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One-Boson-Exchange Model of NN and $N\bar{N}$ Interaction*

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The elastic nucleon-nucleon scattering, due to the exchange of π, η, ρ, ω, φ, and an effective $I=0$ scalar σ meson is calculated using unsubtracted partial-wave dispersion relations with a cutoff. The ρ, ω, and φ vector coupling constants are related by SU_3 to a single constant assuming pure F coupling. The ratio of the vector to tensor coupling of the ρ meson is determined by the $I=1$ charge and anomalous magnetic moment ratio and the tensor couplings of ω and φ are neglected. The η-nucleon axial-vector coupling constant is related to that of the pion by SU_3 with a D/F ratio of $\frac{3}{2}$. The $I=0$ and $I=1$ phase shifts are calculated using a total of four adjustable parameters: the mass and coupling constant of the effective σ meson, the octet-vector coupling constant, and the cutoff parameter. For each of the cutoff values corresponding to laboratory kinetic energies of 600, 700, and 800 MeV, the remaining three parameters are adjusted to fit the $I=1$, ${}^1S_0$, ${}^3P_0$, ${}^3P_1$, and ${}^3P_2$, and the $I=0$, ${}^3S_1$ phase shifts at 25, 50, 95, 142, 210, and 310 MeV. In each of the three cases, a goodness-of-fit parameter is obtained corresponding to a theory with approximately 10% inherent uncertainty. A deuteron pole appears in the solution for the ${}^3S_1$ amplitude corresponding to a binding energy of $\sim$10 MeV. All of the calculated higher partial-wave phase shifts are in good agreement with results of phase-shift analyses. Having obtained a fit to the nucleon-nucleon phase shifts, the nucleon-antinucleon scattering amplitudes are calculated after changing the signs of the odd G-parity exchange terms (π, ω, and φ) but keeping the same values for the four parameters. For each of the three cutoff energies, a bound-state pole is found in the $I=0$, ${}^1S_0$, ${}^3S_1$, and ${}^3P_0$, and the $I=1$, ${}^1S_0$ and ${}^3S_1$ amplitudes. These bound states have the same quantum numbers as the η, ω, σ, π, and ρ, respectively. Although the masses of the bound states are not near those of the physical mesons, it is argued that if the important meson channels (annihilation) were included, the bound-state poles would move toward the physical values. These results lend strong support to the conjecture that the observed mesons are composite particles.

I. INTRODUCTION

THE nucleon-antinucleon interaction due to the exchange of mesons is related by crossing symmetry to similar interactions in the NN system. In principle, information about the $N\bar{N}$ scattering amplitudes can be deduced from the known amplitudes for NN scattering.

The major difference between the NN and $N\bar{N}$ systems is that inelastic channels are closed for low-energy NN scattering whereas multimeson channels coupled to the $N\bar{N}$ system are opened even at the physical $N\bar{N}$ threshold. Nevertheless, it is possible to separate in an approximate fashion the absorptive effects from the two-body potential in both cases. For the $N\bar{N}$ problem, the first attempt along this line was that of Ball and Chew,[1] in which they made use of the fact that the one-pion-exchange potential in the $N\bar{N}$ system is the negative of the same potential in the NN interaction. The absorption (annihilation) in their model was approximated by a black sphere with a radius small compared to the range of the one-pion force. Although the fit to low-energy $N\bar{N}$ scattering with the Ball-Chew theory was quite satisfactory, it would be desirable to improve the black-sphere approximation by making use of our present knowledge of NN and $N\bar{N}$ interactions. In particular, it would be of some interest to separately investigate the effects of short-range (shorter than λ_π) two-body potentials and those of annihilation processes. The present paper is a study of the short-range $N\bar{N}$ forces due to the exchange of various mesons.

Recent theoretical treatments of nucleon-nucleon scattering,[2] such as that of Scotti and Wong,[3] have been quantitatively successful in fitting experimental data solely in terms of an NN interaction which arises from the exchange of pseudoscalar (π,η), vector (ρ,ω,φ), and an effective scalar meson (σ).[4] Because the source of this interaction is meson exchange, the $N\bar{N}$ interaction can be deduced directly from that for NN by simply

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[1] J. S. Ball and G. F. Chew, Phys. Rev. **109**, 1385 (1958).

[2] R. S. McKean, Jr., Phys. Rev. **125**, 1399 (1952); R. A. Bryan, C. R. Dismukes, and W. Ramsay, Nucl. Phys. **45**, 353 (1963). S. Sawada *et al.*, Progr. Theoret. Phys. (Kyoto) **28**, 991 (1962); **32**, 380 (1964).

[3] A. Scotti and D. Y. Wong, Phys. Rev. **138**, B145 (1965), hereafter called SW.

[4] It was shown in Ref. 3 that the exchange of meson systems of $I=0$, $J=0$, $P=+$ must account for the attractive medium-range nuclear force whether the σ meson exists or not. If the σ does not exist, the same effect can still be produced by a low-energy s-wave $\pi\pi$ enhancement as in SW.

changing the sign of the odd G-parity exchange terms. The most significant feature is that the short-range repulsion in the NN system which is produced by the exchange of $I=0$, $G=-1$ vector mesons (ω and φ) becomes a strongly attractive short-range force in the $N\bar{N}$ system. As we shall show in the main text, the net attraction in all four S-wave amplitudes (I and $J=0, 1$) are sufficiently strong to produce bound states. These bound states have exactly the quantum numbers of the η, π, ρ, and ω or φ.[5] From this observation it seems likely that in any dynamical model of these mesons, the $N\bar{N}$ interaction will play an important role. This is particularly true for the η since the only low-mass states that are coupled strongly to the η contain at least four pions. In fact, we find that the energy of the bound state in the $I=0$, ${}^1S_0$ amplitude is in the neighborhood of the η mass while the remaining three S-wave bound states correspond to masses in the neighborhood of 1.5 BeV. In addition to the S states, the ${}^3P_0$ amplitude for $I=0$ has the property that all of the exchange terms add coherently producing a very strong attraction which can overcome the centrifugal barrier to produce binding. Such a bound state has the quantum numbers of the exchanged σ. While no attempt is made to include the multimeson continuum in this work it can be shown that these inelastic contributions are additional attractive interactions which will serve to increase the binding energies of the $N\bar{N}$ bound states. This would be an improvement over the present result which gives too high a mass for the π, ρ, and φ (or ω).

In the present work we will use relativistic dispersion relations to produce unitary scattering amplitudes starting from the sum of single-meson-exchange terms. Our treatment will be similar to that of Scotti and Wong, except for several important differences stated below. Since we are interested in the calculation of the NN and $N\bar{N}$ S-wave scattering amplitudes in terms of the interaction without additional parameters, we cannot employ the subtraction technique used by SW to produce the S-wave scattering lengths. Therefore, we must return to the NN problem, determine what interaction is necessary to fit the NN data and the S-wave scattering lengths without using the subtraction method, and then apply crossing to obtain the $N\bar{N}$ interaction.

Since our treatment of the $N\bar{N}$ system will have considerable uncertainty due to the neglect of the multimeson continuum, it seems unwarranted to attempt to obtain an NN interaction which contains a large number of parameters all delicately fitted to the experimental data. For this reason, we have minimized the number of parameters used to describe the NN interaction to the extent that a reasonably good fit to the NN phase shifts can still be obtained. This simplification is achieved by (i) employing a single sharp cutoff in all dispersion integrals instead of the three Regge-slope parameters in SW; (ii) using SU_3 to relate the coupling constant of the ρ to that of the φ, ω mixture; and (iii) fixing the ratio of the vector coupling of the ρ to the tensor coupling by using the ratio of the charge to magnetic moment isovector form factors. The resulting NN interaction then depends on only four parameters: the cutoff energy s_c, the coupling constant of the nucleon to vector mesons g_v, the coupling constant of the scalar meson (σ meson) to the nucleon g_σ, and the effective mass of the scalar meson m_σ. The four free parameters are sufficient to produce a good fit to the NN phase shifts obtained by phase shift analysis of the data.[6] The only remaining assumption necessary to obtain the $N\bar{N}$ interaction from the NN interaction given this type of parametrization is to relate the cutoff in the NN case to that in the $N\bar{N}$. For simplicity, we use the same cutoff in both cases.

