ment. Now we define Klein-transformed Green components, $\Psi_i^{(\alpha)}(x)$, which are local with respect to all $\varphi_i^{(\alpha)}$,

$$\Psi_i^{(\alpha)}(x)=(-)^{N(i,\alpha)}\varphi_i^{(\alpha)}(x).$$

Here

$$N(i,\alpha)=\sum_{j,\beta}\chi(i,\alpha;\,j,\beta)n(j,\beta),$$

and $n(j,\beta)$ is the number operator for the component field $\varphi_j^{(\beta)}$. Note that

$$[n(j,\beta),\varphi_k^{(\gamma)}(x)]_-=0\,,\ \text{unless}\ j=k\,,\ \text{and}\ \beta=\gamma.$$

Now we are prepared to use a Reeh-Schlieder argument[3] to prove the desired result. Assume that $\mathcal{P}$ is a polynomial of parafields localized in a region B and that

$$\mathcal{P}\Phi_0=0\,, \tag{2}$$

where Φ_0 is the vacuum of the parafield representation $\mathcal{A}$. Note that Φ_0 is also the vacuum of $\mathcal{B}$. Then multiply Eq. (2) by a polynomial $\mathcal{P}'$ of Klein-transformed Green component fields $\Psi_i^{(\alpha)}$ localized in a region B' spacelike to B. Since $\Psi_i^{(\alpha)}$ is local with respect to all $\varphi_j^{(\beta)}$, $\mathcal{P}'$ commutes with $\mathcal{P}$. Therefore we have

$$\mathcal{P}'\mathcal{P}\Phi_0=\mathcal{P}\mathcal{P}'\Phi_0=0. \tag{3}$$

Since $\mathcal{P}'\Phi_0$ is dense in $\mathcal{B}$ because of the Reeh-Schlieder theorem, Eq. (3) implies that $\mathcal{P}$ vanishes acting on $\mathcal{B}$,[4] which is the separating property we wanted to prove.

[3] H. Reeh and S. Schlieder, Nuovo Cimento 22, 1051 (1961). Since a monomial in the $\Psi_i^{(\alpha)}$ acting on Φ_0 equals the same monomial in the $\varphi_i^{(\alpha)}$ acting on Φ_0 up to a constant factor ±1, the known Reeh-Schlieder property for the $\varphi_i^{(\alpha)}$ implies the same property for the $\Psi_i^{(\alpha)}$.

[4] Even for unbounded Bose fields, the Reeh-Schlieder theorem proves that $\mathcal{P}=0$ on the standard polynomial domain of Wightman.

PHYSICAL REVIEW VOLUME 142, NUMBER 4 FEBRUARY 1966

Effect of Intermediate Vector Mesons on Neutrino Production of Lepton Pairs*

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(Received 8 March 1965; revised manuscript received 25 October 1965)

The total cross section for the production of lepton pairs by the scattering of high-energy muon neutrinos and protons is calculated with an assumed weakly interacting vector meson W which has an anomalous magnetic moment κ. We use a modified Weizsäcker-Williams approximation, with added correction terms which change the result about 50%. In addition to these correction terms, we keep t^2, the square of the four-momentum of the exchanged photon, in the denominator of the lepton propagators. This results in an additional deviation of 30% from the standard Weizsäcker-Williams approximation. The cross sections are calculated as a function of κ for meson masses of 2.0, 2.5, and 3.0 BeV, and for neutrino energies above the threshold for W production. Numerical results are presented for $\kappa=1,\ 0,\ -1$. The cross sections for muon-pair and muon-electron production are equal, and larger by a factor of approximately 10^4 than the equivalent cross sections in the four-fermion theory.

1. INTRODUCTION

MANY authors have discussed the idea that the V-A theory of weak interactions is a phenomenological theory, and that the basic theory is one in which a massive vector meson is coupled to parity-nonconserving currents, formed from bilinear combinations of fermion fields.[1,2]

Experiments designed to search for the vector meson, by using high-energy neutrino beams, examine the production of lepton pairs in neutrino-nucleon collisions. If lepton pairs are produced via a four-fermion interaction, then

$$\sigma_{\mu^-l}^{(4F)}\simeq M_pE_\nu G_v^2\alpha^2,$$

where σ_{μ^-l} is the cross section for the process

$$\nu_\mu+p\rightarrow\nu_l+p+\mu^-+l,$$

l is a lepton, either e^+ or μ^+, M_p is the proton mass, E_ν is the incoming neutrino laboratory energy. G_v is the Fermi coupling constant $[G_v=(1.01\times10^{-5})/M_p^2]$, and α is the fine-structure constant ($\alpha=1/137$). For neutrino energies greater than 1 BeV, this estimate yields a cross section approximately equal to 10^{-42} cm^2. This estimate is also applicable even when vector mesons are

* This research was supported in part by the National Science Foundation.

[1] T. D. Lee and C. N. Yang, Phys. Rev. 119, 1410 (1960).

[2] G. Feinberg and A. Pais, Phys. Rev. 133, B477 (1964).

involved, provided that the neutrino energy is below the threshold for real W production. As soon as real W's can be produced, however, the cross section rapidly increases by a factor of about 10^5, since it is then of the order of the cross section for W production times the branching ratio for $W \to$ leptons. For this estimate we obtain $\sigma_{\mu^- l}{}^{(W)} \approx G_v \alpha^2 \approx 10^{-37}$ cm^2. Since the observed $\sigma_{\mu^- l}$ is well below this number, we conclude that if W's exist, their mass must be so high that present-day neutrino beams do not have enough energy to produce them, and this sets a lower limit on the W mass. The neutrino experiments indicate the lower limit on the meson mass is about 2.0 BeV.[3]

Several authors have calculated the pair-production cross section in the four-fermion theory.[4,5] In this paper, we describe a calculation of the cross section in the vector-meson theory for neutrino energies above the W-production threshold. The results bear out the crude estimates given above.

