sum over i_3. Hence the final spin and isospin factor is

$$[(2\Sigma+1)(2\Sigma'+1)(2\tau+1)(2\tau'+1)]^{1/2}W(\tfrac{1}{2}\,\tfrac{1}{2}\,\tfrac{1}{2}\,\tfrac{1}{2};\Sigma\Sigma')$$
$$+W(\tfrac{1}{2}\,\tfrac{1}{2}\,\tfrac{1}{2}\,\tfrac{1}{2};\tau\tau')|\langle\tfrac{1}{2}-\tfrac{1}{2}|\tau\theta\tfrac{1}{2}i_3\rangle|^2. \quad \text{(B4)}$$

The other factors are obtained in corresponding ways.

For the calculation of the Gamow-Teller matrix element one requires a different set of factors. Consider, for example, the B-matrix element shown in Fig. (7c). The isospin part gives

$$\sum_{i_1,i_2\theta\theta'}\langle\tfrac{1}{2}\,\tfrac{1}{2}|\tau\theta\tfrac{1}{2}i_1\rangle\langle\tau\theta|\tfrac{1}{2}\,\tfrac{1}{2}\,\tfrac{1}{2}i_2\rangle$$
$$\times\langle\tau'\theta'|\tfrac{1}{2}-\tfrac{1}{2}\,\tfrac{1}{2}i_2\rangle\langle\tfrac{1}{2}-\tfrac{1}{2}|\tau'\theta'\tfrac{1}{2}i_1\rangle, \quad \text{(B5)}$$

which is not conveniently further reduced, but which sum we do explicitly. For the spin factors we get

$$\sum_{m_1m_2m_3\sigma\sigma'}\langle\tfrac{1}{2}\,\tfrac{1}{2}|\Sigma\sigma\tfrac{1}{2}m_1\rangle\langle\Sigma\sigma|\tfrac{1}{2}m_3\tfrac{1}{2}m_2\rangle$$
$$\times\langle\tfrac{1}{2}\,\tfrac{1}{2}|\Sigma'\sigma'\tfrac{1}{2}m_1\rangle\langle\Sigma'\sigma'|\tfrac{1}{2}m_3\tfrac{1}{2}m_2\rangle m_3, \quad \text{(B6)}$$

where the factor of m_3 comes from the matrix element of σ_z. This sum can be converted to a sum over Clebsch-Gordan coefficients by making use of the identity

$$m_3=(r_3/2)(\langle\tfrac{1}{2}m_3|10\tfrac{1}{2}m_3\rangle) \quad \text{(B7)}$$

we get for the spin sum (B6)

$$[(2\Sigma+1)(2\Sigma'+1)]^{1/2}W(1\Sigma\tfrac{1}{2}\,\tfrac{1}{2};\Sigma'\tfrac{1}{2})W(1\tfrac{1}{2}\Sigma'\tfrac{1}{2};\tfrac{1}{2}\Sigma). \quad \text{(B8)}$$

The other sums for the Gamow-Teller element are done in a similar way.

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Microscopic Theory of the Optical-Model Potential and the Hole-Particle Model in Nuclear Spectroscopy

NGUYEN VAN-GIAI AND J. SAWICKI
Laboratoire Joliot-Curie de Physique Nucléaire, Orsay (Seine et Oise), France

AND

N. VINH MAU
Laboratoire de Physique Théorique, Faculté des Sciences de Bordeaux, Bordeaux, France

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Approximate expressions for the optical-model potential $\mathcal{U}_{\rm opt}$ in terms of the nucleon-nucleon interaction are discussed. The identity of all the $A+1$ nucleons of the scattering problem is approximately taken into account in our final formulas. The imaginary part of $\mathcal{U}_{\rm opt}$ is calculated for the case of ^{12}C and the incident-nucleon energy of 20 MeV. The spherical-model random-phase-approximation (RPA) eigenvalues and eigenvectors are assumed for the complete set of intermediate nuclear states involved. The results are rather insensitive to the exchange-force mixture assumed for our zero-range nucleon-nucleon potential. The antisymmetrization of our V-matrix elements is extremely important as it reduces our $\mathrm{Im}\mathcal{U}_{\rm opt}$ by a factor of 2–3. For a reasonable set of our RPA intermediate eigenvalues and eigenvectors we obtain a semiquantitative agreement with the best phenomenological $\mathrm{Im}\mathcal{U}_{\rm opt}$ available for our case. The most important contribution to $\mathrm{Im}\mathcal{U}_{\rm opt}$ corresponds to the first excited $T=0$, 2^+ state in ^{12}C.

I. INTRODUCTION

A GREAT variety of attempts have been undertaken to calculate the optical-model potential from basic two-body forces. Several independent definitions of the optical potential have been employed which are not exactly equivalent. One common fundamental difficulty of a microscopic derivation of the potential is the exact antisymmetry in the $(A+1)$-particle system, i.e., the Pauli exclusion principle. It leads to many pitfalls. In fact, most early attempts to incorporate the Pauli exclusion principle in the derivation turned out to be failures.[1] In the following we shall not review these attempts, nor shall we discuss all the different approaches to the general problem of the microscopic theory of $\mathcal{U}_{\rm opt}$. Only a few treatments related more closely to our calculations will be mentioned.

