Experimental Electron Energy Distributions for Townsend Discharges in Helium

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The energy distribution of electrons in Townsend discharges in helium has been investigated experimentally for a range of E/N between 1.65×10^{-15} and 6.32×10^{-15} V cm^2, through the use of a retarding-field method. The effects of electron reflection and secondary emission from the surfaces of the retarding-field analyzer are considered; it is shown that these phenomena are relatively unimportant when low-reflectance materials are used to coat analyzer surfaces. Experimental energy distributions exhibit a curtailment of the distribution function near the first excitation potential of helium, except for $E/N=6.32\times10^{-15}$ V cm^2. The low-energy behavior of the experimental energy distribution indicates an increase of the most probable energy with increasing E/N.

I. INTRODUCTION

THE distribution of energy among electrons in an ionized gas has been the subject of many theoretical investigations during the past half-century.[1] For the case in which a uniform electric field of strength E is impressed on a weakly ionized gas of molecular concentration N, with E/N sufficiently small that only elastic collisions of the electrons with the gas molecules need be considered, the studies have commonly yielded distribution functions of the generalized Druyvesteyn form. In recent years there have appeared several reports of theoretical investigations in which the contributions of effects due to inelastic collisions were not neglected.[2–4] The problem is complicated in such cases by the necessity of inclusion of extra terms in the Boltzmann transport equation, whose solution describes the functional dependence of the distribution function on position, velocity, and time. Even when reduced for the special conditions of spatial uniformity and a steady state, the resulting equations are of great complexity. Further, accurate information on the parameters such as collision cross sections, which must be known to make the theory useful, is lacking for many gases.

Among the recent theoretical results, those presented for the noble gases by Heylen and Lewis[5] are of interest because of the unusual form exhibited for the energy distribution function $F(\epsilon)$, the concentration of electrons possessing energy whose value lies in the range $d\epsilon$ at ϵ. For low-intermediate values of E/N, the theory predicts a curtailment of $F(\epsilon)$ in the vicinity of the first excitation potential, thus suggesting that an important effect of the occurrence of inelastic collisions is a reduction in the number of electrons with large energy. Although the relative magnitude of the phenomenon is not predicted to be as great for helium as for some other gases, it is sufficiently large that certain advantages offered by helium make it most attractive for use in an experimental test of this feature of the theory. In particular, the elastic-collision cross section for electrons in helium is reasonably uniform over the important energy range, and the gas purity is relatively simple to control. This article is a report of an investigation whose aim was experimental determination of $F(\epsilon)$ for Townsend discharges in helium, with E/N ranging from 1.65×10^{-15} to 6.32×10^{-15} V cm^2.

II. EXPERIMENTAL METHOD

The underlying elements of the method employed in this experiment may be understood by reference to the schematic diagram presented in Fig. 1. A Townsend discharge is maintained between the plane, parallel electrodes of the centrally located discharge cell. The region exterior to the cell is maintained at a pressure much lower than the discharge pressure; ideally, electrons escaping through the anode perforation make no collisions once outside the cell, so no further alteration of the

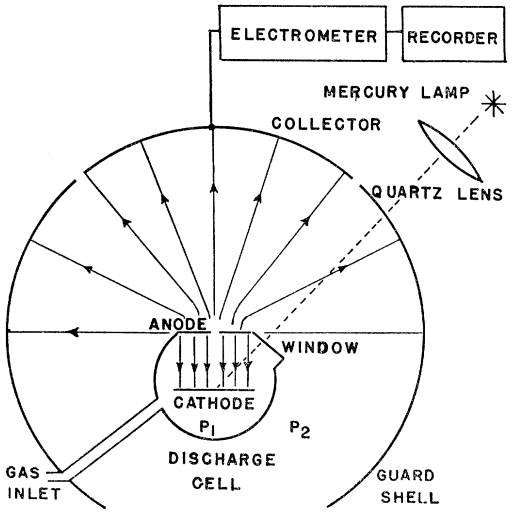

FIG. 1. Schematic diagram of discharge cell and spherical energy analyzer.