In the following section we formulate the partial-wave dispersion relations and the ND^{-1} equations with special attention given to removing a kinematical singularity at zero total energy. In Sec. III the π, η, σ, ρ, ω, and φ exchange contributions to the partial-wave amplitudes are calculated. Section IV contains the application of the ND^{-1} equations to the NN problem together with the resulting fit of the NN phase shifts. The interaction obtained is then converted to the $N\bar{N}$ interaction and the integral equations are solved to obtain the $N\bar{N}$ scattering amplitudes and the masses of the bound states. The last section contains a discussion of the results and possible extensions and improvements of the present calculation. Some remarks are made in support of the composite-particle interpretation of mesons.[7] Explicit formulas for the single-meson-exchange contributions to the partial-wave amplitudes are given in an Appendix.

II. PARTIAL-WAVE DISPERSION RELATIONS

The usual scalar variables s, t, and u are the following functions of the center-of-mass energy, momentum, and scattering angle:

$$s=4E^2=4(p^2+m^2)\,,$$
$$t=-2p^2(1-z)\,,$$
$$u=-2p^2(1+z)\,,$$

where $z=\cos\theta$.

Following the notation of SW, the partial-wave amplitudes are defined in terms of Stapp's nuclear-bar phase shifts[8]:

Singlet:

$$h_J=(E/2imp)[\exp(2i\delta_J)-1]; \tag{1}$$

[5] E. Fermi and C. N. Yang, Phys. Rev. **76**, 1739 (1949); C. H. Albright, R. Blankenbecler, and M. L. Goldberger, Phys. Rev. **124**, 624 (1961); R. C. Arnold, Nuovo Cimento **37**, 589 (1965).

[6] M. H. MacGregor and R. A. Arndt, Phys. Rev. **139**, B362 (1965); H. P. Noyes *et al.*, *ibid.* **139**, B380 (1965).

[7] G. F. Chew, *S-Matrix Theory of Strong Interactions* (W. A. Benjamin, Inc., New York, 1961); and M. Jacob and G. F. Chew, *Strong Interaction Physics* (W. A. Benjamin, Inc., New York, 1964).

[8] H. P. Stapp, T. J. Ypsilantis, and N. Metropolis, Phys. Rev. **105**, 302 (1958).

Uncoupled triplet:

$$h_{JJ}=(E/2imp)[\exp(2i\delta_{JJ})-1]; \qquad (2)$$

Coupled triplet:

$$h_{J-1,J}=(E/2imp)[(\cos 2\epsilon_J)\exp(2i\delta_{J-1,J})-1], \qquad (3)$$

$$h_{J+1,J}=(E/2imp)[(\cos 2\epsilon_J)\exp(2i\delta_{J+1,J})-1], \qquad (4)$$

$$h^J=(E/2mp)\sin 2\epsilon_J \exp[i(\delta_{J-1,J}+\delta_{J+1,J})]. \qquad (5)$$

These expressions hold for $I=0, 1$ $N\bar{N}$ as well as NN amplitudes.

In the NN problem, the h's are related to the invariant helicity scattering amplitudes φ_1, φ_2, φ_3, φ_4, and φ_5 by[3]

$$h_J=\frac{E}{4m}\int_{-1}^{+1} dz\, P_J[\varphi_1-\varphi_2], \qquad (6)$$

$$h_{JJ}=\frac{E}{4m}\int_{-1}^{+1} dz\,[d_{11}{}^J\varphi_3-d_{-11}{}^J\varphi_4], \qquad (7)$$

$$h_{J-1,J}=\frac{1}{2J+1}\frac{E}{4m}\int_{-1}^{+1} dz\,\{JP_J(\varphi_1+\varphi_2)+(J+1)(d_{11}{}^J\varphi_3+d_{-11}{}^J\varphi_4)+4[J(J+1)]^{1/2}d_{10}{}^J\varphi_5\}, \qquad (8)$$

$$h_{J+1,J}=\frac{1}{2J+1}\frac{E}{4m}\int_{-1}^{+1} dz\,\{(J+1)P_J(\varphi_1+\varphi_2)+J(d_{11}{}^J\varphi_3+d_{-11}{}^J\varphi_4)-4[J(J+1)]^{1/2}d_{10}{}^J\varphi_5\}, \qquad (9)$$

$$h^J=\frac{[J(J+1)]^{1/2}}{2J+1}\frac{E}{4m}\int_{-1}^{+1} dz\,\{P_J(\varphi_1+\varphi_2)-(d_{11}{}^J\varphi_3+d_{-11}{}^J\varphi_4)+2/[J(J+1)]^{1/2}d_{10}{}^J\varphi_5\}. \qquad (10)$$

For $N\bar{N}$ scattering, similar expressions hold except that a factor of 2 should be multiplied into the right-hand side of Eqs. (6)–(10) because the Pauli principle does not apply to $N\bar{N}$ scattering.

It was shown by Goldberger, Grisaru, MacDowell, and Wong[9] that the amplitudes $E\varphi_1$, $E\varphi_2$, $E\varphi_3$, $E\varphi_4$, and φ_5 have no kinematical singularity in the complex s-plane ($s=4E^2$). Therefore, it follows from Eqs. (6)–(10) that h_J and h_{JJ} have no kinematical singularities, but the coupled triplet amplitudes $h_{J-1,J}$, $h_{J+1,J}$, and h^J all have a $(s)^{1/2}$ type singularity at $s=0$. In the work of SW, no attempt was made to remove this kinematical singularity in the formulation of dispersion relations because the point $s=0$ is far removed from the region of interest $s\gtrsim 4m^2$. On the other hand, if we consider now the $N\bar{N}$ problem with the expectation of finding strongly bound states, this singularity should no longer be ignored. In our present treatment of partial-wave amplitudes, proper account of this kinematical singularity will be given.

[9] M. L. Goldberger, M. T. Grisaru, S. W. MacDowell, and D. Y. Wong, Phys. Rev. **120**, 2250 (1960); hereafter called GGMW.

A. Singlet Amplitudes

Let us first examine the singlet amplitudes and give a brief review of the formulation of dispersion relations and the ND^{-1} method. From the phase-shift expression given by Eq. (1), one obtains the usual unitarity condition:

$$\mathrm{Im}h_J=(mp/E)|h_J|^2;\quad s\geq 4m^2. \qquad (11)$$

A dispersion relation for h_J can be written in the form

$$h_J(s)=b_J(s)+\frac{1}{\pi}\int_{4m^2}^{\infty} ds'\left(\frac{mp'}{E'}\right)\frac{|h_J(s')|^2}{s'-s-i\epsilon}, \qquad (12)$$

where $b_J(s)$ is a real analytic function containing all the singularities of h_J below $s=4m^2$. As in SW, $b_J(s)$ will be approximated by contributions coming from single-meson-exchange diagrams. One obvious defect of this approximation is that solutions of (12) will certainly not have the required threshold behavior $h_J(s)\simeq(s-4m^2)^J$ for $J>0$, because $b_J(s)$ itself has this behavior while the dispersion integral is positive-definite at threshold. Therefore, some rescattering correction to $b_J(s)$ must be included. We shall modify Eq. (12) by using a similar equation for $\tilde{h}_J$ defined by

$$\tilde{h}_J(s)\equiv(1/s)[(s+s_c)/(s-4m^2)]^J h_J(s), \qquad (13)$$

where s_c is a real parameter.

For this amplitude, the analog of Eq. (12) becomes

$$\tilde{h}_J(s)=\tilde{b}_J(s)+\frac{1}{\pi}\int_{4m^2}^{\infty} ds'\left(\frac{mp's'}{E'}\right)\left(\frac{s'-4m^2}{s'+s_c}\right)^J\frac{|\tilde{h}_J(s')|^2}{s'-s-i\epsilon}, \qquad (14)$$

where $\tilde{b}_J(s)$ is given by

$$\tilde{b}_J(s)=\frac{1}{s}\left(\frac{s+s_c}{s-4m^2}\right)^J b_J(s)+\frac{1}{s}\left(\frac{s_c}{-4m^2}\right)^J(h_J(0)-b_J(0)). \qquad (15)$$

Now, the threshold behavior of $\tilde{b}_J$ from the single-meson-exchange contribution will be like a constant. The solution of (14), if it exists, will also produce a constant threshold behavior for $\tilde{h}_J$, thus the partial-wave amplitude $h_J(s)$ given by the inverse of Eq. (13) will have the proper threshold behavior. However, $h_J(s)$ will now have a Jth-order pole at $s=-s_c$. This singularity is interpreted as an approximate replacement for the singularities produced by rescattering corrections.

