Competition between Neutron and Gamma Emission from Nuclear States with High Spin*

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The effect of the high spin of the compound nuclei on the ratio of the gamma width to the neutron width is studied. It is shown that this ratio increases with the increase of the spin of the compound nucleus. This is attributed to both model-independent and model-dependent factors.

I. INTRODUCTION

N heavy-ion bombardment, states with high spin are populated. The various decay modes of these states can be made to reveal some of the state properties when experimental and theoretical data are compared.

The states excited in heavy-ion bombardment decay predominantly by neutron emission followed by gamma emission. In this paper it is shown that the ratio of the gamma width of Γ_{γ} to neutron width Γ_n increases with the increase of the spin of the compound nucleus. The importance of this ratio is twofold. First, from this ratio information about the model describing these states can be extracted. Second, the knowledge of this ratio is essential for the study of more complicated mixed decays.

It was recognized that the ratio of the gamma width to the neutron width depends on the spin of the compound nucleus. Furthermore, previous information indicates¹⁻⁶ that this ratio increases with an increase of the spin of the compound system. Grover,⁴ in a pioneering work, estimated the competition between neutron and gamma emission from nuclear states with high spin. Grover's calculation assumes that gamma emission is allowed in the last step of a long cascade.

It is shown that the relative probability of gamma emission increases with the increase of the spin of the compound nucleus.

Both deformation-dependent and deformation-independent factors contribute to the gamma-ray emission enhancement. The deformation independent factors are mainly due to statistical considerations. Both neutron and gamma emission depend on the density of final states and on the energy range of these states. Both neutron and gamma emission from states with high spin are reduced. However, neutron emission is more inhibited as the following argument shows. Both neutrons and gamma rays emerge with low spin so that if the initial state has a high spin the final state will also be left with a high spin. However, in neutron emission the final state is at an excitation at which the density of

final states with high spin is very low. This low density of final states inhibits neutron emission considerably. As a matter of fact the final state may be in a region where there are no states with high spin, eliminating the possibility of neutron emission. On the other hand, in gamma emission, for which there is no binding energy restriction, the final state may be one of considerable excitation. In particular, this state may be in an energy region where the density of states with high spin is appreciable. Furthermore, the relative decrease of the range of available final states for gamma emission is considerably smaller than for neutron emission. Therefore, gamma emission from states with high spin is less inhibited than neutron emission from the same states.

The deformation-dependent enhancement is due to the corresponding increase in the nuclear matrix element with deformation. This factor is model-dependent since the deformation is model-dependent. For high spin values the collective rotation can be very well approximated by the rotation of the nucleus as a whole,^{7,8} and the liquid drop model appears to be very appropriate for the description of such rotating states.

In the present calculation no parameters are adjusted to fit experimental data. The calculation proceeds from first principles. Nuclear matrix elements for electromagnetic decay from nuclear states with high spin according to the liquid drop model⁹ are used. Densities of levels as suggested by Lang and LeCouteur¹⁰ are used.

II. THEORY

The relative increase of gamma emission as a function of the spin of the compound nucleus is most easily recognized by means of the function D(E,J) to be defined below. Let S(E,J) be the ratio so that

$$S(E,J) = \Gamma_{\gamma}(E,J) / \Gamma_n(E,J).$$
⁽¹⁾

This ratio is then compared with a similar ratio for zero spin S(E,J=0) by means of a function D(E,J) such that

$$D(E,J) = S(E,J=0)/S(E,J).$$
 (2)

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¹⁰ J. M. B. Lang and K. J. LeCouteur, Proc. Phys. Soc. (London) A67, 585 (1954).

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The function D(E,J), for high spin values, exhibits more explicitly the effects of this high spin.