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[1] A comprehensive history of theoretical developments up to the mid-fifties may be found in L. B. Loeb, *Basic Processes of Gaseous Electronics* (University of California Press, Berkeley, California, 1955).

[2] T. J. Lewis, Proc. Roy. Soc. (London) **A244**, 166 (1958).

[3] H. Dreicer, Phys. Rev. **117**, 343 (1960).

[4] L. S. Frost and A. V. Phelps, Phys. Rev. **127**, 1621 (1962).

[5] A. E. D. Heylen and T. J. Lewis, Proc. Roy. Soc. (London) **A271**, 531 (1963).

equilibrium electron-energy distribution occurs. By changing the intensity of the retarding field and measuring the resulting alteration in the current to the collector, data are obtained which may be related to the electron-energy distribution. The retarding field is applied in a region electrically remote from the discharge, thus reducing the possibility of impairing the uniformity of the discharge field, except in the immediate vicinity of the sampling aperture. The retarding field between the collecting portion of the spherical analyzer and the anode is nearly radial,[6] so escaping electrons whose velocities are directed toward the collector travel parallel to field lines over most of their trajectories, a condition essential for good energy resolution.[7]

When electrons impinge on a metallic surface, some are not collected, but are reflected elastically or inelastically.[8] Further, bombardment of the surface by electrons will cause release of secondary electrons. Thus, some electrons with sufficient energy to overcome the potential difference between the discharge cell anode and collector will not be included in the measured current, and, indeed, the latter may contain a negative component due to secondary emission. To analyze the consequences of these effects we consider the situation in which a retarding potential difference is established between anode and collector,[9] the magnitude being $V<V_m$, where eV_m is the maximum energy of electrons from the discharge. The collecting surface is presented with electron energies ranging from zero to $e(V_m-V)$ electron volts. We define the collection efficiency $T(V)$ such that $nT(V)$ is the number of electrons retained when a number n of monoenergetic electrons of energy eV impinge on the collector surface. The function $T(V)$ is equal to $1-\delta(V)$, where $\delta(V)$ is the total secondary-emission coefficient that is usually presented in reports on electron reflection.[10] When there is a distribution of electron energies present, the portion of $T(V)$ in effect is that portion in the range of V from zero to (V_m-V), and the current $i(V)$ is given by

$$i(V)=K\int_V^{V_m} V'^{1/2}T(V'-V)F(V')dV', \qquad (1)$$

where K is a constant. Equation (1) can be differentiated to give

$$\frac{di}{dV}=-KF(V)T(0)V^{1/2}+K\int_V^{V_m}\frac{\partial}{\partial V}T(V'-V)V'^{1/2}F(V')dV'. \qquad (2)$$

The first term corresponds to the expression commonly employed in experiments which make use of retarding-field energy analyzers, but in which electron reflection is ignored; the second term represents a correction to account for reflection. It is of interest to note that it is the magnitude of the derivative of T, rather than that of T itself, which determines the size of the correction. If T has a relatively small first derivative everywhere, the total correction due to reflection is correspondingly small. Experimental knowledge of the form of T for the collector surface is important to avoid doubts concerning the validity of the results of an experiment in which electron reflection is not taken into account. When T is known, the corrected electron energy distribution may be obtained from the experimental data of i versus V in conjunction with the solution of Eq. (1).