The following is a short outline of the calculation: because of its short lifetime ($<10^{-17}$ sec), the W is treated in the standard resonance approximation.[6] The electromagnetic interactions with the proton are calculated using the recent experimentally determined form factors.[7] The electromagnetic interaction of the W is the usual one which includes the effect of an anomalous magnetic moment.[8] Finally, a modified version of the Weizsäcker-Williams approximation (MWW) is employed which includes additional terms that change the result about 50%. In the calculation of the cross section for $\gamma+\nu_\mu \to \nu_l+\mu^-+l$, which is needed for the Weizsäcker-Williams approximation, we do not set $t^2=0$ in the denominators of the lepton propagators (t^2 is the square of the four-momentum of the photon). This results in an additional deviation of 30% from the standard Weizsäcker-Williams approximation.

Since we are using a resonance approximation, it is obviously necessary to have a reasonably accurate value for the total decay rate of $W \to$ everything. We use the values $\Gamma_l/\Gamma=0.65$ for $M_W=2.5$ BeV, and $\Gamma_l/\Gamma=0.72$ for $M_W=3.0$ BeV, where $\Gamma_l=\Gamma_{\mu\nu}+\Gamma_{e\nu}=G_v M_W{}^3/3\pi\sqrt{2}$.[1] For $M_W=2.0$ BeV, we use $\Gamma_l/\Gamma=1.0$. A detailed calculation of Γ for $M_W=2.5$ and 3.0 BeV has been done by Carhart and Dooher.[9] The value chosen for $M_W=2.0$ BeV is justified by noting that phase-space factors depress the rate for $W \to 2$ baryons, and form factors depress the rates for $W \to$ mesons if we assume that the form factors are dominated by low-lying resonances.[9]

[3] R. Burns, K. Goulianos, E. Hyman, L. Lederman, W. Lee, N. Mistry, J. Rettberg, M. Schwartz, J. Sunderland, and G. Danby, Phys. Rev. Letters 15, 42 (1965).
[4] G. N. Stanciu, Phys. Rev. Letters 13, 288 (1964).
[5] W. Czyz, G. C. Sheppey, and J. D. Walecka, Nuovo Cimento 34, 404 (1964).
[6] Z. Fraenkel, Nuovo Cimento 30, 513 (1963).
[7] J. R. Dunning, Jr., K. W. Chen, A. A. Cone, G. Hartwig, N. F. Ramsey, J. K. Walker, and R. Wilson, Phys. Rev. Letters 13, 631 (1964).
[8] T. D. Lee and C. N. Yang, Phys. Rev. 128, 885 (1962).
[9] R. Carhart and J. Dooher, Phys. Rev. 142, 1214 (1966).

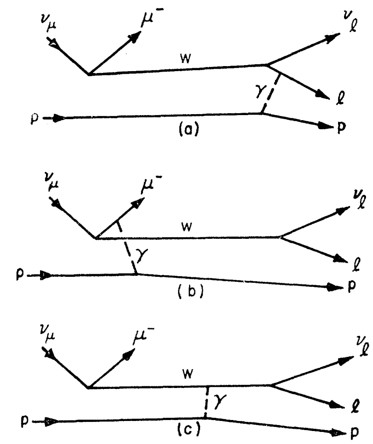

FIG. 1. Feynman diagrams for the process $\nu_\mu+p\to\nu_l+p+\mu^-+l$ with an assumed intermediate vector meson W. Here l is a lepton, either e^+ or μ^+.

Hence, we assume that Γ is almost entirely leptonic for $M_W=2.0$ BeV.

Finally, we indicate that it is not immediately obvious that MWW is a good approximation in this problem. The use of MWW is justified in the following section by a detailed examination of the contributions to the cross section from the different regions of phase space.

2. DIFFERENTIAL CROSS SECTION FOR $\nu_\mu+p\to\nu_l+p+\mu^-+l$; A MODIFIED WEIZSÄCKER-WILLIAMS APPROXIMATION

The relevant Feynman graphs for the process $\nu_\mu+p\to\nu_l+p+\mu^-+l$ are shown in Fig. 1. The cross section can be written as

$$\sigma=\frac{1}{(2\pi)^8}\frac{M_p{}^2}{|(p_1\cdot k_1)|}\int\frac{d^3p_2}{E_2}\int\frac{d^3p^+}{\epsilon_+}\int\frac{d^3p^-}{\epsilon_-}\int\frac{d^3k_2}{\epsilon_2}\times\delta^4(p^++p^-+p_2+k_2-p_1-k_1)\tfrac{1}{2}\sum_{\text{spins}}|\mathfrak{M}|^2,\quad(1)$$

where $p_{1\mu}$, $k_{1\mu}$, are the four-momenta of the incoming proton and neutrino, respectively, and $p_{2\mu}$, $k_{2\mu}$, $p_\mu{}^-$, $p_\mu{}^+$ are the four-momenta of the outgoing proton, neutrino, muon, and positively charged lepton, respectively. $\mathfrak{M}$ is the usual Feynman matrix element. In the lowest order in g, (the weak coupling of the vector meson to fermion currents), and α, in which the process can occur, we can write $\mathfrak{M}$ as follows:

$$\mathfrak{M}=(\bar{U}_2 J_\mu U_1)(t^{-2}A_\mu),\quad(2)$$

where U_1 and U_2 are the proton spinors, $t^{-2}A_\mu$ includes everything else, and t^2 is the square of the momentum transfer to the proton.[10]

The photon-proton vertex can be written

$$\bar{U}_2 J_\mu U_1=ie\gamma_\mu F_1(t^2)-i\eta(e/2M_p)\sigma_{\mu\nu}t_\nu F_2(t^2),\quad(3)$$

[10] We are using the metric corresponding to $A\cdot B=A_\mu B_\mu=\mathbf{A}\cdot\mathbf{B}-A_0\cdot B_0$. We use $\hbar=c=1$.

where we have included the effects of the strong interactions in the form factors F_1 and F_2. η is the anomalous magnetic moment of the proton ($\eta=1.79$).

