New Total-Cross-Section Measurements for the Reactions $^{14}N(^{14}N,^{13}N)^{15}N$ and $^{10}B(^{14}N,^{13}N)^{11}B$†

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Thick targets of AgCN and ^{10}B were bombarded with ^{14}N ions accelerated in the Oak Ridge Tandem Van de Graaff. The amount of ^{13}N present in each irradiated sample was determined by the detection of the 10-min positron activity characteristic of the isotope. In this manner cross sections were measured for the neutron-transfer reactions $^{14}N(^{14}N,^{13}N)^{15}N$ and $^{10}B(^{14}N,^{13}N)^{11}B$. The (assumed equal) neutron reduced widths in ^{14}N and ^{15}N were determined by applying the tunneling theory to the ^{14}N-on-^{14}N measurements. The reduced width in ^{11}B was then calculated from the cross-section data for the reaction of ^{14}N on ^{10}B by the use of the now measured reduced width in ^{14}N. The ^{14}N and ^{11}B reduced widths were found to agree with the same values as determined from the bound-state, single-particle, radial wave functions that have been calculated by using a Woods-Saxon potential. In resolving the experimental decay curves, 2- and 20-min components were found for the ^{14}N and ^{10}B targets, respectively. These activities were ascribed to the ^{15}O and ^{11}C which result from the proton-transfer reactions $^{14}N(^{14}N,^{15}O)^{13}C$ and $^{10}B(^{14}N,^{13}C)^{11}C$; cross sections for these reactions were also measured.

I. INTRODUCTION

EARLY determinations[1–3] of angular distributions of ^{13}N particles resulting from the transfer reaction $^{14}N(^{14}N,^{13}N)^{15}N$ have indicated disagreement between experiment and the tunneling theory[4] despite the fact that experimental conditions satisfied most of the conditions set by the theory.[5] The reaction involves the transfer of a *p*-shell neutron; the *Q* value is almost zero, and the measurements were performed at energies slightly above and below the Coulomb barrier. The differential cross section was found to rise less steeply with increasing angle than predicted by the theory. The indication from these and other transfer-reaction studies[6–8] was, however, that the agreement with theory became better as the incident energy was lowered. Recently, the same reaction has been studied for a variety of energies.[9,10] These new data show that at energies far below the Coulomb barrier the experimental angular distributions are in agreement with the tunneling theory. As the incident energy approaches the barrier energy the fit with theory becomes progressively worse.

The disagreement with the tunneling theory is now thought[10] to be due to the inception of nuclear absorption of the incident ^{14}N ions at some critical distance of closest approach. This distance corresponds to an angle of observation which, for classical Rutherford orbits, becomes progressively smaller as the incident energy is raised. When nuclear absorption is present the transfer cross section is decreased; the increase of the experimentally determined differential cross section with angle is less than that calculated from the tunneling theory, since the latter does not take nuclear absorption into account.

At incident energies where the tunneling theory correctly describes the angular distributions, Becker and McIntyre[9] have determined the product of the neutron reduced widths in ^{14}N and ^{15}N from their total-cross-section measurements for the ground-state transfer reaction $^{14}N(^{14}N,^{13}N)^{15}N$. They have also calculated the reduced widths for the neutron in ^{14}N and ^{15}N by assuming that the two values were the same. However, their cross-section measurements[9] and those of Hiebert, McIntyre, and Couch[10] disagreed over a large energy range with the measurements of Reynolds and Zucker.[1] If ^{14}N-induced transfer reactions are to be used for reduced width determinations then the ^{14}N (or ^{15}N) value must be known accurately; this of course necessitates a correct cross-section measurement. The present study was undertaken to provide an additional check of the total reaction cross section.

The reaction $^{10}B(^{14}N,^{13}N)^{11}B$ was also investigated to determine the neutron reduced width in ^{11}B. This determination is possible once the reduced width for the neutron in ^{14}N is known. In addition, it was felt that [with the exception of $^{14}N(^{14}N,^{13}N)^{15}N$] the reaction probably comes closest to satisfying the conditions set by the tunneling theory. The transfer is that of a *p*-shell neutron; the *Q* value is 0.91 MeV, and the cross section is large enough to be measured, even for energies far below the Coulomb barrier. Excitation functions

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[1] H. L. Reynolds and A. Zucker, Phys. Rev. **101**, 166 (1956).
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[9] L. C. Becker and J. A. McIntyre, Phys. Rev. **138**, B339 (1965).
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for the proton-transfer reactions $^{14}N(^{14}N,^{15}O)^{13}C$ and $^{10}B(^{14}N,^{13}C)^{11}C$ were also obtained.

