Bosons, A Parity, and G_2†

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The quantum number A is re-examined in the framework of G_2. The only two outstanding difficulties of the A selection rule are shown in this picture to be removed. Further consequences of A invariance and the assignment of bosons to G_2 multiplets are examined.

FOR many years, and in many different ways, theoreticians have listed the predictions which follow from what has been called "hypercharge symmetry.[1]" One of the more recent contributions has been that of Bronzan and Low[2] who consider the operation of charge conjugation times hypercharge symmetry which they call A. They devote their attention to the boson spectrum and note the rather good experimental evidence in support of such a selection rule. Their approach is simply based on an A-allowed or A-forbidden criterion for certain boson processes.

However, one of their predictions (Dii), that $\Gamma(\rho^0\rightarrow\pi^0+\gamma)/\Gamma(\omega^0\rightarrow\pi^0+\gamma)\ll 1$, coupled with the assumption of dominance of one-pion exchange (OPE) in ρ^0 and ω^0 photoproduction, seems to be in disagreement with the experiments of Crouch *et al.*[3] Moreover, the A selection rule taken with the assumption of an equal ω and ρ coupling to nucleons also seems to disagree with the analyses of the contribution of the ω resonance to the isoscalar nucleon electromagnetic form factors.[4]

On the other hand, it is interesting to note that there also exists evidence for the symmetry CA in the asymmetry parameters of the $\Xi^-\rightarrow\Lambda+\pi^-$ and $\Lambda\rightarrow p+\pi^-$ decays and in the ratio of the rates for $\Xi^-\rightarrow\Lambda+e^-+\nu$ and $\Lambda\rightarrow p+e^-+\nu$.[5,6] Moreover, recent evidence indicates that the $K\bar{K}$ mode of the A_2 is, indeed, suppressed over the $\rho\pi$ mode,[7] suggesting that A_2 is an eigenstate with $A=-1$.

In this note, we wish to point out that if the theoretical basis for the empirical A-selection rule is taken to be the symmetry G_2, then, while all the other predictions of Bronzan and Low are still valid, the predictions concerning $\omega\rightarrow\pi+\gamma$ and the nucleon form factors are changed so that they are no longer in disagreement with the above experiments. We also examine the consequences of the present A-parity assignments on part of the boson spectrum when interpreted *à la* G_2.

Let us first discuss A parity in the context of G_2. The usual G operator is defined as $Ce^{i\pi I_2}$, where C is the charge conjugation operator and I_2 is the isotopic spin operator which induces a rotation around the second axis of the three-dimensional isospin space. In addition to the isospin rotations induced by I_1, I_2, and I_3, the group G_2 contains another three-dimensional rotation subgroup which commutes with the isospin rotations.[8] This is the group of hypercharge rotations with generators K_1, K_2, K_3 ($K_3\equiv\frac{1}{2}Y$). The existence of the generator K_2 allows us to form a generalized G operator for G_2: the operator $A=Ge^{-i\pi K_2}$.

It is usually assumed that the strong interactions break into two parts: one part, $\mathcal{H}_s$, is invariant under the transformations of G_2 while the other, the symmetry-breaking part $\mathcal{H}_{\rm S.B.}$, is a tensor which transforms as the hypercharge generator Y (here taken as $2K_3$).[9] Since I_2 and K_2 are both generators of G_2 and since the strong interactions are C invariant, it follows immediately that

$$A\mathcal{H}_sA^{-1}=\mathcal{H}_s.$$

Moreover, since $H_{\rm S.B.}$ transforms as K_3 (invariant under I and C), it also follows that

$$A\mathcal{H}_{\rm S.B.}A^{-1}=-\mathcal{H}_{\rm S.B.}.$$

Clearly, A does not commute with baryon number, so that eigenstates of A must have $B=0$. Geometrically, CA reflects the weight diagrams through the origin, i.e., it satisfies $CAL_a(CA)^{-1}=-\tilde{L}_a$ where L_a are the generators of G_2. Since $(CA)^2=1$ for the seven-dimensional basis, any product representation will also have $(CA)^2=1$. The eigenvalues of A, then, are ±1. Since the charge operator is $Q=I_3+K_3$, the operator CA anticommutes with Q. The electromagnetic interaction is then invariant under A provided the photon is taken as an eigenstate with eigenvalue $A=+1$.

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[1] Some of the earlier work which contains this symmetry is A. Salam and J. C. Polkinghorne, Nuovo Cimento **2**, 685 (1955); D. C. Peaslee, *ibid.* **6**, 1 (1957). G. Feinberg and R. E. Behrends, Phys. Rev. **115**, 745 (1959). See also S. Okubo and R. E. Marshak, Nuovo Cimento **28**, 56 (1963).