The next step in the calculation approximates

$$\int\frac{d^3p^+}{\epsilon_+}\int\frac{d^3p^-}{\epsilon_-}\int\frac{d^3k_2}{\epsilon_2}\times\delta^4(p^++p^-+k_2+p_2-p_1-k_1)\sum_{\text{spins}}|\mathfrak{M}|^2$$

by the use of a modified version of the Weizsäcker-Williams approximation derived as follows. (For a discussion of the standard covariant Weizsäcker-Williams approximation, see Refs. 11–13.) We can write

$$\sum_{\text{spins}}|\mathfrak{M}|^2=\frac{J_{\mu\nu}T_{\mu\nu}'}{t^4},\tag{4}$$

where

$$J_{\mu\nu}=\sum_{\substack{\text{proton}\\\text{spins}}}(\bar{U}_2J_\mu U_1)(\bar{U}_1J_\nu{}^*U_2),$$

and[14]

$$T_{\mu\nu}'=\sum_{\substack{\text{lepton}\\\text{spins}}}A_\mu A_\nu{}^*.$$

Gauge invariance implies that $t_\mu T_{\mu\nu}'=t_\nu T_{\mu\nu}'=0$.

Let us integrate the differential cross section over all the final momenta except that of the proton. We obtain

$$d\sigma=\frac{1}{2}\frac{1}{(2\pi)^8}\frac{M_p{}^2}{|(p_1\cdot k_1)|}\frac{J_{\mu\nu}T_{\mu\nu}}{t^4}\frac{d^3p_2}{E_2},\tag{5}$$

where

$$T_{\mu\nu}=\int\frac{d^3p^+}{\epsilon_+}\int\frac{d^3p^-}{\epsilon_-}\int\frac{d^3k_2}{\epsilon_2}\times\delta^4(p^++p^-+k_2+p_2-p_1-k_1)T_{\mu\nu}'.$$

The most general gauge-invariant $T_{\mu\nu}$ is

$$-T_{\mu\nu}=a[(t^2/(k_1\cdot t))k_{1\mu}k_{1\nu}+(k_1\cdot t)\delta_{\mu\nu}-k_{1\mu}t_\nu-k_{1\nu}t_\mu]+b[t^2\delta_{\mu\nu}-t_\mu t_\nu],\tag{6}$$

where a and b are functions of $k_1\cdot t$ and t^2, and have the dimensions of a cross section, i.e., (energy)$^{-2}$.

It can be shown that the functions a and b are of the same order of magnitude.[15] We note that $a(t^2=0)=2(2\pi)^5\sigma_p$, where σ_p is the cross section for the process $\gamma+\nu_\mu\rightarrow\nu_l+\mu^-+l$.[11] For $J_{\mu\nu}$, we obtain the following:

$$J_{\mu\nu}=(p_{2\mu}p_{1\nu}+p_{1\mu}p_{2\nu})\times\left[G_E{}^2+\frac{t^2}{4M_p{}^2}G_M{}^2\right]\left[M_p{}^2\left(1+\frac{t^2}{4M_p{}^2}\right)\right]^{-1}+\delta_{\mu\nu}\frac{t^2}{2M_p{}^2}G_M{}^2+\text{terms which are proportional to } t_\mu \text{ and } t_\nu.\tag{7}$$

The terms proportional to t_μ and t_ν will not contribute to σ because of the gauge-invariance requirement. G_E and G_M are the electric and magnetic form factors formed from linear combinations of F_1 and F_2.[16] In this paper we use

$$G_M=\mu G_E=\frac{\mu e}{(1+1.23t^2/M_p{}^2)^2},$$

which are obtained from the most recent high-energy electron-proton scattering data.[7] μ is the magnetic moment of the proton ($\mu=2.79$).

Using the previous results, we can obtain an expression in terms of a and b for the differential cross section, $\partial^2\sigma/\partial(\omega^2)\partial(t^2)$, where ω is the center-of-mass energy of the outgoing particles μ^-, l, and ν_l. This is done by using the transformation

$$\frac{d^3p_2}{E_2}=\frac{\pi dt^2d\omega^2}{2(k_1\cdot p_1)}\tag{8}$$

(the angular variable has been integrated over), and substituting Eqs. (6) and (7) into Eq. (5). From this expression we see that $\partial^2\sigma/\partial(\omega^2)\partial(t^2)$ falls off rapidly with increasing t^2 and ω^2, if we assume that a varies with ω as $\sigma_p(\omega)$, and has no more than a quadratic dependence with t_μ. For an incoming neutrino energy E_ν of 10 BeV, and a meson mass of 2.0 BeV, $\partial^2\sigma/\partial(\omega^2)\partial(t^2)$ at $\omega^2\approx9.0$ BeV^2 is about $\frac{1}{5}$ of its value at $\omega^2=4.4$ BeV^2, which is the threshold of the resonance region. For $\omega^2<4.4$ BeV^2, the differential cross section is smaller by a factor of 10^4, since the resonance does not occur in this region. Therefore, the region 4.4 BeV$^2\leq\omega^2\leq9.0$ BeV^2 yields the most important contribution to the total cross section σ. Since $\partial^2\sigma/\partial(\omega^2)\partial(t^2)$ falls off very rapidly with t^2, due to the proton form factors and the t^{-4} factor from the exchanged photon, we set $t^2\approx(t^2)_{\min}$, where $(t^2)_{\min}$ is found from the kinematics[17]:

$$\frac{(t^2)_{\max,\min}}{M_p{}^2}=\frac{2}{[2(E_\nu/M_p)+1]}\left\{\left(\frac{E_\nu}{M_p}\right)^2-\frac{1}{2}\left(\frac{E_\nu}{M_p}+1\right)\frac{\omega^2}{M_p{}^2}\pm\frac{E_\nu}{M_p}\left[\left(\frac{E_\nu}{M_p}\right)^2-\left(\frac{E_\nu}{M_p}+1\right)\frac{\omega^2}{M_p{}^2}+\frac{\omega^4}{M_p{}^4}\right]^{1/2}\right\};\tag{9}$$

