The α, $\beta=1, 2, 3$ are the unitary spin indices, i, $j=1, 2$ are the spin indices, g and f are the coupling constant, φ and ω represent physical particles, and the ω-φ mixing angle $\theta=\cos^{-1}(\sqrt{\tfrac{2}{3}})$ is taken. The spatial dependence that is not needed has been suppressed and units $\hbar=c=1$ are used.

Put Eqs. (2) and (3) into Eq. (1), and we obtain

$$\begin{aligned}A=gT_1{}^1[&V_\gamma{}^1P_\delta{}^\gamma P_1{}^\delta+P_\gamma{}^1V_\delta{}^\gamma P_1{}^\delta+P_\gamma{}^1P_\delta{}^\gamma V_1{}^\delta\\&+V_1{}^\gamma P_\gamma{}^\delta P_\delta{}^1+P_1{}^\gamma V_\gamma{}^\delta P_\delta{}^1+P_1{}^\gamma P_\gamma{}^\delta V_\delta{}^1\\&\times\tfrac{2}{3}(V_\gamma{}^\mu P_\delta{}^\gamma P_\mu{}^\delta+P_\gamma{}^\mu V_\delta{}^\gamma P_\mu{}^\delta+P_\gamma{}^\mu P_\delta{}^\gamma V_\mu{}^\delta)]\sigma_1{}^2\\&+fT_1{}^1V_1{}^1P_\delta{}^\gamma P_\gamma{}^\delta\sigma_1{}^2.\end{aligned}\tag{5}$$

The factor $\sigma_1{}^2$ indicates that the Z component of vector-meson spin is $S_Z=1$.

The following relations among the decay amplitudes $V\to P+P'+\gamma$ up to a constant factor are obtained from Eqs. (4) and (5).

$$\begin{aligned}&(\rho^0|\pi^+\pi^-\gamma)\\&\quad=(\omega|\pi^0\pi^0\gamma)=(\omega|\pi^+\pi^-\gamma)=\sqrt{2}(g+f),\\&(\rho^0|\pi^+\pi^0\gamma)\\&\quad=\sqrt{3}(\rho^+|\pi^+\eta\gamma)=\sqrt{3}(\rho^0|\pi^0\eta\gamma)\\&\quad=(K^{*+}|K^+\pi^0\gamma)=(K^{*0}|K^0\pi^0\gamma)\\&\quad=(-1/\sqrt{2})(\varphi|K_1{}^0K_1{}^0\gamma)=(-1/\sqrt{2})(\varphi|K_2{}^0K_2{}^0\gamma)\\&\qquad\qquad=(1/\sqrt{3})(\omega d\pi^0\eta\gamma)=\sqrt{2}g,\\&(\rho^0|\pi^0\pi^0\gamma)=\sqrt{2}(3g+f),\end{aligned}\tag{6}$$

$$\begin{aligned}(K^{*+}|\pi^+K^0\gamma)&=(K^{*0}|\pi K^+\gamma)=(\varphi|K^*K^-\gamma)\\&=(\varphi|K_1{}^0K_2{}^0\gamma)=(\varphi|\eta\pi^0\gamma)\\&=(\varphi|\pi^+\pi\gamma)=(\varphi|\pi^0\pi^0\gamma)=0.\end{aligned}\tag{7}$$

The relations in Eqs. (6) and (7) except for the φ and ω decays can also be obtained from those[5] of $SU(3)$ in Ref. 3. To obtain the φ and ω decays from Ref. 3, one must consider the decay $\omega^0\to P+P'+\gamma$ and then form the suitable combination of ω^0 and φ^0 decays with the mixing angle $\theta=\cos^{-1}(\sqrt{\tfrac{2}{3}})$.