One of these is the Watson multiple-scattering formalism. The corresponding solution constructed for $\mathcal{U}_{\rm opt}$ is a rather complicated infinite series of terms, and only partly considers the indistinguishability of the projectile ("0") from the target nucleons.[2] This approach employs the concept of the two-nucleon t matrix, which we shall refer to in our treatment. Most applications and

[1] J. S. Bell, in *Lectures on the Many-Body Problem*, edited by E. R. Caianiello (Academic Press Inc., New York, 1962), p. 91; in this reference the following papers are criticized: (a) F. Coester and H. Kümmel, Nucl. Phys. **9**, 225 (1958); (b) H. Rollnik, Z. Naturforsch. **13a**, 59 (1958); (c) L. M. Frantz and R. L. Mills, Nucl. Phys. **15**, 16 (1960); (d) L. M. Frantz, R. L. Mills, R. G. Newton, and A. M. Sessler, Phys. Rev. Letters **1**, 340 (1959).

[2] K. M. Watson, Phys. Rev. **89**, 575 (1953); (a) N. C. Francis and K. M. Watson, *ibid.* **92**, 291 (1953); (b) G. Takeda and K. M. Watson, *ibid.* **97**, 1336 (1955); K. M. Watson, *ibid.* **105**, 1388 (1957).

elaborations of the Watson theory refer to infinite nuclear matter.[3] The Watson formula is particularly useful in the high-energy region, where it reduces to the impulse approximation.[3,4]

Another approach is based on the unified theory of nuclear reactions of Feshbach.[5] This method is based on a solution of the coupled-channel Schrödinger equation. Here the formula for $\mathcal{U}_{\rm opt}$ does not incorporate the Pauli principle for the projectile relative to the target nucleons. Extensions of this treatment[6,7] so as to include complete antisymmetry in the corresponding $\mathcal{U}_{\rm opt}$ quite generally appear to be unsatisfactory.

The method of Green's functions has the obvious advantage of automatically securing the identity of all the particles and the Pauli principle, due to the use of second quantization with the fermion operators in the Heisenberg representation, i.e., with the total Hamiltonian H.[8]

In this method the optical potential is given essentially by the continuum part of the spectrum of the Dyson "mass operator" (or "self-energy operator") Σ. Such a definition of $\mathcal{U}_{\rm opt}$ has been discussed by Bell and Squires[9] who, however, work with the interaction representation of fermion operators. A practical calculation of the self-energy operator follows rather the methods of Martin and Schwinger[10] and Puff[11] (cf. also, e.g., Reiner[12] and Koltun and Wilets[13]). The convergence of this method is closely related to the goodness of the independent-pair approximation (IPA) (cf., e.g., Brenig[14]).

In our work we are interested in finite light and medium-heavy nuclei, and especially in seeing how various single-particle and collective levels contribute, particularly to the imaginary part of $\mathcal{U}_{\rm opt}$. For this reason, we choose the complete set of intermediate states in our formula to be model states given by products of plane waves and the target excited states given by the Tamm-Dancoff (TD) or the random-phase approximation (RPA). The eigenvectors of such models do indeed form complete and orthogonal sets. The TD or RPA eigenstates are well known to be rather successful in explaining the properties of low-lying states of even-even light nuclei which contribute the most to $\mathcal{U}_{\rm opt}$.

Higher order terms contain higher order correlation corrections and some "target exchange terms" (cf., Appendix I).

II. DISCUSSION OF GENERAL FORMULAS

If one applies the Martin-Schwinger Green's-function formalism,[10–13] one can develop useful approximate formulas for the optical-model potential. The exact expression for the self-energy operator Σ is (cf., e.g., Baym and Kadanoff[15])

$$\Sigma(0,0')=-iV(0,1)G_2(01^-,\bar{0}1^+)[G_1(\bar{0},0')]^{-1}, \quad (1)$$

where

$$G_2(01^-,\bar{0}1^+)\equiv\lim_{\Delta\to 0} G_2(0,\mathbf{x}_1,t_1-\Delta;\bar{0},\mathbf{x}_1,t_1+\Delta).$$

Physically Σ corresponds to the sum of all the "proper" (irreducible) diagrams (cf., Refs. 1 and 2). A chain of successive approximations for Σ can be obtained from Eq. (1) by appropriate iterative solutions for the respective functions G_1 and G_2.

