Reciprocity, Normality of K^0-$\bar{K}^0$ Mass Matrix, and CP Violation in Weak Interactions

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We propose that the reciprocity principle may be relevant in weak interactions as a replacement for time-reversal invariance. Various forms of this principle are examined and their consequences are noted. When applied to the K^0-$\bar{K}^0$ system, they all imply that the two observed kaon states are orthogonal, and a sum rule emerges, which relates the rates and the CP-violating parameters of different neutral-kaon decay modes. Using the preliminary experimental data, we predict specifically the amount of CP violation in $K_l \to 2\pi^0$. Finally, we discuss the possible implications of reciprocity for various interactions.

I. INTRODUCTION

THE question of CP or T invariance has been raised by the following pertinent experimental observations:

(a) It has been found[1] that the long-lived component K_l of K^0 decays into $\pi^+\pi^-$, thus suggesting that CP is violated in the K^0 decay. If CPT invariance is assumed then this implies that time-reversal invariance is also violated. The experiment gives a decay rate[2] of

$$|\Gamma(K_l^0 \to \pi^+ + \pi^-)/\Gamma(K_s^0 \to \pi^+ + \pi^-)|^{1/2} = (1.8 \pm 0.1) \times 10^{-3}, \quad (1.1)$$

where K_s^0 is the short-lived component of K^0.

(b) Preliminary experiments[3–5] on $K^0 \to \pi^\pm e^\mp \nu$ and $K^0 \to \pi^+\pi^-\pi^0$ indicate that CP violation in these processes is quite large, though the statistics as yet are poor.

(c) Violation of time-reversal invariance, if any, is small in the processes $n \to pe\nu$ and $\Lambda \to \pi^- p$. Within errors, the experiments are consistent with the assumption of time-reversal invariance for these processes.

The above observations naturally lead to the questions: If time reversal had to be violated, why is the violation so small in the $\pi^+\pi^-$ decay of K^0? Why is it so large in the leptonic and $\pi^+\pi^-\pi^0$ decays? Why is time reversal nearly invariant in the processes $n \to pe\nu$ and $\Lambda \to \pi^- p$? There have been various attempts[6–12] from various directions to answer some of these questions. It is of great interest to find a unified approach to answer all these questions.

In this work we postulate that while time-reversal invariance is not valid, reciprocity relations may be true. By reciprocity we mean the relation

$$|\langle A|M|B\rangle| = |\langle B^T|M|A^T\rangle|, \quad (1.2)$$

where M is the transition matrix and A^T and B^T are the time-reversed states of A and B, respectively. The two matrix elements in (1.2) would be equal, including the phase, if time-reversal invariance were valid.

In most cases the reciprocity relation (1.2) deals directly with measurements and hence is physically rather appealing. In the following sections we will show that while preserving many of the features of time-reversal invariance, reciprocity does allow for its violation in neutral K decays. It implies that the total mass matrix for K^0-$\bar{K}^0$ mixing is normal and hence the physical states K_s^0 and K_l^0 are orthogonal to each other. It also provides a sum rule for the violations of time-reversal invariance in various decays of K^0, which seems to be satisfied by the preliminary data on leptonic and 3π decays of the neutral K. It also provides an explanation for the fact that the processes $n \to pe\nu$ and $\Lambda \to \pi^- p$ are nearly time-reversal invariant.

In the following, we will first define reciprocity precisely and consider its general implications. Then we will apply the reciprocity relations to the problem of K^0-$\bar{K}^0$ mixing and describe the various consequences. Finally we will discuss the possible relevance of reciprocity to the strong and the electromagnetic interactions as well as to the weak interactions.

II. RECIPROCITY

It is perhaps an historical accident which leads us to equate the reciprocity principle to time-reversal invariance. In the consideration of time-reversed processes, we start from the assumption of reciprocity (1.2). If, furthermore, it is assumed that these relations hold true for any arbitrary states A and B, and that[13]

$$|\langle A|B\rangle| = |\langle A^T|B^T\rangle|$$

* Work supported by National Science Foundation.

[1] J. H. Christenson, J. W. Cronin, V. L. Fitch, and R. Turlay, Phys. Rev. Letters 13, 138 (1964).

[2] We use the recent value $R = (K_l \to \pi^+ + \pi^-)/(K_l \to \text{all charged}) = (1.87 \pm 0.2) \times 10^{-3}$. Taking into account all the neutral modes, we obtain the value quoted. We thank Professor V. L. Fitch for this communication.

[3] B. Aubert *et al.*, Phys. Letters 17, 59 (1965).

[4] M. Baldo-Ceolin *et al.*, Nuovo Cimento 38, 684 (1965).

[5] J. Anderson *et al.*, Phys. Rev. Letters 14, 475 (1965).

[6] M. T. Burgy, V. E. Krohn, T. B. Novey, G. R. Ringo, and V. L. Telegdi, Phys. Rev. 120, 1829 (1960).

[7] J. W. Cronin and O. E. Overseth, Phys. Rev. 129, 1795 (1963).

[8] R. G. Sachs and S. B. Treiman, Phys. Rev. Letters 8, 137 (1962); R. Sachs, *ibid.* 13, 286 (1964).

[9] N. Truong, Phys. Rev. Letters 13, 358 (1964).

[10] J. Bernstein, G. Feinberg, and T. D. Lee, Phys. Rev. 139, B1650 (1965).