The only experimental decay mode of $V\to P+P'+\gamma$ known presently has the partial decay width[6]

$$\Gamma(\omega|\pi^+\pi\gamma)\leq0.3\text{ MeV},$$

from which one can obtain the following relations among the partial decay widths:

$$\begin{aligned}&\Gamma(\rho^+|\pi^+\pi^0\gamma)\\&\quad=3\Gamma(\rho^+|\pi^+\eta\gamma)=3\Gamma(\rho^0|\pi^0\eta\gamma)=\Gamma(K^{*+}|K^+\pi^0\gamma)\\&\quad=\Gamma(K^{*0}|K^0\pi^0\gamma)=\tfrac{1}{2}\Gamma(\varphi|K_1{}^0K_1{}^0\gamma)\\&\quad=\tfrac{1}{2}\Gamma(\varphi|K_2{}^0K_2{}^0\gamma)=\tfrac{1}{3}\Gamma(\omega|\pi^0\eta\gamma),\\&\Gamma(\rho^0|\pi^+\pi^-\gamma)\\&\qquad=2\Gamma(\omega|\pi^0\pi^0\gamma)=\Gamma(\omega|\pi^+\pi^-\gamma)\leq0.3\text{ MeV}.\end{aligned}\tag{8}$$

The assumptions underlining Eq. (8) in addition to the normal ones are the validity of $SU(6)$ (broken by electromagnetism) in which case the pseudoscalar mesons in $V\to P+P'+\gamma$ are in a symmetric $J^P=0^+$ state so that the decays proceed as a electric dipole transition, and that the ω-φ mixing angle is $\theta=\cos^{-1}(\sqrt{\tfrac{2}{3}})$

Some of the relations in Eq. (8) may perhaps be checked experimentally to see, whether $SU(6)$ symmetry is valid for the decay mode $V\to P+P'+\gamma$.

ACKNOWLEDGMENT

The author would like to thank D. G. Sutherland for pointing out an error in the original manuscript.

[5] Relations (3) are obtained (up to an irrelevant over-all phase in some cases) by putting $a_1=a_1{}^s+\sqrt{3}a_2{}^s=-(2\sqrt{2}/\sqrt{3})g$, $b_1=(2\sqrt{2}/\sqrt{3})(g-f)$, and $a_2=b_2=0$ in Ref. 3.

[6] A. H. Rosenfeld, A. Barbaro-Galtieri, W. H. Barkas, P. L. Bastien, J. Kirz, and M. Roos, Rev. Mod. Phys. **36**, 977 (1964).

PHYSICAL REVIEW VOLUME 142, NUMBER 4 FEBRUARY 1966

Regge-Pole Contribution to Vector-Meson Production*

M. Barmawi†

The Enrico Fermi Institute for Nuclear Studies and the Department of Physics, The University of Chicago, Chicago, Illinois

(Received 15 September 1965)

A Regge-pole model of vector-meson production is considered and applied to ρ and ω production. The Regge-pole contribution is computed explicitly, without use of the asymptotic forms. The residues are related to the well-known coupling constants. The only parameter is the slope of the trajectory, which is estimated to be 0.64/GeV^2. Good results for the differential cross section are obtained. The question of the decay density matrices is discussed briefly.

At present, it is fashionable to explain the peripheral production of resonances on the basis of an absorption model or one-particle-exchange models with form factors.[1] On the other hand, many of the desirable features of both models are automatically contained in

* Work supported in part by the U. S. Atomic Energy Commission.

† Indonesian Fellow, on leave of absence from Bandung Institute of Technology, Indonesia.

[1] See, for example, J. D. Jackson Rev. Mod. Phys. **37**, 484 (1965). This survey paper contains further references.

a Regge-pole exchange picture,[2] which is also very helpful in connection with the unitarity restrictions. Of course, the Regge-pole model has to be handled with some care at nonasymptotic energies, where it also has a somewhat different meaning. Although simple Regge-pole exchange does not seem to be a good approximation in elastic channels at higher energies,[3] it is possible that the accumulations of branch points, which seem to complicate the situation in the angular-momentum plane of a crossed channel with the quantum numbers of the vacuum, may not be very relevant in channels with different quantum numbers and at lower energies. We think, therefore, that it may be reasonable to explore the "naive" Regge-pole model for *inelastic* reactions.