Aside from the Jth-order pole at $s=-s_c$, we have also introduced a $(1/s)$ factor in the definition of $\tilde{h}_J$. Of course, if $b_J(s)$ and $h_J(0)$ were known exactly, there would be no point in considering the amplitude $\tilde{h}_J$ instead of h_J. However, with a given approximation for $b_J(s)$, the solution of Eq. (12) for the partial-wave amplitude may be improved by using the above manipulation in (13)–(15) provided $h_J(0)$ can be obtained by an independent method. It was shown in GGMW that $h_J(0)$ is in fact related to a combination of other partial-wave amplitudes at $s=0$. Namely,

$$h_J=h_{J+2}+\left(\frac{J}{J+1}\right)\left(\frac{1}{2J+1}\right)\times\{(J+1)h_{J-1,J}+Jh_{J+1,J}-2[J(J+1)]^{1/2}h^J\}+\frac{2J+3}{(J+1)(J+2)}h_{J+1,J+1}-\left(\frac{J+3}{J+2}\right)\left(\frac{1}{2J+5}\right)\times\{(J+3)h_{J+1,J+2}+(J+2)h_{J+3,J+2}-2[(J+2)(J+3)]^{1/2}h^{J+2}\}. \quad (16)$$

For $J=0$, the second term in Eq. (16) vanishes and we obtain a relation between the singlet S-wave amplitude and a combination of P and higher partial waves. Since our single-meson-exchange model of NN and $N\bar{N}$ interaction will be more reliable for higher partial waves, Eq. (16) will probably yield a better determination of $h_0(0)$ than the corresponding quantity obtained through the dispersion relation without the $(1/s)$ factor in (13). In practice, it is sufficient to approximate the right-hand side of (16) by using dispersion relations analogous to Eq. (14), but with $|\tilde{h}_J(s')|^2$ replaced by $|\tilde{b}_J(s')|^2$. For $J>0$ in Eq. (15), we shall simply approximate $h_J(0)$ by $b_J(0)$.

Once $\tilde{b}_J(s)$ is given, Eq. (14) can be solved by the familiar N/D method provided $\tilde{b}_J(s)$ vanishes faster than $(\ln s)^{-1}$ as $s\to+\infty$. This asymptotic behavior is in fact not satisfied due to the logarithmic divergence produced by the exchange of vector mesons. On the other hand, if a cutoff is imposed on the dispersion integral, then a solution can be obtained. For simplicity, we will impose the cutoff at $s=s_c$, where s_c is the same parameter which enters into the Jth-order pole at $s=-s_c$. This cutoff procedure is considerably simpler than the Regge-pole approximation of SW and reduces the three Regge-slope parameters to a single cutoff parameter for the present calculation.

The N/D equations are obtained as follows. First, we express $\tilde{h}_J$ in the form of a quotient

$$\tilde{h}_J(s)\equiv N_J(s)/D_J(s) \quad (17)$$

and impose the condition that N_J is real above $s=4m^2$ and D_J is real below $s=4m^2$. From the unitarity condition (11), we obtain

$$\mathrm{Im}D_J(s)=-\left(\frac{s-4m^2}{s+s_c}\right)^J\left(\frac{mps}{E}\right)N_J(s);\quad s\geq 4m^2 \quad (18)$$

and the dispersion relation

$$D_J(s)=1-\frac{1}{\pi}\int_{4m^2}^{s_c}ds'\left(\frac{s'-4m^2}{s'+s_c}\right)^J\left(\frac{mp's'}{E'}\right)\frac{N_J(s')}{(s'-s)}. \quad (19)$$

For the N function, it must contain all the singularities of $\tilde{b}_J(s)D_J(s)$ below $s=4m^2$, but must be pure real above the threshold. Hence the expression for N_J reads

$$N_J(s)=\tilde{b}_J(s)D_J(s)-\frac{1}{\pi}\int_{4m^2}^{s_c}ds'\frac{\tilde{b}_J(s')\,\mathrm{Im}D_J(s')}{s'-s}. \quad (20)$$

After substituting Eq. (19) into Eq. (20), we obtain the integral equation

$$N_J(s)=\tilde{b}_J(s)+\frac{1}{\pi}\int_{4m^2}^{s_c}ds'[\tilde{b}_J(s')-\tilde{b}_J(s)]\times\left(\frac{s'-4m^2}{s'+s_c}\right)^J\left(\frac{mp's'}{E'}\right)\frac{N_J(s')}{(s'-s)}. \quad (21)$$

Equation (21) is a regular Fredholm equation of the second kind which possesses a unique solution for a given $\tilde{b}_J(s)$. This equation can be solved by straightforward numerical methods. Having solved Eq. (21) for the N functions, the D functions can be evaluated by using Eq. (19). For a $\tilde{b}_J(s)$ corresponding to a strong attractive interaction, the D function will pass through zero at a point below the threshold. This zero corresponds to a bound-state pole in the partial-wave amplitude. The square of the mass of the bound state is equal to the value of s at the pole.

B. Uncoupled Triplet Amplitudes

For the partial-wave amplitudes $h_{JJ}(s)$, the orbital angular momentum is equal to J and is greater than zero. Therefore, no advantage will be gained by making use of relations at $s=0$ such as those given by Eq. (16). The problem of threshold behavior is, however, handled in the same way as in the singlet case. We define

$$\tilde{h}_{JJ}(s)=\left(\frac{s+s_c}{s-4m^2}\right)^J h_{JJ}(s), \quad (22)$$

$$\tilde{b}_{JJ}(s)=\left(\frac{s+s_c}{s-4m^2}\right)^J b_{JJ}(s), \quad (23)$$

and use the same N/D equations as (19) and (21) except that the $(mp's'/E')$ factors are now replaced by

(mp'/E'). Here again b_{JJ} denotes the meson-exchange contribution to h_{JJ}.

C. Coupled Triplet Amplitudes

For any given total angular momentum J, let us define h to be the 2×2 matrix

$$h=\begin{pmatrix} h_{J-1,J} & h^J \\ h^J & h_{J+1,J} \end{pmatrix}. \quad (24)$$

The unitarity condition can then be expressed as

$$\mathrm{Im}h^{-1}=-\left(\frac{mp}{E}\right)\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}; \quad s>4m^2. \quad (25)$$

If it were not for the $(s)^{1/2}$ kinematical singularity, we could immediately write down ND^{-1} equations in the matrix form[10] as long as $\mathrm{Im}h^{-1}$ is a known function in the physical region $s>4m^2$.

As we shall see below, the $(s)^{1/2}$ singularity appears in a simple form in the helicity partial-wave amplitudes given by

$$H\equiv\begin{pmatrix} H_{11}{}^J & H_{12}{}^J \\ H_{21}{}^J & H_{22}{}^J \end{pmatrix}=X^T hX, \quad (26)$$

where

$$X=\begin{bmatrix} \left(\frac{J}{2J+1}\right)^{1/2} & \left(\frac{J+1}{2J+1}\right)^{1/2} \\ \left(\frac{J+1}{2J+1}\right)^{1/2} & -\left(\frac{J}{2J+1}\right)^{1/2} \end{bmatrix}. \quad (27)$$

Explicitly, the relation is

$$H_{11}{}^J=\frac{1}{2J+1}\{Jh_{J-1,J}+(J+1)h_{J+1,J}+2[J(J+1)]^{1/2}h^J\}, \quad (28)$$

$$H_{12}{}^J=H_{21}{}^J=\frac{1}{2J+1}\{[J(J+1)]^{1/2}\times(h_{J-1,J}-h_{J+1,J})+h^J\}, \quad (29)$$

$$H_{22}{}^J=\frac{1}{2J+1}\{(J+1)h_{J-1,J}+Jh_{J+1,J}-2[J(J+1)]^{1/2}h^J\}. \quad (30)$$

By making use of (28)–(30) and the relation between partial-wave amplitudes and invariant scattering amplitudes given by Eqs. (6)–(10), one can easily verify that $H_{11}{}^J$ and $H_{22}{}^J$ are given in terms of $E\varphi_1$, $E\varphi_2$, $E\varphi_3$, and $E\varphi_4$ while $H_{12}{}^J(H_{21}{}^J)$ involves only $E\varphi_5$. Therefore, the $(s)^{1/2}$ singularity only appears in $H_{12}{}^J(H_{21}{}^J)$, thus this kinematical singularity can be removed by simply dividing $H_{12}{}^J(H_{21}{}^J)$ by $(s)^{1/2}$. Unfortunately, the task of formulating the ND^{-1} equations is a somewhat complicated matter. Not only do we want to remove the kinematic singularity from the partial-wave amplitudes, but we must also perform a transformation similar to Eq. (22) to produce the proper threshold behavior. Since the helicity amplitudes $H_{11}{}^J$, $H_{22}{}^J$, and $H_{12}{}^J$ are each a combination of $h_{J-1,J}$, $h_{J+1,J}$, and h^J, there are three algebraic relations at the threshold which must be maintained in the ND^{-1}-type equations. For this reason we introduce a new set of amplitudes given by

$$A\equiv\begin{pmatrix} A_{11}{}^J & A_{12}{}^J \\ A_{21}{}^J & A_{22}{}^J \end{pmatrix}=Y^T HY, \quad (31)$$

where

$$Y=\frac{\sqrt{2J+1}}{p^2}\begin{bmatrix} m(J+1)^{1/2}\left(\frac{s+s_c}{p^2}\right)^{(J-1)/2} & p^2(J+1)^{1/2}\left(\frac{s+s_c}{p^2}\right)^{(J+1)/2} \\ -EJ^{1/2}\left(\frac{s+s_c}{p^2}\right)^{(J-1)/2} & -p^2J^{1/2}\left(\frac{m}{E}\right)\left(\frac{s+s_c}{p^2}\right)^{(J+1)/2} \end{bmatrix}. \quad (32)$$