II. EXPERIMENTAL TECHNIQUE

The target materials used in this investigation were silver cyanide and boron enriched in ^{10}B (92%). Targets were prepared by compressing the powdered materials under a pressure of 5 tons/in.[2] into brass molds $\frac{3}{4}$ in. in diameter. These targets, thicker than the range of the nitrogen ions, presented a hard and uniform surface to the incident particles. Bombardments were made in a Faraday cup assembly, and beam currents up to 200 nA were recorded. The energy of the triply ionized nitrogen ions, accelerated in the Oak Ridge Tandem Van de Graaff, was varied from 8.8 to 20.0 MeV. While the beam energy was known to ± 100 keV, the beam resolution was about 10 keV.

After the bombardment the targets were counted in a fixed geometry in a low-level gas-flow beta detector with a background of ~ 0.25 counts/min. Decay curves were resolved into their components and the presence of ^{11}C, ^{13}N, and ^{15}O positron activities in each target was established by means of their half-lives. Bombardment times for energies >13.0 MeV were 5 min. Below this energy, the time was increased to 10 and then 20 min to compensate partially for the rapid decrease in the yield of ^{13}N (the product of chief interest) with decreasing incident ^{14}N energy. A computer program was used to give least-squares fits to the decay curves and to determine the magnitudes of the various decay curve components extrapolated to time zero, i.e., the time at which the beam was turned off. The program also supplied the standard deviations for the time-zero magnitudes.

The low-level counter was calibrated absolutely for the particular geometry used by means of a Ra *DEF* source of known strength. The counting efficiency was found to be 27.5%; the probable error in this number is estimated to be $\pm 15\%$. The counting rates at time zero, as obtained from the computer fit, were then corrected by this efficiency to obtain the absolute yields per incident particle. The probable error in these yields results mainly from the uncertainty in the counter efficiency determination and is, therefore, estimated to be $\pm 15\%$.

Smooth curves were drawn through the thick-target yield points and these curves were then differentiated to obtain the excitation functions. For this determination the stopping power of AgCN and ^{10}B for ^{14}N ions had to be known. It was calculated from the known stopping power of nickel for nitrogen[11] and the relative stopping power for nickel and AgCN and ^{10}B for protons of the same velocity as the nitrogen ions. The proton stopping powers were taken from Allison and Warshaw.[12] The

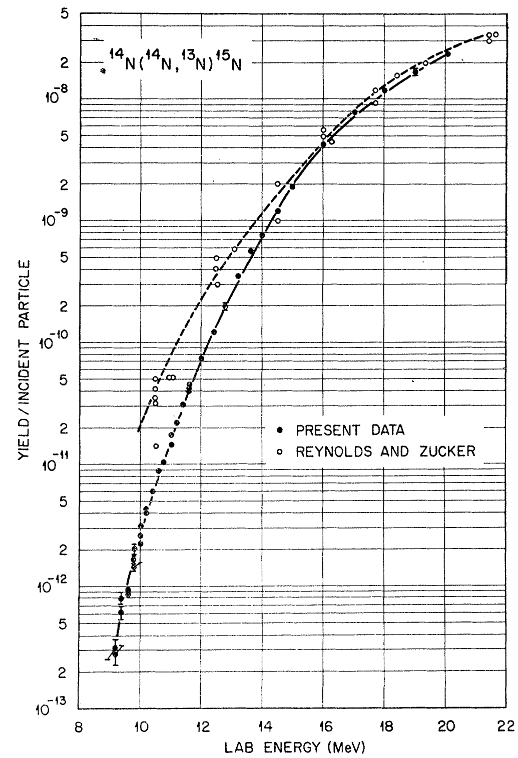

FIG. 1. Yield per incident particle as a function of energy for the reaction $^{14}N(^{14}N,^{13}N)^{15}N$.

probable errors in the absolute cross sections are estimated to be $\pm 30\%$ and are attributed to errors arising from uncertainties in the counter efficiency, the slope of the yield curves, and the stopping power.

Reactions with silver in AgCN did not have to be considered owing to the high Coulomb barrier which this element presents to the incident ≤ 20-MeV nitrogen beam. The ^{14}N threshold energies for the production of ^{13}N and ^{15}O by means of ($^{14}N,^{13}N$) and ($^{14}N,^{15}O$) reactions on ^{12}C are 12.1 and 18.8 MeV, respectively. Therefore, in the energy range investigated one can conclude that the observed ^{15}O activity is due to reactions on ^{14}N and not ^{12}C. The cross section for the production of ^{13}N from ^{12}C has been measured to be 0.2 mb[13] at a nitrogen energy of 27 MeV. Recent measurements[14] on the same reaction extending to 39 MeV confirm the 27-MeV value. In addition, they indicate that the $^{12}C(^{14}N,^{13}N)^{13}C$ excitation function has a shape similar to that observed for other transfer reactions, i.e., the cross section above a certain energy (in this case ~ 32 MeV) remains almost constant and decreases sharply below that energy. This drop in the cross section has been found for other low-*Z* targets[1,15] to be about a factor of 100 for a decrease of

[11] H. L. Reynolds, D. W. Scott, and A. Zucker, Phys. Rev. **95**, 671 (1954).