[11] S. Glashow, Phys. Rev. Letters 14, 35 (1965).

[12] N. Cabibbo, Phys. Letters 12, 137 (1964); T. D. Lee and L. Wolfenstein, Phys. Rev. 138, B1490 (1965); and others.

[13] E. P. Wigner, *Group Theory* (Academic Press Inc., New York, 1959).

then there exists an antiunitary operator T such that

$$T|A\rangle=|A^T\rangle,\quad T^{-1}MT=M^\dagger,$$

and consequently

$$\langle A|M|B\rangle=\langle B^T|M|A^T\rangle, \tag{2.1}$$

which defines time-reversal invariance.

It is clear that reciprocity deals more directly with measurement. We also note that, in the course of discussion, if the generality of the states A, B, etc. is limited, then time-reversal invariance need not follow.[14] It is based on this observation that we shall formulate different reciprocity principles, especially in view of the present status of weak interactions. We will discuss only the following more interesting cases[15]:

(1) *Weak reciprocity* is the one in which A, B, etc. are states with definite total isospin for the participating hadrons.[16] Since the weak interaction discriminates between different isospin states, a principle of this nature is no more mysterious than, say, the $\Delta I=\frac{1}{2}$ rule in nonleptonic weak decays. If this is the case, and if only one isospin state dominates the final state of a decay process, then the effect of CP or T violation is small in this process. To see this,[14,17] we note that different angular-momentum amplitudes of an isospin assignment have the same CP-violating phase. Since CP-violating effects can be observed only through interference between different isospin states, our conclusion follows. Thus, assuming the $\Delta I=\frac{1}{2}$ rule, we can have little CP violation in $\Lambda\to N+\pi$ decay. Similarly, in any leptonic process, if the hadrons in the final product conspire to give only one isospin state, the CP violating effects are small. However, there are two possible final isospin states ($I=\frac{1}{2}, I=\frac{3}{2}$) in $\Sigma^+\to N+\pi$ decays, even under the assumption of the $\Delta I=\frac{1}{2}$ rule, and they may be comparable in magnitude. The CP-violating effects here may therefore be large. In passing, we remark that CP violation in the K^0-$\bar{K}^0$ system due to mixing is allowed.

(2) *Strong reciprocity* is the case in which A, B, etc. are the charged eigenstates of the strong-interaction Hamiltonian. For example, in Λ decays, they are $n\pi^0$, $p\pi^-$, etc. If this is the situation, then there can be no other observable CP-violating effects[17] than those due to K^0-$\bar{K}^0$ mixing. We shall use the Λ decays as an example for a proof of this statement.

[14] T. D. Lee, Columbia University report (unpublished).

[15] We shall assume the validity of the CPT theorem in our discussion. Thus, we shall use CP and T equivalently. Also, T invariance is assumed for strong interactions in Secs. II and III.

[16] If A, B, etc. are eigenstates of the strong S matrix, then reciprocity gives different T-violating phases to various angular-momentum matrix elements in decay processes, and therefore T violation can be large. This is, however, contrary to the results of experiment c mentioned in Sec. 1.

[17] Also refer to case (2). Proofs will be given to first order in H_W in cases (1) and (2).

Define $\langle(\pi^-p)_{\rm out}|$ as the outgoing state of π^-p, etc. By reciprocity,

$$\langle(\pi^-p)_{\rm out}|H_W|\Lambda\rangle=\langle(\pi^-p)_{\rm in}{}^T|H_W|\Lambda^T\rangle^*e^{i\phi},$$

and

$$\langle(\pi^0n)_{\rm out}|H_W|\Lambda\rangle=\langle(\pi^0n)_{\rm in}{}^T|H_W|\Lambda^T\rangle^*e^{i\phi'}. \tag{2.2}$$

We write

$$\langle(\pi N)^{(I)}{}_{\rm out}|H_W|\Lambda\rangle=\bar{u}_N(e^{i\delta_s(I)}a_s(I)+\boldsymbol{\sigma}\cdot\hat{p}e^{i\delta_p(I)}a_p(I))u_\Lambda$$

in the rest frame of Λ, where $\hat{p}$ is a unit vector along the π momentum, the u's are the spinor bases, I is the particular isospin channel, s and p stand for the s wave and p wave, respectively, and the δ's are the strong phase shifts in different channels of the final state. Equation (2.2) leads to

$$(\surd\tfrac{1}{3})e^{i\delta_l(1/2)}a_l{}^{(1/2)}-(\surd\tfrac{2}{3})e^{i\delta_l(3/2)}a_l{}^{(3/2)}=[(\surd\tfrac{1}{3})e^{i\delta_l(1/2)}a_l{}^{(1/2)*}-(\surd\tfrac{2}{3})e^{i\delta_l(3/2)}a_l{}^{(3/2)*}]e^{i\phi},$$

and

$$(\surd\tfrac{2}{3})e^{i\delta_l(1/2)}a_l{}^{(1/2)}+(\surd\tfrac{1}{3})e^{i\delta_l(3/2)}a_l{}^{(3/2)}=[(\surd\tfrac{2}{3})e^{i\delta_l(1/2)}a_l{}^{(1/2)*}+(\surd\tfrac{1}{3})e^{i\delta_l(3/2)}a_l{}^{(3/2)*}]e^{i\phi'},$$

$l=s,\ p$.

