and

$$\langle nlj\|\mathbf{M}^{(q)}\|n'l'j'\rangle$$
$$=(2q+1)^{1/2}W(1j(q-1)j':jq)\{j(j+1)(2j+1)\}^{1/2}$$
$$\times\langle nlj\|\mathbf{Q}^{(q-1)}\|n'l'j'\rangle. \quad (18b)$$

Here, $W(abcd:ef)$ is a Racah coefficient. Equations (9) and (10) then yield (in proper units)

$$f_1\langle 2\|\mathbf{M}_c^{(1)}\|2\rangle=-3.101;\quad f_2\langle 2\|\mathbf{Q}_c^{(2)}\|2\rangle=0.182;$$
$$f_3\langle 2\|\mathbf{M}_c^{(3)}\|2\rangle=-0.160;\quad f_4\langle 2\|\mathbf{Q}_c^{(4)}\|2\rangle=0.033.$$

This would imply nonvanishing values of magnetic octupole and electric hexadecapole moments for the 2^+ state in Si^{28}. Since experimental data on the static moments of excited states in nuclei are scarce, this fact may not be useful in testing the validity of Eq. (16). A systematic study of a number of nuclei could be helpful in this direction by providing information on the consistency of the values of the parameters.

We would like to say, in conclusion, that the remarks made in this section concerning the nature of the core-particle interaction are of a speculative nature and would require further investigation in order to establish the validity or otherwise of Eq. (16). We have tried to show that Eq. (16) would not be unreasonable from the point of view of the core-particle interactions that have been used previously as well as from the point of view of the results for Al^{27} discussed in this paper.

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Lithium-Lithium Scattering*

L. L. Pinsonneault† and J. M. Blair
School of Physics, University of Minnesota, Minneapolis, Minnesota

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Elastic-scattering experiments have been performed with Li^6 on Li^6 from 3.2 to 7.0 MeV and with Li^7 on Li^7 from 4.0 to 6.5 MeV. In the lower portion of this energy range the observations follow Mott's scattering formula but at higher energies fall below these predictions. An analysis of the data in terms of the rounded-cutoff Blair model resulted in a set of parameters which was not sharply defined and probably not unique but which gave curves which reproduced satisfactorily the fluctuations of cross section with angle. An analysis in terms of one parameter, the interaction distance, gave values of $r_0=(1.38\pm0.03)\times10^{-3}$ cm for Li^6 and $r_0=(1.52\pm0.03)\times10^{-13}$ cm for Li^7.

INTRODUCTION

ONE of the most productive sources of information concerning the characteristics of nuclei has been elastic-scattering experiments. In this category the scattering of moderately heavy identical nuclei, for example: C^{12} by C^{12}, and O^{16} by O^{16} by Bromley, Kuehner, and Almqvist[1] and N^{14} by N^{14} by Reynolds and Zucker,[2] have brought out a number of interesting features, due to the complex structure of the particles, which do not appear in the scattering of simpler nuclei. As a part of our program for the investigation of the reactions produced by lithium ions we have studied the scattering of Li^6 by Li^6 over the energy range from 3.2 to 7.0 MeV and the scattering of Li^7 by Li^7 from 4.0 to 6.5 MeV.

Because of the identity of the target and incident nuclei one would expect the scattering cross sections to follow the well-known Mott equation[3] in the absence of nuclear effects. In the carbon, oxygen, and nitrogen scattering experiments referred to above the departures from the predictions of the Mott equation have been interpreted in terms of the sharp-cutoff model discussed by J. S. Blair and others.[4,5] The results of these analyses have been the determination of radii within which nuclear effects become important. We anticipated that measurements of this type performed with lithium would provide some insight into the structure and interactions of these light and relatively simple nuclei.

EQUIPMENT AND PROCEDURE

Lithium ions having energies up to 7 MeV were obtained from the Minnesota Van de Graaff machine,

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* Supported in part by the U. S. Office of Naval Research.

† Now at the Esso Production Research Company, Houston, Texas.

[1] D. A. Bromley, J. A. Kuehner, and E. Almqvist, in *Reactions Between Complex Nuclei*, edited by A. Zucker, F. Howard, and E. Halbert (John Wiley & Sons, Inc., New York, 1960), p. 151.