Now an equation for the function D(E,J) is derived. First the ratio S(E,J) of Γ_{γ} to Γ_n is calculated. Let $\Gamma_{\gamma}(E_i,J_i)$ and $\Gamma_n(E_i,J_i)$ be the gamma and neutron width, respectively, of an initial state at an excitation E_i and with a high spin J_i . Then

$$\Gamma_n(E_i,J_i) = K(E_i,J_i) \sum_{J_f} \int_0^{E_i - B} T_n(E_i,J_i;E_f,J_f) dE_f, \quad (3a)$$

$$\Gamma_{\gamma}(E_i, J_i) = K(E_i, J_i) \sum_{J_f} \int_0^{E_i} T_{\gamma}(E_i, J_i; E_f, J_f) dE_f. \quad (3b)$$

In Eq. (3), $K(E_i,J_i)$ is a proportionality constant depending only on the population of states with E_i and spin J_i . The transition probabilities T_n and T_{γ} are discussed later. Finally, B is the neutron binding energy, and E_i is the maximum energy available to the gamma rays where $E_i - B$ is the maximum energy available to the neutrons. The ratio $S(E_i, J_i)$ becomes

$$S(E_{i},J_{i}) = \frac{\Gamma_{\gamma}(E_{i},J_{i})}{\Gamma_{n}(E_{i},J_{i})} = \frac{\sum_{J_{f}} \int_{0}^{E_{i}} T_{\gamma}(E_{i},J_{i};E_{f},J_{f})dE_{f}}{\sum_{J_{f}} \int_{0}^{E_{i}-B} T_{n}(E_{i}J_{i};E_{f}J_{f})dE_{f}}.$$
 (4)

The effect of the high spin will be recognized by studying the ratio $D(E_i,J_i)$ of $S(E_i,J_i=0)$ to $S(E_i,J_i)$. Now the function $S(E_i,J_i)$ for high spin values is evaluated. First the neutron width for states with high spin is calculated. The probability for neutron emission has been discussed extensively. Here an approach to determine the probability similar to the one used by Thomas,^{11,12} of neutron emission from states with high spin is adopted. Accordingly, the rate for neutron emission $R_n(E_i,J_i; E_f)$ may be written as

$$R_{n}(E_{i},J_{i}; E_{f}) = \sum_{J_{f}} T_{n}(E_{i}J_{i}; E_{f}J_{f})$$

$$= \frac{v}{R} \frac{\hbar}{(\tau^{J_{f}})^{3/2}} \rho(E_{f}) \sum_{J_{f}} (2J_{f}+1) \left\{ \exp\left[\frac{-(J_{f}+\frac{1}{2})^{2}}{2c\tau^{J_{f}}}\right] \right\} \epsilon(J_{f})$$

$$\times \sum_{S=|J_{f}-\frac{1}{2}|}^{J_{f}+\frac{1}{2}} \sum_{l=|J_{f}-S|}^{J_{i}+S} T_{l}(E_{i}-B-E_{f}). \quad (5)$$

Here v is the average neutron velocity in the nucleus, and R is the nuclear radius. In Eq. (5), $\rho(E_f)$ is the spin-independent part of the density of levels so that

$$\rho(E) = C(E+t)^{-5/4} \exp(2aE)^{1/2}, \qquad (6)$$

 τ^{J} is the nuclear spin-dependent temperature defined by

$$\frac{1}{\tau} = d \ln \rho(E,J) / dE = (a/E)^{1/2} - \frac{5}{2} \left[\frac{1}{(2E+t)} \right],$$

$$\frac{1}{\tau^{J}} = \frac{1}{\tau} + \left(\frac{(J+\frac{1}{2})^{2}}{2c\tau} - \frac{3}{2} \right) \frac{d \ln \tau}{dE},$$
 (7)

and t is the thermodynamic nuclear temperature. In Eq. (5), c is related to the moment of inertia \mathcal{S} by

$$c = \mathcal{G}/\hbar^2, \tag{8}$$

and the function $\epsilon(J_f)$ is defined by

$$\epsilon(J_f) = 1, \quad J_f \leq J_M \tag{9a}$$

$$\epsilon(J_f) = 0, \quad J_f > J_M. \tag{9b}$$

Here J_M is defined by

$$\hbar^2 J_M^2 / 2\mathfrak{G} = E. \tag{10}$$

The necessity of the inclusion of the term $\epsilon(J_f)$ follows from the fact that for every value of the spin there is an energy value below which there are no levels with that spin or higher spin. Grover⁴ and Sperber¹³⁻¹⁶ have shown that the lowest energy E of a nucleus with a spin J_M is given by Eq. (10).