One such procedure outlined in Appendix I leads simply to a certain T-matrix approximation:

$$\Sigma(0,0')=-i\langle 01|T|0'1'\rangle G_1(1',1), \quad (2)$$

where

$$\langle 01|T|0'1'\rangle\equiv V(0,\bar{1})\delta(\bar{1}-1)\langle 0\bar{1}|\Omega|0'1'\rangle \quad (3)$$

and

$$\langle 01|\Omega|\bar{0}\bar{1}\rangle=(\delta(0-\bar{0})\delta(1-\bar{1})-\text{ex}) -iG_1(0,0'')G_1(1,2)V(0'',2)\langle 20''|\Omega|\bar{0}\bar{1}\rangle. \quad (4)$$

$V(0,1)$ is a (local) two-particle potential, and "ex" stands for exchange terms. This gives a complicated nonlinear integral equation for Σ; in fact, our matrices of Eqs. (3) and (4) are "exact" and involve Σ via the function G_1.

In practice one has to apply either some model assumption which eliminates this nonlinearity or some iteration procedure which possibly leads to a self-consistent determination of Σ. We note that the matrix Ω is antisymmetrized. In Appendix I, we discuss also several correction terms, some of which contain contributions of the "target exchange" type (in the terminology of Takeda and Watson[2] and of one of us[16]). The "target exchange" terms are shown in Ref. 16 to be only very small corrections at incident energies $\omega_0\gtrsim 50$ MeV.

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[3] See, e.g., K. A. Brueckner, R. J. Eden, and N. C. Francis, Phys. Rev. **100**, 891 (1955); K. A. Brueckner, *ibid.* **103**, 172 (1956); L. Verlet and J. Gavoret, Nuovo Cimento **10**, 505 (1958); J. Dabrowski and J. Sawicki, Nucl. Phys. **13**, 621 (1959); J. Sawicki and S. A. Moszkowski, *ibid.* **21**, 456 (1960); J. Sawicki, Nuovo Cimento **15**, 504 (1960); J. Dabrowski and J. Sawicki, Nucl. Phys. **22**, 318 (1961); G. L. Shaw, Ann. Phys. (N. Y.) **8**, 509 (1959); C. B. Duke, Phys. Rev. **136**, B59 (1964).

[4] W. B. Riesenfeld and K. M. Watson, Phys. Rev. **102**, 1157 (1956).

[5] H. Feshbach, Ann. Rev. Nucl. Sci. **8**, 49 (1958); Ann. Phys. (N. Y.) **5**, 357 (1958); **19**, 287 (1962).

[6] R. Lipperheide, Ann. Phys. (N. Y.) **17**, 114 (1962).

[7] R. H. Lemmer and C. M. Shakin, Ann. Phys. (N. Y.) **27**, 13 (1964).

[8] Just the use of second quantization alone does not guarantee a correct construction of $\mathcal{U}_{\rm opt}$, as one may deal with states which correspond to unphysical initial conditions; this was the case in the work of Coester and Kümmel [Ref. 1(a)] and Lipperheide (Ref. 6).

[9] J. S. Bell and E. J. Squires, Phys. Rev. Letters **3**, 96 (1959).

[10] P. C. Martin and J. Schwinger, Phys. Rev. **115**, 1342 (1959).

[11] R. D. Puff, Ann. Phys. (N. Y.) **13**, 317 (1961).

[12] A. S. Reiner, Phys. Rev. **129**, 889 (1963); **133**, B1105 (1964).

[13] D. S. Koltun and L. Wilets, Phys. Rev. **129**, 880 (1963).

[14] W. Brenig, Nucl. Phys. **13**, 333 (1959).

[15] G. Baym and L. P. Kadanoff, Phys. Rev. **124**, 287 (1961).

[16] J. Sawicki, Nuovo Cimento **15**, 606 (1960); the evaluation of the "target exchange" corrections as to the scattering amplitude for an infinite matter with correlation is correct but the construction of $\mathcal{U}_{\rm opt}$ cited in this paper is that of Rollnik criticized in Ref. 1.

The T matrix of Eqs. (2)–(4) resembles essentially that of the Brueckner theory with the propagator containing "exact" single-particle energies to be determined self-consistently. It is a formulation obviously appropriate to finite nuclei as well as to infinite nuclear matter. It is much easier in practice to work with a T matrix which would correspond to replacing the $\langle|\Omega|\rangle$ on the right-hand side of Eq. (9) by the corresponding delta-function and exchange terms[17]; let us call it $t^{(+)}$.

We choose now a zero-order single-particle (shell-model) Hamiltonian $h_0(\mathbf{0})$. The energy denominator of each G_1 contains the operator $h(\mathbf{0},\mathbf{0}';\hat{\omega}_0)=h_0(\mathbf{0})\delta(\mathbf{0}-\mathbf{0}')+\Sigma(\mathbf{0},\mathbf{0}';\hat{\omega}_0)$, where $\hat{\omega}_0=\omega_0+\mu$, in which μ is the chemical potential; and we work now with the time Fourier transforms of all our quantities. We can now apply the usual convolution formula to the G_1G_1 propagator in the equation for our $t^{(+)}$ (cf., e.g., Refs. 10–13 and Klein and Prange[18]); Galitskii[19] performs the integration over the fourth component of the four-momentum of the relative motion in the case of infinite matter. The final propagator is

$$\Lambda(\mathbf{01},\mathbf{0'1'};E)=[E-h(\mathbf{0},\mathbf{0}';E-\hat{\omega}_1')-h(\mathbf{1},\mathbf{1}';\hat{\omega}_1')+i\delta]^{-1}, \quad (5)$$

where E is the total "exact" energy of the interacting pair (0,1). With this we can write our $t^{(+)}$ as

$$\langle\mathbf{0'1'}|t^{(+)}(E)|\mathbf{01}\rangle=\bar{V}(\mathbf{0,1})\delta(\mathbf{0}-\mathbf{0}')\delta(\mathbf{1}-\mathbf{1}')+V(\mathbf{0}',\mathbf{1}')\Lambda(\mathbf{01},\mathbf{0'1'};E)\bar{V}(\mathbf{0,1}), \quad (6)$$

