Al^{27} and the Excited-Core Model*

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The nuclear properties of the low-lying levels in Al^{27} are discussed in terms of a model in which a proton hole is coupled to the observed levels of Si^{28}. The calculated electric $E2$ transition probabilities are found to be in good agreement with the experimental values, but the agreement between theory and experiment for $M1$ transition probabilities does not appear to be so good. It is found that the core-particle interaction required to fit the observed levels of Al^{27} contains multipoles of order up to four. A possible explicit form for this interaction is suggested.

I. INTRODUCTION

EXPERIMENTS on the inelastic scattering of deuterons[1] and alpha particles[2] seem to indicate that the low-lying levels of Al^{27} might be described as a proton hole coupled to the levels of Si^{28}. The formalism for a model in which the energy levels of an odd-mass nucleus are envisaged as resulting from the coupling of a particle or hole to the observed levels of the neighboring even-even nucleus (the core) has been discussed by Lawson and Uretsky[3] and de-Shalit.[4] The interaction between the core and the particle can, in general, be represented by a sum of scalar products of irreducible tensor operators:

$$H_{\text{int}}=-\sum_r f_r(\mathbf{T}_c^{(r)}\cdot\mathbf{t}_p^{(r)}). \quad (1)$$

In Eq. (1), $\mathbf{T}_c^{(r)}$ operates only on the degrees of freedom of the core while $\mathbf{t}_p^{(r)}$ operates only on those of the particle. The parameter f_r denotes the strength of the interaction. Recently, Thankappan and True[5] have employed a core-particle interaction of the form

$$H_{\text{int}}=-\xi(\mathbf{J}_c^{(1)}\cdot\mathbf{j}_p^{(1)})-\eta(\mathbf{Q}_c^{(2)}\cdot\mathbf{Q}_p^{(2)}), \quad (2)$$

in order to explain the observed properties of the levels of Cu^{63} within the framework of the excited-core model. In Eq. (2), $\mathbf{J}_c^{(1)}$ and $\mathbf{Q}_c^{(2)}$ are, respectively, the angular-momentum operator and the quadrupole-moment operator for the core, while $\mathbf{j}_p^{(1)}$ and $\mathbf{Q}_p^{(2)}$ are similar operators for the particle. In the present paper we would like to examine the validity of the excited-core model with a core-particle interaction represented by Eq. (2) in describing the properties of the low-lying levels of Al^{27}.

II. CALCULATIONS AND RESULTS

Since it appears from experiments on $(p,2p)$ reactions[6] that the single-particle levels in the region of mass number 28 are well separated (by >3 MeV), we include in our calculations only the lowest level, viz., the $1d_{5/2}$ level. Moreover, the core states are restricted to the ground state and the first excited state. Thus the levels of Al^{27} are assumed to be given by the coupling of a proton hole in the $1d_{5/2}$ subshell to the 0^+ ground state and the 2^+ first excited state at 1.77 MeV in Si^{28} (see Fig. 1). The method of calculation is described in detail in Ref. 5 and we will follow the notations of that paper. A state of total spin I obtained by coupling the $d_{5/2}$ proton hole to a core state of spin J_c is denoted by $|J_c,d_{5/2}^{-1}:IM\rangle$, where, M is the z component of I. The parameters of the model are ξ, $\chi_1=\eta\langle 0\|\mathbf{Q}_c^{(2)}\|2\rangle$ and $\chi_2=\eta\langle 2\|\mathbf{Q}_c^{(2)}\|2\rangle$.

The experimental data on Si^{28} and the observed levels of Al^{27} relevant to our discussion are shown in Fig. 1. The data on Si^{28} are taken from Skorka and Retz-Schmidt.[7] The mean life of 7×10^{-13} sec for the 2^+ state is also consistent with the result of Robinson *et al.*[8] The level energies, spin, and parity assignments for Al^{27}, are from Endt and Van der Leun,[9] Towle and Gilboy,[10] and Ophel and Lawergren.[11] The reduced matrix element $\langle d_{5/2}^{-1}\|\mathbf{Q}_p^{(2)}\|d_{5/2}^{-1}\rangle$ is obtained from the relation (see Ref. 12, Chap. 22)

$$\langle d_{5/2}^{-1}\|\mathbf{Q}_p^{(2)}\|d_{5/2}^{-1}\rangle=-\langle d_{5/2}\|\mathbf{Q}_p^{(2)}\|d_{5/2}\rangle,$$

and the oscillator parameter ν that occurs in the evaluation of this matrix element[5] is taken to be $\nu=0.333$ F^{-2}.