Writing $a_l{}^{(I)}=|a_l{}^{(I)}|e^{i\theta_l(I)}$, we have

$$(\surd\tfrac{1}{3})e^{i\delta_l(1/2)}|a_l{}^{(1/2)}|\sin(\theta_l(\tfrac{1}{2})-\tfrac{1}{2}\phi)=(\surd\tfrac{2}{3})e^{i\delta_l(3/2)}|a_l{}^{(3/2)}|\sin(\theta_l(\tfrac{3}{2})-\tfrac{1}{2}\phi),$$

and

$$(\surd\tfrac{2}{3})e^{i\delta_l(1/2)}|a_l{}^{(1/2)}|\sin(\theta_l(\tfrac{1}{2})-\tfrac{1}{2}\phi')=-(\surd\tfrac{1}{3})e^{i\delta_l(3/2)}|a_l{}^{(3/2)}|\sin(\theta_l(\tfrac{3}{2})-\tfrac{1}{2}\phi').$$

We shall assume that

$$\delta_l(1/2)\not\equiv\delta_l(3/2)\quad(\mathrm{mod}\,\pi),$$

then both sides of the above equations must vanish and we have

$$\phi\equiv\phi'(\mathrm{mod}\,2\pi)=2\theta_l(I). \tag{2.3}$$

Therefore, the $a_l{}^{(I)}$'s are relatively real; this means that there is no CP violation in Λ and, likewise, in Σ decays. This approach can be applied to include all presently observed decay processes. (In $K^+\to 3\pi$ decay, we assume s-wave totally-symmetric-state dominance.) Thus, except for mixing phenomena, which we shall discuss in the next section, strong reciprocity is equivalent to time-reversal invariance, at least to first-order perturbation of the weak interaction.

Finally, we add the strongest form of reciprocity:

(2′) A, B, etc. are arbitrary linear combinations of states, permissible by the superselection rules of strong interactions.[18] If this is the situation, there will be no CP-violating effects for any decay processes to all orders in H_W, except in K^0-$\bar{K}^0$ mixing. In addition, the inclusion of electromagnetism will not alter the situation.

[18] Thus, states with different charges are not to be superposed; but we propose that states with different parities can be.

Proof: Let A, or B_1, B_2, etc. be the incoming or outgoing states which correspond to the eigenstates of the strong S matrix. Consider the process $A \to B_{(x)}$, where

$$B_{(x)} = x_1 B_1 + x_2 B_2 + \cdots,$$

with x_1, x_2, $\cdots$ arbitrary. Then, reciprocity applied to $A \to B_{(x)}$ gives

$$\langle B_1|M|A\rangle = \langle A^T|M|B_1^T\rangle e^{i\phi},$$
$$\langle B_2|M|A\rangle = \langle A^T|M|B_2^T\rangle e^{i\phi}, \text{ etc.}$$

Since there is only one arbitrary common phase for all the amplitudes, the results are the same as those due to time-reversal invariance.

At present, we have no prejudice for any of the above forms of reciprocity, though philosophically the strong reciprocity is the more attractive since it would approximate time-reversal invariance more closely. The observation of the presence or the absence of CP violation in any nonmixing phenomena such as K^+, Λ, or Σ decays would determine a choice.

III. THE K^0-$\bar{K}^0$ MIXING AND NORMAL MASS MATRIX

In the following analysis we will assume the Weisskopf-Wigner method[19] of solving the time-dependent Schrödinger equation. The physical states K_s^0 and K_l^0 are coherent mixtures of K^0 and $\bar{K}^0$ states and are defined by

$$(\Gamma + iM)|K_{s,l}^0\rangle = \lambda_{s,l}|K_{s,l}^0\rangle, \tag{3.1}$$

where $(\Gamma + iM)$ is the total mass matrix, Γ and M being two 2×2 Hermitian matrices,

$$\Gamma = \Gamma^\dagger, \quad M = M^\dagger \tag{3.2}$$

and λ_s and λ_l are the two eigenvalues of the total mass matrix. The matrix elements of Γ and M are given by

$$\Gamma_{ij} = 2\pi \sum_n \langle a_i|H_W|n\rangle\langle n|H_W|a_j\rangle \tag{3.3}$$

$$M_{ij} = 2\sum_n \frac{\langle a_i|H_W|n\rangle\langle n|H_W|a_j\rangle}{E_j - E_n}, \tag{3.4}$$

where $a_1 = K^0$, $a_2 = \bar{K}^0$ and n is an intermediate state. By CPT invariance, we have[20]

$$\Gamma_{11} = \Gamma_{22}, \quad M_{11} = M_{22}. \tag{3.5}$$

We now apply the reciprocity relations to K^0-$\bar{K}^0$ mixing. In Appendix A, it is shown that the requirement of reciprocity for the transition matrix implies "reciprocity" for the mass matrix, so that

$$|\Gamma_{12} + iM_{12}| = |\Gamma_{21} + iM_{21}|. \tag{3.6}$$

Therefore by (3.2), (3.5), and (3.6) it follows that matrices Γ and M commute; i.e., the total mass matrix $(\Gamma + iM)$ is normal[21] and can be diagonalized by a unitary transformation. The K_s^0, K_l^0 states are then the orthogonal eigenstates, simultaneously, of Γ and M. Therefore, they can be written in the form

$$K_{s,l}^0 = (1/\sqrt{2})[K^0 \pm e^{i\theta}\bar{K}^0], \tag{3.7}$$

where

$$e^{i\theta} = (\Gamma_{21}/\Gamma_{21}^*)^{1/2} = (M_{21}/M_{21}^*)^{1/2}. \tag{3.8}$$