The integration over E_f in Eq. (3a) and Eq. (3b) has been carried out numerically. It is convenient to break the integrand $R_n(E_i, J_i; E_f)$ into two parts. The first part consists of

$$v/R(\hbar/(\tau^{J_f})^{3/2})\rho(E_f);$$

the second part consists of

$$\sum_{J_f} (2J_f + 1) \epsilon(J_f) \left\{ \exp\left[-\frac{(J_f + \frac{1}{2})^2}{2c\tau^{J_f}} \right] \right\} \\ \times \sum_{S = |J_f - \frac{1}{2}|}^{J_f + \frac{1}{2}} \sum_{l = |J_i - S|}^{J_i + S} T_l(E_i - B - E_f) \right\}$$

The first part of the integrand was calculated in a straightforward way. The second part was calculated in two different ways. First an approximate formula for this second factor was obtained. For simplicity this approximation is now derived neglecting its intrinsic spin dependence. The inclusion of spin causes only small alterations. In the numerical calculation, spin-dependent factors were included. Let

$$N = \sum_{J_f} (2J_f + 1) \left\{ \exp\left[-\frac{(J_f + \frac{1}{2})^2}{2c\tau^{J_f}} \right] \right\} \epsilon(J_f) \sum_{l=|J_i - J_f|}^{J_i + J_f} T_l.$$
(11)

¹⁶ D. Sperber, Phys. Rev. 138, B1028 (1965).

¹¹ T. D. Thomas, Nucl. Phys. 53, 558 (1964).

¹² T. D. Thomas, Nucl. Phys. 53, 577 (1964).

 ¹³ D. Sperber, thesis, Princeton University, 1960 (unpublished).
 ¹⁴ D. Sperber, Phys. Rev. 130, 468 (1963).

¹⁵ D. Sperber, in Proceedings of the Third Conference on Reactions Between Complex Nuclei, edited by A. Ghiorso, R. M. Diamond, and H. E. Conzett, (University of California Press, Berkeley, 1963), p. 379.

In the approximate formula for N, the values of the by a similar approximation F becomes base on the sharp cut-off approximation were used so that

$$T_l = 1, \quad l \leq L$$
 (12a)

$$T_l = 0, \ l > L.$$
 (12b)

$$L = r_0 A^{1/2} (2mE/\hbar^2)^{1/2}.$$
(13)

First the summation over J_f in Eq. (11) was performed Now the summation over l has to be performed: for a particular value of *l*. Let

$$F(J_{i}, J_{M}, l) = \sum_{J_{f}} (2J_{f} + 1) \epsilon(J_{f}) \left\{ \exp \left[-\frac{(J_{f} + \frac{1}{2})^{2}}{2c\tau^{J_{f}}} \right] \right\} T_{l}.$$
(14)

Three cases have to be distinguished. First

$$J_M < J_i - l. \tag{15}$$

In this case whenever ϵ differs from zero T vanishes and whenever T differs from zero ϵ vanishes so that

$$F_1(J_i, J_M, l) = 0.$$
 (16)

$$J_i - l < J_M < J_i + l. \tag{17}$$

In this case ϵ for values of J_f larger than J_M vanishes, so that . . .

$$F_{2}(J_{i}, J_{M}, l) = \sum_{J_{i}=l}^{J_{M}} (2J_{f}+1) \left\{ \exp\left[-\frac{(J_{f}+\frac{1}{2})^{2}}{2c\tau^{J_{f}}}\right] \right\} T_{l}.$$
 (18)

The summation in Eq. (18) can be approximated by integration so that

$$F_{2}(J_{i}, J_{M}, l) = T_{l}(E)(2c\tau^{J_{f}}) \\ \times \left\{ \exp\left[-\frac{(J_{i}-l+\frac{1}{2})^{2}}{2c\tau^{J_{f}}}\right] - \exp\left[-\frac{(J_{M}+\frac{1}{2})^{2}}{2c\tau^{J_{f}}}\right] \right\}.$$
(19)