The parameters ξ and χ_2 are determined from the experimental separation of the levels $\frac{1}{2}^+$, $\frac{7}{2}^+$, and $\frac{9}{2}^+$ in Al^{27}. The value of χ_1 is then obtained from the observed separation of the ground state, $\frac{5}{2}^+$, and the first excited state, $\frac{1}{2}^+$. The values so obtained are[13]

$$\xi=-0.179\text{ MeV};\quad \chi_1=0.507\text{ MeV F}^{-2}; \quad (3a)$$
$$\chi_2=0.014\text{ MeV F}^{-2}.$$

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[1] H. Niewodniczanski *et al.*, Nucl. Phys. **55**, 386 (1964).
[2] J. Kokame, K. Fukunaya, and H. Nakamura, Phys. Letters **14**, 234 (1965).
[3] R. D. Lawson and J. L. Uretsky, Phys. Rev. **108**, 1300 (1957).
[4] A. de-Shalit, Phys. Rev. **122**, 1530 (1960).
[5] V. K. Thankappan and W. W. True, Phys. Rev. **137**, B793 (1965).
[6] G. Tibell, O. Sundberg, and P. U. Rendberg, Arkiv Fysik **25**, 433 (1964).
[7] S. J. Skorka and T. W. Retz-Schmidt, Nucl. Phys. **46**, 225 (1963).
[8] S. W. Robinson, R. D. Bent, and T. R. Canada, Bull. Am. Phys. Soc. **10**, 525 (1965).
[9] P. M. Endt and C. Van der Leun, Nucl. Phys. **34**, 1 (1962).
[10] J. H. Towle and W. B. Gilboy, Nucl. Phys. **39**, 300 (1962).
[11] T. R. Ophel and B. T. Lawergren, Nucl. Phys. **52**, 417 (1964).
[12] A. de-Shalit and I. Talmi, *Nuclear Shell Theory* (Academic Press, Inc., New York, 1963).
[13] The sign of χ_1 is chosen such that the calculated ground-state quadrupole moment has the same sign as the observed one.

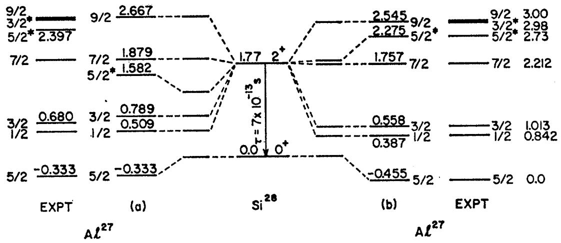

FIG. 1. The low-lying positive parity levels of Al^{27} obtained by coupling a proton hole to the lowest two levels of Si^{28}, using a core-particle interaction (a) given by Eq. (2) and (b) given by Eq. (1), in the text. The observed levels of Al^{27} are shown at the extremes. The experimental information in this figure has been obtained from Refs. 7, 9, 10, and 11.

The excitation energies of the $\frac{3}{2}$ and $\frac{5}{2}*$ levels[14] are calculated using these values of the parameters. The resulting level scheme for Al^{27} is shown in Fig. 1(a). The position of the $\frac{3}{2}$ level is in good agreement with experiment, but the $\frac{5}{2}*$ level is predicted at 1.915 MeV while the observed level is at 2.73 MeV. Ciuffolotti and DeMichelis[15] have reported levels at 1.65 and 1.83 MeV and Winterhalter[16] has observed a level at 1.9 MeV; but these findings have not been confirmed by other experiments. The $\frac{3}{2}*$ level at 2.98 MeV is unaccounted for in the present model. The wave functions of the $\frac{5}{2}$ and $\frac{5}{2}*$ levels are given by

$$\Psi(\tfrac{5}{2})=0.9088\,|0,d_{5/2}^{-1};\tfrac{5}{2}\rangle+0.4172\,|2,d_{5/2}^{-1}:\tfrac{5}{2}\rangle, \quad (4a)$$
$$\Psi(\tfrac{5}{2}*)=-0.4172\,|0,d_{5/2}^{-1}:\tfrac{5}{2}\rangle+0.9088\,|2,d_{5/2}^{-1}:\tfrac{5}{2}\rangle.$$

