Mod. Phys. **36**, 977 (1964).

[15] L. Kirsch *et al.*, Phys. Rev. Letters **13**, 35 (1964).

(2) and the results of Kirsch *et al.* are compatible within the quoted errors.

More restrictive values for x have been recently given as a result of twin experiments by Baldo-Ceolin *et al.*[11] and by Aubert *et al.*[12] Writing $x=|x|e^{i\theta}$, Baldo-Ceolin and collaborators find $|x|=0.44_{-0.24}^{+0.19}$, $\theta=\pm(82°_{-26°}^{+68°})$ while Aubert *et al.* quote $|x|=0.22_{-0.11}^{+0.16}$, $\theta=79°_{-27°}^{+37°}$. We note that the solution obtained on the basis of Eq. (2) is nearly compatible within the quoted errors with the first result but it is in disagreement with the second. Considering the difficulties involved in the $(K^0)_{e3}$ experiments, we feel that more experimental work is needed before the parameter x can be regarded as reliably and precisely determined.

Regarding the $\Sigma^+\to n+l^++\nu$ processes, only two possible candidates have been reported, one $\Sigma^+\to n+\mu^++\nu$ and one $\Sigma^+\to n+e^++\nu$.[16] An upper limit of 10^{-4} has been given for the branching ratio of $\Sigma^+\to n+e^++\nu$.[14] As the solution of Eq. (2) is compatible with a value of $|x|<1$, we see from Eq. (4a) that the present model is compatible with such an upper limit. The smallest value for $\Sigma^+\to n+e^++\nu$ is obtained in this model for one of the real solutions of Eq. (2), namely $x=-0.75\pm0.14$ which gives

$$B(\Sigma^+\to n+e^++\nu)=(0.46\pm0.17)\times10^{-4}, \quad (5a)$$

$$B(\Xi^0\to\Sigma^-+e^++\nu)=0.49\times10^{-5}. \quad (5b)$$

For complex values of x, $|x|$ is larger than in the case of the real solution and we expect somewhat larger values for the branching ratios. It is clear that in order to test the predictions of this model on the $\Delta Q=-\Delta S$ processes the present upper limit on $\Sigma^+\to n+e^++\nu$ should be significantly lowered.

We further note that in the present model the processes $\Sigma^+\to n+l^++\nu$ and $\Xi^0\to\Sigma^-+l^++\nu$ should only occur via the vector interaction. If these processes are found to exist, this prediction could be checked by studying the spectra, angular correlations and asymmetries.

Complex values of x imply CP noninvariance in the $(K^0)_{e3}$ decays. In particular, in his argument to explain the smallness of the CP violation in $K_2{}^0\to\pi^++\pi^-$ decay, Sachs[2] has proposed the possibility that x is almost purely imaginary and of order 1. The present model, on the other hand, favors a value of x with $|x|<1$ and a negative real part. Nonetheless, considering the uncertainties in the connection between the self-energy matrix, which involve divergent integrals, and the leptonic rates we believe that the present model is qualitatively consistent with Sachs' argument.

A further prediction can be obtained on the basis of the G_2 vector currents by computing the vector contribution to $\Sigma^-\to n+e^-+\bar\nu$ in the symmetric limit and then determining the corresponding axial-vector coupling from the experimental rate. In this rather indirect manner we obtain for this decay $|G_A/G_V|_{\Sigma^-n}=1.48\pm0.18$. A similar method could be used in the case of $\Xi^-\to\Sigma^0+e^-+\bar\nu$ and $\Xi^0\to\Sigma^++e^-+\bar\nu$ when these rates become experimentally known. These indirect determinations are, of course, not completely satisfactory from the theoretical point of view but they have the advantage of being fairly reliable because the vector matrix elements are not affected strongly by symmetry-breaking effects. In the same vein we note that the vector contributions provide reliable "lower limit" predictions for the rates of $\Xi^-\to\Sigma^0+e^-+\bar\nu$ and $\Xi^0\to\Sigma^++e^-+\bar\nu$. On the basis of the G_2 vector currents we obtain that the branching ratios for these decays should be larger than 1.5×10^{-5} and 1.2×10^{-5}, respectively.