It is the purpose of this note to discuss the Regge-pole contribution to vector-meson production.[4] The Regge formalism is applied to ρ and ω production. We show that, under reasonable assumptions, the damping of the differential cross section at large values of momentum transfer can be well described by Regge pole terms alone. Furthermore, we find that the Regge ansatz gives the right order of magnitude for the cross sections. Of special interest is the amplitude for ω production, because in this case only ρ^+-meson exchange is allowed, and the relevant coupling constants are well known.

Let us consider the reactions $\pi N\to VN$, where $V=\omega,\rho$. The Regge-pole contribution can be calculated using the helicity representation in the channel $N\bar{N}\to\pi V$ together with the crossing relations.[4] It can be shown[5] that the Regge-pole term in the crossed-channel amplitude is given by

$$\begin{aligned}\langle\lambda_V|F(s,t)|\lambda_1,\lambda_2\rangle=&-(2\alpha+1)N(\alpha)\\ &\times\pi\alpha'(0)(e^{-i\pi\beta(s)}\pm1)/2\sin\pi\beta(s)\\ &\times[4p_sk_s/2M(m_\pi m_V)^{1/2}]^{\beta(s)}P_{\beta(s)}{}^{(a,b)}(z)\\ &\quad\times[(1-z)/2]^{a/2}[(1+z)/2]^{b/2}\times C_{\lambda_V,\lambda_1\lambda_2},\end{aligned}\qquad(1)$$

where $\lambda=\lambda_1-\lambda_2$; $\lambda_{\max,\min}=\max,\min(\lambda,\lambda_V)$; $a=\lambda_{\max}-\lambda_{\min}$; $b=\lambda_{\max}+\lambda_{\min}$; p_s,k_s=initial, final momenta in the crossed-channel c.m. system, $z=\cos\theta_s$, $\alpha(s)$=Regge trajectory, $\beta(s)=\alpha(s)-\lambda_{\max}$, M=nucleon mass, and

$$N(\alpha)=\left[\frac{\Gamma(\alpha+\lambda_{\max}+1)\Gamma(\alpha-\lambda_{\max}+1)}{\Gamma(\alpha+\lambda_{\min}+1)\Gamma(\alpha-\lambda_{\min}+1)}\right]^{\frac{1}{2}}\times(-1)^{\lambda_{\min}-\lambda}.$$

The momentum transfer distribution is then

$$\frac{d\sigma}{ds}=\frac{M^2}{16\pi p_t t}\times\frac{1}{2}\times\sum_{\lambda_V,\lambda_1\lambda_2}|\langle\lambda_V|F(s,t)|\lambda_1\lambda_2\rangle|^2,\qquad(2)$$

where p_t=initial momentum in the production channel.

The factors in Eq. (1) have the following interpretations: $(2\alpha+1)N(\alpha)$ is a normalization factor arising from the partial-wave decomposition. The second term is the signature factor, where $\pi\alpha'(0)$ has been introduced explicitly, so that the term reduces to a simple pole as $\sin\pi\beta\to0$, which happens for $s\to m_{\mathrm{ex}}{}^2$ (m_{ex}=mass of exchanged particle). The signature factor, therefore, has the same effect as a pole[3] for small values of the momentum transfer, and it is expected to play an important role. The third factor is the orbital factor, which is mainly determined by the orbital part $\beta(s)$ of the total angular momentum. This interpretation is consistent with the appearance of $\beta(s)$ in the threshold behavior $(4p_sk_s)^{\beta(s)}$. For Re$z>0$, the Jacobi function can be represented by a hypergeometric series as

$$P_\beta{}^{(b,a)}(z)=\binom{\beta+b}{b}\left(\frac{1+z}{2}\right)^\beta\times F\left(-\beta,-\beta-a;b+1;\frac{z-1}{z+1}\right).\qquad(3)$$

Since for $s\leq1$ (GeV)^2, z is roughly a constant and F is a slowly varying function of s, we see that

$$[(1-z)/2]^{a/2}[(1+z)/2]^{b/2}P_\beta{}^{(a,b)}(z)$$

behaves roughly like $t^{\alpha(s)}$, which produces the damping at large momentum transfer. Finally, there is the factor $C_{\lambda_V,\lambda_1\lambda_2}$, which is determined by the type of interaction; it contains the coupling constants and some kinematical factors.