[2] H. L. Reynolds and A. Zucker, Phys. Rev. **102**, 1378 (1956).

[3] N. F. Mott and H. S. W. Massey, *Theory of Atomic Collisions* (Oxford University Press, New York, 1949), 2nd ed., p. 19.

[4] J. S. Blair, Phys. Rev. **95**, 1218 (1954).

[5] J. A. McIntyre, K. H. Wang, and L. C. Becker, Phys. Rev. **117**, 1337 (1960).

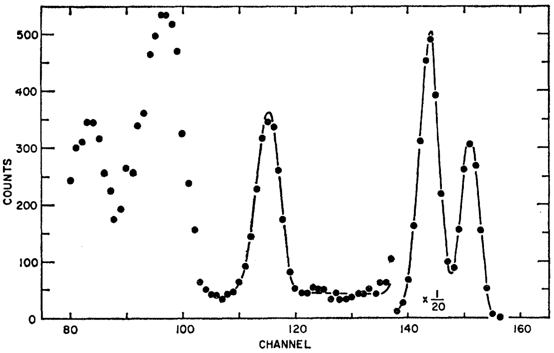

FIG. 1. Typical pulse spectrum. Li^6 on Li^6 at a laboratory energy of 4.2 MeV. Observed at a laboratory angle of 35°. From right to left peaks are: Li^6 scattered from O, Li^6 from C, Li^6 from Li^6, recoil C, and recoil O. Curve is the nonlinear least-squares fit to the data.

which has a maximum terminal potential of 4 MV. The singly charged ions from the ion source were accelerated to approximately 1 MeV at which point they passed through a carbon film of 1-μg/cm^2 thickness. This film was supported in the accelerating tube on a fine molybdenum screen. Passing through this film removed an additional electron from some of the lithium ions so that the remaining 3 MV of accelerating potential resulted in an additional 6 MeV of beam energy. The screen supporting the carbon film was mounted in a field-free region 6 in. long to protect it from disruptive electrostatic forces. The supporting structure also contained permanent magnets to suppress the emission of secondary electrons which would have had a deleterious effect on the performance of the accelerating tube. A beam of 0.1 μA could be obtained from the accelerator for 40 h before significant deterioration of the stripping foil set in.

The ion beam was analyzed by a 90° magnet and focused on the target by electrostatic quadrupole lenses. The target chamber was a 7-in.-diam cylinder, the top of which could be rotated to move the detector. Two rectangular slits 33 in. apart defined the incoming beam. The first aperture was 0.125 in. wide by 0.50 in. high. The second aperture was 0.020 in. wide by 0.187 in. high. Just in front of the silicon surface-barrier detector was a defining aperture 0.0211 in. wide by 0.300 in. high which was 2.160 in. from the target at the center of the chamber. The angular position of the detector could be determined to 0.1°. Measurements of small-angle scattering from the target determined the zero of the angular scale with an error of 0.1°.

Pulses from the detector were amplified by a charge-sensitive preamplifier and a linear amplifier of standard design using single delay line clipping. Tests using 8.78-MeV alpha particles gave a peak with a full width at half-maximum of 30 keV. The pulse spectrum was analyzed in a 512-channel pulse-height analyzer. The whole electronic system was found to be stable to within ±1 channel in 400 during a day's operation.

Targets used in this work were prepared by evaporating lithium metal onto thin carbon backings. The Li^7 used was natural lithium, the Li^6 had the composition 95.9% Li^6 atoms and 4.1% Li^7 atoms. From the weight of metal evaporated it was estimated that the Li^7 target contained no more lithium than 2.5 μg/cm^2 and the Li^6 target contained no more than 1.5 μg/cm^2. During the transfer of the targets from the evaporation vacuum system to the target chamber the thin lithium films became oxidized. All of the final data runs were taken using one target for each isotope.

After passing through the target, the ion beam was collected in an insulated cup connected to a current integrator. Loss or gain of secondary electrons was prevented by magnetic fields around the mouth of the cup. From information obtained in this laboratory and elsewhere[6] it was known that the average charge carried by the lithium ions entering the current collection cup increased with increasing ion energy, varying from 2.7 to 2.9. The actual values of these factors are of less importance than the relative variation, since the absolute cross sections obtained in this work were based upon the observation of pure Mott scattering at the low end of the energy range. The calibration of the current integrator was checked periodically and found to be constant to within 0.3%.