We note also that since the K_{e3} decays proceed through the vector currents, the predictions

$$\frac{R(K^+\to\pi^0+e^++\nu)}{R(K^+\to\pi^0+\mu^++\nu)}=1.55\,,\quad \frac{R(K_2{}^0\to\pi^\pm+e^\mp+\nu)}{R(K_2{}^0\to\pi^\pm+\mu^\mp+\nu)}\sim1.5$$

are expected to hold when first-order symmetry-breaking effects are included. The present experimental values are 1.4 ± 0.1 and 1.3 ± 0.2, respectively.[14] These last predictions follow, of course, in any scheme in which the vector currents are assumed to be partially conserved.

IV. AXIAL-VECTOR AMPLITUDES

The axial-vector currents, not being conserved quantities in the symmetric limit, are expected in general to have matrix elements which are renormalized to first order in the symmetry-breaking interaction. In order to estimate the size of these effects, one might look at the first-order effect in the mass splittings of baryons, i.e.,

$$[m_\Xi-\tfrac{1}{2}(m_N+m_\Xi)]/\tfrac{1}{2}(m_N+m_\Xi)=(m_\Xi-m_N)/(m_\Xi+m_N)=0.168\,,$$

which is nearly a 17% effect. If the symmetry-breaking interactions transform as the hypercharge generator of G_2, it is possible to prove that the first-order symmetry-breaking effects are, for example, opposite in $\Xi^-\to\Lambda+l^-+\nu$ and $\Lambda^0\to p+l^-+\bar\nu$ and zero in other processes such as $\Sigma\to\Lambda+l^-+\nu$. "Symmetry predictions" involve, in the simplest cases, the ratio of the rates of two physical processes. If the first-order symmetry-breaking effects to the axial-vector matrix elements are characterized by a parameter similar to the one given above, it is plausible to expect errors of order $(1+0.17)^2\approx1.37$ in some "rate" predictions and of order $(1+0.17)^2/(1-0.17)^2\approx2$ in others. On the other hand vector currents, being conserved in the symmetric limit, have matrix elements which are renormalized to second order in the symmetry-breaking interactions. If these

[16] A. Barbaro-Galtieri *et al.*, Phys. Rev. Letters **9**, 26 (1962): $\Sigma^+\to n+\mu^+\nu$; U. Nauenberg *et al.*, *ibid.* **12**, 679 (1964): $\Sigma^+\to n+e^++\nu$.

second-order effects are characterized by a parameter $(0.17)^2 \approx 0.03$ the same argument would lead to deviations of order 1.06 to 1.12 in "rate" predictions involving only the vector amplitudes. Of course, it may happen that for some special reason the first-order corrections to the axial-vector matrix elements are smaller than the corresponding contributions to the mass splittings. The above argument suggests, however, that it is entirely possible that the rate contributions of the vector currents can be computed in a fairly reliable manner with errors $\leq 10\%$ while the axial-vector contributions may involve errors as large as 40% to 100%.

For this reason, in the present paper we have placed particular emphasis on general predictions and properties of the model which are not affected by symmetry-breaking effects (Sec. II) and predictions involving only the vector amplitudes (Sec. III). Nonetheless, it is also of interest to study the quantitative predictions obtained from the axial currents in the symmetric limit.

In G_2 there are essentially two different ways in which one can construct seven-dimensional currents by taking bilinear combinations of the Λ, N, Ξ and Σ fields. One way is to combine the Λ field, which belongs to a one-dimensional representation, with the N, Ξ, and Σ which belong to a septet. Thus, for example, the $\Delta S=0$ and $\Delta S=1$ members of such a septet may be taken to be

$$j'^{(7)}=\bar{\Sigma}^+\Lambda+\bar{\Lambda}\Sigma^-, \tag{6a}$$

$$s'^{(7)}=\bar{p}\Lambda+\bar{\Lambda}\Xi^-. \tag{6b}$$

One may also consider the currents $(1/\sqrt{2})(\bar{\Sigma}^+\Lambda-\bar{\Lambda}\Sigma^-)$ and $(1/\sqrt{2})(\bar{p}\Lambda-\bar{\Lambda}\Xi^-)$ but this septet is ruled out by assumption (c). One can also consider septets constructed by considering bilinear expressions in the N, Ξ, and Σ fields. In fact, the product representation $7\otimes7=1\oplus7\oplus14\oplus27$ contains the seven-dimensional representation once. For example the $\Delta S=0$ and $\Delta S=1$ components of such a septet may be taken to be

$$j^{(7)}=\bar{p}n+\bar{\Xi}^0\Xi^-+(1/\sqrt{2})(\bar{\Sigma}^+\Sigma^0-\bar{\Sigma}^0\Sigma^-), \tag{7a}$$

$$s^{(7)}=-(1/\sqrt{2})\bar{p}\Sigma^0-\bar{n}\Sigma^--\bar{\Sigma}^+\Xi^0+(1/\sqrt{2})\bar{\Sigma}^0\Xi^-. \tag{7b}$$

These two septets can in principle be differentiated by an elementary symmetry reflection of the free Lagrangian which does not belong, however, to the symmetry group. This is simply the reflection $\Lambda\rightarrow-\Lambda$, $\Psi_i\rightarrow\Psi_i$, where the Ψ_i stand for the N, Σ and Ξ fields.