Because the energies we consider are not very high, we do not make use of the asymptotic form for the Jacobi function, but we compute it explicitly.[6] The background integral is neglected, because its largest contribution is near the forward direction, where the pole at $s=m_{\mathrm{ex}}{}^2$ is most effective in enhancing the Regge-pole term.

[2] M. M. Islam, Nuovo Cimento **30**, 579 (1963); M. M. Islam and R. Piñon, *ibid.* **30**, 837 (1963); H. Überall, *ibid.* **30**, 366 (1963). K. Gottfried and J. D. Jackson, *ibid.* **33**, 309 (1964); Phys. Letters **8**, 144 (1964). M. S. Marinov, Zh. Eksperim. i Teor. Fiz. **46**, 947 (1964) [English transl.: Soviet Phys.—JETP **19**, 646 (1964)]. V. Barger, Nuovo Cimento **35**, 700 (1965). R. J. N. Phillips and W. Rarita, Phys. Rev. **40**, B200 (1965).

[3] R. Oehme, in *Strong Interaction in High Energy Physics*, edited by R. G. Moorhouse (Oliver and Boyd, Edinburgh, 1964).

[4] See K. Gottfried, J. D. Jackson, and M. S. Marinov, Ref. 2.

[5] F. Calogero, J. M. Charap, and E. J. Squires, Ann. Phys. (N. Y.) **25**, 325 (1964).

[6] The asymptotic form of the Jacobi function is proportional to

$$\frac{\Gamma(2\alpha+1)}{\Gamma(\alpha+\lambda_{\max}+1)\Gamma(\alpha-\lambda_{\min}+1)}\left(\frac{z-1}{2}\right)^{\alpha-\lambda_{\max}}\times\left[\frac{1+(-\alpha+\lambda_{\max})(-\alpha+\lambda_{\min})}{(-2\alpha)}\frac{2}{1-z}+O\left(\frac{2}{1-z}\right)^2\right]+\frac{\Gamma(-2\alpha-1)}{\Gamma(-\alpha+\lambda_{\max})\Gamma(-\alpha-\lambda_{\min})}\left(\frac{z-1}{2}\right)^{-\alpha-\lambda_{\max}-1}\left[1+O\left(\frac{2}{1-z}\right)\right]$$

which reduces to Eq. (A2) of Ref. 5 for $z\gg1$, and it is valid for $|z-1|>2$. In order to see the reason for the explicit valuation of the Jacobi function, we give the following numbers: In the case of ρ production we have at the peak of $d\sigma/ds$ the value $z=1.9$ (3.8) at 4.0 (8.0) GeV/c incident π momentum, while for $s<-0.2$ (GeV)^2 the value $z\sim4$. (9.). In the case of ω production for $s<-0.2$ (GeV)^2, we have $z\sim3$. These numbers indicate that we are below or slightly above the bound for the validity of the asymptotic expansion.