The angular range over which useful data could be obtained and the low-energy limits of the measurements were determined by the effective separation of the lithium-lithium scattering peak from other peaks in the pulse spectrum. Figure 1 shows a pulse spectrum obtained from Li^6 on Li^6 at a laboratory angle of 35° and energy of 4.2 MeV. The peaks shown are, from right to left, Li^6 scattered from O, Li^6 scattered from C, Li^6 scattered from Li^6, recoil C atoms, and recoil O atoms. The line is the result of the nonlinear least-squares analysis described below. The widths of the peaks agree with those expected when the angular spread of the incident beam, the angular range of scattered particles accepted by the detector, and the spread due to multiple scattering are taken into account. At larger angles of observation the Li-Li peak moved closer to the peak caused by recoil C atoms. At laboratory angles greater than 45° the two could not be satisfactorily resolved. This determined the high-angle limit to the data. At lower angles the Li-Li peak moved closer to that due to Li scattered from C. For Li^6 the minimum useful laboratory angle was 17.5°, while for Li^7 it was 20°. The laboratory energies for which Li^6-Li^6 scattering data were obtained were 3.2, 3.4, 3.6, 3.8, 4.0, 4.4, 4.6, 4.8, 5.0, 5.25, 5.5, 5.75, 6.0, 6.5, and 7.0 MeV. In the Li^7-Li^7 work the peaks were closer together due to kinematic effects and the multiple scattering effects were more pronounced due to the lower ion velocities for a given energy. Consequently, the useful

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[6] Ia. A. Teplova, I. S. Dmitriev, V. S. Nikolaev, and L. N. Fateeva, Zh. Eksperim. i Teor. Fiz. 32, 974 (1957) [English transl.: Soviet Phys.—JETP 5, 797 (1957)].

data were obtained at energies of 4.0, 4.2, 4.4, 4.6, 4.8, 5.0, 5.25, 5.5, 5.75, 6.0, and 6.5 MeV.

At a laboratory angle of 35° the peaks of interest could be resolved at lower beam energies, so observations could be extended down to 2.4 MeV for Li^6-Li^6 and down to 2.6 MeV for Li^7-Li^7. In both cases the ratio of the observed yields to the Mott scattering cross section remained constant to within 1% for Li^7-Li^7 and 2% for Li^6-Li^6 over the lowest 400 keV of the useful ranges. In view of the fact that these energies fall well below the Coulomb barrier height and the consistency of the ratio of observed scattering yield to the cross sections calculated by Mott's equation, it was assumed that the observed scattering was Mott scattering only. These results were used to normalize the yield to absolute cross sections. For higher energies the normalization factor was corrected for target-thickness effects and changes in the average charge per ion entering the current collector. During the course of the experiment checks were made to detect target-deterioration effects but none were observed.

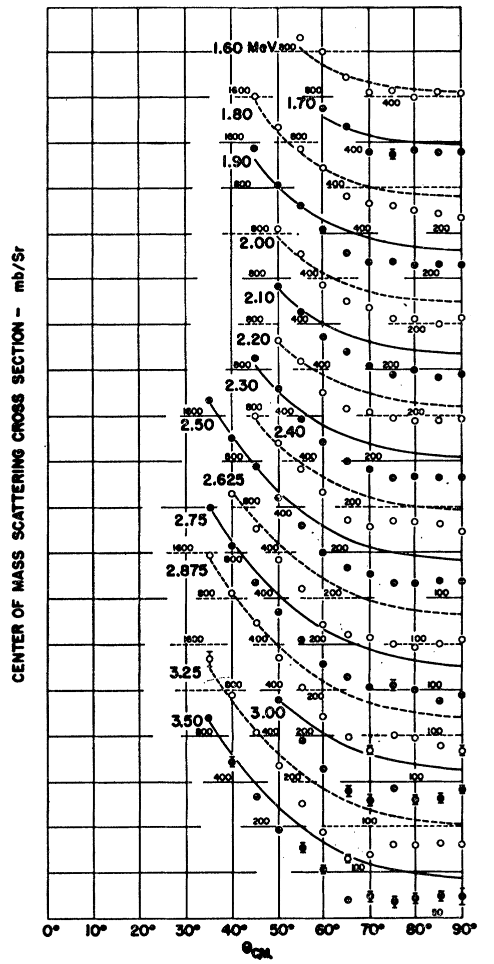

FIG. 2. Scattering of Li^6 by Li^6. Center-of-mass cross section versus center-of-mass angle for center-of-mass energies from 1.60 to 3.50 MeV. The ordinate scale for each curve is shifted by a factor of two from the curve above it.