We also note that the currents of Eqs. (6) and (7) transform in the same manner under an inner automorphism of the group which reflects the weight diagram through the origin. This automorphism is a generalization of the charge-symmetry transformation. If I_1, I_2, and I_3 are the infinitesimal generators of the isotopic-spin group, then there also exists in G_2 a set of 3 infinitesimal generators K_1, K_2, and $K_3\equiv\frac{1}{2}Y$ of the hypercharge group which commute with the I_i. The charge symmetry operation is $e^{i\pi I_2}$. A generalization of this is $U=e^{-i\pi K_2}e^{i\pi I_2}$.[17] It is easy to see that U induces the reflection:

$$p\rightleftarrows-\Xi^-,\quad n\rightleftarrows\Xi^0,\quad \Sigma^{+,-,0}\rightleftarrows-\Sigma^{-,+,0},\quad \Lambda\leftrightarrow\Lambda.$$

Thus, under U the currents of Eqs. (6) and (7) satisfy

$$UjU^{-1}=-j^\dagger, \tag{8a}$$

$$UsU^{-1}=-s^\dagger. \tag{8b}$$

The 14-dimensional currents[1] transform in identical manner under U. Therefore, if we call j the complete hadronic current it satisfies:

$$UjU^{-1}=-j^\dagger. \tag{8c}$$

There is no *a priori* reason why any of the two septets in Eqs. (6) and (7) should play a more fundamental role in the present model. For this reason we assume that the axial currents are linear combinations of the members of the two septets. In this case the axial-vector contributions to the leptonic decays can be written as

$$\mathfrak{L}^{(A)}=[\rho j_{5\mu}^{(7)}+\rho' j_{5\mu}'^{(7)}+\alpha s_{5\mu}^{(7)}+\alpha' s_{5\mu}'^{(7)}]l_\mu+\text{H.c.}, \tag{9}$$

where l_μ stands for the lepton current and ρ, ρ', α, and α' are parameters characterizing the coupling of the axial currents to the leptons. Equation (9) leads to the relation:

$$R(\Xi^-\rightarrow\Lambda^0+e^-+\bar{\nu})=2.13R(\Lambda^0\rightarrow p+e^-+\bar{\nu}). \tag{10a}$$

Inserting the experimental results for the lifetimes of Λ^0 and Ξ^- and the branching ratio for $\Lambda^0\rightarrow p+e^-+\bar{\nu}$ one obtains in the symmetric limit the prediction

$$B(\Xi^-\rightarrow\Lambda^0+e^-+\bar{\nu})=1.24\times10^{-3} \tag{10b}$$

while experimentally this branching ratio is $(3.0\pm1.7)\times10^{-3}$. Taking into account the structure of both the vector and axial-vector currents one can also derive the relations:

$$(G_A/G_V)_{\Xi^0\Sigma^+}=-2(G_A/G_V)_{\Xi^-\Sigma^0}=(G_A/G_V)_{\Sigma^-n}. \tag{11}$$

In Sec. III we obtained an estimate for $|G_A/G_V|_{\Sigma^-n}$ on the basis of the calculated vector contributions and the experimental rate of $\Sigma^-\rightarrow n+e^-+\bar{\nu}$. Using this value and Eq. (11) we can now obtain the following predictions for the branching ratios:

$$B(\Xi^-\rightarrow\Sigma^0+e^-+\bar{\nu})=(3.99\pm0.61)\times10^{-5}, \tag{12a}$$

$$B(\Xi^0\rightarrow\Sigma^++e^-+\bar{\nu})=(9.22\pm1.97)\times10^{-5}, \tag{12b}$$

where the quoted errors correspond only to the error in $|G_A/G_V|_{\Sigma^-n}$. The above are all the basic predictions that can be made without further specifying the cou-

[17] R. E. Behrends, L. Landovitz, and B. Tunkelang, preceding paper, Phys. Rev. **142**, 1092 (1966).