Finally, for

$$J_i + l < J_M, \tag{20}$$

$$F_{3} = T_{l}(E)(2c\tau^{J_{f}}) \left\{ \exp\left[-\frac{(J_{i} + \frac{1}{2} - l)^{2}}{2c\tau^{J_{f}}}\right] - \exp\left[-\frac{(J_{i} + \frac{1}{2} + l)^{2}}{2c\tau^{J_{f}}}\right] \right\}.$$
 (21)

$$N = \sum_{l=0}^{L} F(J_{i}, J_{M}, l).$$
 (22)

Here four different cases have to be considered. First

$$J_M < J_i - L. \tag{23}$$

In this case Eq. (22) reduces to

$$N_1 = \sum_{l=0}^{L} F_1(J_i, J_M, l) = 0.$$
 (24)

Second,

$$J_i - L < J_M < J_i. \tag{25}$$

In this case Eq. (22) reduces to

$$N_{2} = \sum_{l=0}^{J_{i}-J_{M}} F_{1}(J_{i},J_{M},l) + \sum_{J_{i}-J_{M}}^{L} F_{2}(J_{i},J_{M},l)$$
$$= \sum_{l'=0}^{L-J_{i}+J_{M}} \left\{ \exp\left[-\frac{(J_{M}-l'+\frac{1}{2})^{2}}{2c\tau^{J_{f}}}\right] \right\}$$
$$-\exp\left[-\frac{(J_{M}+\frac{1}{2})^{2}}{2c\tau^{J_{f}}}\right]. \quad (26)$$

Since for the case of interest

$$l^2/2c\tau^{J_f}\ll 1$$
, (27)

the sum in Eq. (26) may very well be approximated by

$$N_{2} = (2c\tau^{J_{f}}) \exp\left[-\frac{(J_{M} + \frac{1}{2})^{2}}{2c\tau^{J_{f}}}\right] \left[\frac{1 - \exp\left[2(J_{M} + \frac{1}{2})(L - J_{i} + J_{M} + 1)/2c\tau^{J_{f}}\right]}{1 - \exp\left[2(J_{M} + \frac{1}{2})/2c\tau^{J_{f}}\right]} - (L - J_{i} + J_{M} + 1)\right].$$
(28)

Third,

$$J_i < J_M < J_i + L. \tag{29}$$

In this case, using a similar approximation to the previous one, we get

$$N_{3} = \sum_{l=0}^{J_{M}-J_{i}} F_{3}(J_{i},J_{M},l) + \sum_{J_{M}-J_{i}}^{L} F_{2}(J_{i},J_{M},l) = (2c\tau^{J_{f}}) \left\{ \exp\left[-\frac{(J_{i}+\frac{1}{2})^{2}}{2c\tau^{J_{f}}}\right] \right\} \\ \times \left\{ \frac{\left\{ \exp\left[(2J_{i}+1)(J_{M}-J_{i}+1)/2c\tau^{J_{f}}\right]\right\} - 1}{\left\{ \exp\left[(2J_{i}+1)/2c\tau^{J_{f}}\right]\right\} - 1} - \frac{\left\{ \exp\left[-(2J_{i}+1)(J_{M}-J_{i}+1)/2c\tau^{J_{f}}\right]\right\} - 1}{\left\{ \exp\left[-(2J_{i}+1)/2c\tau^{J_{f}}\right]\right\} - 1} \right\} \\ + (2c\tau^{J_{f}}) \left\{ \exp\left[-\frac{(2J_{i}-J_{M}+\frac{1}{2})^{2}}{2c\tau^{J_{f}}}\right] \right\} \left\{ \frac{\left\{ \exp\left[2(2J_{i}-J_{M}+\frac{1}{2})(L-J_{M}+J_{i}+1)/2c\tau^{J_{f}}\right]\right\} - 1}{\left\{ \exp\left[2(2J_{i}-J_{M}+\frac{1}{2})/2c\tau^{J_{f}}\right]\right\} - 1} - (L-J_{M}+J_{i}+1) \exp\left[-\frac{(J_{M}+\frac{1}{2})^{2}}{2c\tau^{J_{f}}}\right] \right\}.$$
(30)