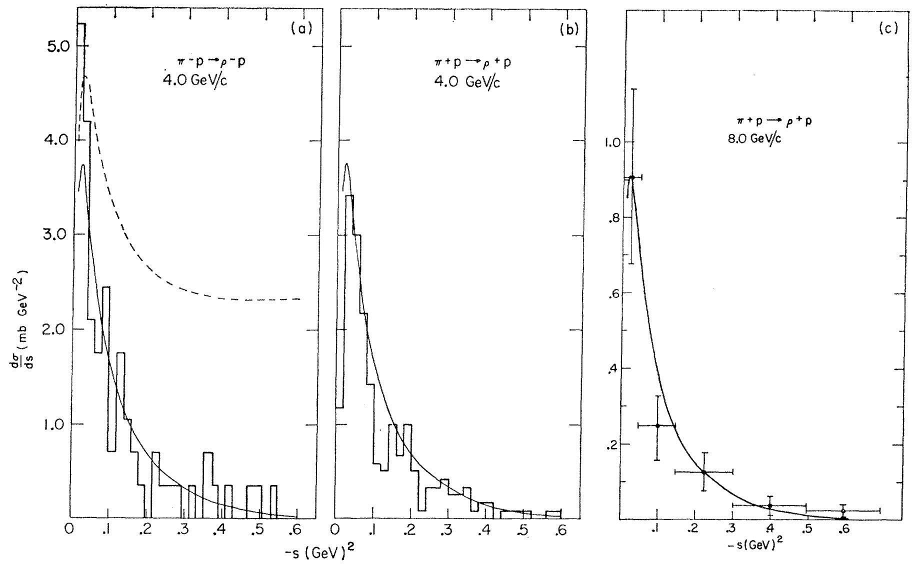

FIG. 1. Comparison between π Regge-pole contribution to $d\sigma/ds$ with the experimental data. Coupling constants: $f_{\rho\pi\pi}^2/4\pi=2.0$; $g_{NN\pi}^2/4\pi=14.5$. Total number of events included in Figs. 1(a) and 1(b) are, respectively, 61 and 220. The dashed curve is calculated from simple π-pole model which is reduced by the Regge-pole damping to the solid curve.

It may be useful to discuss briefly the energy dependence of a superposition of Regge poles. For a qualitative discussion we neglect spin effects and write $St^{\alpha(0)+s\alpha'(0)}$ (S=signature factor). We assume that all the trajectories are parallel. Since the relevant Regge poles in the present problem are those of P ($P=0^-$ meson) and V ($V=1^-$ meson), we have the superposition

$$[t^{\alpha_P(0)}S_P(s)+t^{\alpha_V(0)}S_V(s)]t^{s\alpha'(0)}.$$

Because of $\alpha_P(0)<0$ and $\alpha_V(0)>0$, we see that, as the energy increases, the P pole will be suppressed and the V pole enhanced. This has the following consequences at nonasymptotic energies: At lower energies $t^{\alpha(0)}$ does not modify the pole effects drastically. Consequently, the highest lying trajectory does not have to be the dominant one. As the energy is increased, there will then be a transfer of domination from the P to the V term. In our model, the π meson has the strongest pole effect (the π pole is closest to the physical region). When π exchange is dominant, the induced peak is sharp. On the other hand, the V peak is much broader and therefore the transfer leads to a widening of the distribution and to deviations from the logarithmic shrinking law in cases where π and V exchange are allowed by the selection rules. The rate of transfer depends on the relative magnitude of the coupling constants.

In the application to ρ and ω production, we retain the assumption that the trajectories are approximately straight lines and approximately parallel. As a rough guess for the slope we take the vacuum trajectory passing through $\alpha=2$ at the f^0 resonance and $\alpha=1$ at $s=0$, which gives $\alpha'(0)=0.64$. We use the approximation where the factors $C_{\lambda_V,\lambda_1\lambda_2}$ are determined by the condition that Eq. (1) reduces to the simple Born term for $s\to m_{\rm ex}^2$ and that the residue of the Regge pole in the l plane is independent of s.[7] The result of the matching at the pole is presented in Table I, where the notation of Jackson and Pilkuhn[8] is used. The residues are determined by the decay width (see figure captions for explicit values).

The result of the calculation of $d\sigma/ds$ for ρ production is shown in Fig. 1, where it is compared with the

TABLE I. $C_{\lambda_V,\lambda_1\lambda_2}$. $f_{P,V}$=meson-vertex coupling constants for P,V exchange. $g_{P,V}$=nucleon-vertex coupling constants for P,V exchange. $\gamma=g_T/g_V$.