TABLE I. Summary of leptonic-decay predictions.

Reaction	Interaction	Branching ratio
$\Lambda^0 \to p+e^-+\nu$	Pure A	Input
$\Sigma^- \to n+e^-+\nu$	$\lvert G_A/G_V\rvert = 1.48\pm0.18$	Input
$\Sigma^+ \to n+e^++\nu$	Pure V	$0.81\lvert x\rvert^2\times10^{-4}$ [Eq. (4a)]
$\Sigma^- \to \Lambda^0+e^-+\nu$	Pure A	$\cdots$
$\Xi^- \to \Lambda^0+e^-+\bar{\nu}$	Pure A	1.24×10^{-3}
$\Xi^- \to \Sigma^0+e^-+\bar{\nu}$	$(G_A/G_V)_{\Xi^-\Sigma^0} = -\frac{1}{2}(G_A/G_V)_{\Sigma^-n}$	$(3.99\pm0.61)\times10^{-5}$
$\Xi^0 \to \Sigma^++e^-+\bar{\nu}$	$(G_A/G_V)_{\Xi^0\Sigma^+} = (G_A/G_V)_{\Sigma^-n}$	$(9.22\pm1.97)\times10^{-5}$
$\Xi^0 \to \Sigma^-+e^++\nu$	Pure V	$0.87\lvert x\rvert^2 10^{-5}$

pling parameters ρ, ρ', α, and α'.[18] In Table I we collect the various predictions on baryonic rates and G_A/G_V values obtained on the basis of Secs. III and IV.

V. NONLEPTONIC DECAYS

As was mentioned in Sec. IV, in the present model the complete hadronic current J satisfies Eq. (8c). It then follows that the nonleptonic Lagrangian which, according to our assumptions, is made of $J^\dagger J$ terms, is invariant under U. As was pointed out by Marshak and Okubo[19] this can lead to certain relations between the decays $\Xi^- \to \Lambda+\pi^-$ and $\Lambda \to p+\pi^-$. In fact, let us consider the matrix elements

$$\langle p\pi^- | \mathcal{H}_w | \Lambda\rangle = \bar{\mu}_p(a_\Lambda + b_\Lambda\gamma_5)\mu_\Lambda, \qquad (13)$$

$$\langle \Lambda\pi^- | \mathcal{H}_w | \Xi^-\rangle = \bar{\mu}_\Lambda(a_\Xi + b_\Xi\gamma_5)\mu_{\Xi^-}. \qquad (14)$$

If we take the point of view that the symmetry predictions are to be applied to the invariant amplitudes a's and b's, we obtain in the symmetric limit the relations:

$$a_\Xi = a_\Lambda^*, \quad b_\Xi = -b_\Lambda^*. \qquad (15)$$

The S and P amplitudes are given by

$$A = [((m_1+m_2)^2 - m_\pi^2)/4m_1m_2]^{1/2}a, \qquad (16a)$$

$$B = [((m_1-m_2)^2 - m_\pi^2)/4m_1m_2]^{1/2}b, \qquad (16b)$$

where m_1 and m_2 are the masses of the initial and final baryons. For these amplitudes we obtain on the basis of Eqs. (15) and (16) the relations:

$$A_\Xi = 1.00A_\Lambda^*, \quad B_\Xi = -1.16B_\Lambda^*. \qquad (17)$$

In terms of the amplitudes A and B, the total rate R and the asymmetry parameters are given by

$$R = (1/2\pi)(m_2/m_1)|\mathbf{p}_2|[|A|^2 + |B|^2], \qquad (18a)$$

$$\alpha = 2\,\mathrm{Re}(AB^*)/(|A|^2+|B|^2),$$
$$\beta = 2\,\mathrm{Im}(AB^*)/(|A|^2+|B|^2), \qquad (18b)$$
$$\gamma = (|A|^2-|B|^2)/(|A|^2+|B|^2),$$

where $\mathbf{p}_2$ is the momentum of the final baryon. Equations (17) imply the predictions

$$|A_\Xi|/|A_\Lambda| = 1.00, \qquad |B_\Xi|/|B_\Lambda| = 1.16,$$
$$\mathrm{Re}(AB^*)_\Xi/\mathrm{Re}(AB^*)_\Lambda = -1.16, \qquad \mathrm{Sign}\,\beta_\Xi = \mathrm{Sign}\,\beta_\Lambda. \qquad (19)$$