also

$$\omega_{\max}=[2E_\nu M_p+M_p{}^2]^{1/2}-M_p.\tag{10}$$

[11] V. N. Gribov, V. A. Kolkunov, L. B. Okun, and V. M. Shekhter, Zh. Eksperim. i Teor. Fiz. **41**, 1839 (1961) [English transl.: Soviet Phys.—JETP **14**, 1308 (1962)].
[12] V. Gorgé, M. Locher, and H. Rollnik, Nuovo Cimento **27**, 928 (1963).
[13] V. Gorgé, Nuovo Cimento **35**, 545 (1965).
[14] We use the notation: $A_\mu{}^*=(-1)^{\eta(\mu)}A_\mu{}^\dagger$, where $\eta(\mu)=0$ for $\mu=1, 2, 3$, and $\eta(4)=1$.
[15] E. P. Shabalin, Zh. Eksperim. i Teor. Fiz. **93**, 175 (1962) [English transl.: Soviet Phys.—JETP **16**, 124 (1963)].
[16] L. N. Hand, D. G. Miller, and R. Wilson, Rev. Mod. Phys. **35**, 335 (1963).
[17] G. F. Chew and F. E. Low, Phys. Rev. **113**, 1640 (1959).

From Eq. (9) we can show that in the region

$$4.4\ \mathrm{BeV}^2 \leq \omega^2 \leq 9.0\ \mathrm{BeV}^2,$$

$$\frac{(t^2)_{\min}}{\omega^2} < 0.1, \quad \text{and} \quad \frac{(t^2)_{\min}}{M_W^2} \leq 0.1.$$

Although the previous statements were made for $E_\nu \approx 10$ BeV, they can be shown to hold in essence for any $E_\nu > W$ threshold.

We can use the following approximations derived from the previous discussion that for E_ν above the threshold for W production:

$$\omega^2 \gg t^2, \quad M_p E_\nu \gg t^2,$$

since

$$\omega_{\max}{}^2 \approx 2M_p E_\nu, \quad \text{and} \quad \omega^2 \gg t^2.$$

We shall consider only energies $E_\nu \gg M_p$. We obtain

$$\frac{\partial^2\sigma}{\partial(\omega^2)\partial(t^2)} = \frac{\alpha}{\pi}\tilde{\sigma}_p(\omega^2,t^2)F(\omega^2,t^2), \qquad (11)$$

where

$$t^4\left(1+1.23\frac{t^2}{M_p{}^2}\right)^4 F(\omega^2,t^2)$$
$$= \left(1+\mu\frac{t^2}{4M_p{}^2}\right)t^2(t^2+\omega^2)^{-1}\left(1+\frac{t^2}{4M_p{}^2}\right)^{-1}$$
$$-\tfrac{1}{4}E_\nu{}^{-2}\left(1+\mu\frac{t^2}{4M_p{}^2}\right)(t^2+\omega^2)\left(1+\frac{t^2}{4M_p{}^2}\right)^{-1}$$
$$-\tfrac{1}{2}E_\nu{}^{-1}M_p{}^{-1}\left(1+\mu\frac{t^2}{4M_p{}^2}\right)t^2\left(1+\frac{t^2}{4M_p{}^2}\right)^{-1}$$
$$+\tfrac{1}{8}\mu^2 M_p{}^{-2}E_\nu{}^{-2}(t^2+\omega^2)t^2,$$

where

$$\tilde{\sigma}_p(\omega^2,t^2) = \frac{a(\omega^2,t^2)}{2(2\pi)^5}.$$

We note that

$$\tilde{\sigma}_p(\omega^2,\, t^2=0) = \sigma_p(\omega^2).$$

We have dropped the b terms in deriving Eq. (11) because b is the same order of magnitude as a and the ratio of the expression multiplying b to that multiplying a is less than M_p/E_ν.

The standard Weizsäcker-Williams approximation consists in replacing $\tilde{\sigma}_p(\omega^2,t^2)$ by its value at the photon pole, $\sigma_p(\omega^2)$. Also the approximation $M_pE_\nu \gg \omega^2$ is usually made.[11] These approximations lead to the equation

$$\frac{\partial^2\sigma}{\partial(\omega^2)\partial(t^2)} = \frac{\alpha}{\pi}\sigma_p(\omega^2)\left(1+\frac{\mu t^2}{4M_p{}^2}\right)t^{-2}(t^2+\omega^2)^{-1}$$
$$\times\left(1+\frac{t^2}{4M_p{}^2}\right)^{-1}\left(1+1.23\frac{t^2}{M_p{}^2}\right)^{-4}. \qquad (12)$$

We find that the approximation $M_pE_\nu \gg \omega^2$ is not valid in this calculation because the region $\omega^2 > \omega_t{}^2$ yields the most important contribution to the total cross section σ, as previously stated. ω_t is the threshold for the resonance region:

$$\omega_t = (M_W + m_\mu). \qquad (13)$$

Since $M_W > 2.0$ BeV, relatively large values of ω^2 contribute to the calculation of σ. Therefore the terms neglected in the derivation of Eq. (12) contribute 50% to the total result in our problem.

The replacement of $\tilde{\sigma}_p(\omega^2,t^2)$ by its value at the photon pole is also a poor approximation in our problem. The reason for this is that for a large region of phase space, $t^2 > m_\mu{}^2$, and $t^2 > m_l{}^2$. Since t^2 occurs in the denominators of the lepton propagators, the position of the poles of the propagators are much different for $t^2=0$ and $t^2 \neq 0$. The terms involving the lepton propagators which contribute to σ can differ by a factor of 2 for $t^2=0$ and $t^2\neq 0$. This is the most important difference between $\tilde{\sigma}_p(\omega^2,t^2)$ and $\sigma_p(\omega^2)$. Contributions to the ratio $[\sigma_p(\omega^2)-\tilde{\sigma}_p(\omega^2,t^2)]/\sigma_p(\omega^2)$, other than those which arise by including the correct t^2 dependence in the denominators of the lepton propagators, are either of the order of t^2/ω^2 or $t^2/M_W{}^2$; therefore these contributions may be neglected. The result of this analysis is that $\tilde{\sigma}_p(\omega^2,t^2)$ can be approximated by calculating $\sigma_p(\omega^2)$ and putting the correct t^2 dependence in the denominators of the lepton propagators.