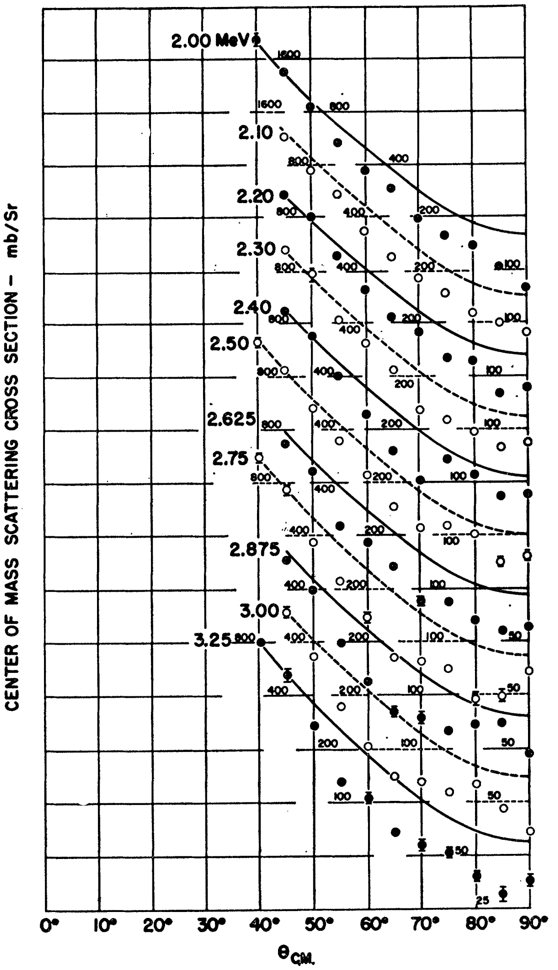

FIG. 3. Scattering of Li^7 by Li^7. Center-of-mass cross section versus center-of-mass angle for center-of-mass energies from 2.00 to 3.25 MeV. The ordinate scale for each curve is shifted by a factor of two from the curve above it.

PROCESSING OF DATA

Before any comparison with theory could be made the raw data required considerable treatment. The major problem was the determination of the total number of scattered particles which contributed to the peak of interest in each spectrum of pulses. This was complicated by the fact, illustrated in Fig. 1, that the spectrum peak due to lithium-lithium scattering was much smaller than the neighboring peaks and at many angles the lithium-lithium peak was on the tail of an adjacent peak.

A careful consideration of peak shape in 30 pulse spectra where peaks were well separated showed that an isolated peak was a slightly skewed Gaussian over the upper half and down to two standard deviations below the median. Below that, for four standard deviations,

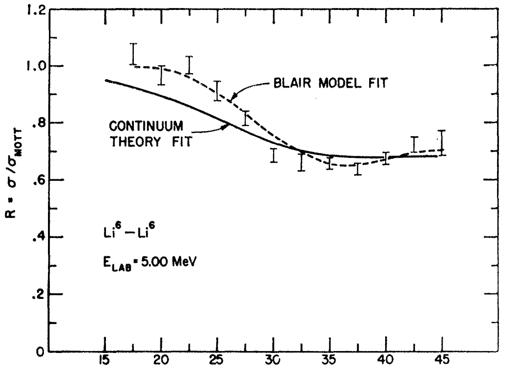

FIG. 4. Ratio of observed scattering to Mott scattering for Li^6 on Li^6 at 5.00 MeV. Curves were calculated by nonlinear least-squares procedures based on continuum theory and the rounded cutoff Blair model.

the observed curve could be well fitted by an exponential tail. The existence of the exponential tail was of importance at forward angles where the lithium-lithium peak approached the peak due to lithium scattered from carbon, which was typically 20 times larger. The system of peaks was superimposed upon a constant background due to slit-edge and residual-chamber-gas scattering.