(here the repeated variables are not integrated over) where

$$\bar{V}(\mathbf{0,1})=V(\mathbf{0,1})(1-P_{01}),$$

the antisymmetrized V operator.

We can go over to the representation in which h and G_1 are diagonal with the eigenvalues $\{\epsilon_n(\hat{\omega})\}$. In this representation the single-particle density matrix is diagonal: $\rho_{(1)}(n_0,n_0')=\delta_{n_0n_0'}\rho_{(1)}(n_0,n_0)$.

If we neglect the imaginary part of Σ (of h) in Λ, we we can apply the usual methods,[10–13] and we find for a $(\mathbf{k}_0,\mathbf{k}_0')$ Fourier component of our Σ of Eq. (2)

$$\Sigma(\mathbf{k}_0,\mathbf{k}_0';\hat{\omega}_0)\cong\sum_{n_0}\langle\mathbf{k}_0',n_0|t^{(+)}(\tilde{E})|\mathbf{k}_0,n_0\rangle\times\rho_{(1)}(n_0,n_0)=\mathrm{Tr}(t^{(+)}\rho_{(1)}), \quad (7)$$

where $\tilde{E}=\omega_0+E_{n_0}$, E_{n_0} is the n_0th root of the equation $\hat{\omega}_1=\epsilon_n(\hat{\omega}_1)$, and $\int d\mathbf{1'}h(\mathbf{1},\mathbf{1}';\hat{\omega}_1)\psi_n(\mathbf{1}',\hat{\omega}_1)=\epsilon_n(\hat{\omega}_1)\psi_n(\mathbf{1},\hat{\omega}_1)$; $\psi_n(\mathbf{1},\hat{\omega}_1)$ is the corresponding eigenfunction. In terms of $\bar{V}$, we rewrite Eq. (7) as

$$\Sigma(\mathbf{k}_0,\mathbf{k}_0';\hat{\omega}_0)\cong\sum_{n_0}\langle\mathbf{k}_0',n_0|\bar{V}(0,1)|\mathbf{k}_0,n_0\rangle\rho_{(1)}(n_0,n_0)+\sum_{n_0}\sum_{k,n}\frac{\langle\mathbf{k}_0',n_0|\bar{V}(0',1')|k,n\rangle\langle k,n|\bar{V}(0,1)|\mathbf{k}_0,n_0\rangle}{\tilde{E}-E_k-E_n+i\delta}\rho_{(1)}(n_0,n_0). \quad (8)$$

In the zero-order representation in which h_0 is diagonal, we can write

$$\Sigma(\mathbf{k}_0,\mathbf{k}_0';\hat{\omega}_0)\cong\sum_{n_0}\sum_{\nu\nu'}\langle\mathbf{k}_0'\nu'|\bar{V}(0,1)|\mathbf{k}_0\nu\rangle\rho_{(1)}{}^{(n_0)}(\nu\nu')\rho_{(1)}(n_0,n_0)+\sum_{n_0}\sum_{kn}\sum_{\substack{\nu\nu'\kappa\kappa'\\ \mu\mu'}}\frac{\langle\mathbf{k}_0'\nu'|V|\kappa'\mu'\rangle\rho_{(2)}{}^{(k,n)}(\kappa\mu,\kappa'\mu')\langle\kappa\mu|\bar{V}|\mathbf{k}_0\nu\rangle\rho_{(1)}{}^{(n_0)}(\nu\nu')\rho_{(1)}(n_0,n_0)}{\tilde{E}-E_k-E_n+i\delta}, \quad (9)$$

where $\rho_{(1)}{}^{(n_0)}(\nu\nu')\equiv\langle\nu|n_0\rangle\langle n_0|\nu'\rangle$, and a Fourier component of the two-particle density matrix is

$$\rho_{(2)}{}^{(k,n)}(\kappa\mu,\kappa'\mu')\equiv\langle\kappa'\mu'|kn\rangle\langle kn|\kappa\mu\rangle.$$

Our shell-model potential $u(\mathbf{0})$ contained in $h_0(0)$ has to be added on to Σ to give the total optical potential (cf., Bell[1] and Bell and Squires[9])

$$\mathcal{U}_{\mathrm{opt}}(\mathbf{0},\mathbf{0}';\hat{\omega}_0)\equiv u(\mathbf{0})\delta(\mathbf{0}-\mathbf{0}')+\Sigma(\mathbf{0},\mathbf{0}';\hat{\omega}_0). \quad (10)$$