For the individual elements of the 2×2 matrix, the above transformation gives

$$A_{11}{}^J=\left(\frac{2J+1}{p^4}\right)\left(\frac{s+s_c}{p^2}\right)^{J-1}\{(J+1)m^2H_{11}{}^J+JE^2H_{22}{}^J-2[J(J+1)]^{1/2}mEH_{12}{}^J\}, \quad (33)$$

$$A_{12}{}^J=A_{21}{}^J=\left(\frac{2J+1}{p^2}\right)\left(\frac{s+s_c}{p^2}\right)^{J}\{(J+1)mH_{11}{}^J+JmH_{22}{}^J-[J(J+1)]^{1/2}(E^2+m^2)H_{12}{}^J/E\}, \quad (34)$$

$$A_{22}{}^J=(2J+1)\left(\frac{s+s_c}{p^2}\right)^{J+1}\{(J+1)H_{11}{}^J+Jm^2H_{22}{}^J/E^2-2[J(J+1)]^{1/2}mH_{12}{}^J/E\}. \quad (35)$$

[10] J. D. Bjorken, Phys. Rev. Letters 4, 473 (1960).

It is easily seen that these amplitudes have no branch point at $s=0$ and all behave like a constant near the threshold. The inverse transformation from A to h will give the proper threshold behavior for each element of h.

At this point, we can derive the ND^{-1} equations for the 2×2 matrix A. First, the unitarity condition is given by

$$\begin{aligned}\rho &\equiv \mathrm{Im}A^{-1}=\mathrm{Im}(Y^TX^ThXY)^{-1}\\ &=(Y^{-1})(X^{-1})(\mathrm{Im}h)(X^T)^{-1}(Y^T)^{-1}\\ &=-\left(\frac{mp}{E}\right)\left(\frac{p^2}{s+s_c}\right)^{J-1}\left[\frac{1}{J(J+1)}\right]\begin{bmatrix}\dfrac{(J+1)E^2+Jm^2}{(2J+1)} & -\left(\dfrac{mE^2}{s+s_c}\right)\\ -\left(\dfrac{mE^2}{s+s_c}\right) & \dfrac{JE^2+(J+1)m^2}{(2J+1)}\left(\dfrac{E}{s+s_c}\right)^2\end{bmatrix},\quad s\geq 4m^2.\end{aligned}\tag{36}$$

Expressing A in the form

$$A=ND^{-1}$$

and making the usual requirement that the N matrix is real on the right $s\geq 4m^2$ and the D matrix is real on the left $s<4m^2$, we obtain

$$D(s)=I-\frac{1}{\pi}\int_{4m^2}^{s_c}ds'\frac{\rho(s')N(s')}{(s'-s)},\tag{37}$$

where the product ρN is of course understood as a matrix product. Finally, we denote the meson-exchange contribution to A^J by B^J and obtain the integral equation for N as before:

$$N(s)=B^J(s)+\frac{1}{\pi}\int_{4m^2}^{s_c}ds'\times\frac{1}{s'-s}[B^J(s')-B^J(s)]\rho(s')N(s').\tag{38}$$

Returning now to the definition of A_{11}^J, A_{12}^J, and A_{22}^J given by Eqs. (33)–(35), we see that, in general, A_{22}^J has a pole at $s=0$ given by

$$A_{22}^J\underset{s\to 0}{=}\{4J(2J+1)m^2(-s_c/m^2)^{J+1}H_{22}^J(0)\}/s.\tag{39}$$

In order to take proper account of this pole, the $B_{22}^J(s)$ element appearing in Eq. (38) must be replaced by

$$B_{22}^J(s)\to B_{22}^J(s)+\{4J(2J+1)m^2(-s_c/m^2)^{J+1}H_{22}^J(0)-[sB_{22}^J(s)]_{s\to 0}\}/s.\tag{40}$$

For $J=1$, we make use of Eq. (16) to obtain the following expression for $H_{22}^J(0)$ in terms of P and higher partial waves:

$$H_{22}^1=2h_1-2h_3-(5/3)h_{2,2}+(8/3)H_{22}^3.\tag{41}$$

As in the case of the singlet S amplitude we approximate the right-hand side of (41) by the meson-exchange contribution plus a dispersion integral obtained from the first iteration of the meson-exchange terms. For $J>1$, the quantity appearing in the bracket of Eq. (40) will be neglected.

Now we proceed to the calculation of the meson-exchange contribution to $b_J(s)$, $b_{JJ}(s)$, and $B^J(s)$.

III. SINGLE-MESON-EXCHANGE CONTRIBUTION

In the following, we write down explicitly the t-channel meson-exchange contribution to the NN helicity amplitudes $\varphi_1,\cdots,\varphi_5$ in the $I=0$ state. The u-channel contribution gives rise to a factor of 2 which must be supplied to the right-hand side of Eqs. (6)–(10) for all partial-wave amplitudes that are not excluded by the Pauli principle. For the $N\bar{N}$ partial waves, there is only one crossed channel having baryon number zero. Therefore, one needs not supply a factor of two in the calculation of meson-exchange contribution. However, a factor of two is already present in the relation between partial-wave amplitudes and the invariant scattering amplitudes as we noted earlier. The net result is that the magnitude of each meson-exchange contribution is the same for a given $N\bar{N}$ and NN partial wave provided that the NN state is not excluded by the Pauli principle. As one can verify by general arguments, the odd G-parity meson exchanges (π,ω,φ) have the opposite sign in NN compared to $N\bar{N}$, and the even G-parity mesons have the same sign. Hence, all of the meson-exchange contributions to the $N\bar{N}$ partial-wave amplitudes can be inferred directly from those of the NN partial-wave amplitudes.

To avoid ambiguities in the definition of coupling constants, we shall write down the conventional Lagrangians which will give rise to the following invariant amplitudes in the first-order perturbation expansion.

A. π Meson

The Lagrangian is given by

$$\mathcal{L}=(4\pi)^{1/2}g_\pi\bar{\psi}\gamma_5\boldsymbol{\tau}\cdot\boldsymbol{\varphi}_\pi\psi.\tag{42}$$

The $I=0$ helicity amplitudes corresponding to the

t-channel single-pion-exchange diagram are

$$(E/m)\varphi_1^{(0)}=0, \quad (43)$$

$$(E/m)\varphi_2^{(0)}=(3g_\pi^2/4m)t/(\mu_\pi^2-t), \quad (44)$$

$$(E/m)\varphi_3^{(0)}=0, \quad (45)$$

$$(E/m)\varphi_4^{(0)}=-(3g_\pi^2/4m)(1-z)2p^2/(\mu_\pi^2-t), \quad (46)$$

$$(1/\sin\theta)\varphi_5^{(0)}=0. \quad (47)$$

We shall use

$$g_\pi^2=13$$

in agreement with the coupling constant determined by most nucleon-nucleon phase-shift analysis.[6]

B. η Meson

The Lagrangian is

$$\mathcal{L}=(4\pi)^{1/2}g_\eta\bar{\psi}\gamma_5\varphi_\eta\psi. \quad (48)$$

The $I=0$ amplitudes can be obtained from π-exchange terms by replacing g_π^2 by $(-g_\eta^2/3)$ and μ_π by μ_η. The coupling constant g_η^2 can be obtained from g_π^2 assuming SU_3 symmetry provided the (D/F) ratio is given. The relation is

$$g_\eta^2=\tfrac{1}{3}(1-4F/(D+F))^2g_\pi^2. \quad (49)$$

In the work of Martin and Wali,[11] they find that

$$F/(D+F)\simeq 0.25 \quad (50)$$

would give a good fit to the masses and coupling constants of the baryon decuplet members N^*, Y_1^*, Ξ^*, and Ω. This would yield a very small value for g_η^2. On the other hand, if one assumes the approximate SU_6 symmetry, then one finds

$$F/(D+F)=0.4. \quad (51)$$

However, Eq. (49) should now be applied to the axial-vector coupling constants rather than the pseudoscalar coupling constants. One obtains, then,

$$g_\eta^2\simeq(m_\pi^2/m_\eta^2)(1-1.6)^2g_\pi^2\simeq 0.3. \quad (52)$$

We note that our final solutions for the NN and $N\bar{N}$ amplitudes are quite insensitive to the value of g_η^2. For example, setting $g_\eta^2=12$ as in SW will require a small modification of the scalar and vector coupling constants, but has a rather insignificant effect on the fit to the nucleon-nucleon data. The bound-state energies in the $N\bar{N}$ problem are also insensitive to the variation in g_η^2.