Here

Second

929

J

Finally,

930

$$_{i}+L
(31)$$

In this case using an approximation similar to those for the two previous cases

$$N_{4} = \sum_{l=0}^{L} F_{3}(J_{i}, J_{M}, l) = (2c\tau^{J_{f}}) \exp\left[-\frac{(J_{i} + \frac{1}{2})^{2}}{2c\tau^{J_{f}}}\right] \\ \times \left[\frac{\left\{\exp\left[(2J_{i} + 1)(L + 1)/2c\tau^{J_{f}}\right]\right\} - 1}{\left\{\exp\left[(2J_{i} + 1)/2c\tau^{J_{f}}\right]\right\} - 1} - \frac{1 - \left\{\exp\left[-(2J_{i} + 1)(L + 1)/2c\tau^{J_{f}}\right]\right\}}{1 - \left\{\exp\left[-(2J_{i} + 1)/2c\tau^{J_{f}}\right]\right\}}\right].$$
(32)

In the previous equations L is determined by Eq. (11), where E is the energy of the outgoing neutron and J_M is determined by

$$J_{M} = (1/\hbar) [2(E_{n} - E)g]^{1/2} = (1/\hbar) [2(E_{i} - B - E)g]^{1/2}.$$
(33)

Here E_n is the maximum energy available to a neutron. Depending on the relation between J_i , J_M , and L the appropriate form for N was chosen. Using the function N integrals of the type

$$I = \int_{0}^{E_{i}-B} \frac{v}{R} \frac{\hbar}{(\tau^{J_{f}})^{3/2}} \rho(E_{f}) N(J_{i}, J_{M}, E_{f}, L(E_{i}-B-E_{f})) dE_{f}$$
(34)

were evaluated numerically.

To justify the use of the approximate form for N in Eqs. (22), (28), (30), and (32) an exact numerical calculation was performed for a few cases. The transmission coefficients for this calculation were taken from the work of Auerbach and Perey.¹⁷ Auerbach and Perey calculated transmission coefficients using an optical model with the inclusion of spin orbit interactions. Their transmission coefficients are limited to neutron energies of up to 5 MeV. A comparison of results using the approximate formula and optical model calculation indicates that for higher neutron energies the present approximation is sufficient. Therefore, for higher neutron energies only the approximation formula was used.

Second, the gamma width for emission from nuclear states with high spin is evaluated. First the transition probability is broken into a sum of transition probabilities of definite multipolarity so that the sum over J_f in Eq. (3b) may be rewritten as

$$\sum_{J_f} T_{\gamma}(E_i, J_i; E_f, J_f) = \sum_{J_f} \sum_{\substack{l=|J_i-J_f| \\ l=|J_i-J_f|}}^{J_i+J_f} T_{\gamma}^{l}(E_i, J_i; E_f, J_f).$$
(35)

In Eq. (35) the $T_{\gamma}^{l}(E_{i},J_{i};E_{f},J_{f})$ are the transition probabilities of electric radiation of order l, which can be written as

$$T_{\gamma}{}^{l}(E_{i},J_{i};E,J_{f}) = \frac{1}{(\tau^{J_{f}})^{3/2}} \frac{1}{l} \frac{8\pi(l+1)}{[(2l+1)!!]^{2}} \rho(E_{f}) \left(\frac{E_{i}-E_{f}}{\hbar v_{l}}\right)^{2l+1} \frac{1}{(2J_{i}+1)} (2J_{f}+1) \times \left\{ \exp\left[\frac{-(J_{f}+\frac{1}{2})^{2}}{2c\tau^{J_{f}}}\right] \right\} \sum_{M_{i}M_{f}m} |\langle J_{i}M_{i}|Q_{m}{}^{l}|J_{f}M_{f}\rangle|^{2}.$$
(36)

Here v_l is the velocity of light.

Now the deformation dependent factor is calculated. This factor is attributed to the increase of the nuclear matrix element with deformation The nuclear matrix element is calculated according to the liquid drop model. In this case the nuclear matrix element in Eq. (36) has to be replaced by Q_m^2 , where

$$\langle Q_m^2(J)\rangle = \int \rho_Z Y_m^2(\mathbf{r}) d^3 \mathbf{r} \,.$$
 (37)

In Eq. (37), $\rho_Z(\mathbf{r})$ is the charge density and $Y_m^2(\mathbf{r})$ are the solid spherical harmonics of order 2.