If we calculate σ for $t^2=0$ in the lepton propagators, we obtain results that are approximately 30% larger than those with t^2 in the propagators.

3. TOTAL CROSS SECTION FOR $\gamma+\nu_\mu \to \nu_l+\mu^-+l^+$

The Feynman graphs pertinent to the calculation of the total cross section of the process $\gamma+\nu_\mu \to \nu_l+\mu^-+l$

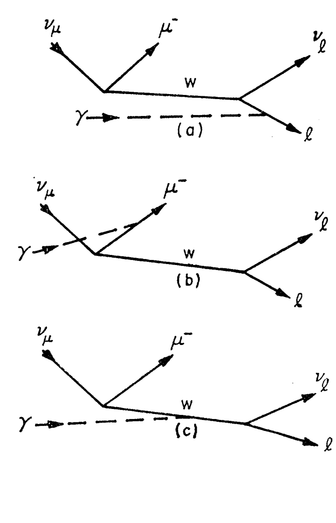

FIG. 2. Feynman diagrams for the process $\gamma+\nu_\mu \to \nu_l+\mu^-+l$ with an assumed intermediate vector meson W. Here l is a lepton, either e^+ or μ^+.

are shown in Fig. 2.

$$\sigma_p=\frac{1}{(2\pi)^5}\frac{1}{2}\frac{1}{|(k_1\cdot t)|}\int\frac{d^3p^+}{\epsilon_+}\int\frac{d^3p^-}{\epsilon_-}\int\frac{d^3k_2}{\epsilon_2}\times\delta^4(p^++p^-+k_2-t-k_1)\tfrac{1}{2}\sum_{\text{spins}}|\mathfrak{M}_p|^2. \quad (14)$$

t_μ is now the four-momentum of the incoming photon. For $\mathfrak{M}_p$, we use the Feynman rules. The lepton-photon vertex is the usual one.

For a vector-meson–lepton vertex, we have

$$\mathfrak{M}_{p,\nu_l,W}=ig\phi_\mu{}^m\bar{\nu}_{\mathbf{k}}(1-\gamma_5)\gamma_\mu u_{s,\mathbf{p}}. \quad (15)$$

This corresponds to the reaction $l\to\nu_l+W$. $\phi_\mu{}^m$ is the polarization vector of the W. $\bar{\nu}_{\mathbf{k}}=\nu_{\mathbf{k}}{}^\dagger\gamma_4$ is the spinor for a neutrino with momentum $\mathbf{k}$. $u_{s,\mathbf{p}}$ is the spinor for a lepton with spin s and momentum $\mathbf{p}$. $g^2=G_vM_W{}^2/\sqrt{2}$.

The propagator of the vector meson is given by

$$-i[\delta_{\mu\nu}+q_\mu q_\nu/M_W{}^2](q^2+M_W{}^2)^{-1},$$

where q_μ is the four-momentum of the vector meson.

The photon-meson vertex, corresponding to $W\to W+\gamma$, is given by[8]

$$\begin{aligned}\mathfrak{M}_{\gamma W}&=\phi_\alpha{}^m\phi_\beta{}^n\epsilon_\mu{}^rV_{(\mu)\alpha\beta},\\ V_{(\mu)\alpha\beta}&=ie[\delta_{\alpha\beta}(q_1+q_2)_\mu-\delta_{\alpha\mu}(-\kappa q_2+q_1+\kappa q_1)_\beta\\&\quad-\delta_{\beta\mu}(-\kappa q_1+q_2+\kappa q_2)_\alpha].\end{aligned} \quad (16)$$

$q_{1\mu}$, $\phi_\mu{}^m$ are the momentum and the polarization vectors of the incoming meson. $q_{2\mu}$, $\phi_\mu{}^n$ are the momentum and polarization vectors of the outgoing vector meson. $\epsilon_\mu{}^r$ is the polarization vector of the photon. κ is the anomalous magnetic moment of the vector meson.

Using the Feynman rules, we obtain

$$\begin{aligned}\mathfrak{M}_p=&\,ig^2e\frac{\bar{u}_{\mathbf{p}^-}\gamma_\alpha(1+\gamma_5)\nu_{\mathbf{k}_1}\bar{\nu}_{\mathbf{k}_2}(1-\gamma_5)\gamma_\alpha(t\!\!\!/-p\!\!\!/^+)\gamma\cdot\epsilon^rv_{\mathbf{p}^{+e}}}{[q_1{}^2+M_W{}^2](-2p^+\cdot t+t^2)}+ig^2e\frac{\bar{u}_{\mathbf{p}^-}\gamma\cdot\epsilon^r(p\!\!\!/^--t\!\!\!/)\gamma_\alpha(1+\gamma_5)\nu_{\mathbf{k}_1}\bar{\nu}_{\mathbf{k}_2}(1-\gamma_5)\gamma_\alpha v_{\mathbf{p}^{+e}}}{(-2p^-\cdot t+t^2)[q_2{}^2+M_W{}^2]}\\&-ig^2e\frac{\bar{u}_{\mathbf{p}^-}\gamma_\alpha(1+\gamma_5)\nu_{\mathbf{k}_1}\bar{\nu}_{\mathbf{k}_2}(1-\gamma_5)\gamma_\alpha v_{\mathbf{p}^{+e}}[(q_1+q_2)\cdot\epsilon^r]}{[q_1{}^2+M_W{}^2][q_2{}^2+M_W{}^2]}-i(1+\kappa)g^2e\frac{\bar{u}_{\mathbf{p}^-}\gamma\cdot\epsilon^r(1+\gamma_5)\nu_{\mathbf{k}_1}\bar{\nu}_{\mathbf{k}_2}(1-\gamma_5)t\!\!\!/v_{\mathbf{p}^{+e}}}{[q_1{}^2+M_W{}^2][q_2{}^2+M_W{}^2]}\\&+i(1+\kappa)g^2e\frac{\bar{u}_{\mathbf{p}^-}t\!\!\!/(1+\gamma_5)\nu_{\mathbf{k}_1}\bar{\nu}_{\mathbf{k}_2}(1-\gamma_5)\gamma\cdot\epsilon^rv_{\mathbf{p}^{+e}}}{[q_1{}^2+M_W{}^2][q_2{}^2+M_W{}^2]},\end{aligned} \quad (17)$$