C. ρ Meson

The Lagrangian includes the vector coupling constant $g_{\rho1}$ and the tensor coupling constant $g_{\rho2}$:

$$\mathcal{L}=i(4\pi)^{1/2}(g_{\rho1}+g_{\rho2})\bar{\psi}\gamma_\nu\boldsymbol{\tau}\cdot\boldsymbol{\varphi}_\rho{}^\nu\psi -(4\pi)^{1/2}(g_{\rho2}/2m)(p+p')_\nu\bar{\psi}\boldsymbol{\tau}\cdot\boldsymbol{\varphi}_\rho{}^\nu\psi. \quad (53)$$

The $I=0$ NN helicity amplitudes corresponding to the t-channel diagram are

$$\frac{E}{m}\varphi_1^{(0)}=\left(+\frac{3g_{\rho1}^2}{m}\right)\left[\frac{2p^2+\frac{1}{2}m^2(1+z)}{m_\rho^2-t}\right]+\left(\frac{3g_{\rho2}^2p^2}{4m}\right)\left[\frac{3-4z+z^2}{m_\rho^2-t}\right]-\left(\frac{3g_{\rho1}g_{\rho2}}{m}\right)\left[\frac{t}{m_\rho^2-t}\right], \quad (54)$$

$$\frac{E}{m}\varphi_2^{(0)}=\left(-\frac{3g_{\rho1}^2}{2m}\right)\left[\frac{1-z}{m_\rho^2-t}\right]+\left(\frac{3g_{\rho2}^2p^2}{4m^3}\right)\left[\frac{-3p^2-m^2+2p^2z+(p^2+m^2)z^2}{m_\rho^2-t}\right]-\left(\frac{3g_{\rho1}g_{\rho2}}{2m}\right)\left(\frac{t}{m_\rho^2-t}\right), \quad (55)$$

$$\frac{E}{m}\varphi_3^{(0)}=\left(+\frac{3g_{\rho1}^2}{m}\right)\left[\frac{(1+z)(p^2+\frac{1}{2}m^2)}{m_\rho^2-t}\right]+\left(\frac{3g_{\rho2}^2p^2}{4m}\right)\left[\frac{(1+z)(z-1)}{m_\rho^2-t}\right], \quad (56)$$

$$\frac{E}{m}\varphi_4^{(0)}=\left(\frac{3g_{\rho1}^2}{m}\right)\left[\frac{(1-z)m^2}{m_\rho^2-t}\right]+\left(\frac{3g_{\rho2}^2p^2}{4m^3}\right)\left[\frac{(1-z)(3p^2+m^2+p^2z+m^2z)}{m_\rho^2-t}\right]-\left(\frac{3g_{\rho1}g_{\rho2}}{m}\right)\left[\frac{(1-z)p^2}{m_\rho^2-t}\right], \quad (57)$$

$$\frac{1}{\sin\theta}\varphi_5^{(0)}=\left(-\frac{3g_{\rho1}^2}{2m}\right)\left(\frac{m^2}{m_\rho^2-t}\right)+\left(\frac{3g_{\rho2}^2p^2}{4m}\right)\left[\frac{(1-z)}{m_\rho^2-t}\right]+\left(\frac{3g_{\rho1}g_{\rho2}}{m}\right)\left(\frac{p^2}{m_\rho^2-t}\right). \quad (58)$$

We shall assume that the electromagnetic form factors of the nucleon are dominated by the contribution of the vector-meson pole. We then have

$$\frac{e}{2}=(g_{\rho1}g_{\rho\gamma}/m_\rho^2), \quad (59)$$

$$\mu_v(e/2m)=(g_{\rho2}g_{\rho\gamma}/2mm_\rho^2), \quad (60)$$

where $\mu_v=1.83$ is the gyromagnetic ratio of the isovector anomalous moment. From (59) and (60), we obtain

$$g_{\rho2}^2=4\mu_v^2g_{\rho1}^2=13.4g_{\rho1}^2. \quad (61)$$

D. ω and φ Mesons

The Lagrangians for the ω and φ interactions are the same as that for ρ except for the replacement of $(\boldsymbol{\tau}\cdot\boldsymbol{\varphi})$ by φ. As in SW, we omit the $g_{\omega2}$ and $g_{\varphi2}$ coupling in view of the extremely small isoscalar anomalous mag-

[11] A. W. Martin and K. C. Wali, Phys. Rev. **130**, 2455 (1963).

netic moment. In the present work, we also assume that the vector-baryon coupling is primarily in the form of a pure F-type octet. We can then obtain all the coupling constants in terms of one parameter g_v:

$$g_{\rho 1}^2=g_{\omega 1}^2=\tfrac{1}{2}g_{\varphi 1}^2=g_v^2;\qquad g_{\rho 2}^2=13.4g_v^2,\quad g_{\omega 2}=g_{\varphi 2}=0. \tag{62}$$

Here we also use the ω-φ mixing hypothesis to obtain the ratio between $g_{\omega 1}$ and $g_{\varphi 1}$. In the actual calculation, we will set all the vector-meson masses equal to an average value, hence the results are independent of the ω-φ mixing ratio.

E. $I=0$ Scalar Meson (σ)

Following SW, we approximate the contribution of the $I=0$, $J=0$, $P=+$multimeson continuum by an effective scalar particle of mass m_σ and coupling constant g_σ. The Lagrangian reads

$$\mathcal{L}=(4\pi)^{1/2}g_\sigma\bar{\psi}\varphi_\sigma\psi.$$

The $I=0$, NN amplitudes are

$$\frac{E}{m}\varphi_1^{(0)}=\left(\frac{g_\sigma^2}{2m}\right)\left[\frac{m^2(1+z)}{m_\sigma^2-t}\right], \tag{63}$$

$$\frac{E}{m}\varphi_2^{(0)}=\left(\frac{g_\sigma^2}{2m}\right)\left[\frac{-(p^2+m^2)(1-z)}{m_\sigma^2-t}\right], \tag{64}$$

$$\frac{E}{m}\varphi_3^{(0)}=\left(\frac{g_\sigma^2}{2m}\right)\left[\frac{(1+z)m^2}{m_\sigma^2-t}\right], \tag{65}$$

$$\frac{E}{m}\varphi_4^{(0)}=\left(\frac{g_\sigma^2}{2m}\right)\left[\frac{(1-z)(p^2+m^2)}{m_\sigma^2-t}\right], \tag{66}$$

$$\frac{1}{\sin\theta}\varphi_5^{(0)}=\left(\frac{g_\sigma^2}{2m}\right)\left[\frac{-m^2}{m_\sigma^2-t}\right]. \tag{67}$$

IV. NUMERICAL RESULTS

A. Nucleon-Nucleon Scattering

Having obtained the formulas for the single-meson-exchange contribution to the helicity amplitudes as given in the previous section, one can apply Eqs. (6)–(10) to evaluate the partial-wave projection of these amplitudes. These results are explicitly given in the Appendix. Some of the parameters appearing in the single-meson-exchange terms are measurable quantities which will be taken with fixed values, namely,

$$m=938\text{ MeV},$$
$$m_\pi=140\text{ MeV},$$
$$m_\eta=548\text{ MeV},$$
$$g_\pi^2=13.$$

For simplicity we will use an average mass for the vector mesons ρ, ω, and φ:

$$m_v^2=\tfrac{1}{4}(m_\rho^2+m_\omega^2+2m_\varphi^2)\simeq(6.45m_\pi)^2. \tag{68}$$

The weight is taken according to the coupling strength

$$g_{\rho 1}^2=g_{\omega 1}^2=\tfrac{1}{2}g_{\varphi 1}^2=g_v^2. \tag{69}$$

The tensor coupling constants $g_{\omega 2}$ and $g_{\varphi 2}$ are set equal to zero and $g_{\rho 2}$ is determined by the isovector anomalous magnetic moment to charge ratio:

$$g_{\rho 2}^2=13.4g_{\rho 1}^2=13.4g_v^2. \tag{70}$$

The η coupling constant given by Eq. (52) is

$$g_\eta^2=0.3.$$

The only additional parameter is the cutoff s_c which enters into the ND^{-1} equations.