A formula closely related to our Eq. (9) is obtained by the usual Martin-Schwinger-Puff approximation.[10–13] In this case, one uses simply $\Omega^{(0)}$ and $T^{(0)}$, which involve $G_1{}^{(0)}$, in the place of G_1 in Eqs. (3) and (4). If $G_1{}^{(0)}$ of $\Omega^{(0)}$ corresponds to the Hamiltonian h_0 for a finite system with the single-particle energies $\{E_\nu{}^{(0)}\}$, we obtain

$$\langle\mathbf{k}_0'\nu|T^{(0)}(\hat{\omega}_0+\hat{\omega}_1)|\mathbf{k}_0\nu\rangle=\langle\mathbf{k}_0'\nu|\bar{V}|\mathbf{k}_0\nu\rangle+\sum_{\kappa\mu}\frac{\langle\mathbf{k}_0'\nu|V|\kappa\mu\rangle\langle\kappa\mu|T^{(0)}(\hat{\omega}_0+\hat{\omega}_1)|\mathbf{k}_0\nu\rangle}{\hat{\omega}_0+\hat{\omega}_1-E_\kappa{}^{(0)}-E_\mu{}^{(0)}+2\mu+i\delta}. \quad (11)$$

This can be brought to a form similar to that of Eqs. (8) and (9).

One unfortunate feature of the T matrix of Eq. (3) is that it does not automatically exclude from the intermediate states (k,n) in Eq. (8) those occupied by the remaining $A-1$ nucleons in the ground state. In order to secure this exclusion one has to modify the T matrix by introducing an appropriate projection operator. The effect of the difference between such a new matrix T' and T should be then estimated together with the other higher order nonlinear terms.

Another approach to the problem in which one calculates $\mathcal{U}_{\mathrm{opt}}$ directly *without* employing the concept of the proper self-energy operator Σ is that of Takeda and

[17] This is rather close to the approximate T matrix with the functions G_1 replaced by $G_1{}^{(0)}$, called $T^{(0)}$ (cf., Appendix I).

[18] R. Prange and A. Klein, Phys. Rev. **112**, 1008 (1958).

[19] V. M. Galitskii, Zh. Eksperim. i Teor. Fiz. **34**, 151 (1958) [English transl.: Soviet Phys.—JETP **7**, 104 (1958)].

Watson.[2] In particular, their construction, although approximate, is free from the last complication mentioned above. If one expresses $\mathcal{U}_{\rm opt}$ in terms of $V=\sum_{i=1}^{A} V(0,i)$ and retains in it all such terms only up to second order, one finds[20]

$$\mathcal{U}_{\rm opt}(\mathbf{k}_0,\mathbf{k}_0';\omega_0)\cong\langle\mathbf{k}_0'\psi_0|\sum_{i=1}^{A}\bar{V}(0,i)|\mathbf{k}_0\psi_0\rangle + \sum_{k}\sum_{n\neq 0}\frac{\langle\mathbf{k}_0'\psi_0|\sum_{i=1}^{A}\bar{V}(0,i)|\psi_{kn}\rangle\langle\psi_{kn}|\sum_{j=1}^{A}\bar{V}(0,j)|\mathbf{k}_0,\psi_0\rangle}{\omega_0-\epsilon_k-\Delta E_n+i\delta}. \quad (12)$$

Here ψ_0 represents the ground state of the A system of energy E_0. The model states $\{\psi_{kn}\}$ refer to the $(A+1)$ system (particles $0,1,2,\cdots,A$); the state of the particle "0" labeled "k" has the unperturbed single-particle energy ϵ_k, while the label "n" refers to the excitation of the remaining A system $(1,2,\cdots,A)$ relative to its ground state ψ_0; the states ψ_0 and ψ_{kn} must be orthogonal; the excitation energy $\Delta E_n=E_n-E_0$ does not involve any "extra" rearrangement energy. In addition $\{\psi_{kn}\}$ is a complete set excluding the ψ_0 component.

Equation (12) is also a Feshbach-type formula[5] (of the coupled-channel reaction theory).[21] One essential difference is the presence of the pair-antisymmetrization operator $(1-P_{01})$ in the V-matrix elements. Note that, in contrast to Eq. (12), only one of the two V-matrix elements in the $\sum_{kn}$ term in Eq. (8) was antisymmetrized. This antisymmetrization is only part of the effect of the Pauli principle. Unfortunately, the "target-exchange" terms as incorporated in an extension of the Feshbach formula by Coester and Kummel [Ref. 1(a)] or by Lipperheide[6] are not correct, nor does the version given by Lemmer and Shakin[7] appear to be satisfactory.

If the energy ϵ_k of the particle "0" is indeed "free," our Eq. (12) represents essentially the ordinary second-order perturbation theory (a difference remains only in the restrictions on k, n).