We shall relate θ to the various decay widths of K^0. We first note that by CPT invariance

$$|\langle i|H_W|K^0\rangle| = |\langle CPT(i)|H_W|\bar{K}^0\rangle|. \tag{3.9}$$

We choose the phase of K^0 and $\bar{K}^0$ so that

$$\langle 2\pi(I=0)_{\text{st}}|H_W|K^0\rangle = \langle 2\pi(I=0)_{\text{st}}|H_W|\bar{K}^0\rangle \tag{3.10}$$

and real which one can always do, where the subscript st represents a standing wave. Then from Eq. (3.8) we get

$$\tan\theta = -\sum_i \Gamma_i r_i \sin\phi_i / (\Gamma_0 + \sum_i \Gamma_i r_i \cos\phi_i), \tag{3.11}$$

where

$$\Gamma_0 = 2\pi|\langle 2\pi(I=0)|H_W|K^0\rangle|^2 \rho_{2\pi}, \tag{3.12}$$

$$\Gamma_i = 2\pi|\langle i|H_W|K^0\rangle|^2 \rho_i, \tag{3.13}$$

ρ being the final state density, and ϕ_i and r_i are defined by

$$\langle i_{\text{st}}|H_W|\bar{K}^0\rangle = r_i e^{i\phi_i}\langle i_{\text{st}}|H_W|K^0\rangle. \tag{3.14}$$

The index i runs over all the states into which K^0 can decay except the $I=0$ state of 2π; of these the only significant ones are the 3π, leptonic, and 2π $(I=2)$ states.

Experimentally we know that Γ_0 is much larger than $\Gamma_i r_i$ so that

$$\theta = -(1/\Gamma_0)\sum_i \Gamma_i r_i \sin\phi_i. \tag{3.15}$$

Thus we see that θ is necessarily small, of the order of the branching ratio of K^0 decay into the 3π or leptonic mode to that into the 2π mode. Of course, if time-reversal invariance were valid, all $\phi_i = 0$ and CP would be conserved. We will now discuss the sum rule (3.15) term by term.

(1) *3π decay:* In general there are several 3π final isospin states allowed, for each of which, by CPT invariance (3.9), $r_i = 1$. For simplicity of analysis however, we will assume that only one of the final isospin states dominates,[22] or if there are more than one important final states we will assume that ϕ is the same for each of these states. (The latter is the case if the strong reciprocity is assumed.) This assumption is not essential but it greatly simplifies our analysis of the

[19] V. F. Weisskopf and E. P. Wigner, Z. Physik **63**, 54 (1930); **65**, 18 (1930).

[20] T. D. Lee, R. Oehme, and C. N. Yang, Phys. Rev. **106**, 340 (1957).

[21] A matrix A is normal if $[A, A^\dagger] = 0$, and a normal matrix can always be diagonalized by a unitary transformation.

[22] The experimental value of the branching ratio $\Gamma(K^+ \to \pi^+\pi^+\pi^-)/\Gamma(K^+ \to \pi^+\pi^0\pi^0)$ is consistent with the assumption that the totally symmetric state with $I=1$ or the 3π system dominates $K^+ \to 3\pi$ decay.

experimental data. Under this assumption, we have $r_{3\pi}=1$ and we will evaluate $\phi_{3\pi}$ by using the experimental data[5] for $\pi^+\pi^-\pi^0$ decay of K^0. From the experiment we have

$$\left(\frac{\Gamma_{K_s\to\pi^+\pi^-\pi^0}}{\Gamma_{K_l\to\pi^+\pi^-\pi^0}}\right)^{1/2}=\left|\frac{1+e^{i\phi_{3\pi}}}{1-e^{i\phi_{3\pi}}}\right|=1.03\pm0.65\,, \quad (3.16)$$

so that

$$\phi_{3\pi}=-88°\pm36°. \quad (3.17)$$

The sign of $\phi_{3\pi}$ depends upon the sign of $m_{K_s}-m_{K_l}$ and the above value is for $m_{K_s}-m_{K_l}$ being negative.[23] Comparing the rate of $K_l\to3\pi$ with $K_s\to2\pi$, we get

$$\Gamma_{K_l\to3\pi}/\Gamma_{K_s\to2\pi}=\Gamma_{3\pi}(1-\cos\phi_{3\pi})/2\Gamma_0 = (6.0\pm0.8)\times10^{-4}. \quad (3.18)$$

Therefore,[24]

$$\Gamma_{3\pi}\sin\phi_{3\pi}/\Gamma_0=-(1.2\pm0.8)\times10^{-3}. \quad (3.19)$$

(2) *Leptonic decays:* In leptonic decays of neutral K, if the $\Delta S=\Delta Q$ rule holds, then r_l is either zero or infinite, depending on the charge of the lepton. In either case the leptonic decay mode does not contribute to the sum rule (3.15), nor does it exhibit any effects of CP violation. However, recent analyses[3,4] do indicate that r_l is not zero and hence $\Delta S=-\Delta Q$ decay may be allowed. Thus for $\pi^-e^+\nu$ decay,

$$r_l=0.33\pm0.17 \quad\text{and}\quad \phi_l=-80°{}_{-52°}^{+26°}. \quad (3.20)$$