To summarize, we have a total of four adjustable parameters: g_v, m_σ, g_σ, and s_c. For a given value of s_c, we vary g_v, m_σ, g_σ to obtain a best fit to the pp: $^1S_0$, $^3P_0$, $^3P_1$, $^3P_2$ and np: $^3S_1$ phase shifts at 25, 50, 95, 142, 210, and 310 MeV.[6] Results for the cutoff s_c corresponding to laboratory kinetic energies of 600, 700, and 800 MeV are presented in Table I. It is apparent that the fit is not very sensitive to the value of the cutoff in this region. In terms of the "goodness-of-fit" parameter, all of these fits are consistent with a four-parameter theory having an inherent uncertainty of approximately 10%. The best values of the three physical parameters g_σ^2, m_σ^2, g_v^2 for each value of the cutoff are given in Table II.

By calculating the D function (the determinant of the D matrix in the coupled triplet case), we find that a bound-state pole appears in the $I=0$ coupled triplet $J=1$ amplitude. This pole corresponds to a deuteron with binding energy in the neighborhood of 10 MeV. The amount of discrepancy between this value and the true binding energy of $\sim$2.2 MeV is not surprising in view of the simplicity of our four-parameter theoretical model.

We note that it is not possible to obtain a reasonably good fit with a cutoff below 400 MeV or above 1200 MeV.

For each set of parameters corresponding to Table II, we also calculated the D and higher partial-wave phase shifts. These results are comparable to those obtained by SW and they are in fairly good agreement with those given by phase-shift analyses.[6]

B. Nucleon-Antinucleon Bound States

After fitting the NN scattering phase shifts with the four adjustable parameters, we solve the ND^{-1} equations for $N\bar{N}$ scattering without changing the values of the parameters. The only modification needed is to change the sign of the (π,ω,φ) exchange contributions as required by crossing symmetry. Here, we also calculate all of the partial-wave amplitudes excluded by the Pauli

TABLE I. Comparison of theoretical and experimental nuclear bar phase shifts in degrees.

			Cutoff (MeV)	Lab kinetic energy in MeV 25	50	95	142	210	310
pp	$^1S_0$	Expt.	···	50.2±0.4	37.7±0.6	25.1±2.4	16.6±0.7	5.1±0.6	−6.9±1.6
		Theor.	600	52.0	42.0	28.6	17.4	3.6	−15.1
			700	47.8	39.2	26.9	16.6	3.8	−13.2
			800	45.1	37.3	25.9	16.1	4.1	−11.5
	$^3P_0$	Expt.	···	5.6±0.9	12.0±0.8	12.8±1.9	6.3±0.6	−0.7±0.6	−11.3±1.7
		Theor.	600	9.0	12.6	12.0	7.9	−0.1	−12.8
			700	8.6	12.0	11.2	7.0	−0.6	−12.9
			800	8.7	12.1	11.3	7.4	0.0	−11.6
	$^3P_1$	Expt.	···	−3.5±0.4	−8.0±0.3	−13.0±0.5	−17.1±0.4	−21.6±0.6	−28.5±1.3
		Theor.	600	−4.8	−7.8	−11.4	−14.4	−18.4	−24.8
			700	−4.9	−8.2	−12.2	−15.4	−19.9	−25.5
			800	−5.0	−8.2	−12.3	−15.7	−20.2	−26.7
	$^3P_2$	Expt.	···	2.0±0.2	6.1±0.2	10.6±0.5	13.7±0.2	15.9±0.3	16.4±0.7
		Theor.	600	2.4	5.8	11.3	15.4	18.7	20.2
			700	2.0	4.9	9.6	13.0	15.9	17.4
			800	1.9	4.5	8.6	11.7	14.1	15.2
np	$^3S_1$	Expt.	···	78.7±5.2	60.8±2.7	44.5±1.7	29.6±0.9	17.6±2.4	−1.0±5.2
		Theor.	600	95.7	76.4	56.4	42.0	25.9	5.8
			700	95.4	76.4	56.4	42.3	26.3	7.0
			800	95.2	76.2	56.4	42.2	26.7	8.1

principle in the NN problem. For each of the three sets of parameters given above, we find that there are five and only five bound-state poles in the partial-wave amplitudes. They are the four S-wave amplitudes having the quantum number of η, π, ω (or φ), and ρ, and the $I=0$ $^3P_0$ amplitude having the quantum number of the σ. Numerical results are tabulated in Table III.

V. REMARKS

As we have stated before, the main objective of the present work in regard to NN scattering is to use a minimum number of phenomenological parameters in as much as an over-all fit to all the $I=0$, 1 phase shifts is possible. The results given above indicate that the two-body nuclear forces are, to a good approximation, dominated by π, σ, η, ρ, ω, and φ exchange.

Although a reasonable fit to the $^3S_1$ phase shift will guarantee the occurrence of the deuteron pole, it is rather encouraging that our calculation yields a binding energy within 10 MeV of the physical deuteron in spite of the fact that the potentials due to an individual meson is typically several hundred MeV in strength.

From a pragmatic standpoint, the question of whether a particle is composite can be answered by an S-matrix calculation using our knowledge of the strong interaction at any given stage. If the calculated S matrix agrees with the scattering data to the expected accuracy and contains a pole with the proper mass and the proper sign of the residue, then this pole corresponds to a composite particle. A physical particle must be found with the same quantum numbers and approximately the same mass and coupling constant, otherwise, the fit to the scattering data would be invalidated. In the case of the deuteron, experience has strongly supported the composite-particle interpretation and we have only added one more claim along that line. Presumably, nuclei with baryon number greater than two are also composite in the same sense. The more interesting questions concern particles of baryon number one and zero.

For the baryon number one, many authors[12] have contributed works showing that the baryons and the baryon resonances are composite particles consisting of mesons and baryons. However, the knowledge of the forces, the S-matrix method, and the scattering data

TABLE II. Masses (in pion units) and coupling constants of the effective σ meson and coupling constants of octet vector meson obtained by fitting nucleon-nucleon scattering phase shifts.

Cutoff	600 MeV	700 MeV	800 MeV
g_σ^2	5.15	4.80	4.15
m_σ	3.90	3.95	3.85
g_v^2	1.36	1.41	1.41

TABLE III. Square of the masses of the nucleon-antinucleon bound states (in pion units) having the quantum numbers of η, π, ω, ρ, and σ.

Bound states \ Cutoff	600 MeV	700 MeV	800 MeV
$I=0$, $^1S_0$	71	−10	−75
$I=1$, $^1S_0$	171	171.6	172.8
$I=0$, $^3S_1$	172.8	173	173.4
$I=1$, $^3S_1$	155.3	150	146.4
$I=0$, $^3P_0$	172.3	169.9	168.0

[12] G. F. Chew, Phys. Rev. Letters 9, 233 (1962); E. Abers and C. Zemach, Phys. Rev. 131, 2305 (1963); J. S. Ball and D. Y. Wong, *ibid.* 133, B179 (1964).

are all less reliable than those in the NN problem. Nevertheless, from the point of view discussed above, it is fair to say that the accumulated evidence is in favor of all baryon and baryon resonances being composite.

When one examines particles with baryon number zero (mesons), the question of compositeness is still more dubious. The most frequently discussed problem is that of the ρ meson.[13] In all of the works without the $N\bar{N}$ channel, a very short-range force of phenomenological nature either in the form of a cutoff or in the form of a distant unphysical singularity has to be included in order to produce the physical ρ meson as a composite particle. On the other hand, our present work shows that the $N\bar{N}$ channel alone is capable of producing a bound particle in the $I=1$, triplet $J=1$ state without using a distant cutoff. The fact that the same nuclear forces used in the NN problem do produce the bound state in the $N\bar{N}$ system can be taken to be a strong evidence that this composite state is associated with a physical particle. Due to the omission of the low-lying $\pi\pi$, $K\bar{K}$ channels, it is to be expected that the bound state we produced is substantially more massive than the physical ρ meson. It seems rather likely that the combination of these meson channels together with the $N\bar{N}$ system can yield a fairly realistic picture of the ρ meson.