III. APPARATUS AND DATA

A block diagram of the apparatus is shown in Fig. 2. The gas used was research-grade Airco helium in 1-liter Pyrex flasks sealed onto the system. The gas was treated by cataphoresis, passed over hot Ti-Zr shavings to remove oxygen, and through a conventional liquid-N_2 cold trap before being admitted to the discharge cell. Helium escaping through the perforation in the anode was pumped out of the chamber by a 1400-liter-per-sec oil diffusion pump. Backstreaming of oil into the chamber was reduced by a liquid-nitrogen-cooled baffle between pump and chamber. The discharge cell was mounted just above the baffle to permit full utilization of the

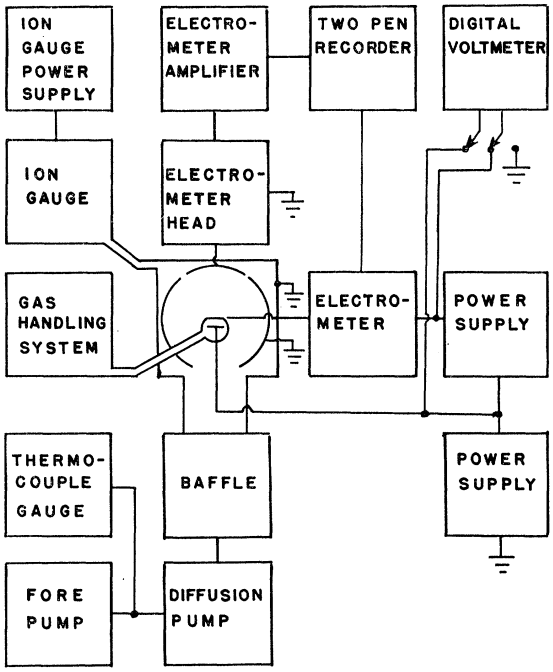

FIG. 2. Block diagram of apparatus.

[6] W. R. Smythe, *Static and Dynamic Electricity* (McGraw-Hill Book Company, Inc., New York, 1939).
[7] J. A. Simpson, Rev. Sci. Instr. **32**, 1283 (1961).
[8] H. E. Farnsworth, Phys. Rev. **20**, 358 (1922).
[9] The remainder of the spherical analyzer surface is also maintained at the collector potential.
[10] I. H. Khan, J. P. Hopson, and R. A. Armstrong, Phys. Rev. **129**, 1513 (1963).

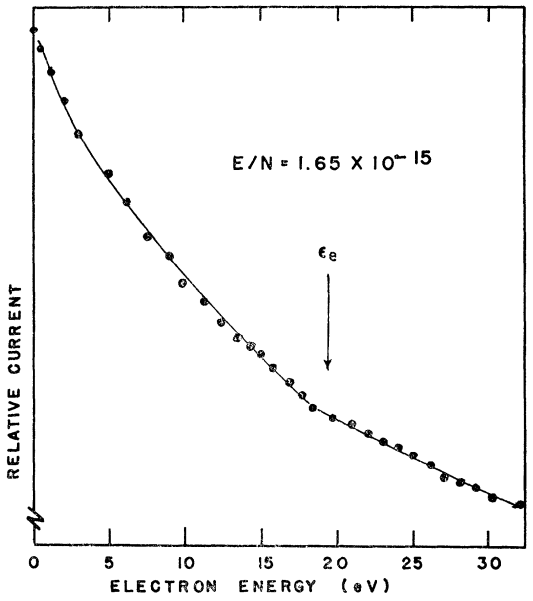

FIG. 3. Collector current versus repelling potential for $E/N=1.65\times10^{-15}$ V cm².

high speed of the pump. Chamber pressure was measured with an ionization gauge located to the side of the chamber and connected to the chamber by a $1\frac{1}{2}$-in. piece of $\frac{3}{4}$-in. Pyrex-Kovar tubulation. Chamber pressure during a run was typically 5×10^{-6} Torr or less, while the pressure within the discharge cell was established at values ranging from 1 to 1.5 Torr.