From the experimental values $\gamma_\Lambda = 0.78\pm0.06$, $\gamma_\Xi = 0.85\pm0.04$, and $\tau_\Xi/\tau_\Lambda = 0.66\pm0.02$ we obtain[20]

$$[|A_\Xi|/|A_\Lambda|](\mathrm{expt}) = 1.30\pm0.03,$$
$$[|B_\Xi|/|B_\Lambda|](\mathrm{expt}) = 1.05\pm0.20. \qquad (20a)$$

Moreover, using the experimental values $\alpha_\Lambda = 0.64\pm0.06$ and $\alpha_\Xi = -0.48\pm0.05$ we find

$$[\mathrm{Re}(AB^*)_\Xi/\mathrm{Re}(AB^*)_\Lambda](\mathrm{expt}) = -1.22\pm0.17. \qquad (20b)$$

Thus, the ratios $|B_\Xi|/|B_\Lambda|$ and $\mathrm{Re}(AB^*)_\Xi/\mathrm{Re}(AB^*)_\Lambda$ are in good agreement with the theoretical predictions. On the other hand, the prediction for $|A_\Xi|/|A_\Lambda|$ differs by about 30% from the experimental value, a difference which does not appear to be serious in view of the

[18] In the particular cases: $\alpha'=\rho'=0$ or $\alpha=\rho=0$ or $\alpha'/\rho'=\alpha/\rho$, the $\Delta S=1$ and $\Delta S=0$ axial currents transform as components of a single septet and one can obtain two further predictions relating the axial-vector contributions to $\Sigma^-\to n+e^-+\bar{\nu}$ and $\Sigma^-\to\Lambda^0+e^-+\bar{\nu}$ and the $K^+\to\mu^++\nu$ and $\pi^+\to\mu^++\nu$ rates. With this particular choice of parameters one obtains in the symmetric limit $B(\Sigma^-\to\Lambda+e^-+\bar{\nu}) = (1.78\pm0.16)\times10^{-4}$ and $B(\Sigma^-\to n+e^-+\bar{\nu}) = 4.2\times10^{-3}$ while the experimental numbers are $(0.75\pm0.28)\times10^{-4}$ and $(1.4\pm0.3)\times10^{-3}$, respectively (Ref. 14). Thus, we conclude that in the present model, both septets contribute and $\alpha'/\rho' \neq \alpha/\rho$.

[19] S. Okubo and R. E. Marshak, Nuovo Cimento **28**, 56 (1963).

[20] The experimental values for α_Λ, β_Λ, and γ_Λ have been taken from J. W. Cronin and O. E. Overseth, Phys. Rev. **129**, 1795 (1963) and Lind *et al.*, Ref. 8, footnote 6. The experimental values for α_Ξ, β_Ξ, and γ_Ξ have been taken from H. K. Ticho, International Conference on Fundamental Aspects of Weak Interactions, Brookhaven National Laboratory Report No. BNL 837(C-39), p. 410, 1963 (unpublished).

approximations involved. We also note that the prediction $\text{Sign}\beta_\Xi = \text{Sign}\beta_\Lambda$ is quite consistent with the experimental values $\beta_\Lambda = 0.18 \pm 0.24$ and $\beta_\Xi = 0.13 \pm 0.17$.

VI. A BRIEF SUMMARY OF SU_3 RESULTS

It is perhaps convenient to recall briefly some of the comparable results of the well known theory constructed by Cabibbo on the basis of the SU_3 algebra.[21] This theory predicts the rule $\Delta Q = \Delta S$, [for example $x=0$, $R(\Sigma^+ \to n + l^+ + \nu) = R(\Xi^0 \to \Sigma^- + l^+ + \nu) = 0$] and the relation $R_2 = R_+$. The fact that in this theory $x=0$ implies that no observable violation of CP can be found in the time distribution of the $(K^0)_{e3}$ decays. In addition the Cabibbo theory leads to the following ratios for G_A/G_V in leptonic decays[22]:

$$(G_A/G_V)_{\Lambda^0 p} = -0.68\,,$$
$$(G_A/G_V)_{\Sigma^- n} = 0.305\,,$$
$$(G_A/G_V)_{\Xi^- \Lambda^0} = -0.19\,,$$
$$(G_A/G_V)_{\Xi^- \Sigma^0} = (G_A/G_V)_{\Xi^0 \Sigma^+} = -1.18\,.$$