In the following we shall employ a model for $\{\psi_{kn}\}$ which excludes ψ_0 automatically. We choose it to be products of plane waves $\chi_{\sigma_0 z}(\sigma_0)\eta_{\tau_0 z}(\tau_0)\exp(i\mathbf{k}\cdot\mathbf{r}_0)$ for the particle "0" with TD or RPA eigenvectors $|\psi_n\rangle$ (of a shell-model configuration mixing) corresponding to the excitations of the A system (target) given by $\Delta E_n\equiv E_{kn}-\epsilon_k$. Through the E_{kn} and the above choice of $\{\psi_{kn}\}$ we have destroyed the symmetry (equivalence) of the particle "0" relative to the other A particles.

In the language of second quantization we would keep, in calculating $\langle\mathbf{k}_0'\psi_0|V|\psi_{kn}\rangle$, the fermion operators relative to the particle "0" as fixed (uncontractable with any others). In this sense, we consider V as a one-particle operator, and, consequently, we can write in the place of our $\langle\psi_{kn}|\sum_{i=1}^{A}\bar{V}(0,i)|\mathbf{k}_0\psi_0\rangle$ of Eq. (12)

$$\sum_{\nu\neq\nu'}\langle\mathbf{k}\nu'|\bar{V}|\mathbf{k}_0\nu\rangle\langle\psi_n{}^{\rm TD(RPA)}|C_{\nu'}{}^{\dagger}C_{\nu}|\psi_0\rangle = \sum_{\nu\neq\nu'}\langle\mathbf{k}\nu'|\bar{V}|\mathbf{k}_0\nu\rangle\chi_n(\nu\nu')^{\rm TD(RPA)}, \quad (13)$$

where $\{\chi_n(\nu\nu')^{\rm TD(RPA)}\}$ are components of the eigenvector of the nth TD (RPA) state. In the TD case $|\psi_0\rangle$ is replaced by $|\phi_0\rangle$ the ground state of the h_0 model (no correlations; a Slater determinant of h_0 eigenfunctions) and so no components of the "backward-going graphs" appear. It should be pointed out that the complete set of eigenstates of RPA contains also the unphysical states of negative excitation energies $(-|\Delta E_n|)$; these we shall leave out somewhat arbitrarily as if we were working with the TD case throughout.

It is just the purpose of this paper to examine how various eigenstates n of different spins, parities, and isobaric spins contribute to $\mathcal{U}_{\rm opt}$. We have chosen just TD or RPA eigenstates as affording a quite succesful version of the shell-model configuration mixing in explaining properties of low-lying, especially collective, states of light and medium-heavy nuclei.[22]

Particularly interesting is the imaginary part of $\mathcal{U}_{\rm opt}$. With Eq. (13) we can write the second term of the right-hand side of Eq. (12) as

$$\sum_{\mathbf{k}n}\sum_{\nu\nu',\mu\mu'}\frac{\chi_n{}^*(\nu\nu')\langle\mathbf{k}_0'\nu|\bar{V}|\mathbf{k}\nu'\rangle\chi_n(\mu\mu')\langle\mathbf{k}\mu'|\bar{V}|\mathbf{k}_0\mu\rangle}{\omega_0-\epsilon_{\mathbf{k}}-\Delta E_n+i\delta}. \quad (14)$$

From Eq. (14) we find the imaginary part as

$$\mathrm{Im}\mathcal{U}_{\rm opt}=\mathrm{Im}\Sigma(\mathbf{k}_0,\mathbf{k}_0';\omega_0)=-\pi\sum_{\mathbf{k}n}\delta(\omega_0-E_{kn}) \times\sum_{\nu\nu',\mu\mu'}\chi_n{}^*(\nu\nu')\langle\mathbf{k}_0'\nu|\bar{V}|\mathbf{k}\nu'\rangle\chi_n(\mu\mu')\langle\mathbf{k}\mu'|\bar{V}|\mathbf{k}_0\mu\rangle. \quad (15)$$

Our present notation is appropriate rather to deformed nuclei; for spherical nuclei one has to go through the Racah algebra; the corresponding final formulas are given in Sec. III.

In Appendix II, we also give the corresponding formulas for the case of the infinite nucleus (nuclear matter).

[20] In principle, the single-particle state k of the particle "0" should lie above all the single-particle states occupied in "n" of the A system [single-particle state ordering in the double sum of Eq. (12)]. This is not exactly so in our subsequent model approximation; what we actually calculate numerically below is essentially the second-order [in $V=\Sigma_{i=1}^{A}V(0,i)$] perturbation theory.

[21] The formula for $\mathcal{U}_{\rm opt}$ given by Feshbach (Ref. 5) for the non-antisymmetrized case is

$$\mathcal{U}_{\rm opt}(E)=\langle\psi_0|V+VQ_0(E-H_0-Q_0VQ_0+i\delta)^{-1}Q_0V|\psi_0\rangle,$$

where Q_0 is the projector excluding the ground state ψ_0, $V=\Sigma_{i=1}^{A}V(0,i)$, and H_0=kinetic energy of the particle "0" plus H_A, the total Hamiltonian of the target.