The discharge cell was constructed from Kovar-sealing glass. A thin disk of diameter 1.9 cm was sealed to the top of the truncated spherical cell.[11] The disk, which served as the Townsend discharge anode, was ground flat, after sealing, to a thickness of about 0.2 mm. The anode perforation was 0.12 mm in diameter. Several considerations entered into choice of the size of the sampling aperture. On the one hand, the aperture must be made large enough to permit adequate collector current for good signal-to-noise ratios in the data. However, it is also desirable to keep the aperture as small as possible to insure (a) satisfactory differential pressure, (b) minimum distortion of the discharge field, and (c) effusive flow of electrons. The aperture size chosen met these criteria adequately for cell pressures of about one Torr; the largest dimension of the aperture was then of the order of $(NQ)^{-1}$, where Q is the collision cross section for electrons on helium. Under these conditions the gas flow through an aperture has been shown to be predominately effusive.[12]

The anode-cathode spacing in the cell was typically about 2.2 cm. A quartz window port was placed close to the anode to permit ultraviolet light to fall on the activated (Ba, Sr) O surface of the photocathode. The spherical shell, consisting of the collector portion and the guard portion, was 19 cm in diameter. The collector portion subtended a solid angle of $0.15\times4\ \pi$ sr at the sampling aperture.

The spherical shell, anode, and all leads were gold-plated to minimize difficulties which might be encountered from the possible establishment of large contact differences of potential. The inner surface of the sphere was coated with gold black, a porous, filamentary deposit of gold metal with a stable, well-behaved, low total secondary-emission coefficient. The collection efficiency $T(V)$ for our particular collector surface was determined in a separate experiment[13] in which the discharge cell was replaced by an electron gun.

The current in the collector circuit was measured with a vibrating-reed electrometer, the electrometer head being supported in an inverted position on the top of the vacuum chamber by a short cylindrical electrostatic shield. The total length of the current lead from collector to electrometer head was about 4 in. This arrangement proved to be stable and permitted rate-of-charge measurements to be made near the useful sensitivity limit of the electrometer. The discharge current was measured with an electrometer, and the outputs of both electrometers were recorded simultaneously by a two-pen recorder. This procedure permitted direct correlation between fluctuations in discharge current and collector current. The net average collector current for any particular value of retarding potential was divided by the discharge current to normalize for effects of fluctuations in intensity and cathode efficiency. Collector currents were corrected for background and leakage by taking the difference of measurements made with and without the ultraviolet light illuminating the photocathode.

Figures 3 and 4 show the characteristic curves of collector current vs repelling potential differences for two

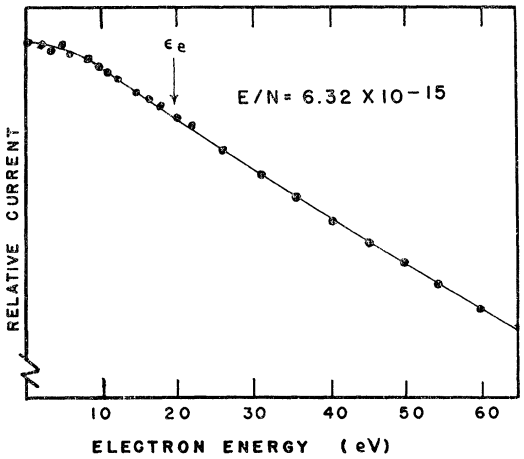

FIG. 4. Collector current versus repelling potential for $E/N=6.32\times10^{-15}$ V cm².

[11] The cell was constructed spherical in shape to increase the path length of undesirable wall currents.

[12] M. Knudsen, Ann. Physik 28, 75 (1909).

[13] T. D. Roberts and D. S. Burch, Rev. Sci. Instr. 35, 1067 (1964).

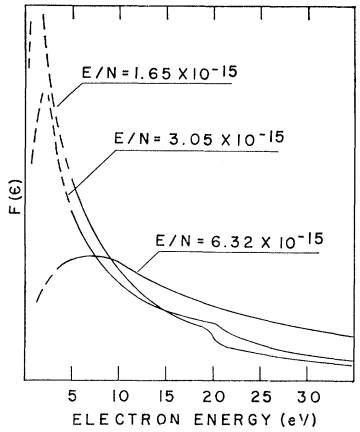

FIG. 5. Experimental electron energy distributions for helium.

settings of E/N in the Townsend discharge. The currents in the latter were of the order of 10^{-9} amperes and the corresponding maximum collector currents were about 10^5 times smaller, which is consistent with the geometry of the apparatus.