According to the liquid drop model, nuclei with high spin have been very well approximated by spheroidal shapes.¹⁸⁻²⁰ Obviously these spheroidal shapes have vanishing dipole moment, but a nonvanishing quad-

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¹⁷ E. H. Auerbach and F. G. J. Perey, Brookhaven National Laboratory Report, 765 (T-286), 1962 (unpublished).

¹⁸ R. Beringer and W. N. Knox, Phys. Rev. 121, 1195 (1961). ¹⁹ B. C. Carlson and Pao Lu, in Proceedings of the Rutherford Jubilee International Conference 292, edited by J. B. Birks (Aca-demic Press Inc., New York, 1961).
 ²⁰ S. Cohen, F. Plasil and W. J. Swiatecki, in Proceedings of the Third Conference on Reactions Between Complex Nuclei, edited

by A. Ghiorso, R. M. Diamond, and H. E. Conzett (University of California Press, Berkeley, 1963), p. 325.

	neutron binding energy equalling 7 MeV.											
	E=8	E = 10	E = 12	E = 14	E=16	E = 18	E = 20	E = 22	E = 24	E = 26	E = 28	E = 30
$\overline{J = 15}$	0.0000	0.6651	0.8323	0.9231	0.9481	0.9627	0.9717	0.9778	0.9821	0.9853	0.9878	0.9895
J = 30	0.0000	0.0000	0.2988	0.6014	0.7407	0.8182	0.8654	0.8965	0.9176	0.9335	0.9445	0.9532
J = 45	0.0000	0.0000	0.0000	0.0000	0.0000	0.3630	0.5679	0.6771	0.7529	0.7903	0.8421	0.8698

TABLE I. The function D(E,J) for a nucleus of A = 200 with a = 20 MeV⁻¹ and the neutron binding energy equalling 7 MeV.

rupole moment. Such rotating liquid drops mainly emit E2 radiation. Beringer and Knox¹⁸ calculated the shapes of spheroidal nuclei with high spin. For spins lower than a critical value of the spin, they find that the equilibrium shapes are oblate spheroids with this axis of rotation coinciding with the axis of cylindrical symmetry. However, for nuclei with spins exceeding this critical value the shapes of equilibrium are prolate spheroids rotating around one of their minor axes. A simple calculation shows⁹ that for the first type of shapes the components of the quadrupole tensor are

$$\langle Q_0^2 \rangle = \frac{1}{4} (5/\pi)^{1/2} \frac{1}{5} ZeR_0^2 (2/\eta^{2/3})(\eta^2 - 1),$$
 (38a)

$$\langle Q_{\pm 1}^2 \rangle = \langle Q_{\pm 2}^2 \rangle = 0, \qquad (38b)$$

and for the second type of configuration

$$\langle Q_0^2 \rangle = \frac{1}{4} (5/\pi)^{1/2} \frac{1}{5} ZeR_0^2 (1/\eta^{4/3})(\eta^2 - 1),$$
 (39a)

$$\langle Q_{\pm 1}^2 \rangle = 0, \qquad (39b)$$

$$\langle Q_{\pm 2}^2 \rangle = \frac{1}{4} (15/\pi 2^{1/2} \frac{1}{5} ZeR_0^2 (1/\eta^{4/3})(\eta^2 - 1).$$
 (39c)

In Eqs. (38) and (39) the z axis has been chosen as the axis of rotation and η is the ratio of the minor to major axis. The previous equations for the gamma Γ_{γ} and for the neutron width Γ_n allow one to calculate the ratio S(E,J) defined in Eq. (1). To obtain the function D(E,J) the knowledge of S(E, J=0) is required. However, it should be borne in mind that the previous approximations for the neutron width are valid only for high spin values. Therefore, other approximations are required for the evaluation of S(E,J) for low spin values. For this purpose the existing standard²¹ forms for the neutron width were used.