[22] Cf., e.g., (a) V. Gillet, esthis, University of Paris, 1962 (unpublished); (b) N. Vinh Mau, thesis, University of Paris, 1963 (unpublished); V. Gillet and N. Vinh Mau, Nucl. Phys. **54**, 321 (1964); V. Gillet and E. A. Sanderson, *ibid.* **54**, 472 (1964); J. Sawicki and T. Soda, *ibid.* **28**, 270 (1961); S. G. Nilsson, J. Sawicki, and N. K. Glendenning, *ibid.* **33**, 239 (1962); R. Arvieu, E. Baranger, M. Vénéroni, M. Baranger, and V. Gillet, Phys. Letters **4**, 119 (1963).

One further point discussed, e.g., in Refs. 1 and 6 is the problem of the origin of the system of reference. Indeed, our coordinates refer to some rather arbitrarily fixed origin in the laboratory frame. In our case, we think of the center of mass of the target nucleus as if it were infinitely heavy. Thereby, we would completely neglect the recoil of this nucleus in a calculation of the scattering amplitude with our $\mathcal{U}_{\text{opt}}$. This general ambiguity is dangerous also for the convergence of our successive-approximation series.[1] On the other hand, a formulation of the problem in terms of physical coordinates (including the recoil effect) with the elimination of the center-of-mass coordinate of the entire $(A+1)$ system is possible, although complicated. Such a construction is given in Ref. 6 [although some of the formulas corresponding to these of Ref. 11 are subject to the criticism of Ref. 1; cf., e.g., the "equivalent potentials" of Eq. (2.3.37) of Ref. 6]. On the other hand, Eq. (2.3.37) of Ref. 6 reduces in the case of $A\gg 1$ to Eq. (2.3.38) of the same, which essentially corresponds to our $\langle\Psi_{\mathbf{k}'n'}|\bar{V}|\Psi_{\mathbf{k}n}\rangle$. Unfortunately, our numerical example involves a nucleus not sufficiently heavy. However, we believe that the inclusion of the recoil corrections should not destroy the essential features of our only semiquantitative conclusions, even in the case of lighter nuclei. This question is also raised in Refs. 23 and 24.

III. CALCULATION OF OPTICAL POTENTIAL FOR SPHERICAL NUCLEI AND THE TD OR RPA APPROXIMATIONS

Before going to the calculation of our optical potential, the zero-order single-particle Hamiltonian $h_0(\mathbf{0})$ has to be specified. We are dealing with spherical nuclei and we are using the TD or RPA nuclear wave functions Ψ_n. Then $h_0(\mathbf{0})$ is the isotropic harmonic-oscillator Hamiltonian, whose basis wave functions are well known and easy to use.

In this section, we should like to obtain the final formulas for $\mathcal{U}_{\text{opt}}$, the optical potential, step by step. We will start by considering only the nonantisymmetrized optical potential, from which the antisymmetrized one will be deduced very simply at the end of this section. Furthermore, in the first step we consider the unrealistic case of a pure Wigner force: $V(0,1)=V(|\mathbf{r}_0-\mathbf{r}_1|)=V(r_{01})$. In a second step we will include the Bartlett, Majorana, and Heisenberg exchange forces; then in a last one we will introduce the antisymmetrization operator P_{01}.

Nevertheless, to render the introduction of antisymmetrization easy, one has to assume that V_{01} is a zero-range potential. Since this approximation is necessary only at that stage, we start with the most general Wigner force.

(A) Wigner Force

We assume that

$$V_{01}=V(|\mathbf{r}_0-\mathbf{r}_1|)=V(r_{01}).$$

Let us call the corresponding optical potential $\mathcal{U}_0$. We can separate $\mathcal{U}_0$ into two parts:

$$\mathcal{U}_0=\mathcal{U}_0^{(1)}+\mathcal{U}_0^{(2)},$$

where

$$\mathcal{U}_0^{(1)}=\langle\mathbf{k}_0,\psi_0|\sum_{i=1}^{A}V(r_{0i})|\mathbf{k}_0',\psi_0\rangle, \tag{16}$$

$$\mathcal{U}_0^{(2)}=\frac{2m}{\hbar^2}\sum_n\sum_{\sigma_{0z}'\sigma_{0z}''\tau_{0z}'\tau_{0z}''}S_{\sigma_{0z}\tau_{0z}}\int d\mathbf{k}_0''\times\langle\mathbf{k}_0'\psi_0|\sum_{i=1}^{A}V(r_{0i})|\mathbf{k}_0''\psi_n\rangle\times\langle\mathbf{k}_0''\psi_n|\sum_{j=1}^{A}V(r_{0j})|\mathbf{k}_0\psi_0\rangle(Z_n^2-k_0''^2+i\delta)^{-1}. \tag{17}$$

$\sigma_0\sigma_{0z}$, $\tau_0\tau_{0z}$, $\sigma_0'\sigma_{0z}'$, $\tau_0'\tau_{0z}'$, $\sigma_0''\sigma_{0z}''$, and $\tau_0''\tau_{0z}''$ are, respectively, the spins and isospins of initial, final, and intermediate nucleons. S means the average over spin and isospin of the initial nucleon; $Z_n^2=k_0^2-2M\Delta E_n/\hbar^2$, since we consider the approximation where $\epsilon_{\mathbf{k}}$ is simply the kinetic energy, neglecting in the denominator operator the interaction term between nucleon 0 and nucleons of the nucleus. This approximation is consistent with our choice of $\Psi_{\mathbf{k}n}$ as a simple product of a 0 wave function and a nucleus wave function.