where $A\!\!\!/=\gamma_\mu A_\mu$ and $v_{\mathbf{p}^{+e}}$ is the spinor for the positively charged lepton. ω is now the center-of-mass energy of the incoming photon and neutrino. We have kept t^2 in the denominators of the lepton propagators, as indicated in the preceding discussion.

Since we are doing the calculation in the resonance region $\omega>\omega_t$, we may neglect the lepton masses in the numerator terms of $\sum_{\text{spins}}|\mathfrak{M}_p|^2$.

The resonance is treated by replacing M_W by $M_W-\frac{1}{2}i\Gamma$.[6] Γ is the total decay width of the vector meson. We have $M_W\gg\Gamma$ and $M_W\Gamma\gg\Gamma^2$.[1] The resonance occurs at $q_2{}^2=-M_W{}^2$. Therefore the terms with $q_2{}^2+M_W{}^2$ in the denominator will be the most important ones. When $\sum_{\text{spins}}|\mathfrak{M}_p|^2$ is computed, and we integrate to obtain $\tilde{\sigma}_p$, the squares of the resonant terms are the most significant by a factor of 10^4 [Graphs (b) and (c) in Fig. 2].[18] For $\tilde{\sigma}_p$, in the region $\omega>\omega_t$, we obtain

$$\tilde{\sigma}_p=\sigma_0{}^p[\kappa^2\beta_2{}^p+\kappa\beta_1{}^p+\beta_0{}^p], \quad (18)$$

where

$$\sigma_0{}^p=6\sqrt{2}\alpha G_v\left(\frac{m_\mu}{M_W}\right)^2\frac{\Gamma_l}{\Gamma}.$$

$$\beta_2{}^p=R_2(\omega)\ln\left(\frac{x}{\lambda}\right)^2+S_2(\omega)[z_0{}^2-1]^{1/2},$$

where

$$x=\frac{\omega}{m_\mu},\quad\lambda=\frac{M_W}{m_\mu},\quad z_0=\frac{x^2+1-\lambda^2}{2x}.$$

$$R_2(\omega)=\frac{\lambda^2}{6}-\frac{1}{6}\frac{\lambda^4}{x^2}+\frac{2}{3}\frac{\lambda^2}{x^2}-\frac{1}{6}\frac{\lambda^4}{x^4},$$

$$S_2(\omega)=\frac{\lambda^2}{8x}+\frac{1}{3}\frac{\lambda^4}{x^3}+\frac{1}{24}\frac{\lambda^6}{x^5}-\frac{25}{24}\frac{1}{x},$$

$$\beta_1{}^p=R_1(\omega)\ln\left(\frac{x}{\lambda}\right)^2+S_1(\omega)[z_0{}^2-1]^{1/2},$$

$$R_1(\omega)=-\frac{\lambda^2}{3}-\frac{\lambda^4}{3x^2}+\frac{\lambda^2}{3x^2}-\frac{2}{3}\frac{\lambda^4}{x^4},$$

$$S_1(\omega)=\frac{31}{12}\frac{\lambda^2}{x}+\frac{\lambda^4}{x^3}+\frac{1}{12}\frac{\lambda^6}{x^5}-\frac{49}{12}\frac{1}{x},$$

$$\beta_0{}^p=R_0(\omega)\ln\left(\frac{x}{\lambda}\right)^2+S_0(\omega)[z_0{}^2-1]^{1/2}+T_0(\omega)\times\ln\left\{\frac{z_0(1-y^2/x^2)+y^2/x+[z_0{}^2-1]^{1/2}(1+y^2/x^2)}{z_0(1-y^2/x^2)+y^2/x-[z_0{}^2-1]^{1/2}(1+y^2/x^2)}\right\},$$

where $y^2=t^2/m_\mu{}^2$. The term involving $T_0(\omega)$ includes the effect of keeping t^2 in the denominator of the muon

[18] The full details of the calculations of σ_p are given in J. Dooher, thesis, Stevens Institute of Technology, 1965 (unpublished).

TABLE I. The numerical values of the total cross section for the process $\nu_\mu+p\rightarrow\nu_l+p+\mu^-+l$; l is a lepton, either e^+ or μ^+. An intermediate vector meson is assumed with a mass of 2.0, 2.5, and 3.0 BeV.

Neutrino energy (BeV)	M_W (BeV)	Γ_l/Γ	β_2	β_1	β_0	σ ($\kappa=1$) (cm²)	σ ($\kappa=0$) (cm²)	σ ($\kappa=-1$) (cm²)
4.37				Threshold for W production				
5.63			0.268	0.949	3.75	8.94×10^{-39}	6.75×10^{-39}	5.52×10^{-39}
9.38	2.0	1.0	4.63	15.9	52.2	1.31×10^{-37}	9.40×10^{-38}	7.37×10^{-38}
13.1			12.9	41.1	128.7	3.29×10^{-37}	2.32×10^{-37}	1.80×10^{-37}
16.9			23.6	70.8	215.7	5.58×10^{-37}	3.88×10^{-37}	3.03×10^{-37}
20.6			35.7	102.0	304.6	7.96×10^{-37}	5.48×10^{-37}	4.29×10^{-37}
5.99				Threshold for W production				
7.50			0.180	0.659	2.97	2.85×10^{-39}	2.22×10^{-39}	1.86×10^{-39}
12.2	2.5	0.65	4.27	15.4	57.4	5.76×10^{-38}	4.29×10^{-38}	3.46×10^{-38}
16.9			13.0	43.8	153.0	1.57×10^{-37}	1.14×10^{-37}	9.14×10^{-38}
21.6			24.9	79.2	267.3	2.78×10^{-37}	2.00×10^{-37}	1.59×10^{-37}
8.25				Threshold for W production				
10.3			0.155	0.585	2.97	2.13×10^{-39}	1.71×10^{-39}	1.46×10^{-39}
14.1	3.0	0.72	2.00	7.65	34.0	2.51×10^{-38}	1.96×10^{-38}	1.63×10^{-38}
17.8			6.18	22.7	94.3	7.08×10^{-38}	5.42×10^{-38}	4.47×10^{-38}
21.6			12.4	43.8	173.3	1.32×10^{-37}	9.96×10^{-38}	8.14×10^{-38}

propagator [Graph (b) in Fig. 2].