Now the effect of the deformation on the ratio $S(E,J_1)$ to $S(E,J_2)$ for two high spin values is calculated. This result is limited to high spin values since rotational states with low spin values cannot be described by the liquid drop model. Let $S_u(E,J)$ and $S_d(E,J)$ be the values of the function S(E,J) excluding and including the effect of spin-dependent deformation, respectively. Then

$$S_d(E,J_1)/S_d(E,J_2) = \alpha(J_1,J_2)S_u(E,J_1)/S_u(E,J_2). \quad (40)$$

Using Eqs. (36) and (37), the correction function $\alpha(J_1, J_2)$ becomes

$$\alpha(J_1,J_2) = \sum_m \left| \left\langle Q_m^2(J_1) \right\rangle \right|^2 / \sum_m \left| \left\langle Q_m^2(J_2) \right\rangle \right|^2.$$
(41)

III. DISCUSSION

First the function D(E,J), defined in Eq. (1) of Sec. II, was calculated disregarding model dependent factors. In other words, the change of the nuclear matrix element with deformation is neglected. A sample calculation was performed for a nucleus with A = 200, a neutron binding energy of 7 MeV, a value of the parameter a=20 MeV⁻¹, a rigid body moment of inertia, for J=15, 30, and 45, and for energies of the compound nucleus ranging from 0 to 30 MeV. The results of this calculation are summarized in Table I. The following conclusions may be drawn from inspection of Table I and Fig. 1: (a) the relative probability for gamma emission increases with the increase of the spin of the compound nucleus [note that D(E,J) is proportional to the reciprocal of this probability]; (b) the relative increase of gamma emission is less marked and disappears with very high energy; (c) the energy at which the effect disappears increases with the spin. [For a spin of 15, $D(13,15) \ge 0.9$ at 13 MeV; but for a spin of 30 this value of 0.9 is reached at 23 MeV; and for a spin of 45, even at 30 MeV, D(E,J) is still less than 0.9.]

The vanishing values of D(E,J) for low energies (see Table I) can be attributed to the fact that from states with high spin and low energy, neutron emission is not possible. This is due to the lack of availability of appropriate final states.

Second, the effect of deformation on the probability of gamma emission from nuclear states with high spin

D (E,J)

FIG. 1. The function D(E,J) for a nucleus of A = 200 with a = 20 MeV⁻¹ and the neutron binding energy 7 MeV. The energy E is in MeV.

²¹ For example, see J. M. Blatt, and V. F. Weisskopf, *Theoretical Nuclear Physics* (John Wiley & Sons, Inc., New York, 1952), p. 367.



FIG. 2. The function $\alpha(J_1, J_2)$ for Cu⁶³.

was calculated. The calculation is based on the liquid drop model.

The correction function $\alpha(J_1, J_2)$, which is defined in Eq. (40), was calculated for Cu⁶³ with $J_2=30$ and

TABLE II. The correction function $\alpha(J_1, J_2 = 30)$ for Cu⁶³.

J_1	30	40	50	60	70	80
$\alpha(J_1, J_2 = 30)$	1.00	4.93	17.17	46.81	70.85	129.77

 $30 \le J_1 \le 80$. For this region of spins in Cu⁶³, Beringer and Knox¹⁸ calculated the dependence of η on J. This dependence was used to calculate $\alpha(J_1, J_2 = 30)$ for Cu⁶³. This correction function is shown in Table II and in Fig. 2. The rapid increase of $\alpha(J_1, J_2 = 30)$ with J_2 indicates the enhanced relative probability of gamma emission from states with high spin due to spin-dependent nuclear deformations.

Therefore, a slow experimentally measured increase in the ratio $S(E,J_1)/S(E,J_2)$ indicates that the nuclear matrix element does not increase with the nuclear spin and the transition can be described by a single particle type of transition. For single particle decay the nuclear matrix element changes only slightly with the nuclear spin. Therefore, the expected increase in $S(E,J_1)/S(E,J_2)$ will be almost the same as the one predicted by $S_u(E,J_1)/S_u(E,J_2)$. On the other hand, a detected, much faster, increase in this ratio indicates the increase of the nuclear matrix element with deformation. Such an increase in the nuclear matrix element attributes the origin of radiation to the nucleus as a whole.