[2] J. B. Bronzan and F. E. Low, Phys. Rev. Letters **12**, 522 (1964).

[3] H. R. Crouch *et al.*, Phys. Rev. Letters **13**, 640 (1964).

[4] See, for example, J. S. Levinger and C. P. Wang, Phys. Rev. **138**, B1207 (1965) and E. B. Hughes *et al.*, *ibid.* **139**, B458 (1965).

[5] R. E. Behrends and A. Sirlin, Phys. Rev. **142**, 1095 (1966), (following paper).

[6] Bronzan and Low reject CA symmetry for baryons because of the incorrect electromagnetic mass differences it predicts. These incorrect predictions are in the symmetric limit and, in fact, are radically changed by the symmetry-breaking effects (see R. E. Behrends, following paper). The baryon spectrum might be another argument against CA symmetry [specifically the $N^*(1238)$ and $\Xi^*(1530)$ have different isotopic spin]. This situation will be considered in a forthcoming paper.

[7] S. U. Chung, *et al.*, Phys. Rev. Letters **15**, 325 (1965): the branching ratios are $B(A_2\rightarrow\rho\pi)=0.91_{-0.10}^{+0.04}$, $B(A_2\rightarrow K\bar{K})=0.055\pm0.015$.

[8] See, for example, R. E. Behrends, J. Dreitlein, C. Fronsdal and W. Lee, Rev. Mod. Phys. **34**, 1 (1962).

[9] R. E. Behrends and L. F. Landovitz, Phys. Rev. Letters **11**, 296 (1963).

TABLE I. The λ, δ, ϵ members of $M_{0-}{}^{14}$.

Particle	$I,\lvert Y\rvert$	Mass (MeV)	Principal decay mode	Order of forbiddenness
λ	1, 0	$m_\lambda \gtrsim 420$	3π	1
		$280 \lesssim m_\lambda \lesssim 420$	$\pi\pi\gamma$	$\alpha\,\beta$
			$\pi\gamma\gamma$	α^2
		$140 \lesssim m_\lambda \lesssim 280$	$\pi\gamma\gamma,\ \lambda^0 \to \gamma\gamma$	α^2
δ	$\frac{3}{2}$, 1	$m_\delta \gtrsim 775$	$K\pi\pi$	1
		$635 \lesssim m_\delta \lesssim 775$	$K\pi\gamma$	$\alpha\,\beta$
			$\delta^+ \to K^+\gamma\gamma,\ \delta^0 \to K^0\gamma\gamma$	α^2
		$495 \lesssim m_\delta \lesssim 635$	$\delta^+ \to K^+\gamma\gamma,\ \delta^0 \to K^0\gamma\gamma$	α^2
			$\delta^{++} \to \pi^+\pi^+,\ \delta^- \to \pi^-\pi^0$	weak
ϵ	0, 2	$m_\epsilon \gtrsim 1125$	$KK\pi$	1
		$990 \lesssim m_\epsilon \lesssim 1125$	$KK\gamma$	$\alpha\,\beta$
		$m_\epsilon \lesssim 990$	$K\pi$	weak

For the mesonic states (each N-dimensional representation will be an eigenstate of A) we are led to the following assignments for the eigenvalues of A: For the 1-dimensional representation, $A=G$; for the 7- and 14-dimensional representations, A equals the G parity of the $I=1$, $K_3=0$ member of the supermultiplet. Accordingly, we find from the empirical A-parity assignments of Bronzan and Low (we label the representations $M_{JPA}{}^N$)

$$M_{0-}{}^7:\ \pi, K \qquad M_{0-}{}^{14}:\ \eta, \cdots \qquad M_{1-+}{}^{14}:\ \gamma$$

$$M_{1-+}{}^7:\ \rho, K^* \qquad M_{1-+}{}^{14}:\ \varphi, \cdots \qquad M_{1-}{}^1:\ \omega.$$

If β is a measure of the coupling strength of the symmetry-breaking interaction $\mathcal{H}_{\text{S.B.}}$, then A-parity violation is of order $O(\beta)$.