The reduced transition probability $B(E2)$ for an $E2$ transition is related to the transition energy E_γ(MeV) and the mean life $\tau(E2)$ by

$$B(E2)=[8/\tau(E2)E_\gamma^5]\times10^{-10}\,e^2\,\mathrm{F}^4. \quad (5)$$

TABLE I. $M1$ and $E2$ reduced transition probabilities for the levels in Al^{27}. See the text for the explanation of columns (i) and (ii).

Initial state	Final state	$B(E2)/e^2\mathrm{F}^4$ (i)	(ii)	$b(M1)$
$\frac{1}{2}$	$\frac{5}{2}$	67.225	78.955	
$\frac{3}{2}$	$\frac{5}{2}$	41.341	45.197	0.668
$\frac{3}{2}$	$\frac{1}{2}$	3.535	6.062	2.489
$\frac{7}{2}$	$\frac{5}{2}$	65.059	58.916	0.358
$\frac{7}{2}$	$\frac{3}{2}$	1.112	3.905	
$\frac{5}{2}*$	$\frac{5}{2}$	17.164	17.529	0.157
$\frac{5}{2}*$	$\frac{1}{2}$	0.685	0.008	
$\frac{5}{2}*$	$\frac{3}{2}$	18.642	14.644	2.025
$\frac{5}{2}*$	$\frac{7}{2}$	3.960	8.706	4.379
$\frac{9}{2}$	$\frac{5}{2}$	58.232	62.378	
$\frac{9}{2}$	$\frac{7}{2}$	2.607	0.238	1.178
$\frac{9}{2}$	$\frac{5}{2}*$	7.111	4.233	

[14] The asterisk denotes the second level of a given spin.
[15] L. Ciuffolotti and F. DeMichelis, Nucl. Phys. **39**, 252 (1962).
[16] D. Winterhalter, Nucl. Phys. **39**, 535 (1962).

This gives for the 1.77-MeV 2^+ to 0^+ transition in Si^{28} the result

$$B(E2:2^+\rightarrow0^+)\approx65\,e^2\,\mathrm{F}^4. \quad (6)$$

The ground-state quadrupole moment and the reduced transition probabilities for Al^{27} are calculated using this value. The calculated value of the ground-state quadrupole moment is +0.167 b which is to be compared with the experimental value[17] of +0.152 b. The calculated reduced transition probabilities are listed in Table I, column 3.

If we use the free-nucleon value for the gyromagnetic ratio of the odd proton, viz., $g_p=1.9162$, the ground-state magnetic moment is predicted to be 4.578 nm, where the gyromagnetic ratio for the core is taken to be $g_c=Z/A=0.5$. This is considerably larger than the observed ground-state magnetic moment[17] of 3.63 nm. An exact agreement with the observed moment can be obtained by empirically reducing the proton gyromagnetic ratio to the value 1.5108 (i.e., by a factor of ~0.8). The $M1$ transition probabilities are calculated using this reduced gyromagnetic ratio. We define the quantity $b(M1)$ by

$$b(M1)=(4\pi/3\mu_0^2)B(M1), \quad (7)$$

where, $B(M1)$ is the $M1$ reduced transition probability[5] and μ_0 is the nuclear magneton. The radiative width, $\Gamma(M1)$, for an $M1$ transition of energy E_γ(MeV) is then given by

$$\Gamma(M1)=4.26h\times10^{12}(E_\gamma)^3b(M1)$$
$$=2.804\times10^{-3}(E_\gamma)^3b(M1)\text{ eV}. \quad (8)$$

The calculated values of $b(M1)$ for the various transitions in Al^{27} are listed in the last column of Table I.