The number of counts under the peak of interest was obtained by using a computer program which fitted the curve described above to the observed data using the Gauss iterative least-squares method described elsewhere.[7–9] The application of this procedure permitted the evaluation of peaks which were much too close to neighboring peaks to be treated in any simpler manner. In a number of cases there were peaks which were sufficiently isolated to permit analysis by hand in addition to being treated by means of the computer program. In these cases the two methods agreed typically to within one quarter of the estimated error. Usually the error estimate obtained during the least-squares process was larger than the error assigned to the hand analysis of the peaks from consideration of the square root of the number of counts.

The result of the least-squares analysis was corrected for the dead time of the pulse-height analyzer (estimated probable error 0.5%), for the finite geometry of the beam and detector (0.5% error), and for the variation in average charge per ion as a function of ion energy (0.6% error). The normalization of the yield to Mott scattering at low energies was estimated to be in error by less than 2%.

RESULTS AND ANALYSIS

The result of the data analysis described above is the ratio of the cross section for elastic scattering to that for Mott scattering as a function of angle and bombarding energy. Since this ratio is the same in both laboratory and center-of-mass (c.m.) coordinate systems, the c.m. cross section was obtained by multiplying this ratio by the theoretical Mott cross section. Figures 2 and 3 present the c.m. differential cross section versus c.m. angle as a function of c.m. energy for Li^6-Li^6 and Li^7-Li^7 scattering. The curves represent the predictions of the Mott scattering formula.

In the range of this experiment the Mott cross section shows little interference modulation as compared with other heavy-ion cross sections which have been studied.[1] This happens because the Coulomb parameter, $\eta=(ze)^2/\hbar v$, is small and also the spin factor reduces the amplitude of the modulation by a factor of 3 or 4 from that of C-C or O-O scattering [see Eq. (2) in Ref. 1]. In the present case the Mott curves are monotonic up to 90° c.m. The measured angular distributions show somewhat more structure than this, although there appears to be little strong variation such as might be attributable to resonance phenomena.

The results of the measurements were analyzed using both continuum theory[10] and the rounded-cutoff Blair model.[5] Fitting the data to this modification of the Blair cutoff model involved the evaluation of three parameters: l_A, the limiting value of orbital angular momentum for absorption of incident particles; Δl_A, the range of values of angular momentum related to the thickness of the region in which the nuclear reaction can take place between the colliding nuclei without the destruction of the identity of either particle; and δ, which introduces the strength of the nuclear phase shift. Initially these parameters were calculated, using the nonlinear least-squares procedure, for each angular distribution. The results were not consistent, due to the fact that the three parameters were highly correlated. Then a consistent set of parameters which fit reasonably well all of the data for each of the iso-

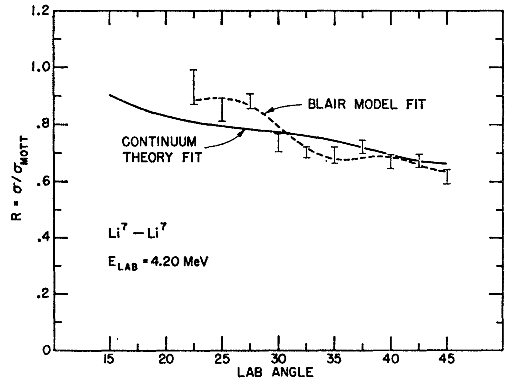

FIG. 5. Ratio of observed scattering to Mott scattering for Li^7 on Li^7 at 4.20 MeV. Curves were calculated by nonlinear least squares procedures based on continuum theory and the rounded cutoff Blair model.

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[7] R. H. Moore and R. K. Zeigler, Los Alamos Scientific Laboratory Report LA-2367, 1959 (unpublished).

[8] P. McWilliams, W. S. Hall, and H. E. Wegner, Rev. Sci. Instr. 33, 70 (1962).

[9] R. K. Hobbie and L. Pinsonneault, Rev. Sci. Instr. 34, 1445 (1963).