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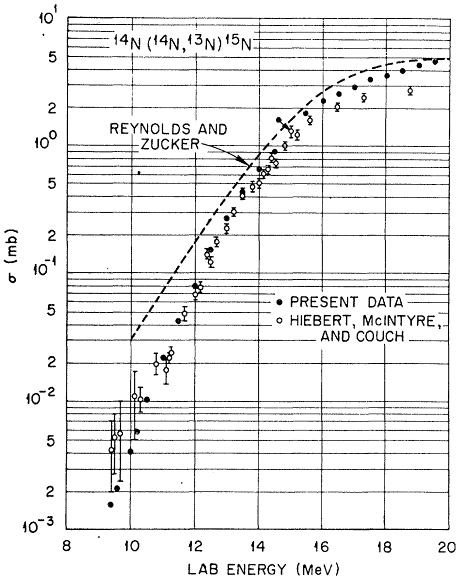

FIG. 2. Cross-section data for the reaction $^{14}N(^{14}N,^{13}N)^{15}N$. The present results are compared with those from Refs. 1 and 10.

5 or 6 MeV in the ^{14}N energy. One might, therefore, expect the $^{12}C(^{14}N,^{13}N)^{13}C$ cross section at 20 MeV to be ~0.002 mb. At the same energy, the $^{14}N(^{14}N,^{13}N)^{15}N$ cross section is 3–5 mb.[1,10] we conclude that the production of ^{13}N from carbon is negligible.

III. RESULTS

A. $^{14}N(^{14}N,^{13}N)^{15}N$

The yield per incident particle as a function of energy for the reaction $^{14}N(^{14}N,^{13}N)^{15}N$ is shown in Fig. 1. The data are compared with the earlier results of Reynolds and Zucker,[1] who also bombarded thick AgCN targets but with nitrogen ions accelerated in the ORNL 63-in. cyclotron. The curve through their yield points is intended to be a reasonable duplicate of the curve drawn by the authors; it was this curve that they differentiated to obtain their cross-section data. In the region ≥16 MeV, where the yield is leveling off, the two sets of data agree well. Where the yield varies rapidly with energy, however, the earlier data points are consistently higher than the yields measured in the present study; the discrepancy becomes progressively greater with decreasing energy. The reason for the disagreement at low energies is due most probably to the energy spread in the 63-in. cyclotron ^{14}N beam. The full width at half-maximum of that beam was found[7] to be ~1 MeV and its total spread ±~1 MeV. With this beam width one includes the yield at energies 1 MeV higher than the average beam energy at which the measurement is being made. If the yield is rapidly varying with incident energy, this will result in a value substantially higher than the true one. Because the 63-in. cyclotron energy was varied by the use of nickel absorber foils, additional spread was introduced into the beam due to straggling in the nickel. It should be noted that some of the low-energy points obtained by Reynolds and Zucker are in better agreement with the present measurements than the yield curve that was drawn and differentiated by them.

The cross-section data are compared with those of Reynolds and Zucker[1] and of Hiebert *et al.*,[10] in Fig. 2. The data of Reynolds and Zucker are indicated by the dashed curve, the results of Hiebert *et al.*, by open circles, and the present cross-section determinations by closed circles. An absolute uncertainty (not indicated in the diagram) of ~±30% is estimated for our measurements. Reynolds and Zucker quote an uncertainty of ±25% in their cross-section determinations. As in the case of the thick-target yields, the present cross-section measurements are consistently below those determined by Reynolds and Zucker, though the two sets of data, within experimental errors, do not disagree for energies above ~12 MeV. Within errors, the data of Hiebert *et al.*, are in agreement with ours; there is an indication, however, that their cross sections above 16 MeV are lower and below 11 MeV are higher than ours. In the intermediate region, where the tunneling theory has been normalized to their data, the agreement between the two sets of results is excellent.

In Fig. 3 the data of Becker and McIntyre[9] shown as open circles are compared with the present results. The sets of data are in agreement though the experimental errors involved in the measurements of Becker and McIntyre are extremely large for energies below 11 MeV.