λ_V \ λ_1, λ_2	$+\frac{1}{2}$, $+\frac{1}{2}$	$+\frac{1}{2}$, $-\frac{1}{2}$
$+1$	$\dfrac{f_Vg_V}{3m}k_ss^{1/2}\left(1+\gamma\dfrac{s}{4M^2}\right)$	$-\dfrac{f_Vg_V}{3\sqrt{2}mM}(1+\gamma)k_ss$
0	$\dfrac{f_Pg_P}{2Mm_V}k_ss$	0
-1	$-\dfrac{f_Vg_V}{3m}k_ss^{1/2}\left(1+\gamma\dfrac{s}{4M^2}\right)$	$+\dfrac{f_Vg_V}{3\sqrt{2}mM}(1+\gamma)k_ss$

[7] This model was suggested by Professor Oehme to the author in the early stage of the present work. The motivation for this model is to assign a definite physical interpretation to the residue of the Regge pole in the l-plane, which in this model is assumed to be energy-independent. (See Ref. 3, p. 170). For the ρ Regge pole, the analysis of R. K. Logan [Phys. Rev. Letters **14**, 414 (1965)] also indicates the constancy of this residue, and with our value of the slope, this model gives a reasonable description of π-p charge-exchange scattering. The paper of Phillips and Rarita [Phys. Rev. **139**, B1336 (1965)] uses Regge poles in a way which is quite different from our approach. Therefore, their parametrization should not be directly compared with ours.

[8] J. D. Jackson and H. Pilkuhn, Nuovo Cimento **33**, 906 (1964).

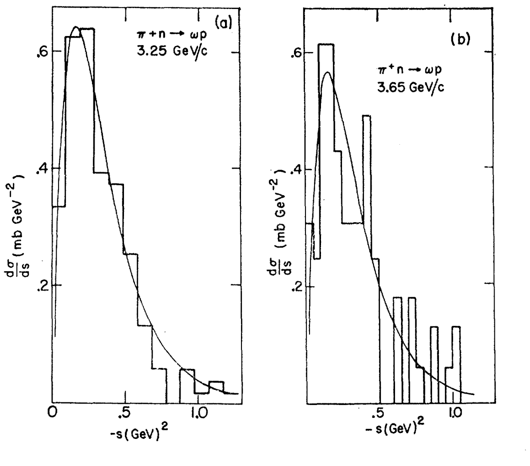

FIG. 2. Comparison between $d\sigma/ds$, calculated from ρ^+ Regge pole exchange, with the experimental data. Coupling constants: $f_{\omega\rho\pi}^2/4\pi=10.5$; $g_V^2/4\pi=2.2$, for ρ^+ exchange it is twice this value; $g_T/g_V=2.98$. At 3.25 GeV/c (Ref. 9a) the histogram is normalized to the calculated cross section up to $-s=0.80$ (GeV)^2 and at 3.65 GeV/c (Ref. 9b) up to $-s=0.50$ (GeV)^2. Total number of events included in Fig. 2(a): 151 and 6 events outside the range; in Fig. 2(b): 75 events and 1 event is outside the range.

experimental data.[9] In this reaction, also ω exchange is permitted by the selection rules, but $g_{NN\omega}$ is not very well known.[10] With the present value of $\alpha'(0)$, the result in Fig. 1 will not be changed drastically as long as $g_{NN\omega}^2/4\pi\lesssim 2$.[11] In the calculation of $d\sigma/ds$ for ω production the ratio $g_T/g_V=2.98$ is used, which is based on the electromagnetic-form-factor data.[12] The result is shown and compared with the data at 3.25[13a] and 3.65 GeV/c[13b] in Fig. 2. The total cross section is estimated to be 0.27 mb at 3.25 GeV/c and 0.24 mb at 3.65 GeV/c incident π^+ momenta. Preliminary results at 3.65 GeV/c[13b] give (0.35 ± 0.06) mb, indicating that our result is of the right order of magnitude.