In Table II, we compare the available experimental information on the transition probabilities in Al^{27} with the theoretical values. $\Gamma(I_i\rightarrow I_f)$ represents the sum of the $E2$ and $M1$ radiative widths for a transition from an initial state I_i to a final state I_f, while $\Gamma(I)$ is the total width of the level with spin I. δ denotes the $E2/M1$ amplitude mixing ratio. The experimental values of the energies, shown in Fig. 1, are used in these calculations except for the $\frac{5}{2}*$ level which is assumed to be at 1.9 MeV. It is seen that the agreement between theory and experiment is good for the $E2$ transitions and branching ratios, but $M1$ transitions are not in such good agreement. Leaving aside the $\frac{5}{2}*$ level, the main discrepancy is in the transition $\frac{3}{2}\rightarrow\frac{5}{2}$. This might be due to the influence of the $\frac{3}{2}*$ level at 2.98 MeV, which is not accounted for in the present calculations.

The serious discrepancy of the model is, however, in the prediction of the $\frac{5}{2}*$ level at 1.9 MeV instead of at 2.73 MeV. One of two explanations may be offered for this. The first is that there is indeed a level at about 1.9 MeV as has been observed by Ciuffolotti and

[17] *Nuclear Data Sheets*, compiled by K. Way *et al.* (Printing and Publishing Office, National Academy of Sciences—National Research Council, Washington, D. C.).

TABLE II. Comparison of the calculated and the experimental values for quantities related to the electromagnetic properties of the levels in Al^{27}. Columns (i) and (ii) and the symbols in column (1) are explained in the text.

Quantity	Theory (i)	Theory (ii)	Experiment	Ref.
$\tau(E2:\frac{1}{2}\to\frac{5}{2})$	2.81×10^{-11} sec	2.39×10^{-11} sec	$(3.2\pm1.0)\times10^{-11}$ sec	a
$\tau(E2:\frac{3}{2}\to\frac{5}{2})$	1.81×10^{-11} sec	1.69×10^{-11} sec	$(1.3\pm0.4)\times10^{-11}$ sec	b
$\tau(E2:\frac{7}{2}\to\frac{5}{2})$	2.34×10^{-13} sec	2.59×10^{-13} sec	$(2.33\pm0.36)\times10^{-13}$ sec	c
$\tau(E2:\frac{5}{2}^*\to\frac{5}{2})$	1.68×10^{-12} sec	3.01×10^{-13} sec	$(2.7_{-0.9}^{+2.6})\times10^{-13}$ sec	d
$\tau(E2:\frac{9}{2}\to\frac{5}{2})$	5.65×10^{-14} sec	5.28×10^{-14} sec	$(6.35\pm0.7)\times10^{-14}$ sec	d
$\Gamma(M1:\frac{3}{2}\to\frac{5}{2})$	0.195×10^{-2} eV	0.184×10^{-2} eV	$(0.36\pm0.15)\times10^{-3}$ eV	b
$\Gamma(M1:\frac{7}{2}\to\frac{5}{2})$	1.081×10^{-2} eV	1.034×10^{-2} eV	$(1.22\pm0.12)\times10^{-2}$ eV	c
$\Gamma(M1:\frac{5}{2}^*\to\frac{5}{2})$	0.302×10^{-2} eV	0.864×10^{-2} eV	$(0.3$ to $3.0)\times10^{-2}$ eV	b
$\delta(\frac{3}{2}\to\frac{5}{2})$	-0.135	-0.145	-0.29 ± 0.04	e
$\delta(\frac{3}{2}\to\frac{1}{2})$	0.003	0.046	0 to 0.16 or	f
			-2.3 to -1.2	
$\delta(\frac{7}{2}\to\frac{5}{2})$	0.510	0.496	-0.46 ± 0.04	b
			0.37 to 0.50	c, d
$\delta(\frac{5}{2}^*\to\frac{5}{2})$	-0.341	-0.503	-0.038 ± 0.055	b
$\delta(\frac{5}{2}^*\to\frac{3}{2})$	0.046	0.079	-0.226 ± 0.064	b
$\delta(\frac{9}{2}\to\frac{7}{2})$	0.020	0.006	-0.03 ± 0.03	g
$\Gamma(\frac{3}{2}\to\frac{1}{2})/\Gamma(\frac{3}{2})$	1.73%	1.82%	(2.4 ± 0.3)%	f
$\Gamma(\frac{7}{2}\to\frac{5}{2})/\Gamma(\frac{7}{2})$	98.97%	99.94%	$\gtrsim$98%	b, c
$\Gamma(\frac{5}{2}^*\to\frac{3}{2})/\Gamma(\frac{5}{2}^*)$	54.12%	71.17%	(80 ± 5)%	b
			70%	h
$\Gamma(\frac{9}{2}\to\frac{7}{2})/\Gamma(\frac{9}{2})$	12.34%	11.63%	(20 ± 10)%	g