$$R_0(\omega)=\frac{\lambda^2}{6}+\frac{1}{2}\frac{\lambda^4}{x^2}-\frac{4}{3}\frac{\lambda^6}{x^4}+\frac{4}{3}\frac{\lambda^8}{x^6}+\frac{2}{3}\frac{\lambda^2}{x^2}-\frac{5}{6}\frac{\lambda^4}{x^4}-\frac{10}{3}\frac{\lambda^6}{x^6},$$

$$S_0(\omega)=\frac{1}{8}\frac{\lambda^2}{x}+\frac{1}{3}\frac{\lambda^4}{x^3}+\frac{65}{24}\frac{\lambda^6}{x^5}-\frac{73}{24}\frac{1}{x},$$

$$T_0(\omega)=\frac{2}{3}\frac{\lambda^4}{x^2}-\frac{4}{3}\frac{\lambda^6}{x^4}+\frac{4}{3}\frac{\lambda^8}{x^6}-\frac{4}{3}\frac{\lambda^2}{x^2}+\frac{16}{3}\frac{\lambda^4}{x^4}+8\frac{\lambda^6}{x^6}.$$

In the derivation of these expressions we have used $M_W\gg\Gamma$ and $M_W\Gamma\gg\Gamma^2$.

For the evaluation of a typical resonant integral, see the Appendix.

4. TOTAL CROSS SECTION FOR $\nu_\mu+p\rightarrow\nu_l+\mu^-+l+p$

To obtain σ, we integrate $\partial^2\sigma/\partial(t^2)\partial(\omega^2)$, as given by Eq. (11), numerically over the region $\omega_t^2\leq\omega^2\leq\omega_{\max}^2$, $(t^2)_{\min}\leq t^2\leq(t^2)_{\max}$. The final cross section takes the form

$$\sigma=\sigma_0(\kappa^2\beta_2+\kappa\beta_1+\beta_0). \tag{19}$$

β_2,β_1, and β_0 are functions of E_ν and M_W. They are given in Table I.

$$\sigma_0=8.14\left(\frac{M_p}{M_W}\right)^2\frac{\Gamma_l}{\Gamma}\times10^{-39}\ \text{cm}^2.$$

The values used for Γ_l/Γ have been discussed in the Introduction. The numerical results are summarized in Table I.

5. DISCUSSION OF RESULTS

Deriving the final expression for MWW, we have neglected terms which are smaller by factors of t^2/ω^2, t^2/M_W^2, t^2/M_pE_ν, or M_p/E_ν than those terms which we have included. t^2/ω^2, t^2/M_W^2, and t^2/M_pE_ν are all less than 0.1 in the regions of t^2 and ω^2 which contribute to σ for $E_\nu>W$ threshold. For $E_\nu\geq10$ BeV, we have $M_p/E_\nu<0.1$. Therefore, for $E_\nu\geq10$ BeV we believe that 10% is a reasonable upper limit on the error in σ. For 5 BeV$\leq E_\nu\leq$10 BeV the error may be slightly larger.

By including the correct t^2 dependence in the denominators of the lepton propagators, we can extend the validity of MWW for our problem over a wide region of the range of the variables t^2 and ω^2. The only requirement is that the most important contributions of phase space to the cross section occur in the region $t^2/\omega^2\ll1$, $t^2/M_W^2\ll1$. In its usual form, the Weizsäcker-Williams approximation (WW) is good only if the positions of the poles of the lepton propagators are not much different for $t^2=0$ and $t^2\neq0$.[19] This will be true only if $t^2\ll m_l^2$, where l corresponds to the particular lepton propagator being considered. This condition is extremely restrictive and is not satisfied in our problem. However, the conditions $t^2/\omega^2\ll1$ and $t^2/M_W^2\ll1$, are satisfied. In general, for a process involving exchanged photon and lepton propagators, the requirements for the validity of MWW are easiest to meet than those for WW.

We see from Table I that for neutrino energies varying from 5 to 20 BeV, the cross section varies from 10^{-39} to 10^{-37} cm². Calculations using the four-fermion theory show that in a similar energy range, the cross sections vary from 10^{-44} to 10^{-43} cm².[4,5] The results are in accord with the crude estimates discussed in the Introduction. We also note that the rate for μ^-, μ^+ production is equal to that for μ^-, e^+ production in the vector-meson theory. In the four-fermion theory, however, μ^-, e^+ production

[19] V. Gorgé, M. Locher, and H. Rollnik, in passing, mention the effect of lepton propagators on the regions of phase space which contribute to the cross section of a process such as $e^-+p\rightarrow e^-+p+\gamma$ (Ref. 12). See Ref. 13 by Gorgé for a longer discussion.

is larger by a factor of 2 to 3 for neutrino energies from 5 to 20 BeV.[4,5] This can be understood by noting that $\sigma_{\mu^-\mu^+}{}^{(W)} \approx \sigma_{W\,\text{production}} \times (\Gamma_{\mu\nu}/\Gamma)$, and that $\sigma_{\mu^-e^+}{}^{(W)} \approx \sigma_{W\,\text{production}} \times (\Gamma_{e\nu}/\Gamma)$. For a large-mass boson, we have $\Gamma_{\mu\nu} = \Gamma_{e\nu}$. Hence the cross sections are equal in the vector-meson theory. This result corresponds to the fact that we neglected graph (a) in Fig. 2, and neglected the lepton masses in numerator terms. The only place where lepton masses are significant is in the denominators of lepton propagators. In graph (b) only the muon propagator is involved. In graph (c) no lepton propagators are involved.