These values are the average values of those quoted in Refs. 3 and 4. We shall assume the same values for μ-leptonic decays, which is justifiable under the assumptions of μ-e universality and conserved vector current. However, it would be of interest to determine these parameters experimentally for μ-leptonic decays. Then we have

$$\frac{\Gamma_{K_l\to l}}{\Gamma_{K_s\to2\pi}}=\frac{\Gamma_l(1+r_l^2-2r_l\cos\phi_l)}{4\Gamma_0}=(1.0\pm0.2)\times10^{-3}. \quad (3.21)$$

Therefore,

$$\Gamma_l r_l\sin\phi_l/\Gamma_0=(-1.3\pm0.7)\times10^{-3} \quad (3.22)$$

(3) *2π decays:* The ratio $\Gamma_{2\pi(I=2)}/\Gamma_0=\Gamma_2/\Gamma_0$ can be estimated by

$$\Gamma_2/\Gamma_0=\tfrac{4}{3}f(\Gamma_{K^+\to\pi^+\pi^0})/(\Gamma_{K_s\to2\pi}) =f(2.0\pm0.4)\times10^{-3}, \quad (3.23)$$

where

$$f=\tfrac{3}{2}[\Gamma_{K_0\to2\pi(I=2)}]/[\Gamma_{K^+\to\pi^+\pi^0}]$$
$=1$ if the $\Delta I=\frac{3}{2}$ component dominates in $K\to2\pi$ $(I=2)$ decay,
$=(9/4)$ if the $\Delta I=\frac{5}{2}$ component dominates.

Clearly by CPT invariance we have $r_2=1$.

Using this information, we get the estimate of the mixing angle θ:

$$\theta=(2.5\pm1.1)\times10^{-3}-(2.0\pm0.4)\times10^{-3}f\sin\phi_2. \quad (3.24)$$

The angles θ and ϕ_2 are related to the ratios of the amplitudes $a_{K_{l,s}\to2\pi}$ of $K_{l,s}\to2\pi$ decays as[25]

$$\eta_{+-}=(a_{K_l\to\pi^+\pi^-})/(a_{K_s\to\pi^+\pi^-})=-\tfrac{1}{2}i\theta -i(\sqrt{2})^{-1}\exp[i(\delta_2-\delta_0)](\Gamma_2/\Gamma_0)^{1/2}\sin\tfrac{1}{2}\phi_2 \quad (3.25a)$$

and

$$\eta_{00}=(a_{K_l\to\pi^0\pi^0})/(a_{K_s\to\pi^0\pi^0})=-\tfrac{1}{2}i\theta +i\sqrt{2}\exp[i(\delta_2-\delta_0)](\Gamma_2/\Gamma_0)^{1/2}\sin\tfrac{1}{2}\phi_2\,, \quad (3.25b)$$

where δ_2 and δ_0 are the s-wave phase shifts of $I=2$ and $I=0$ states of 2π system. From these, we get a sum rule

$$\eta_{00}+2\eta_{+-}=-\tfrac{3}{2}i\theta. \quad (3.26)$$

The magnitude of η_{+-} has been measured by experiment[1] and is given in Eq. (1.1). The phase of η_{+-}, and thus ϕ_2 through Eqs. (3.24) and (3.25b), can be obtained by the interference experiment on the $K_0\to\pi^+\pi^-$ decay, as has been done[3–5] in the experiments on $K_0\to3\pi$ or K_0 leptonic decays.

If we require the strong reciprocity, we obtain

$$\phi_2=0\,,$$

since the CP-violating angles of $K^0\to2\pi(I=0)$ and $K^0\to2\pi(I=2)$ are equal, according to an argument similar to that in the preceding section, and we have chosen the convention (3.10). Then the sum rules (3.24) and (3.25) lead to

$$2\eta_{+-}=2\eta_{00}=-i\theta=-i(2.5\pm1.1)\times10^{-3}. \quad (3.27)$$

The prediction is about one standard deviation off the experimental value (1.1).

If we assume weak reciprocity, ϕ_2 is not zero, in general. On the assumption that f in Eq. (3.23) is of the order of unity, the angle ϕ_2 can at most be $\sim\frac{1}{5}$ according to Eqs. (1.1), (3.24), and (3.25a). Together with (3.26), we obtain the inequalities

$$(0.2\pm1.7)\times10^{-3}\leq|\eta_{00}|\leq(7.4\pm1.7)\times10^{-3}. \quad (3.28)$$

If, however, the condition $|\sin(\delta_2-\delta_0)|\ll1$ is assumed, which seems reasonable, we have the prediction that $|\eta_{00}|$ has the value of either of the boundary values of (3.28). The present experiment on $|\eta_{00}|$, which gives an upper bound[25] $|\eta_{00}|\leq\sim2\times10^{-2}$, is consistent with both predictions (3.27) and (3.28). It would be pertinent

[23] The relative sign of y and $m_{K_s}-m_{K_l}$ in Ref. 5 isin correct. We thank Professor F. Crawford for this communication [Phys. Rev. Letters 15, 645 (1965)].