[a] See Ref. 9.
[b] See Ref. 11.
[c] V. J. Vanhuyse and G. J. Vanpraet, Nucl. Phys. 45, 602 (1963).
[d] See Ref. 21.
[e] G. McCallum, Phys. Rev. 123, 568 (1961); T. R. Ophel and B. T. Lawergren, Nucl. Phys. 30, 215 (1962).
[f] E. Almqvist *et al.* Nucl. Phys. 19, 1 (1960).
[g] B. T. Lawergren, Nucl. Phys. 53, 417 (1964).
[h] R. D. Bent and W. W. Eidson, Phys. Rev. 122, 1514 (1961).

DeMichelis[15] and Winterhalter.[16] The levels at 2.73 and 2.98 MeV, as well as the probable level[15] at 1.65 MeV, will then be explained as due to the coupling of the proton configuration $(d_{5/2})^{-2}s_{1/2}$ to the levels of Si^{28}. This would require the spacing between the $1d_{5/2}$ and $2s_{1/2}$ subshells to be smaller than that indicated by $(p,2p)$ reactions, and even then it is doubtful that a quantitative fit with the observed levels of Al^{27} could be obtained. Moreover, this explanation will not be consistent with the fact[11] that the 2.98-MeV level decays almost entirely to the ground state. It should also be mentioned here that a search by Ophel and Lawergren[18] for the two levels reported in Ref. 15 had failed to detect them.

The second, and more probable, explanation is that the core-particle interaction is not adequately represented by Eq. (2) and therefore the more general Eq. (1) has to be used. Leaving out the $r=0$ term, which causes a shift in the excited states of the core,[4] there will be five parameters in this case, which we define as

$$\epsilon_r=f_r\langle 2\|\mathbf{T}_c^{(r)}\|2\rangle\langle d_{5/2}^{-1}\|\mathbf{t}_p^{(r)}\|d_{5/2}^{-1}\rangle,\quad r=1\text{ to }4;$$
$$\zeta=f_2\langle 0\|\mathbf{T}_c^{(2)}\|2\rangle\langle d_{5/2}^{-1}\|\mathbf{t}_p^{(r)}\|d_{5/2}^{-1}\rangle. \tag{9}$$

Since there are only five energy-level spacings to be fitted, an exact fit can be obtained, the required values of the parameters being

$$\epsilon_1=-6.3390;\quad \epsilon_3=-2.5299;$$
$$\epsilon_2=1.4275;\quad \epsilon_4=-3.0033;$$
$$\zeta=5.5721. \tag{10}$$

The mode of level splittings in this case is shown in Fig. 1(b). The wave functions of the $\frac{5}{2}$ and $\frac{5}{2}^*$ states are

$$\Psi(\tfrac{5}{2})=0.9130|0,d_{5/2}^{-1}:\tfrac{5}{2}\rangle+0.4081|2,d_{5/2}^{-1}:\tfrac{5}{2}\rangle,$$
$$\Psi(\tfrac{5}{2}^*)=-0.4081|0,d_{5/2}^{-1}:\tfrac{5}{2}\rangle+0.9130|2,d_{5/2}^{-1}:\tfrac{5}{2}\rangle. \tag{4b}$$

If we assume that the $r=1$ and the $r=2$ terms in Eq. (1) can still be represented by Eq. (2), we would obtain

$$\xi=-0.160\text{ MeV},\quad \chi_1=0.711\text{ MeV F}^{-2},$$
and
$$\chi_2=0.182\text{ MeV F}^{-2}. \tag{3b}$$