Finally, we note that even for very high neutrino energies, ($E_\nu \approx 20$ BeV), the cross section falls off as M_W is increased. This occurs because the proton form factors rapidly fall off as the momentum transfer increases. Even though $t^2 \approx (t^2)_{\min}$, $(t^2)_{\min}$ will increase with M_W for fixed E_ν because it increases with ω, and, hence ω_t.

ACKNOWLEDGMENTS

We wish to thank Dr. W. Williamson and Dr. R. R. Lewis for a discussion of their calculation. We are indebted to Dr. R. Wilson for communicating his results on high-energy form factors before publication. Valuable discussions with Dr. G. Feinberg, Dr. D. Yennie, Dr. R. Lapidus, and Dr. A. Layzer are also gratefully acknowledged. We thank Adelphi University for providing computer time. One of us (J. D.) would like to thank Adelphi University for supporting some of this work with funds from a National Science Foundation Institutional grant.

APPENDIX: A TYPICAL FULLY RESONANT INTEGRAL

We consider the following integral typical of a term arising from the contribution of the square of graph (b) in Fig. 2 to $\bar{\sigma}_p$

$$I = \int \frac{d^3p^-}{\epsilon_-} \frac{1}{(-2p^-\cdot t + t^2)} \frac{1}{(q_2^2 + M_W^2)^2 + \Gamma^2 M_W^2}. \quad \text{(A1)}$$

We have done the integration over d^3p^+ and d^3k_2 covariantly using the properties of the energy and momentum delta functions.[20] Letting $x = \omega/m_\mu$, $z = \epsilon_-/m_\mu$, $\lambda = M_W/m_\mu$, $\Gamma_0 = \Gamma/m_\mu$, and $y^2 = t^2/m_\mu^2$, we may write, after the angular integration has been done,

$$I = \frac{2\pi}{m_\mu^3 \omega} \int_1^{x/2} dz (|2xz - 2xz_0|^2 + \Gamma_0^2\lambda^2)^{-1} \times \ln\left\{\frac{z(1-y^2/x^2) + y^2/x + (z^2-1)^{1/2}(1+y^2/x^2)}{z(1-y^2/x^2) + y^2/x - (z^2-1)^{1/2}(1+y^2/x^2)}\right\}, \quad \text{(A2)}$$

where

$$z_0 = (x^2 + 1 - \lambda^2)/2x.$$

[20] J. D. Jackson, in *Brandeis Summer Institute of Theoretical Physics, 1962* (W. A. Benjamin, Inc., New York, 1963).

We have kept y^2 in the logarithm but neglected over-all factors proportional to y^2/x^2.

For $x > x_0 = 1 + \lambda$ we can have $q_2^2 = -M_W^2$ in the region of integration. Using this fact, we shall show that

$$(|2xz - 2xz_0|^2 + \Gamma_0^2\lambda^2)^{-1} = (\pi/2x\Gamma_0\lambda)\delta(z - z_0), \quad \text{(A3)}$$

for $x > 1 + \lambda + 10\Gamma_0$, (i.e., x sufficiently far from the resonance threshold).

Since a relatively large region contributes to σ in the integration of $\partial^2\sigma/\partial(\omega^2)\partial(t^2)$ over ω, an error in the region $M_W + m_\mu \le \omega \le M_W + m_\mu + 10\Gamma$ will be negligible since $M_W \gg \Gamma$. Therefore, once Eq. (A3) is derived we may use it throughout the calculation.

To prove Eq. (A3) we consider the following integral:

$$I_1 = \int_1^{x/2} \frac{z\,dz}{|2xz - 2xz_0|^2 + \Gamma_0^2\lambda^2}. \quad \text{(A4)}$$

This integral is straightforward, and for $x > x_0$ we obtain

$$I_1 = \frac{z_0}{2x\Gamma_0\lambda} \tan^{-1}\left\{\frac{2xz - 2xz_0}{\Gamma_0\lambda}\right\}\Bigg|_1^{x/2}. \quad \text{(A5)}$$

We used $M_W \gg \Gamma$.

For $x > x_0 + 10\Gamma_0$ we may expand the arctangent as follows:

$$\tan^{-1}\left\{\frac{2xz - 2xz_0}{\Gamma_0\lambda}\right\}\Bigg|_1^{x/2} \approx \pi - \Gamma_0\lambda\left[\frac{1}{\lambda^2 - 1} + \frac{1}{x^2 - 2x - (\lambda^2 - 1)}\right]. \quad \text{(A6)}$$

For $x > x_0 + 10\Gamma_0$ we may drop the second term in Eq. (A6). We obtain

$$I_1 = (\pi/2\Gamma_0\lambda)(z_0/x). \quad \text{(A7)}$$

This is the result obtained by using Eq. (3). Equation (A3) can be shown to hold for[18]:

$$I_n = \int_1^{x/2} \frac{z^n dz}{|2xz - 2xz_0|^2 + \Gamma_0^2\lambda^2}, \quad n = 0, 1, \cdots. \quad \text{(A8)}$$

Using Eq. (A3) we obtain for I

$$I = \frac{\pi^2}{m_\mu^2 \omega^2}\left(\frac{1}{\Gamma_0\lambda}\right) \times \ln\left\{\frac{z_0(1-y^2/x^2) + y^2/x + [z_0^2 - 1]^{1/2}(1+y^2/x^2)}{z_0(1-y^2/x^2) + y^2/x - [z_0^2 - 1]^{1/2}(1+y^2/x^2)}\right\}. \quad \text{(A9)}$$

The remaining resonant integrals contributing to $\bar{\sigma}_p$ can be done in the same way.