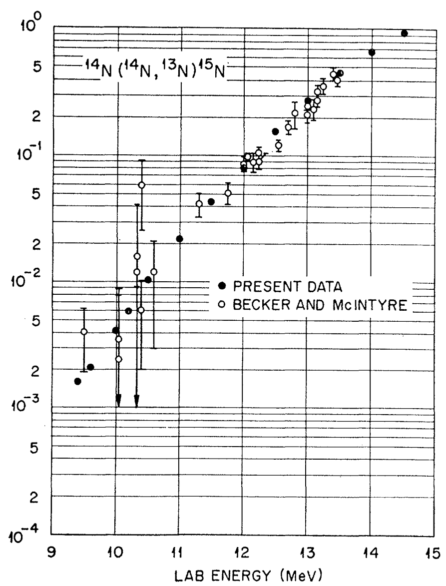

FIG. 3. Cross-section data for the reaction $^{14}N(^{14}N,^{13}N)^{15}N$. The present results are compared with those from Ref. 9.

B. $^{14}N(^{14}N,^{15}O)^{13}C$

The yield data for the reaction $^{14}N(^{14}N,^{15}O)^{13}C$ are shown in Fig. 4, together with the results of Reynolds

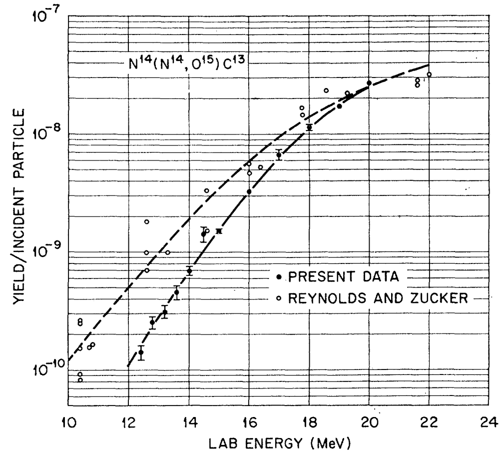

FIG. 4. Yield per incident particle as a function of energy for the reaction $^{14}N(^{14}N,^{15}O)^{13}C$.

and Zucker.[1] The curve through their points is again intended to be a close approximation to the one drawn by those authors. As in the production of ^{13}N the Reynolds and Zucker yield points are consistently higher than ours, probably because of the spread of the ^{14}N beam from the 63-in. cyclotron. The excitation function derived from our data is shown in Fig. 5.

C. $^{10}B(^{14}N,^{13}N)^{11}B$ and $^{10}B(^{14}N,^{13}C)^{11}C$

The thick-target yield data for the ^{10}B reactions are shown in Fig. 6 with the corresponding excitation functions being displayed in Fig. 7. Cross sections for the two reactions have been measured previously.[15] In the earlier study, however, the targets consisted of boron with a natural isotopic content (19.78% ^{10}B). The yields from the two investigations should not be the same and they are, therefore, not compared in Fig. 6. On the other hand, the cross sections are expected to be the same. Portions of the previously reported[15] excitation functions (up to 17 MeV) are shown in Fig. 7. As in the case of the reactions on ^{14}N the earlier cross sections, particularly for the reaction $^{10}B(^{14}N,^{13}C)^{11}C$, were measured to be higher than those determined in the present investigation. For the neutron-transfer reaction $^{10}B(^{14}N,^{13}N)^{11}B$ good agreement exists between the two studies for energies above 13 MeV. But, for the proton-transfer reaction $^{10}B(^{14}N,^{13}C)^{11}C$ the disagreement between the two measurements is substantial, even for energies where the excitation functions have leveled off and therefore cannot be ascribed to large energy spread of the ^{14}N beam which was utilized in the earlier study.

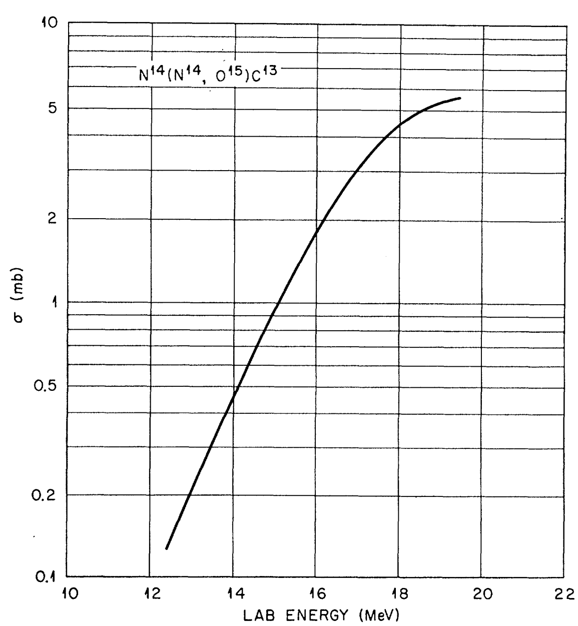

FIG. 5. Excitation function for the reaction $^{14}N(^{14}N,^{15}O)^{13}C$.