The transition probabilities can be then calculated as before and the results are shown in Table I, column 4 and Table II, column 3. The calculated value of the ground-state quadrupole moment is 0.163 b which is a little nearer to the observed value of 0.152 b than in the previous case. The transition probabilities are not appreciably different from the previous values. This is to be expected as the wave functions are nearly the

[18] T. R. Ophel and B. T. Lawergren, Phys. Letters 6, 230 (1963).

same in both cases. The disagreement with regard to the $M1$ component of the transition $\frac{3}{2}\rightarrow\frac{5}{2}$ continues. Otherwise, the major disagreement with experiment concerns the $E2/M1$ amplitude mixing ratio for the decay of the 2.73-MeV level. This is rather disturbing as the decay properties of this level would be crucial in identifying it as a member of the core multiplet. Since, however, one cannot be too confident of the predictions of the excited-core model for $M1$ transitions, it would be desirable to have more accurate measurements on the electric transition probabilities of this level.

We can calculate the quadrupole moment of the 2^+ state of the core (Si^{28}) using the $B(E2)$ value for the transition $2^+\rightarrow 0^+$ and the values of the parameters χ_1 and χ_2 (see Ref. 5). The result, using the values given by Eqs. (6) and (3b), is $Q(2^+)=0.035$ b. The level structure of the Si^{28} nucleus does not fit with the predictions of any of the simple collective models. However, the asymmetric rotational model of Davydov and Filippov[19] might be a good approximation, even though this model would predict a 2^+ level at about 4.5 MeV which has not as yet been observed. Assuming that this model is valid for Si^{28}, we can calculate the value of the asymmetry parameter γ from the following relations:

$$B(E2:2^+\rightarrow 0^+)=(e^2Q_0^2/32\pi)[1+(3-2\sin^2 3\gamma)/(9-8\sin^2 3\gamma)^{1/2}], \quad (11)$$

$$|Q(2^+)|=6|Q_0|\cos 3\gamma/7(9-8\sin^2 3\gamma)^{1/2}, \quad (12)$$

where Q_0 is the intrinsic quadrupole moment which is related to the deformation parameter β, the atomic number Z, and the radius of the nuclear charge distribution R_0 by[20]

$$Q_0\approx[3/(5\pi)^{1/2}]ZR_0^2\beta\approx 0.0109ZA^{2/3}\beta\text{b} \quad (13)$$

if we take $R_0=1.2A^{1/3}$ F. We obtain $\gamma\approx 28.5°$, $|Q_0|=0.533$b, and $|\beta|\approx 0.38$. The value of γ obtained from the ratio of the excitation energies of the levels in Si^{28} is $\gamma\gtrsim 24°$.

The previous attempts at explaining the properties of Al^{27} have been mostly based on the Nilsson model. For an account of these, the reader is referred to the works of Lombard[21] and Bhatt.[22]

III. DISCUSSION

It then appears that the excited-core model would give a fairly good description of the low-lying levels of Al^{27}, but the core-particle interaction should contain multipoles of order higher than two. A possible explanation of this interaction is that it is the multipole expansion of the usual two-body interaction used in nuclear-structure calculations. The interaction between the ith nucleon in the core and the extra-core nucleon can be written as (Ref. 12, Chap. 21)

$$\begin{aligned}V_{ip}&=V_0(\mathbf{r}_{ip})+V_1(\mathbf{r}_{ip})(\boldsymbol{\sigma}_i\cdot\boldsymbol{\sigma}_p)\\&=\textstyle\sum_k[v_{k0}(\mathbf{r}_i,\mathbf{r}_p)+v_{k1}(\mathbf{r}_i,\mathbf{r}_p)(\boldsymbol{\sigma}_i\cdot\boldsymbol{\sigma}_p)]\\&\qquad\times(\mathbf{C}_i^{(k)}\cdot\mathbf{C}_p^{(k)}),\end{aligned} \quad (14a)$$

where $\mathbf{C}^{(k)}=[4\pi/(2k+1)]^{1/2}\mathbf{Y}_k$ and $\boldsymbol{\sigma}$ is the intrinsic spin operator. If we assume that the radial dependence of the interaction can be represented by that of the multipole force used in collective model calculations,[23,24] we have $v_{ks}(\mathbf{r}_i,\mathbf{r}_p)\propto r_i^k r_p^k$, and