Let us now consider the ω. The process $\omega \to \pi+\gamma$ seems, at first glance, to be allowed by A parity. In reality, however, what A parity tells us is that $\omega \to \pi+\gamma$ proceeds through interactions which contain *even* powers of $\mathcal{H}_{\text{S.B.}}$. What we must determine, therefore, is whether the process is allowed in zeroth order. To do this, we note that the final state is the product of a seven- and a fourteen-dimensional representation. But $7\otimes 14=7\oplus 27\oplus 64$ which does not contain a one-dimensional representation (the ω). It therefore follows that $\omega \to \pi+\gamma$ must proceed through an interaction of order $O(\beta^2)$. Since $\rho^0 \to \pi+\gamma$ proceeds through $O(\beta)$, it then follows that assuming one-pion-exchange dominance in ρ^0 and ω^0 photoproduction, one should expect more ρ^0's than ω^0's, which is the experimental situation.[3]

As to the nucleon form factors, it is clear that the dominant contributions come from the two- and three-pion resonant states, the ρ^0 and the ω^0. It therefore follows that *if* the ρ^0 and the ω^0 are coupled approximately equally to nucleons, the coupling strengths $\rho^0 \to \gamma$ and $\omega^0 \to \gamma$ should also be approximately equal. The argument as to the approximate equality of the coupling strength of the ρ^0 and ω^0 to nucleons usually follows from their being assigned as members of the same supermultiplet. In G_2, however, the ρ^0 is a member of a seven-dimensional representation while the ω^0 is assigned to a one-dimensional representation. There is, therefore, no argument in G_2 for equating, even approximately, the strength of the coupling of these two particles to nucleons. It also follows from arguments similar to those given above for $\omega \to \pi+\gamma$ that the interaction $\rho^0 \to \gamma$ is of order $O(\beta^2)$ while $\omega^0 \to \gamma$ is of order $O(\beta)$. This would suggest, on the basis of the form-factor analyses, that the strength of the ω^0 coupling to nucleons is smaller, by a factor of about β, than the ρ^0 coupling to nucleons. By using this result one can then conclude that even if one-pion exchange is not dominant in ρ^0 and ω^0 photoproduction (as was assumed in the previous paragraph) one still would expect more ρ^0's than ω^0's. The important conclusion, therefore, is that the analysis of nucleon form factors and ρ^0 and ω^0 photoproduction do not rule against the A selection rule when taken in the context of G_2.

Let us now proceed to a further analysis of the boson spectrum according to G_2. We should now ask, where are the other members of the 14-dimensional representation to which the η belongs? With so many members missing, we can only guess at the masses. If we make the *ad hoc* assumption, which seems to be empirically true so far, that levels belonging to different multiplets do not cross each other under the mass-splitting effect, then the $I=1$, $K_3=0$ member (call it λ) should lie between $m_\pi=140$ MeV$\lesssim m_\lambda \lesssim 750$ MeV$=m_\rho$, and the $I=\frac{3}{2}$, $K_3=\pm\frac{1}{2}$ member, call it δ, should lie between $m_K=495$ MeV$\lesssim m_\delta \lesssim 890$ MeV$=m_{K^*}$. We can make no guess about the mass of the $I=0$, $K_3=\pm 1$ member (call it ϵ).

In Table I we have listed the principal decay modes of the λ, δ, and ϵ under various assumptions for their masses. Experimentally, one probably should have seen the fast modes of decay as resonances, provided their production rates are not too small. Since such resonances have not been observed, we would conjecture that these fast modes might be energetically forbidden. If this is so, detection of these particles is made more difficult, experimentally, since their principal modes of decay involve photons. To our knowledge, no experimental evidence exists on such decay modes in these regions (except near the η mass).

An interesting special case is the ϵ when its mass is less than $2m_K$. The only way it is then energetically allowed to decay is by violating strangeness, i.e., through the weak interactions. Assuming the weak decay rule $|\Delta S|\leq 1$, we would expect the weak decay modes $K^+\pi^0$ and $K^0\pi^+$ to be the dominant ones (appearing with the frequency 1:2 if the $|\Delta I|=\frac{1}{2}$ rule applies). Moreover, the decay would only occur through a parity-violating interaction. Such a meson was reported some years ago first by the Dubna group[10] and later by Yamanouchi and Kaplon.[11] This particle was called the D meson. Yamanouchi[12] noted in 1959 that six other anomalous events could be interpreted as D-meson events. From the data, the mass of the D was about 750 MeV. Subsequently, a search was conducted for this meson in the process $K^++p\rightarrow D^++\Sigma^+$ at 1.8 BeV/c with the result that if it were produced, it would appear with a cross section less than 12 μb.[13] Although no reasonable estimate of production cross section can be made, a value of 12 μb, and sometimes even less, is typical of resonance-production cross sections where the number of strange particles in the initial and final states differ (e.g., $K^-+p\rightarrow K^*+\Xi$).

Another interesting special case is the δ when its mass lies between m_K+m_π and m_K+2m_π. For the δ^{++} and the δ^- the main mode of decay would be $K\pi\gamma$ while for the δ^+ and δ^0 the main modes would be $K\pi\gamma$ and $K\gamma\gamma$. To our knowledge, no experiments so far have considered such decay modes in this mass region.