For the ω and φ mesons, we have also found a bound state in the $N\bar{N}$ system having the proper quantum numbers. Again, the inclusion of meson channels such as $K\bar{K}$ will lower the mass of the bound state. However, it is very unlikely that the meson channels will produce an additional composite particle to account for the physically distinct ω and φ. Since the physical ω and φ are commonly believed to be mixtures of a singlet and an octet in the SU_3 scheme, one should naturally take strangeness into consideration. As one can easily see, the baryon-antibaryon system can couple to the singlet as well as the octet states. Generally speaking, the potentials in the singlet state tend to add coherently and is therefore stronger than those in the octet. On the other hand, the existence of the bound state with the quantum number of the ρ indicates that the potential in the octet is already strong and attractive. Hence, one might find that the addition of the $\Lambda\bar{\Lambda}$, $\Sigma\bar{\Sigma}$, and $\Xi\bar{\Xi}$ channels will yield two bound states of $I=0$ and $J=1$ with the singlet particle more tightly bound than the octet. Further addition of the two-pseudoscalar channel will then lower the mass of the octet particle without affecting the singlet since the latter is forbidden by Bose statistics. Of course, the foregoing arguments are speculative and can be substantiated only by calculations. Nevertheless, this seems to constitute a feasible dynamical model of the ω-φ mixing.

For the singlet $J=0$ systems, our result for the mass of the $I=1$ bound state is approximately a factor of 10 heavier than the pion mass. Clearly, the $N\bar{N}$ system is not the dominant channel in making the physical pion. Among the available meson channels, the totally symmetric three-pion system seems to be the most likely candidate to produce a low-lying bound state.[14] It would be of some interest to combine the $N\bar{N}$ channel with the three-pion system and investigate the migration of the bound-state pole. In particular, one can observe whether there is one or more composite particles in the combined system.

As we have shown in the previous section, the bound state in the $I=0$, $J=0$ amplitude is considerably more tightly bound than all of the others. It is also the only bound state that is sensitive to the cutoff parameter. The square of the mass varies from $71m_\pi^2$ at 600-MeV cutoff to the unphysical value of $-75m_\pi^2$ at 800-MeV cutoff. Although these results undoubtedly indicate the inadequacy of the present S-matrix calculation, nevertheless, they also show that the attractive force in this state is clearly stronger than that in the other states. There seems to be no compelling reason to believe that other channels will be important in a realistic calculation of the η meson. Among the meson channels, the lowest lying ones are the $K\bar{K}^*$ channel and the uncorrelated four-pion channel. It is not surprising that the $N\bar{N}$ channel is indeed the dominant one.

For the ${}^3P_0$ amplitude, the correspondence of the $N\bar{N}$ bound state to any physical particle is somewhat dubious because of the lack of clear cut experimental evidence for an $I=0$, $J=0$, $P=+$particle. Theoretically, it will be of some interest to examine the behavior of this bound state under the coupling to the $\pi\pi$ channel, particularly in regard to the question of whether there should be a threshold enhancement or an actual peak in the $\pi\pi$ cross section.

Finally, among the other P-wave states, we find that the strongest attraction appears in the $I=1$, ${}^3P_1$ amplitude. Although no resonance is found, the phase shift is sufficiently large ($\sim 40°$) that a resonance can easily be produced when an additional attractive channel is turned on. This might be a relevant consideration in a dynamical model of the B meson.[15]

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APPENDIX

We now present explicit formulas for the contribution of each type of meson exchange to the amplitudes

[13] For reference to earlier works, see, for example, J. R. Fulco, G. L. Shaw, and D. Y. Wong, Phys. Rev. **137**, B1242 (1965).

[14] A. Ahmadzadeh and J. A. Tjon, Phys. Rev. **139**, B1085 (1965).

[15] M. Abolins *et al.*, Phys. Rev. Letters **11**, 381 (1963).

h_J, h_{JJ}, $h_{J-1,J}$, $h_{J+1,J}$, and h^J. These contributions are denoted b_J, b_{JJ}, $b_{J-1,J}$, $b_{J+1,J}$, and b^J and are the results of performing the appropriate angular projection operations on the φ's as given in Sec. III. Each meson-exchange term is to be multiplied by the isotopic spin crossing matrix. For the t-channel contribution,

$$\begin{pmatrix} I_{NN}=0 \\ I_{NN}=1 \end{pmatrix} = \begin{pmatrix} \frac{1}{2} & -\frac{3}{2} \\ \frac{1}{2} & \frac{1}{2} \end{pmatrix} \begin{pmatrix} I_m=0 \\ I_m=1 \end{pmatrix},$$

where I_m is the isospin of the meson and I_{NN} is the isospin state of the NN. The u-channel contribution gives rise to a factor of 2 for all nonvanishing partial-wave amplitudes. This is included in the expressions below.

A. Pseudoscalar Exchange

The contribution of π exchange and η exchange has the following form:

$$b_J = \frac{g_p^2}{2m}\left\{\left(\frac{J}{2J+1}\right)Q_{J-1}(Z_p) - Q_J(Z_p) + \left(\frac{J+1}{2J+1}\right)Q_{J+1}(Z_p)\right\},$$

$$b_{JJ} = \frac{g_p^2}{2m}\left\{-\left(\frac{J+1}{2J+1}\right)Q_{J-1}(Z_p) + Q_J(Z_p) - \left(\frac{J}{2J+1}\right)Q_{J+1}(Z_p)\right\},$$

$$b_{J-1,J} = \frac{g_p^2}{2m}\left(\frac{1}{2J+1}\right)[Q_{J-1}(Z_p) - Q_J(Z_p)],$$

$$b_{J+1,J} = \frac{g_p^2}{2m}\left(\frac{1}{2J+1}\right)[Q_J(Z_p) - Q_{J+1}(Z_p)],$$

$$b^J = \frac{g_p^2}{2m}\left[\frac{[J(J+1)]^{1/2}}{2J+1}\right](-Q_{J-1}(Z_p) + 2Q_J(Z_p) - Q_{J+1}(Z_p)),$$

where g_p is the pseudoscalar coupling constant, the Q's are Legendre functions of the second kind, and

$$Z_p = 1 + (\mu_p^2/2p^2),$$

μ_p being the mass of the exchanged meson.

B. Scalar Meson Exchange

The scalar-meson-exchange contribution, i.e., the σ meson is as follows:

$$b_J = \left(\frac{g_s^2}{2mp^2}\right)\left[-\left(\frac{J}{2J+1}\right)p^2 Q_{J-1}(Z_s) + (p^2+2m^2)Q_J(Z_s) - \left(\frac{J+1}{2J+1}\right)p^2 Q_{J+1}(Z_s)\right],$$

$$b_{JJ} = \left(\frac{g_s^2}{2mp^2}\right)\left[-\left(\frac{J+1}{2J+1}\right)p^2 Q_{J-1}(Z_s) + (p^2+2m^2)Q_J(Z_s) - \left(\frac{J}{2J+1}\right)p^2 Q_{J+1}(Z_s)\right\},$$

$$b_{J-1,J} = \left(\frac{g_s^2}{2mp^2}\right)\frac{1}{(2J+1)^2}\{[(2J^2+2J+1)(p^2+2m^2) + 4J(J+1)mE]Q_{J-1}(Z_s)$$
$$+ 2J(J+1)(p^2+2m^2-2mE)Q_{J+1}(Z_s) - (2J+1)^2 p^2 Q_J(Z_s)\},$$

$$b_{J+1,J} = \frac{g_s^2}{2mp^2}\left(\frac{1}{2J+1}\right)^2\{2J(J+1)(p^2+2m^2-2mE)Q_{J-1}(Z_s) - (2J+1)^2 p^2 Q_J(Z_s)$$
$$+ [(2J^2+2J+1)(p^2+2m^2) + 4J(J+1)mE]Q_{J+1}(Z_s)\},$$

$$b^J = \frac{g_s^2}{2mp^2}\frac{[J(J+1)]^{1/2}}{(2J+1)^2}\{-p^2-2m^2+2mE\}\{Q_{J-1}(Z_s) - Q_{J+1}(Z_s)\},$$

where g_s is the scalar-meson coupling constant and $Z_s = 1 + (\mu_s^2/2p^2)$, μ_s is the mass of the scalar meson.