$$\begin{aligned}V_{ip}=\textstyle\sum_k[&f_{k0}(\mathbf{Q}_i^{(k)}\cdot\mathbf{Q}_p^{(k)})\\&+f_{k1}(-1)^{k+1}\textstyle\sum_r(-1)^r(\mathbf{U}_i^{(r)k}\cdot\mathbf{U}_p^{(r)k})],\end{aligned} \quad (14b)$$

where

$$\mathbf{Q}_\kappa^{(k)}=r^k\mathbf{Y}_{k\kappa}$$

and

$$\mathbf{U}_\rho^{(r)k}=[\boldsymbol{\sigma}^{(1)}\times\mathbf{Q}^{(k)}]_\rho^{(r)}=\sum_{\mu,\kappa}(1\mu k\kappa|r\rho)\boldsymbol{\sigma}_\mu\mathbf{Q}_\kappa^{(k)}.$$

The Greek subscripts denote the components of the tensors and $(j_1m_1j_2m_2|jm)$ represents a Clebsch-Gordan coefficient. The interaction between the core and the particle is given by

$$H_{\text{int}}=\textstyle\sum_i V_{ip}. \quad (15)$$

Equation (15) would be quite consistent with Eq. (1), but not, in general, with our assumption that the dipole and the quadrupole parts of the interaction be represented by Eq. (2). Since this assumption is important for the calculation of the gamma-ray transition probabilities, it might be more appropriate from the point of view of the excited-core model to introduce the following phenomenological core-particle interaction, in analogy with Eqs. (2) and (14b):

$$H_{\text{int}}=-\textstyle\sum_k f_k(\mathbf{Q}_c^{(k)}\cdot\mathbf{Q}_p^{(k)})-\textstyle\sum_q f_q(\mathbf{M}_c^{(q)}\cdot\mathbf{M}_p^{(q)}). \quad (16)$$

The operator $\mathbf{M}^{(q)}$ for a single particle is defined as

$$\begin{aligned}\mathbf{M}_\nu^{(q)}&=[\mathbf{j}^{(1)}\times\mathbf{Q}^{(q-1)}]_\nu^{(q)}\\&=[q(2q+1)]^{-1/2}[\mathbf{j}^{(1)}\cdot\text{grad}(r^qY_{q\nu})].\end{aligned} \quad (17)$$

The parity of the interaction is thus $(-1)^k=(-1)^{q+1}$. The values of the parameters f_k may depend on the particle state as well as on the core state.

The reduced matrix elements of $\mathbf{Q}_p^{(k)}$ and $\mathbf{M}_p^{(q)}$ can be calculated using the relations (see, for example, Ref. 12, Chap. 15)

$$\begin{aligned}\langle nlj\|\mathbf{Q}^{(k)}\|n'l'j'\rangle\\=(-1)^{3/2+j+k}\langle nl|r^k|n'l'\rangle\{(2j+1)(2j'+1)/4\pi\}^{1/2}\\\times(j\tfrac{1}{2}j'-\tfrac{1}{2}|k0)\times\tfrac{1}{2}[1+(-1)^{l+l'+k}]\end{aligned} \quad (18a)$$

[19] A. S. Davydov and G. F. Filippov, Nucl. Phys. **8**, 237 (1958).
[20] A. Bohr and B. Mottelson, Kgl. Danske Videnskab. Selskab, Mat. Fys. Medd. **27**, No. 16 (1953).
[21] R. Lombard, Ph.D. thesis, University of Paris, 1964 (unpublished).
[22] K. H. Bhatt, Nucl. Phys. **39**, 375 (1962).
[23] R. M. Dreizler, Phys. Rev. **132**, 1166 (1963).
[24] A. M. Lane, *Nuclear Theory* (W. A. Benjamin, Inc., New York, 1964), Part II.

and

$$\langle nlj\|\mathbf{M}^{(q)}\|n'l'j'\rangle$$
$$=(2q+1)^{1/2}W(1j(q-1)j':jq)\{j(j+1)(2j+1)\}^{1/2}$$
$$\times\langle nlj\|\mathbf{Q}^{(q-1)}\|n'l'j'\rangle. \quad (18b)$$