On the other hand, there has been a good deal of controversy over whether a "$K\pi$" resonance at 725 MeV exists with a width that varies from rather narrow to rather broad depending upon the experiment. Moreover, as Goldhaber[14] has pointed out, the evidence is stronger for this in complicated processes leading to five-particle final states than in more simple direct processes. Since the Q value for a $K\pi\gamma$ mode is about 90 MeV, we wonder if it is not possible that the $\kappa\rightarrow K\pi$ mode is in reality a $K\pi\gamma$ mode where, because of the experimental resolution, the low-energy part of the γ-ray spectrum in $K\pi\gamma$ has been included as "$K\pi$" events. If such were the case, the variations of the width, the mass and the very existence of the κ would be correlated with the resolution.[15]

It might be amusing to speculate that both the ϵ and the δ are to be identified with the D (750) and the κ ($\sim$750), respectively. (If the κ decay mode is really $K\pi\gamma$, then its mass will be slightly larger than the reported 725 MeV.) According to G_2, the second order mass relation satisfied by the members of a 14-dimensional representation is

$$3m_\lambda=4m_\delta-m_\epsilon,$$

which would place the λ at about 750 MeV. (It is to be noted that the results would not be changed if a quadratic mass formula were used.) As one sees from the table, its main mode of decay would be 3π. Depending upon the mechanism of production, the cross section for λ's may easily be $\frac{1}{3}$ to $\frac{1}{4}$ of η production in the process $\pi^-+p\rightarrow n+(\eta$ or $\lambda)$. Then, when compared with ω production in the process $\pi^-+p\rightarrow n+\omega$, the λ production would be down by about a factor of about 10, which might well explain its apparent absence in experiments with low statistics.

Moreover, on the basis of our previous arguments that the $\omega\rightarrow\pi^0+\gamma$ mode is inhibited by the symmetry-breaking interaction, it might, then, be conjectured that the neutral decays experimentally observed in missing mass plots and usually attributed to $\omega\rightarrow\pi^0+\gamma$, are, at least in part, the neutral $3\pi^0$ mode of λ decay.

We note, here, that on the basis of a rather large branching ratio of the $X^0(959)$ to $\pi^+\pi^-\gamma$ compared with the main mode $\eta\pi^+\pi^-$,[16] an assignment of $A=+1$ for the X^0 seems entirely reasonable.[17] It would then seem natural to assign the X^0 to either a one-dimensional or a 27-dimensional representation of G_2. In this picture, the X^0 would not be a heavy η but rather would be distinguished from the η by an opposite A parity as well as belonging to a different dimensional representation.

[10] *Proceedings of the 1959 Annual International Conference on High Energy Physics* held in Kiev, USSR (unpublished).

[11] T. Yamanouchi and M. F. Kaplon, Phys. Rev. Letters **3**, 283 (1959).

[12] T. Yamanouchi, Phys. Rev. Letters **3**, 480 (1959).

[13] L. M. Barkov *et al.*, Zh. Eksperim. i Teor. Fiz. **43**, 335 (1962) [English transl.: Soviet Phys.—JETP **16**, 240 (1963)].

[14] G. Goldhaber, University of California Radiation Laboratory Report No. UCRL-11971, (1965) (unpublished).

[15] As to the isospin of the κ, it is amusing to note that if one took the ratio of the κ^+ and K^{*+} production cross section $\sigma(K^+p\rightarrow\kappa^+p\pi^+\pi^-)/\sigma(K^+p\rightarrow K^{*+}p\pi^+\pi^-)$ one might expect this ratio to be the same for the $K^0\pi^+$ decay mode as for the $K^+\pi^0$ decay mode *if the isospin of the* K^+ *and the* κ^+ *were the same.* A rough estimate from the date of M. Ferro-Luzzi *et al.* [Phys. Letters **12**, 255 (1964)], seems however to indicate that this ratio for the $K^+\pi^0$ decay mode is nearly 4 times the ratio for the $K^0\pi^+$ decay modes, just about the result one might expect for an $I=\frac{1}{2}$, K^+ and an $I=\frac{3}{2}$, κ. On the other hand, it is true that these same authors do not observe a sharp peak in the $K^+\pi^+$ mode, which, of course, could as well be a result of the mechanism of production as a result of the isospin of the κ.

[16] A. H. Rosenfeld *et al.*, University of California Radiation Laboratory Report No. UCRL-8030, (1965) (unpublished), where the branching ratios are $B(X^0\rightarrow\eta 2\pi)=0.78\pm0.04$ and $B(X^0\rightarrow\pi^+\pi^-\gamma)=0.22\pm0.04$.

[17] J. Badier *et al.*, Phys. Letters **17**, 337 (1965). These authors suggest $A=+1$ for the same reasons.