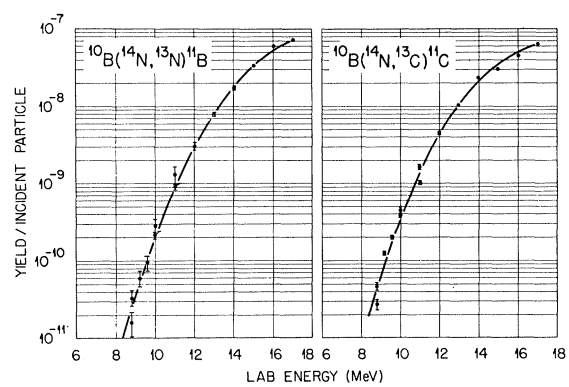

FIG. 6. Yield per incident particle as a function of energy for the reactions $^{10}B(^{14}N,^{13}N)^{11}B$ and $^{10}B(^{14}N,^{13}C)^{11}C$.

IV. DISCUSSION AND DETERMINATION OF REDUCED WIDTHS

The tunneling theory of Breit and collaborators[4] predicts the variation of neutron-transfer cross sections with energy. In the original semiclassical formulation of the theory[4a] the total cross section σ_{SC} varies with energy as

$$\sigma_{SC}=\frac{\Lambda^2}{8}\frac{1}{\alpha^2\lambda\lambda'}\left(\frac{\alpha b_1}{1+\alpha b_1}\right)^2\left(\frac{\alpha b_2}{1+\alpha b_2}\right)^2\exp(X), \qquad (1)$$

where

$$X=(2(2M)^{1/2}/\hbar)[E_s^{1/2}(b_1+b_2)(1-E_B/E)].$$

Here b_1 and b_2 are the radii of initial and final nuclei 1 and 2, respectively; $\Lambda=\hbar/Mv=$ wavelength of the transferred neutron, $v=$ relative velocity of the projectile and the target nucleus$=(2E_{\text{lab}}/M_p)^{1/2}$, and M_p is the projectile mass; $M^2=M_1M_2$, where M_1 and M_2 are the reduced masses of the neutron in the nuclei 1 and 2 respectively; $E_s=$binding energy of the neutron.

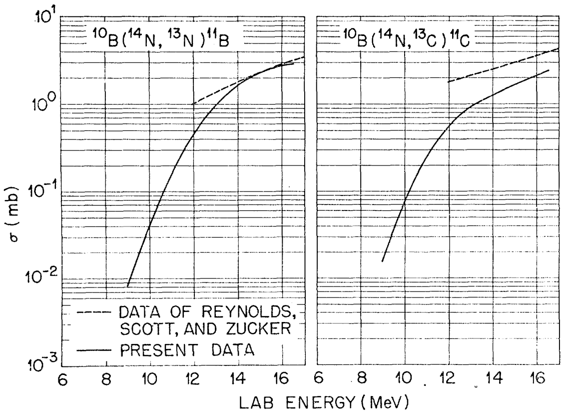

FIG. 7. Excitation functions for the reactions $^{10}B(^{14}N,^{13}N)^{11}B$ and $^{10}B(^{14}N,^{13}C)^{11}C$.

$\alpha=(2ME_s/\hbar^2)^{1/2}$, where α is the average for the initial and final nuclei, i.e., $\alpha=(\alpha_1+\alpha_2)/2$; $E_B=Z_1Z_2e^2/r_0(A_1^{1/3}+A_2^{1/3})=$ Coulomb barrier; and $E=$ center-of-mass energy.

In a recent quantum-mechanically modified version of the tunneling theory, Breit, Chun, and Wahsweiler[16] give the corrected value of the total cross section σ_{QM} as

$$\sigma_{QM}=\sigma_{SC}\exp[8\eta(\mu_0-\tan^{-1}\mu_0)], \qquad (2)$$

where, $\eta=Z_1Z_2e^2/\hbar v$ and $\mu_0=\alpha/2k$, k being the wave number, i.e., $k=2E/\hbar v$.

Besides the various kinematical factors, the cross-section expressions [Eqs. (1) and (2)] contain the product of the reduced widths in the two participating nuclei, $\lambda\lambda'$, where λ refers to the nucleus donating the neutron and λ' refers to the nucleus which has accepted the neutron. The probability of finding a neutron in a shell of unit thickness around one of these nuclei is proportional to the reciprocal of the appropriate λ. The kinematical factors in formulas (1) and (2) are calculable; therefore, the $\lambda\lambda'$ product can be determined if the experimental cross section is known.