[24] If the $\Delta I=\frac{1}{2}$ rule is assumed in $K\to3\pi$ decay, we may use the second alternative experimental value of Ref. 23. This gives a value of $-(1.0\pm0.6)\times10^{-3}$ for Eq. (3.19).

[25] T. T. Wu and C. N. Yang, Phys. Rev. Letters 13, 380 (1964)

to have a more precise measurement of $|\eta_{00}|$ as well as of the phases of η_{+-} and η_{00}.

IV. SUMMARY AND DISCUSSIONS

(1) We have seen that the requirement of reciprocity leads to the normality of the total mass matrix in the K^0-$\bar{K}^0$ mixing problem and thus implies the orthogonality of K_l^0 and K_s^0 states. We note that the requirement is equivalent, in this specific problem, to the variational principle for the total mass matrix

$$\delta(\langle\Psi|\Gamma+iM|\Psi\rangle/\langle\Psi|\Psi\rangle)=0 \tag{4.1a}$$

with

$$\Psi=aK^0+b\bar{K}^0. \tag{4.1b}$$

In fact, when the total mass matrix is normal, the decay rate of K_l^0 is a minimum and the mass difference $|m_{K_l}-m_{K_s}|$ is a maximum; thus normality leads to the most stable solution.

It is perhaps worth noting that even if reciprocity itself is not valid, the mass matrix may yet be normal, in which case the sum rule (3.15) would still be valid. We feel, however, that reciprocity provides an attractive alternative to time-reversal invariance and deserves attention.

(2) Under the assumption of strong reciprocity, we predict the following: ($S1$) No CP or T violation can be observed except in the K^0-$\bar{K}^0$ decay. ($S2$) For $K_{s,l}{}^0$ decay, we have $\eta_{+-}=\eta_{00}=-i\theta/2$. This would imply that the $\Delta I=\frac{1}{2}$ rule is valid for $K_l^0\rightarrow 2\pi$ decay.

The prediction ($S1$) is consistent with experiments (a)–(c) of Sec. I and the prediction ($S2$) deviates by about one standard deviation from the experiment. If consistency is shown by more accurate experiments, we may say that we understand the smallness of the experimental value of $|\eta_{+-}|$.

On the other hand, if we assume the weak form of reciprocity, we have: ($W1$) CP or T violation is small in the decay processes other than K^0-$\bar{K}^0$, if the final state is dominated by one isospin state of hadrons. Therefore most of the leptonic decays and also the nonleptonic decays which obey the $\Delta I=\frac{1}{2}$ rule will not show large CP or T violation. An exception is the Σ^+ decay for which we may observe a significant T violation. ($W2$) The sum rule (3.15) as well as the relations (3.25) must be further examined by experiments.

(3) In view of the significant role of the $\Delta I=\frac{1}{2}$ rule in the nonleptonic weak decays, we may postulate another type of reciprocity.

Decompose the decay transition matrix M into a sum of components $M^{(i)}$ which have definite isospin transformation properties:

$$M=\sum_{i=\Delta I} M^{(i)}. \tag{4.2}$$

Then we require reciprocity by

$$|\langle A|M^{(i)}|B\rangle|=|\langle B^T|M^{(i)}|A^T\rangle|, \tag{4.3}$$

where the states A, B are arbitrary linear combinations of the states permissible by the superselection rules of the strong interaction[18] but are restricted by the isospin property of $M^{(i)}$. If this is the case, we obtain predictions the same as those of the weak reciprocity, except for the $\Sigma^+\rightarrow N\pi$ decay, for which T violation will be small if the $\Delta I=\frac{1}{2}$ rule is valid.

(4) At present, the experiment on $\Sigma\rightarrow N\pi$ decay does not satisfy the $\Delta I=\frac{1}{2}$ rule very well as compared with the case of the other baryonic decays,[26] if we assume T invariance. This might be considered as an indication favoring weak reciprocity. (See, however, Franzini *et al.*,[27] who suggest that the validity of the $\Delta I=\frac{1}{2}$ rule with T invariance in Σ decay is not excluded in the sense of a χ^2 test. It is important to clarify this point in Σ decay.)

(5) Experiments seem to show that the strong interactions as well as the electromagnetic interactions of leptons are invariant under C, P, and T separately.[28] If, however, reciprocity is a more fundamental principle than time-reversal invariance, as was postulated in this article, and we require the strong form of reciprocity, (2′) of Sec. II, for the strong interactions, then it follows that T and CP are invariant except in neutral K decays. Note that CPT invariance, which we assume here, follows from such general principles as Lorentz invariance and local commutativity, etc. Then, restricting ourselves to the strong interactions, we may ask why they are C- and P-invariant separately.

If the basic Hamiltonian is constructed with the observed baryons and mesons, it is hard to find the principles which provide C and P invariance from CP invariance.[29] We note, however, that if the basic Hamiltonian is made up of the unitary (SU_3) triplet quark baryons and a unitary-singlet meson (with spin 0 and/or spin 1), then we can find the principles mentioned above: the assumptions of nonderivative Yukawa-type interactions, isospin invariance, and conservation of currents generated by the spin-1 meson, together with CP invariance, are sufficient to guarantee C and P invariance.[29]

Minimal electromagnetic interactions which are derived by the principle of gauge invariance are C- and P-invariant accordingly.

Thus, in such a model, we have a unified picture for C, P, and T invariance of the strong, electromagnetic, and weak interactions.