IV. RESULTS AND DISCUSSION

The function of $F(\epsilon)$ for $E/N=1.65\times10^{-15}$ V cm^2 was obtained from Eq. (1), which was approximated numerically and solved with the aid of a computer. This solution was compared with the function found by neglecting the integral correction term in Eq. (2), i.e., by differentiation of the i versus V characteristic and multiplication by $\epsilon^{-1/2}$. The two normalized solutions, the former being corrected for electron reflection and the latter obtained ignoring the correction, differed at most by a few percent. Thus use of a collector surface whose $T(V)$ curve varies slowly over most of the significant range of V resulted in an overall effect from electron reflection which was nearly negligible. Utilization of a material such as platinum black,[14] whose collection properties are superior to those of the gold black used in this experiment, would reduce the corrections still further, thus virtually eliminating a disadvantage which has often been associated with the use of surfaces as collecting agents in retarding-field experiments.

As the distribution tail is made to rise, the fraction of the range of electron energies for which $(\partial T/\partial V)$ is significant is made correspondingly smaller. Thus, as E/N is increased the effects of reflection are diminished. Consequently, although the correction was incorporated in reduction of data for $E/N=1.65\times10^{-15}$ V cm^2, it was not included in analysis of the data for larger values of E/N. The experimental results appear in Fig. 5. The theoretical calculations of Heylen and Lewis are shown in Fig. 6.

The comparison of theory and experimental results shows several features of agreement and discrepancy. There appears to be some agreement with respect to the relative magnitude and location of the break along the energy axis of $F(\epsilon)$. As E/N is increased a gradual increase of the value of the most probable energy is observed experimentally. This feature is not present in the theoretical results. Further, the observed energy distribution appears to fall off more sharply with increasing electron energy than the predicted distribution, with the exception of the curve for $E/N=6.32\times10^{-15}$ V cm^2.

Several considerations lead us to believe that the comparison between theory and experiment need not be expected to be excellent. The theoretical results were obtained with the aid of a number of questionable approximations and were calculated from data of uncertain validity. In addition, the possible presence of certain systematic experimental errors requires that the quantitative features of the measured distribution functions be viewed with some caution, particularly those for the energy range about the most probable energy. The derivative curves based on experiment, shown in Fig. 5, were obtained from least squares fitted curves, and are thus understood to represent the derivative behavior of a fitted curve approximating data points obtained from measurement of $i(V)$. The error associated with differentiating smoothed experimental data exhibiting fluctuations about a least-squares-fitted curve can be quite large,[15] and for large values of the slope, small variations in the fitted curve may lead to large changes in the magnitude of the derivative curve. An error in determination of the slope of the $i(V)$ curve will then be magnified for decreasing ϵ when the factor $\epsilon^{-1/2}$ is inserted in the computation of $F(\epsilon)$. The assumption of effusive flow of the electrons may be unjustified for low-energy electrons, for which $(NQ)^{-1}$ is of the order of, or smaller than, the sampling aperture. Atoms streaming through the aperture form a cloud immediately outside the discharge cell, and the effect of collisions between electrons and these atoms may alter the electron energy

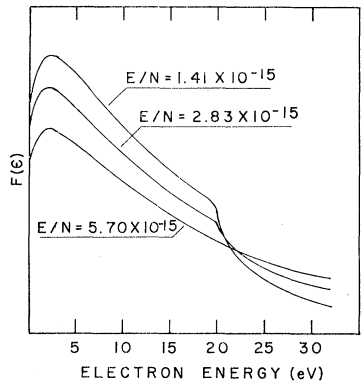

FIG. 6. Heylen and Lewis's theoretical electron energy distributions for helium.