The two $(^{14}N,^{13}N)$ excitation functions are shown in Fig. 8, together with curves that represent the variation of the total cross sections as predicted by the Breit, Chun, and Wahsweiler tunneling theory [Eq. (2)]. The theoretical curves are normalized to the data at 13.5 MeV for the ^{14}N reaction and at 11.5 MeV for the ^{10}B results. The points of normalization were selected at energies sufficiently below the Coulomb barrier, where the theory is expected to fit. For the ^{14}N reaction it was known from recent investigations[9,10] that the tunneling mechanism is a good description of the transfer process for incident energies below $\sim$15 MeV. Since the Coulomb barrier for ^{14}N incident on ^{10}B is about 2 MeV (in the laboratory system) less than that for ^{14}N on ^{14}N, the normalization energy for the ^{10}B data was chosen to be 2 MeV less than that for ^{14}N. The theoretical curves fit the ^{14}N data for energies $\lesssim$15 MeV and the ^{10}B results below $\sim$13 MeV. Above these energies the experimental excitation functions fall more and more below the theoretical curve. This deviation is presumably due to the effects of nuclear absorption.[10] As the ^{14}N energy is increased a greater number of incident particles are absorbed by the target nuclei; this results in a greater probability for compound nucleus reactions and a probability for transfer which is less than that predicted by the tunneling theory, since the latter does not take nuclear absorption into account and assumes that no competing reactions are occurring.

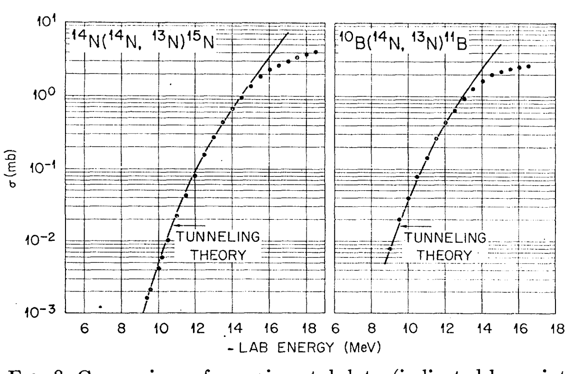

FIG. 8. Comparison of experimental data (indicated by points) with the predictions of the tunneling theory (solid curves) for the reactions $^{14}N(^{14}N,^{13}N)^{15}N$ and $^{10}B(^{14}N,^{13}N)^{11}B$.

Hiebert *et al.*,[10] in their study of the $^{14}N(^{14}N,^{13}N)^{15}N$ reaction, noted that cross sections measured in the energy region 9.4–10.4 MeV fell above the tunneling curve (normalized to their data below 13 MeV by means of a least-squares fit). They suggested that this might indicate that some process, in addition to tunneling (such as virtual Coulomb excitation), could be taking place at these low energies. Becker and McIntyre[9] in their investigation of the same reaction make no statement concerning this possibility since in the same energy region they have just as many data points above as below the theoretical curve. Our data for both targets indicate no significant deviation from the tunneling curve, even at the lowest energy. The indication is then that the contribution to the transfer process from some other mechanism, such as virtual Coulomb excitation, must be quite small.

The products of the neutron reduced widths, $1/\lambda_{^{14}N}\lambda_{^{15}N}$ and $1/\lambda_{^{14}N}\lambda_{^{11}B}$, were determined from the experimental cross sections at the normalization energies: (1) for $^{14}N(^{14}N,^{13}N)^{15}N$, $\sigma_{QM}=0.44$ mb at 13.5 MeV; and, (2) for $^{10}B(^{14}N,^{13}N)^{11}B$, $\sigma_{QM}=0.54$ mb at 11.5 MeV. The actual cross section for the ^{10}B reaction at 11.5 MeV is 0.27 mb (see Figs. 7 and 8). The tunneling theory was formulated, however, to explain the identical-particle transfer reaction $^{14}N(^{14}N,^{13}N)^{15}N$; therefore, it is necessary to double the cross sections for reactions with nonidentical target and projectile nuclei.

The quantity $1/\lambda$ is defined[4,9] as

$$1/\lambda=r^2R^2(r). \qquad (3)$$

[16] G. Breit, K. W. Chun, and H. G. Wahsweiler, Phys. Rev. **133**, B403 (1964).

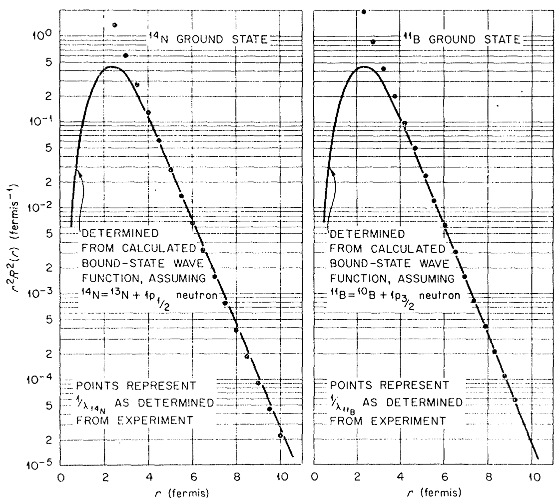

FIG. 9. Comparison of the quantity $r^2R^2(r)$ for the ^{14}N and ^{11}B ground states as determined from experiment and the tunneling theory (represented by points) and from calculated bound-state radial wave functions (represented by solid curves).