[26] For the status of the $\Delta I=\frac{1}{2}$ rule in Σ decays, see R. Dalitz, in Proceedings of the International Conference on Fundamental Aspects of Weak Interactions, Brookhaven National Laboratory Report No. BNL 837 (C–39), 1963, p. 378 (unpublished).

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APPENDIX A: RELATIONS OF THE TRANSITION MATRIX AND THE MASS MATRIX

In this Appendix, we shall derive general expressions for the transition matrix and the mass matrix. We shall see that they are given essentially by the same equation. We shall follow the work of Arnous and Zienau.[30]

The transition matrix $S(t,t_1)$ in the Schrödinger picture satisfies the equation

$$i(\partial/\partial t)S(t,t_1)=(H+H_W)S(t,t_1)\,, \tag{A1}$$

where H includes the free, strong, and electromagnetic interactions, while H_W describes the weak interactions. The initial condition is

$$S(t_1,t_1)=1\,. \tag{A2}$$

The solution of (A1) and (A2) is

$$S(t,t_1)=\exp[-i(H+H_W)(t-t_1)]\,. \tag{A3}$$

Its Fourier decomposition is

$$S(t,t_1)=\int_{-\infty}^{\infty}dE\,\exp\{-iE(t-t_1)\}S(E)\,. \tag{A4}$$

To obtain the solution (A3) for $t>t_1$, we have

$$S(E)=(i/2\pi)(E-H-H_W+i\epsilon)^{-1}\,,$$

or

$$(E-H-H_W+i\epsilon)S(E)=i/2\pi\,. \tag{A5}$$

In order to describe a perturbation problem, with H_W as the perturbation, it is better to write $S(E)$ in a different form. Let us write $S(E)$ as

$$S(E)=\{1+(E-H+i\epsilon)^{-1}M(E)\}\Lambda(E) \tag{A6}$$

and also make the following assumptions:

(1) $\Lambda(E)$ is a diagonal matrix, except in the K^0-$\bar{K}^0$ subspace. There it is a nonsingular 2×2 matrix.

(2) $M(E)$ is a matrix, the diagonal elements and the K^0-$\bar{K}^0$ submatrix of which vanish.

These assumptions are made in anticipation of the final results we want, which are physically well understood.

Substituting (A6) into (A5) and after some rearrangement, we have

$$\{E-H+M(E)-[H_W+H_W(E-H+i\epsilon)^{-1}M(E)]\}\times\Lambda(E)=i/2\pi\,. \tag{A7}$$

[30] E. Arnous and S. Zienau, Helv. Phys. Acta **24**, 279 (1951); H. Umezawa, *Quantum Field Theory* (North-Holland Publishing Company, Amsterdam, 1956), p. 301.

The off-diagonal elements of this equation give

$$M(E)=\{H_W+H_W(E-H+i\epsilon)^{-1}M(E)\}_{\text{n.d.}}\,, \tag{A8}$$

where n.d. (nondiagonal) means

$$\langle j|M(E)|j\rangle=\langle K^0|M(E)|\bar{K}^0\rangle=\langle\bar{K}^0|M(E)|K^0\rangle$$
$$=\langle K^0|M(E)|K^0\rangle=\langle\bar{K}^0|M(E)|\bar{K}^0\rangle=0\,;$$

j is any eigenstate of H.

We now take matrix elements of (A7) between $\langle i|$ and $|i\rangle(\neq|K^0\rangle,\,|\bar{K}^0\rangle)$. It is easily seen that

$$\Lambda(E)=(i/2\pi)\{E-H+\tfrac{1}{2}i\bar{\Gamma}(E)\}^{-1}$$

with

$$\bar{\Gamma}(E)=2i\{H_W+H_W(E-H+i\epsilon)^{-1}M(E)\}_{\text{d.}}\,, \tag{A9}$$

where d. (diagonal) denotes that only $\langle i|\bar{\Gamma}(E)|i\rangle\neq 0$, when $|i\rangle\neq|K^0\rangle,|\bar{K}^0\rangle$.

In the K^0-$\bar{K}^0$ subspace, we have the equation

$$\mathfrak{M}(E)\Lambda(E)=i/2\pi\,,$$

in which

$$\mathfrak{M}(E)=E-H-[H_W+H_W(E-H+i\epsilon)^{-1}M(E)]\,.$$

Let us denote the eigenvalues of $\mathfrak{M}(E)$ by λ_i, the corresponding right eigenvectors by $|b_i\rangle$, i.e.

$$\mathfrak{M}(E)|b_i\rangle=\lambda_i|b_i\rangle\,,\quad i=s,\,l\,,$$

and the corresponding left eigenvectors by $\langle\bar{b}_i|$, i.e.,

$$\langle\bar{b}_i|\mathfrak{M}(E)=\lambda_i\langle\bar{b}_i|\,.$$

$|b_i\rangle$ are the conventionally defined[25] $|K_s\rangle$ and $|K_l\rangle$. The $\langle\bar{b}_i|$ are in general not the complex conjugates[31] of the $|b_i\rangle$. It can be shown that with proper normalization, we have

$$\langle\bar{b}_i|b_j\rangle=\delta_{ij}\,,$$

and

$$\textstyle\sum_i|b_i\rangle\langle\bar{b}_i|=1\,.$$

Some simple algebra then leads to

$$\Lambda(E)=(i/2\pi)\textstyle\sum_i|b_i\rangle(1/\lambda_i)\langle\bar{b}_i|$$
$$=(i/2\pi)\textstyle\sum_i|b_i\rangle[E-m+\tfrac{1}{2}i\bar{\Gamma}_i(E)]^{-1}\langle\bar{b}_i|\,,$$

where

$$H|b_i\rangle=m|b_i\rangle\,,$$

and[32]

$$\bar{\Gamma}_i(E)=2i\langle\bar{b}_i|H_W+H_W(E-H+i\epsilon)^{-1}M(E)|b_i\rangle\,. \tag{A10}$$