C. Vector-Meson Exchange

We will first present the results for the vector meson–nucleon charge coupling, which is applicable to the ρ, ω, and φ exchange. This contribution is

$$b_J=-(g_{V1}^2/mp^2)(2p^2+m^2)Q_J(Z_v),$$

$$b_{JJ}=-\frac{g_{V1}^2}{mp^2}\left[\left(\frac{J+1}{2J+1}\right)p^2Q_{J-1}(Z_v)+(p^2+m^2)Q_J(Z_v)+\frac{J}{(2J+1)}p^2Q_{J+1}(Z_v)\right],$$

$$b_{J-1,J}=-\frac{g_{V1}^2}{mp^2}\left[\frac{1}{2J+1}\right]^2\{[(J+1)^2p^2+(2J^2+2J+1)m^2+2J(J+1)mE]Q_{J-1}(Z_v)$$
$$+(6J^2+5J+1)p^2Q_J(Z_v)+J(J+1)(p^2+2m^2-2mE)Q_{J+1}(Z_v)\},$$

$$b_{J+1,J}=-\frac{g_{V1}^2}{mp^2}\left[\frac{1}{2J+1}\right]^2\{J(J+1)(2m^2+p^2-2mE)Q_{J-1}(Z_v)+(6J^2+7J+2)p^2Q_J(Z_v)$$
$$+[J^2p^2+(2J^2+2J+1)m^2+2J(J+1)mE]Q_{J+1}(Z_v)\},$$

$$b^J=-\frac{g_{V1}^2}{mp^2}\frac{[J(J+1)]^{1/2}}{(2J+1)^2}\{[-(J+1)p^2-m^2+mE]Q_{J-1}(Z_v)+(2J+1)p^2Q_J(Z_v)+(-Jp^2+m^2-mE)Q_{J+1}(Z_v)\},$$

where g_{V1}^2 is the vector-meson charge coupling and in these equations as well as in the following $Z_v=1+(m_v^2/2p^2)$, where m_v is the mass of the vector meson.

For the ρ meson the existence of an anomalous magnetic-moment-type coupling gives rise to two additional contributions, one in which both vertices are pure magnetic coupling and the other, a mixed coupling resulting from charge coupling at one vertex and magnetic at the other. The pure magnetic-coupling terms are

$$b_J=\frac{g_{V2}^2}{4m^3}\left\{\left[\frac{J(J-1)}{(2J+1)(2J-1)}\right]p^2Q_{J-2}(Z_v)+\frac{J}{2J+1}(2p^2+4m^2)Q_{J-1}(Z_v)\right.$$
$$-\left[\frac{2}{(2J-1)(2J+3)}\right][(5J^2+5J-4)p^2+2(2J-1)(2J+3)m^2]Q_J(Z_v)$$
$$\left.+\left(\frac{2J+2}{2J+1}\right)(p^2+2m^2)Q_{J+1}(Z_v)+\left[\frac{J^2+3J+2}{(2J+1)(2J+3)}\right]p^2Q_{J+2}(Z_v)\right\},$$

$$b_{JJ}=\frac{g_{V2}^2}{4m^3}\left\{\left[\frac{(J+1)(J-1)}{(2J+1)(2J-1)}\right]p^2Q_{J-2}(Z_v)+\left[\left(\frac{2J+3}{2J+1}\right)p^2+\left(\frac{2}{2J+1}\right)m^2\right]Q_{J-1}(Z_v)\right.$$
$$\left.-\left[\frac{10J^2+10J-9}{(2J-1)(2J+3)}\right]p^2Q_J(Z_v)+\left[\left(\frac{2J-1}{2J+1}\right)p^2-\left(\frac{2}{2J+1}\right)m^2\right]Q_{J+1}(Z_v)+\left[\frac{J(J+2)}{(2J+1)(2J+3)}\right]p^2Q_{J+2}(Z_v)\right\},$$

$$b_{J-1,J}=\frac{g_{V2}^2}{4m^3(2J+1)}\left\{-\left[\frac{J-1}{(2J+1)(2J-1)}\right][(2J^2+2J+1)(p^2+2m^2)+4J(J+1)mE]\right.$$
$$\times Q_{J-2}(Z_v)+\frac{1}{(2J+1)}[-(4J^2+5J+3)p^2+4J^2m^2+4J(J+1)mE]Q_{J-1}(Z_v)$$
$$-\left[\frac{1}{(2J-1)(2J+3)}\right][(-20J^3-30J^2+7J+9)p^2+(8J^3+4J^2-6J+6)m^2+4J(J+1)mE]Q_J(Z_v)$$
$$+\left(\frac{1}{2J+1}\right)[-(4J^2+3J-1)p^2+4J(J+1)m^2-4J(J+1)mE]Q_{J+1}(Z_v)$$
$$\left.-\left[\frac{2J(J+1)(J+2)}{(2J+1)(2J+3)}\right](p^2+2m^2-2mE)Q_{J+2}(Z_v)\right\},$$

$$b_{J+1,J}=\frac{g_{V2}^2}{4m^3}\frac{1}{(2J+1)}\bigg\{-\bigg[\frac{2J(J-1)(J+1)}{(2J+1)(2J-1)}\bigg](p^2+2m^2-2mE)Q_{J-2}(Z_v)$$
$$-\left(\frac{J}{2J+1}\right)[(4J+5)p^2-4(J+1)m^2+4(J+1)mE]Q_{J-1}(Z_v)$$
$$-\bigg[\frac{1}{(2J-1)(2J+3)}\bigg][(-20J^3-30J^2+7J+8)p^2+2(4J^3+10J^2+5J-4)m^2-4J(J+1)mE]Q_J(Z_v)$$
$$-\left(\frac{1}{2J+1}\right)[(4J^2+3J+2)p^2-4(J+1)^2m^2-4J(J+1)mE]Q_{J+1}(Z_v)$$
$$-\bigg[\frac{J+2}{(2J+1)(2J+3)}\bigg][(2J^2+2J+1)(p^2+2m^2)+4J(J+1)mE]Q_{J+2}(Z_v)\bigg\},$$

$$b^J=\frac{g_{V2}^2}{4m^3}\left(\frac{[J(J+1)]^{1/2}}{(2J+1)}\right)\bigg\{\bigg[\frac{J-1}{(2J+1)(2J-1)}\bigg](p^2+2m^2-2mE)Q_{J-2}(Z_v)$$
$$+\left(\frac{1}{2J+1}\right)(3p^2+4Jm^2+2mE)Q_{J-1}(Z_v)+\frac{[p^2-2(8J^2+8J-7)m^2-2mE]}{(2J+3)(2J-1)}Q_J(Z_v)$$
$$+\frac{[-3p^2+4(J+1)m^2-2mE]}{(2J+1)}Q_{J+1}(Z_v)+\frac{J+2}{(2J+1)(2J+3)}(-p^2-2m^2+2mE)Q_{J+2}(Z_v)\bigg\},$$

where g_{V2}^2 is the tensor coupling constant of the vector meson.

The mixed charge and magnetic coupling gives

$$b_J=\frac{g_{V1}g_{V2}}{m}\bigg[\left(\frac{J}{2J+1}\right)Q_{J-1}(Z_v)-Q_J(Z_v)+\left(\frac{J+1}{2J+1}\right)Q_{J+1}(Z_v)\bigg],$$

$$b_{JJ}=\frac{g_{V1}g_{V2}}{m}\bigg[-\left(\frac{J+1}{2J+1}\right)Q_{J-1}(Z_v)+Q_J(Z_v)-\left(\frac{J}{2J+1}\right)Q_{J+1}(Z_v)\bigg],$$

$$b_{J-1,J}=\frac{g_{V1}g_{V2}}{m(2J+1)^2}\bigg\{\bigg[4J^2+2J+1+4J(J+1)\frac{E}{m}\bigg]Q_{J-1}(Z_v)-(2J+1)(4J+1)Q_J(Z_v)+4J(J+1)\left(1-\frac{E}{m}\right)Q_{J+1}(Z_v)\bigg\},$$

$$b_{J+1,J}=\frac{g_{V1}g_{V2}}{m(2J+1)^2}\bigg\{4J(J+1)\left(1-\frac{E}{m}\right)Q_{J-1}(Z_v)-(2J+1)(4J+3)Q_J(Z_v)$$
$$+\bigg[4J^2+6J+3+4J(J+1)\left(\frac{E}{m}\right)\bigg]Q_{J+1}(Z_v)\bigg\},$$

$$b^J=\frac{g_{V1}g_{V2}}{m}\bigg[\frac{[J(J+1)]^{1/2}}{(2J+1)^2}\bigg]\bigg\{\bigg[2J-1+2\left(\frac{E}{m}\right)\bigg]Q_{J-1}(Z_v)-2(2J+1)Q_J(Z_v)+\bigg[2J+3-2\left(\frac{E}{m}\right)\bigg]Q_{J+1}(Z_v)\bigg\}.$$