Here r is the nuclear radius and $R(r)$ is the radial wave function of the neutron. The variable r in Eq. (3) is the same as the variables b_1 and b_2 that enter into Eqs. (1) and (2). The $1/\lambda\lambda'$ product determined from the experimental cross section measurement is, therefore, a function of the radius r, i.e., $1/\lambda\lambda'$ depends on the values of b_1 and b_2 that are used in Eqs. (1) and (2). For the ^{14}N on ^{14}N reaction it was assumed that b_1(^{14}N radius) $=b_2$(^{15}N radius) and the $1/\lambda_{^{14}\mathrm{N}}\lambda_{^{15}\mathrm{N}}$ product was calculated for a variety of radii, ranging from 2.5 to 10.0 F. The quantities $1/\lambda_{^{14}\mathrm{N}}$ and $1/\lambda_{^{15}\mathrm{N}}$ were then determined by assuming that the two values are the same. This is a reasonable assumption since the last neutron in both ^{14}N and ^{15}N is a $p_{1/2}$ neutron and both neutrons have essentially the same binding energy. For the reaction ^{14}N on ^{10}B the quantity $1/\lambda_{^{14}\mathrm{N}}\lambda_{^{11}\mathrm{B}}$ was calculated as a function of r by assuming that b_2(^{11}B radius) $=(11/14)^{1/3}b_1$(^{14}N radius), that is, the same radius parameter, r_0, was used to calculate both ^{14}N and ^{11}B radii. Since $1/\lambda_{^{14}\mathrm{N}}$ was known, the quantity $1/\lambda_{^{11}\mathrm{B}}$ could also be determined.

For the reduced-width determinations it was assumed that at these low incident energies the two reactions proceed only to the ground states of the residual nuclei. This was based primarily on the work of Becker and McIntyre[9] who concluded from their range measurements at 13.2 MeV that the number of excited-state transfers in the reaction ^{14}N(^{14}N,^{13}N)^{15}N was statistically insignificant. At the investigated energies range measurements are not available for the ^{10}B reaction. We, therefore, normalized the tunneling theory to the data at 11.5 MeV and assumed that as in the ^{14}N on ^{14}N reaction the incident energy was low enough so that transfers to the ^{11}B ground state were relatively more important than transfers to excited states.

The quantities $1/\lambda_{^{14}\mathrm{N}}$ (or $1/\lambda_{^{15}\mathrm{N}}$) and $1/\lambda_{^{11}\mathrm{B}}$ are shown in Fig. 9 as a function of r. They are compared with values of $r^2R^2(r)$ for ^{14}N and ^{11}B determined from shell-model calculations. Bound-state single-particle radial wave functions have been calculated[17] by using a Woods-Saxon potential with parameters derived from scattering experiments. These wave functions are computed as part of the Oak Ridge direct-reaction JULIE program and are wave functions of neutrons in a well:

$$V_0\left[f_s(r)-\lambda_{\mathrm{so}}\left(\frac{\hbar}{\mu_p c}\right)^2\frac{1}{r}\frac{df_s(r)}{dr}\boldsymbol{\sigma}\cdot\mathbf{l}\right], \qquad (4)$$

where μ_p is the proton mass; $f_s(r)$ is the Saxon shape $\{1+\exp[(r-\rho A^{1/3})/a]\}^{-1}$; λ_{so} is a numerical parameter; and V_0 is a real depth automatically adjusted by the code to yield the binding energy, which is inserted as part of the input. The numerical parameters used were $\rho=1.25$ F, $a=0.65$ F, and $\lambda_{\mathrm{so}}=20$. The ^{14}N ground state was assumed to be ^{13}N$+1p_{1/2}$ neutron, while the ^{11}B ground state was taken to be ^{10}B$+1p_{3/2}$ neutron.

The values of $r^2R^2(r)$ as determined from experiment and the tunneling theory agree well for radii $\gtrsim 4$ F with those determined from the calculated bound-state wave functions. The tunneling theory assumes a shape of the neutron-radial wave function which is reasonable only for large r; the deviations between the two sets of $r^2R^2(r)$ for small radii are, therefore, to be expected. The good agreement that is found for large radii is unexpected and is probably accidental because: (1) the experimental uncertainty is 30%; (2) the bound-state wave function $R(r)$ varies for different values of the numerical parameters used in the calculation; and (3) uncertainties arise due to the fact that we have not considered spectroscopic factors in our determinations of the $1/\lambda$ quantities.