In our model we have made a pole approximation in the complex angular-momentum plane, and partially also in the s plane, at least as far as the coefficient $C_{\lambda_V,\lambda_1\lambda_2}$ is concerned. As a consequence of this, we find that the decay density matrices, in the case of pure exchanges, are the same as those obtained in the simple pole model or the peripheral model. These do not agree very well with experiments. In $\rho^\pm$ production, at least part of the discrepancy may be due to the presence of ω exchange, but for ρ^0 production this exchange is forbidden. These discrepancies are not unexpected, because the density matrices are more sensitive to the approximations made than the momentum-transfer distributions.

As we have mentioned before and as may be seen from Table I, some of the helicity components of the Regge-pole terms are zero. This is due to our pole approximation[14] in the *energy plane* resulting in an s-independent residue of the pole *in the l plane*.[3] The constant residue is determined by the vertices (on the mass shell) and therefore it is subject to the selection rules of angular momentum and parity conservation. If we do not make the pole approximation *in the s plane*, then, in general, there will be contributions to the helicity amplitudes other than those which have the pole in s.[15] These contributions, as well as contributions from other poles and cuts *in the angular-momentum plane*, may well modify the density matrices without seriously affecting the distributions. It is quite possible that the discrepancy encountered for the density matrices in ω production indicates the necessity for exchange of another particle of a higher mass.[16]

Finally we mention that, as is well known, the absorption model gives a reasonable description of the decay density matrices. In this paper we are not interested in the phenomenological description, but in the question to what extent a Regge picture may be applicable for inelastic processes, in spite of the difficulties encountered by the model in the elastic case.

The author wishes to express his gratitude to Professor R. Oehme for suggesting this problem, for stimulating guidance and for advice. He is grateful to Professor M. Deutschmann, Dr. N. Gelfand, and G. Benson for information concerning the experimental data. He wishes also to thank C. S. Lai and S. Ragusa for helpful conversations, and M. Parkinson for the generous advice concerning the computer programming.

[9] (a) Aachen-Birmingham-Bonn-Hamburg-London (I. C.)-Munchen collaboration, Nuovo Cimento **31**, 729 (1964) (b) Aachen-Berlin-Birmingham-Bonn-Hamburg-London (I. C.)-Munchen collaboration, Phys. Rev. **138**, B897 (1965); M. Deutschmann (private communication) (c) Aachen-Berlin-CERN collaboration, Phys. Letters **12**, 356 (1964).

[10] K. Kawarabayashi, Phys. Rev. **134**, B877 (1964); J. S. Ball, A. Scotti, and D. Y. Wong (to be published) gives $g_{NN\omega}^2/4\pi=\frac{1}{2}$, $g_{NN\varphi}^2/4\pi=1.4$.

[11] From Fig. 2, using $g_{NN\omega}=g_{NN\rho}$ it can be estimated that the largest deviation in $d\sigma/ds$ due to ρ contribution is of the order of 0.3 mb for ρ production, which is a small correction.

[12] L. N. Hand, D. G. Muller, and R. Wilson, Rev. Mod. Phys. **35**, 335 (1963).

[13] (a) W. Bugg, H. Cohn, G. Condo, N. Gelfand, and G. Lutjens (unpublished); N. Gelfand (private communication). (b) G. Benson, L. Lovell, E. Marquit, B. Roe, D. Sinclair, and J. vander Velde, Bull. Am. Phys. Soc. **10**, 502 (1965); G. Benson (private communication).

[14] This can be shown using the method described in L. Durand III, P. C. de Celles, and R. B. Marr, Phys. Rev. **126**, 1882 (1962). This property is also valid for off-mass-shell correction due to self-energy diagrams and form factors.

[15] Note that the Regge pole in the l plane does not correspond only to a simple pole term in the s plane. Rather it corresponds to the exchange of a particle with given integer spin only for $s=m_{ex}^2$, where m_{ex} is the mass of this particle. For $s\neq m_{ex}^2$ the conservation of angular momentum and parity does not, in general, lead to the structure of Table I.

[16] *Note added in manuscript.* More recent work by the author (M. Barmawi, to be published) shows, that the exchange of an axial-vector meson gives rise to nonzero values for just those components. The inclusion of such a meson makes it possible to account for the experimental results obtained for the decay density matrices, especially in the case of ω production.