[14] R. J. Zollweg, J. Appl. Phys. 34, 2590 (1963).

[15] F. B. Hildebrand, *Introduction to Numerical Analysis* (McGraw-Hill Book Company, Inc., New York, 1956).

distribution. The over-all effect of such collisions is difficult to estimate. Previous calculations[16] concerning molecular beams interacting with the exterior cloud, although not strictly applicable to the experiment, have indicated the correction is probably of the order of only a few percent. To emphasize the above considerations, portions of the experimental curves where some doubt exists concerning the magnitude of $F(\epsilon)$ are indicated by broken lines.

Finally, there remains the question of whether or not the discharge electrons are in equilibrium with the electric field at the values of pressure and electrode spacing used in this experiment. While there is probably not time for elastic collisions alone to cause the distribution function to reach equilibrium, inelastic collisions play an important role. The product of electrode spacing and reduced pressure, $p_0 x = 2.2$ Torr cm, is comparable or greater than the $p_0 x$ for which Chanin and Rork[17] have obtained essentially constant α/p_0 in helium for fixed higher values of E/N. In view of their results, there is strong evidence that the experimental distribution represents an equilibrium situation, especially in the important range $E/N \leq 3\times10^{-15}$ V cm^2.

ACKNOWLEDGMENT

The authors wish to thank Ralph Applebee for his helpful suggestions concerning electron reflection.

[16] I. R. Estermann, O. C. Simpson, and O. Stern, Phys. Rev. 71, 238 (1947).

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Quantum Theory of Photon Interaction in a Plasma

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The quantum theory of interaction of electromagnetic waves in a plasma is formulated from two different points of view. The first is to consider scattering of light off light (in the form of laser beams) with the plasma acting as a mediator of the interaction; the second is to consider scattering of one of the light beams off a system consisting of the plasma and the other laser beam. Based on the first viewpoint, the light-light scattering cross sections, both elastic and inelastic, are calculated in the lowest order. By summing over the final states of one of the photons, we obtain, based on the above results, the lowest order cross section of scattering of the other photon from the photon-plasma system. In the presence of a second stimulating laser beam, this cross section is enhanced. When both laser beams are very intense, the lowest order perturbation treatment is inadequate. The second viewpoint is then conveniently adopted to include the plasma-laser beam interaction to all orders. The results are discussed and compared with those in previous treatments. Finally, a simple model is considered. In this model, the plasmon is treated as the quantum of a harmonic oscillator which is linearly coupled to a system of phonons. All the previous results are explicitly verified in this model, which is solved exactly.

I. INTRODUCTION

TREATMENTS of the interaction of light with light in a plasma, both quantum mechanically[1,2] and classically,[3] have been given. However, the results of Refs. 1 and 2 differ from those of Ref. 3. This difference was finally resolved in a brief communication.[4] The present paper gives a systematic account of the subject.[5]

II. GENERAL THEORY OF LIGHT-LIGHT INTERACTION

We consider the interaction of a plasma with photons 1, 2, 3, and 4. The total Hamiltonian is

$$H = H_0 + H_1 + H_2, \tag{1}$$

[1] P. M. Platzman, S. J. Buchsbaum, and N. Tzoar, Phys. Rev. Letters 12, 573 (1964); also P. M. Platzman and N. Tzoar, Phys. Rev. 136, A11 (1964).

[2] D. F. Dubois and V. Gilinsky, Phys. Rev. 135, A995 (1964).

[3] N. M. Kroll, A. Ron, and N. Rostoker, Phys. Rev. Letters 13, 83 (1964).

[4] H. Cheng and Y. C. Lee, Phys. Rev. Letters 14, 426 (1965).

[5] After the publication of Ref. 4, a letter by D. F. Dubois appeared [Phys. Rev. Letters 14, 818 (1965)] the results of which are essentially contained in Ref. 4 and are roughly equivalent to those of Sec. III in the present paper.