The variation of $R(r)$ with the numerical parameters can be illustrated by examining the ^{14}N wave function at 5 F as calculated[17] for different combinations of ρ, a, λ_{so}. When λ_{so} is changed from 20 to 30 $R(r)$ changes from 3.54×10^{-2} to 3.46×10^{-2} F$^{-3/2}$. When ρ is decreased from 1.25 to 1.22 the wave function becomes 3.45×10^{-2} F$^{-3/2}$. When ρ and a are increased to 1.35 and 0.75, respectively, the wave function increases to 4.25×10^{-2} F$^{-3/2}$. This increase in $R(r)$ is probably due about equally to the increase in ρ and in a. (The estimate is based on the previously noted change in the radial wave function when ρ was decreased to 1.22.) Recently, Lutz *et al.*,[18] have used the optical model to fit their data obtained for the elastic scattering of 14-MeV neutrons on various light target nuclei. For ^{14}N they used $\rho=1.2$ and $a=0.64$; for the natural boron target the two parameters were taken to be 1.35 and 0.55.

The uncertainties due to the spectroscopic factors can be best discussed by relating the quantity $1/\lambda$ to

[17] E. C. Halbert, Oak Ridge National Laboratory (unpublished).

[18] H. F. Lutz, J. B. Mason, and M. D. Karvelis, Nucl. Phys. **47**, 521 (1963).

the more usual definition of the reduced width. The reduced width for the transfer of a nucleon between two states is regarded as a product of two factors and is defined as[19]

$$\theta^2 = S\theta_0^2 . \tag{5}$$

Here S is the spectroscopic factor and θ_0^2 is the single-particle reduced width. It is defined as[19]

$$\theta_0^2 = \tfrac{1}{3} r^3 K^2(r) . \tag{6}$$

Since the transfer-reaction cross section is determined by the value of θ^2, rather than just θ_0^2, one must compare $1/\lambda$ with θ^2 rather than directly with θ_0^2. The tunneling theory was derived, however, for the reaction $^{14}N(^{14}N,^{13}N)^{15}N$. Therefore, the spectroscopic factors for the nitrogen nuclei are presumably taken into account by the theory. (Macfarlane and French[19] estimate the relevant spectroscopic factor for ^{14}N to be ~ 1.3 and for ^{15}N to be ≥ 1.07 or ≤ 1.25.) If the tunneling theory, when applied to the reaction $^{10}B(^{14}N,^{13}N)^{11}B$, does not account for the boron spectroscopic factors then the magnitude of $r^2R^2(r)$ for ^{11}B derived from experiment would have to be reduced by the appropriate value of S. This spectroscopic factor is given as 7/4 by Macfarlane and French.[19]

From the above discussion we note that the uncertainties due to experimental errors and the neglect of spectroscopic factors are on the same order of magnitude as those that enter into the calculated radial wave functions when different combinations of numerical parameters are used. Therefore, while the agreement between the calculated and experimentally determined values of $r^2R^2(r)$ is good, the results shown in Fig. 9 can only be taken to mean that the tunneling theory, when applied to transfers of $1p$ neutrons, yields a reasonable measure of the neutron radial wave function for $r \geq 4$ F.

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$^{12}C(\gamma,n)^{11}C$ Giant Resonance with Gamma Rays*

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The photonuclear reaction $^{12}C(\gamma,n)^{11}C$ was produced in natural carbon by means of the monochromatic gamma rays from $T(p,\gamma)He$ in the range of $21 < E_\gamma < 26.7$ MeV. The reaction was detected by means of the positron radioactivity of ^{11}C using coincidences of the annihilation gamma radiation from the positrons. The cross section was determined absolutely to an accuracy of 10%. The gamma-ray energy width or resolution varied from about 0.1 MeV at $E_\gamma = 22$ MeV to 0.2 MeV at $E_\gamma = 26$ MeV. The peak cross section is 7.8 mb at both 22.15 and 23.0 MeV, and additional structure is observed at 25.6 MeV. The integrated cross section from 20 to 27 MeV is 36 MeV mb. Comparison is made with other reported measurements and with theoretical calculations. Some agreement is found with the deformed-nucleus calculation of Nilsson, Sawicki, and Glendenning. Also, some indication is observed of transition to excited states of ^{11}C suggesting two-hole–two-particle excitation in ^{12}C.

INTRODUCTION

A GREAT number of measurements[1] have been made of the carbon photoneutron reaction since the first observation of the "giant resonance" by Baldwin and Klaiber.[2] The major source of photons has been bremsstrahlung from energetic and essentially monochromatic electrons accelerated by betatrons and lineacs with gradually improving resolution. The heterogeneity of these photons has made the interpretation of yield curves somewhat difficult, particularly with respect to the so-called "breaks." Annihilation radiation from energetic positrons in flight[3] has recently been applied to this measurement with resolution of about 2%.

Indirect but valuable evidence of the cross-section variation may be obtained from high-resolution neutron-energy measurements using bremsstrahlung[4]

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