[10] J. M. Blatt and V. K. Weisskopf, *Theoretical Nuclear Physics* (John Wiley & Sons, Inc., New York, 1952), p. 340.

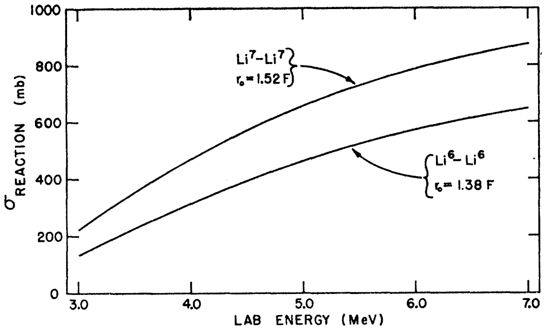

FIG. 6. Total reaction cross sections for Li^6 on Li^6 and Li^7 on Li^7 as a function of energy, computed from the interaction radii determined from the analysis of scattering data.

topes was sought. The resulting set of parameters is not sharply defined and probably not unique. This situation has been noticed in the analysis of other heavy-particle scattering data.[11] The best over-all fit to the Li^6 data was obtained with the set of parameters $\Delta l_A = 1.8$, $\delta = 2.2$, and l_A having that value corresponding to $r_0 = 1.65 \times 10^{-13}$ cm, where $R = 2r_0A^{1/3}$ is the rounded-cutoff Blair model interaction distance. For Li^7 the corresponding parameters were $\Delta l_A = 2.8$, $\delta = -2.8$, and l_A corresponding to $r_0 = 1.70 \times 10^{-13}$ cm. These values of r_0 are consistent with those obtained using the Blair model with the other heavy ions.[1] Typical fits obtained using the rounded-cutoff Blair model are shown in Figs. 4 and 5.

The ambiguity in these parameters is not surprising because, in this case, both l_A and η are too small for the model to be adequate. Values of l_A obtained ranged from 0 to 2, while in a typical satisfactory Blair-model analysis l_A is at least 6 to 10. For l values less than approximately $(l_A - 2)$ the partial waves are essentially completely absorbed. For slightly larger values of l there is a transition region where the absorption drops off to zero. Behavior in this region depends upon the detailed nature of the "tail" of the interaction potential between the two nuclei. However, this region ordinarily contributes only a minor part to the predicted cross sections. Therefore, inadequacies of the model in the transition region do not seriously affect the results. In our case, there is no region of complete absorption. We have nothing but transition region to deal with, which is just where the model is weak. The parameters obtained fit the data fairly well, but their physical significance is obscured.

Continuum theory has only one adjustable parameter, the interaction distance R. The nonlinear method of least squares was used to obtain this quantity from the observed data. Typical fits are also shown in Figs. 4 and 5. Since the continuum model does not provide diffraction effects, the curves developed from it do not fit the observed data as well as those from the Blair model. However, the experimental results and the continuum model predictions agree on the average and the interaction distances obtained were constant to within the estimated errors of the calculations for both the Li^6 and Li^7 data. The resulting values of r_0 were $(1.38 \pm 0.03) \times 10^{-13}$ cm for Li^6 and $(1.52 \pm 0.03) \times 10^{-13}$ cm for Li^7. These values are a bit smaller than those obtained from the Blair model but this is not unexpected since this distance describes slightly different things in the two models. In the Blair model all particles coming inside the radius interact and none outside it interact. In the continuum theory, particles may either come within the radius or tunnel within the radius to produce an interaction. For an observed amount of interaction we should expect the continuum-model radii to be somewhat smaller.

With the parameters determined from the elastic-scattering data one can calculate the total reaction cross section.[12] Using the continuum-model radii obtained above, the curves shown in Fig. 6 for the total reaction cross sections as a functions of the laboratory energy of the lithium ions were obtained. It will be interesting to compare these total cross sections with the results of various lithium-lithium reaction experiments when they become available in this energy range.

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[11] J. Alster and H. E. Conzett, in *Reactions Between Complex Nuclei*, edited by A. Zucker, F. Howard, and E. Halbert (John Wiley & Sons, Inc., New York, 1960), p. 175.

[12] Reference 10, p. 321.