The development from this point on follows strictly that of Arnous and Zienau. It can be shown that[30] for

$$\tilde{S}(t,t_1)=\exp\{iH(t-t_1)\}S(t,t_1)\,,$$

which describes transitions due to H_W, it has matrix

[31] R. Jacob and R. G. Sachs, Phys. Rev. **121**, 350 (1961); R. G. Sachs, Ann. Phys. (N. Y.) **22**, 239 (1963).

[32] $\bar{\Gamma}$ is equal to $(\Gamma+iM)$ of the previous notation.

elements

$$\langle f|\bar{S}(t,-\infty)|i\rangle = -\langle f|M(E_f)|i\rangle / [E_f - E_i + \tfrac{1}{2}i\bar{\Gamma}_i(E_f)], \quad \text{(A11)}$$

where

$|i\rangle \neq |f\rangle$ and $|i\rangle$, $|f\rangle \neq a|K^0\rangle + b|\bar{K}^0\rangle$, $\bar{\Gamma}_i = \langle i|\bar{\Gamma}|i\rangle$,

and E_i, E_f are energies of the initial and the final states, respectively. Also,

$$\langle f|\bar{S}(t,-\infty)|i\rangle = \sum_j \frac{-\langle f|M(E_f)|b_j\rangle\langle\bar{b}_j|i\rangle}{E_f - m + \frac{1}{2}i\bar{\Gamma}_j(E_f)}, \quad \text{(A12)}$$

when $|f\rangle \neq a|K^0\rangle + b|\bar{K}^0\rangle$, but $|i\rangle$ is some linear combination $|K^0\rangle$ and $|\bar{K}^0\rangle$.

From (A11) and (A12), it can be seen[30] that $M(E)$ is the matrix which describes transitions between eigenstates of H, and $\bar{\Gamma}(E)$ is the matrix which gives the self-energy corrections, the mixing of K^0 and $\bar{K}^0$, and the decay widths due to H_W. Looking at (A8), (A9), and (A10), we see that $M(E)$ and $\bar{\Gamma}(E)$ are essentially given by the same matrix operator.

It is clear that this approach can be generalized to mixing problems when there are more than two degenerate levels.

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Vertex Poles and Bound States in the Lee Model*

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We study the $Z=0$ limit of a version of the Lee model, recently introduced and solved by Bronzan, from the point of view of recent work by Gerstein and Deshpande. It is shown how vertex function and inverse propagator poles develop and behave for small vertex renormalization constant, and their connection with the bound-state limit is studied. It is found that the condition of finite mass renormalization in the $Z_V=0$ limit can be satisfied in this model and leads to bootstrap-type results.

I. INTRODUCTION

In a recent article[1] we considered the problem of how to define a bootstrap in the context of Lagrangian field theory. We showed that if in the limit $Z_A=0$, where A is the bootstrapped particle and Z_A its wave-function renormalization constant, we also had finite self-mass, or even only

$$\lim_{Z_A\to 0} Z_A \delta\mu_A = 0, \quad (1)$$

where $\delta\mu_A$ is its mass renormalization, then the solution is identical to that of the usual bootstrap theory based on crossing symmetry, partial-wave dispersion relations and the N/D method.[2]

Explicitly, we wrote the partial-wave scattering aplitude as

$$T(s) = \Gamma(s)\Delta(s)\Gamma(s) + U(s), \quad (2)$$

where $\Gamma(s)$ is the vertex function and $\Delta(s)$ the propagator of the A particle. The first term, the single-particle reducible part (RP), contains all diagrams in which A appears as an intermediate state and we found that in the limit (1) this term vanished and $U(s)$ contained the A-particle pole with the correct residue. The mechanism by which this occurs is that as Z_A approaches zero the vertex function and the inverse propagator develop poles which move down to μ_A in the limit. These poles give rise to a pole in the RP, which however is cancelled by an identical term which appears in $U(s)$.[3] However, in the limit of $Z_A=0$ it also cancels the elementary-particle pole in the RP at μ_A, and this entire term vanishes leaving us with

$$T(s) = U(s). \quad (3)$$

It is clear that this is the only way we can get a bootstrap since the residue of the elementary-particle pole, g^2, is nonzero in the $Z_A=0$ limit and hence this pole must be cancelled if we are to obtain (3) in the limit.

In the present paper we shall study the above mechanism in a soluble model, the version of the Lee model[4] recently introduced and solved by Bronzan.[5] Although this model has no crossing symmetry so that, strictly speaking, we cannot have a bootstrap solution, it is clear from the above that the critical point is obtaining Eq. (3) from Eq. (2). The cancellation of poles and resultant vanishing of the RP are the physical basis of the bootstrap and this can be studied even without crossing; indeed in Ref. (1) we demonstrated that an

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