The Cabibbo theory also predicts the branching ratios

$$B(\Lambda \to p + e^- + \bar{\nu}) = 0.91 \times 10^{-3}\,,$$
$$B(\Sigma^- \to n + e^- + \bar{\nu}) = 1.32 \times 10^{-3}\,,$$
$$B(\Sigma^- \to \Lambda + e^- + \bar{\nu}) = 0.61 \times 10^{-4}\,,$$
$$B(\Xi^- \to \Lambda + e^- + \bar{\nu}) = 0.65 \times 10^{-3}\,.$$

[21] N. Cabibbo, Phys. Rev. Letters **10**, 531 (1963).

[22] W. Willis *et al.*, Phys. Rev. Letters **13**, 291 (1964). We quote solution A(i).

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Electromagnetic Interactions and G_2†

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The electromagnetic predictions of the particle symmetry G_2 with symmetry breaking are given. Since none of these predictions are in conflict with present experimental results, it is concluded that arguments against G_2 based on its electromagnetic predictions without symmetry breaking are invalid.

SOME of the arguments against G_2 as a particle symmetry have been that it predicts, in the symmetric limit, incorrect electromagnetic properties for the particles.[1] For example, G_2 contains CA (charge conjugation times A parity) as an inner automorphism. From CA, it follows, to all orders in the electromagnetic field and to zeroth order in the moderately strong symmetry-breaking interaction $H_{\text{S.B.}}$, that

$$\begin{array}{llll} m_{\Xi^-} - m_{\Xi^0} = m_p - m_n\,, & \text{(a)} & \Gamma_{\Sigma^+} = -\Gamma_{\Sigma^-}\,, & \text{(e)} \\ m_{\Sigma^+} - m_{\Sigma^-} = 0\,, & \text{(b)} & \Gamma_p = -\Gamma_{\Xi^-}\,, & \text{(f)} \\ \Gamma_{\Sigma^0} = 0\,, & \text{(c)} & \Gamma_n = -\Gamma_{\Xi^0}\,, & \text{(g)} \\ \Gamma_\Lambda = 0\,, & \text{(d)} & & \end{array} \quad (1)$$

where Γ_p is the electromagnetic vertex operator for the proton. Moreover, there exist other inner automorphisms of G_2 which, in the same approximations, give the additional results

$$\begin{array}{llll} \Gamma_n = 0\,, & \text{(h)} & m_{\Sigma^+} - m_{\Sigma^0} = m_p - m_n\,. & \text{(j)} \\ \Gamma_p = \Gamma_{\Sigma^+}\,, & \text{(i)} & & \end{array} \quad (1')$$

By using these and other properties of G_2, one can also show, to the same approximation, that the following processes are forbidden

$$\begin{array}{llll} \omega^0 \to \pi^0 + \gamma\,, & \text{(k)} & \rho^0 \to \gamma\,, & \text{(n)} \\ \rho^0 \to \pi^0 + \gamma\,, & \text{(l)} & \Sigma^0 \to \Lambda^0 + \gamma\,. & \text{(o)} \\ \omega^0 \to \gamma\,, & \text{(m)} & & \end{array} \quad (1'')$$

It is immediately apparent that all of these predictions on which data exists are in violent disagreement with experiment. Clearly, *to this degree of approximation* in electromagnetic processes, G_2 is a very poor candiate for a particle symmetry.

Now, it would be convenient for physicists if the particle symmetry of nature were such that all its predictions in the symmetric limit were valid to a high degree of accuracy. Unfortunately, however, nature does not always arrange itself for our mathematical convenience. It thus seems entirely reasonable to examine G_2 and its electromagnetic predictions a little more closely before arriving at any conclusion as to its

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[1] See, for example, G. Feinberg and R. E. Behrends, Phys. Rev. **115**, 745 (1959); R. E. Behrends and A. Sirlin, *ibid.* **121**, 324 (1961). Y. Dothan and H. Harari, Nuovo Cimento **32**, 498 (1964). A. J. Macfarlane, N. Mukunda, and E. C. G. Sudarshan, Phys. Rev. **133**, B475 (1964); N. Mukunda, A. J. Macfarlane, and E. C. G. Sudarshan, *ibid.* **138**, B665 (1965); J. B. Bronzan and F. E. Low, Phys. Rev. Letters **12**, 522 (1964); S. Okubo and R. E. Marshak, Nuovo Cimento **28**, 56 (1963).