Here, $W(abcd:ef)$ is a Racah coefficient. Equations (9) and (10) then yield (in proper units)

$$f_1\langle 2\|\mathbf{M}_c^{(1)}\|2\rangle=-3.101; \quad f_2\langle 2\|\mathbf{Q}_c^{(2)}\|2\rangle=0.182;$$
$$f_3\langle 2\|\mathbf{M}_c^{(3)}\|2\rangle=-0.160; \quad f_4\langle 2\|\mathbf{Q}_c^{(4)}\|2\rangle=0.033.$$

This would imply nonvanishing values of magnetic octupole and electric hexadecapole moments for the 2^+ state in Si^{28}. Since experimental data on the static moments of excited states in nuclei are scarce, this fact may not be useful in testing the validity of Eq. (16). A systematic study of a number of nuclei could be helpful in this direction by providing information on the consistency of the values of the parameters.

We would like to say, in conclusion, that the remarks made in this section concerning the nature of the core-particle interaction are of a speculative nature and would require further investigation in order to establish the validity or otherwise of Eq. (16). We have tried to show that Eq. (16) would not be unreasonable from the point of view of the core-particle interactions that have been used previously as well as from the point of view of the results for Al^{27} discussed in this paper.

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Lithium-Lithium Scattering*

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Elastic-scattering experiments have been performed with Li^6 on Li^6 from 3.2 to 7.0 MeV and with Li^7 on Li^7 from 4.0 to 6.5 MeV. In the lower portion of this energy range the observations follow Mott's scattering formula but at higher energies fall below these predictions. An analysis of the data in terms of the rounded-cutoff Blair model resulted in a set of parameters which was not sharply defined and probably not unique but which gave curves which reproduced satisfactorily the fluctuations of cross section with angle. An analysis in terms of one parameter, the interaction distance, gave values of $r_0=(1.38\pm0.03)\times10^{-3}$ cm for Li^6 and $r_0=(1.52\pm0.03)\times10^{-13}$ cm for Li^7.

INTRODUCTION

ONE of the most productive sources of information concerning the characteristics of nuclei has been elastic-scattering experiments. In this category the scattering of moderately heavy identical nuclei, for example: C^{12} by C^{12}, and O^{16} by O^{16} by Bromley, Kuehner, and Almqvist[1] and N^{14} by N^{14} by Reynolds and Zucker,[2] have brought out a number of interesting features, due to the complex structure of the particles, which do not appear in the scattering of simpler nuclei. As a part of our program for the investigation of the reactions produced by lithium ions we have studied the scattering of Li^6 by Li^6 over the energy range from 3.2 to 7.0 MeV and the scattering of Li^7 by Li^7 from 4.0 to 6.5 MeV.

Because of the identity of the target and incident nuclei one would expect the scattering cross sections to follow the well-known Mott equation[3] in the absence of nuclear effects. In the carbon, oxygen, and nitrogen scattering experiments referred to above the departures from the predictions of the Mott equation have been interpreted in terms of the sharp-cutoff model discussed by J. S. Blair and others.[4,5] The results of these analyses have been the determination of radii within which nuclear effects become important. We anticipated that measurements of this type performed with lithium would provide some insight into the structure and interactions of these light and relatively simple nuclei.

EQUIPMENT AND PROCEDURE

Lithium ions having energies up to 7 MeV were obtained from the Minnesota Van de Graaff machine,

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[1] D. A. Bromley, J. A. Kuehner, and E. Almqvist, in *Reactions Between Complex Nuclei*, edited by A. Zucker, F. Howard, and E. Halbert (John Wiley & Sons, Inc., New York, 1960), p. 151.
[2] H. L. Reynolds and A. Zucker, Phys. Rev. **102**, 1378 (1956).
[3] N. F. Mott and H. S. W. Massey, *Theory of Atomic Collisions* (Oxford University Press, New York, 1949), 2nd ed., p. 19.
[4] J. S. Blair, Phys. Rev. **95**, 1218 (1954).
[5] J. A. McIntyre, K. H. Wang, and L. C. Becker, Phys. Rev. **117**, 1337 (1960).