For now only the imaginary part of $\mathcal{U}_0$ will be calculated and $\mathcal{U}_0^{(1)}$ can be left out since it contributes only to the real part of $\mathcal{U}_0$.

We assume that the 0 nucleon is represented by a plane wave

$$|\mathbf{k}_0\rangle\equiv e^{i\mathbf{k}_0\cdot\mathbf{r}_0}\chi_{\sigma_{0z}}(\sigma_0)\eta_{\tau_{0z}}(\tau_0)$$

and the intermediate nuclear-state wave functions are those of, "Approximation I", (the TD approximation) or "Approximation II" (the RPA approximation) of the hole-particle model. Then

$$\langle\mathbf{k}_0'\psi_0|\sum_{i=1}^{A}V(r_{0i})|\mathbf{k}_0''\psi_n\rangle=(2\pi)^{-3}\sum_{a,b}\chi_n^*(a,b)\times\sum_{m_a,m_b,\tau_{az},\tau_{bz}}\eta(a,b)\langle j_aj_b,m_a-m_b|J_nM_n\rangle\times\langle\tfrac{1}{2}\tfrac{1}{2},\tau_{az}-\tau_{bz}|T_n0\rangle\langle\mathbf{k}_0'b|V(r_{01})|\mathbf{k}_0''a\rangle, \tag{18.1}$$

where a and b represent all quantum numbers of particle and hole states. In Approximation I, a is an unoccupied state and b an occupied state of the shell-model ground state. In Approximation II, a and b are respectively either unoccupied and occupied states (corresponding to the "forward-going graphs") or occupied and unoccupied states (corresponding to the "backward-going graphs"). $\chi_n(ab)$ are the eigenvectors, e.g., of Ref. 22(b). $\eta(ab)$ is the phase defined in the same reference. J_n, M_n, and T_n are the quantum numbers of the nth nuclear state.

[23] A. K. Kerman, H. McManus, and R. M. Thaler, Ann. Phys. (N. Y.) 8, 551 (1959).

[24] D. J. Thouless, Rept. Progr. Phys. 27, 53 (1964); also the general method of applying the propagators to the study of $\mathcal{U}_{\text{opt}}$ is outlined in this reference.

Similarly one writes the second matrix element

$$\langle \mathbf{k}_0''\psi_n|\sum_{i=1}^{A} V(r_{0i})|\mathbf{k}_0\psi_0\rangle=(2\pi)^{-3}\sum_{\alpha\beta}\chi_n(\alpha,\beta)\times\sum_{m_\alpha,m_\beta,\tau_{\alpha z},\tau_{\beta z}}\eta(\alpha,\beta)\langle j_\alpha j_\beta, m_\alpha-m_\beta|J_nM_n\rangle\times\langle\tfrac{1}{2}\tfrac{1}{2},\tau_{\alpha z}-\tau_{\beta z}|T_n0\rangle\langle\mathbf{k}_0''\beta|V(r_{01})|\mathbf{k}_0\alpha\rangle. \quad (18.2)$$

The operator $V(r_{01})$ acts only on space variables and the spin and isospin parts of (18.1) and (18.2) can be performed immediately. Going over from j-j coupling to L-S coupling through the usual transformation, we obtain

$$\mathcal{V}_0{}^{(2)}=\sum_n \mathcal{V}_{0n}{}^{(2)}\delta_{T_n,0},$$

where

$$\mathcal{V}_{0n}{}^{(2)}=\frac{8m}{(2\pi)^6\hbar^2}\sum_{ab\alpha\beta}(-1)^{j_b+j_\beta+1}\chi_n{}^*(a,b)\chi_n(\alpha,\beta)\hat{j}_a\hat{j}_b\hat{j}_\alpha\hat{j}_\beta\hat{J}_n{}^2\times\begin{Bmatrix} l_a & \frac{1}{2} & j_a\\ l_b & \frac{1}{2} & j_b\\ J_n & 0 & J_n\end{Bmatrix}\begin{Bmatrix} l_\alpha & \frac{1}{2} & j_\alpha\\ l_\beta & \frac{1}{2} & j_\beta\\ J_n & 0 & J_n\end{Bmatrix}\int\frac{A_{ab\alpha\beta}(\mathbf{k}_0'')d\mathbf{k}_0''}{Z_n{}^2-k_0''{}^2+i\delta}\equiv\int_0^\infty F(k_0'')k_0''{}^2dk_0'', \quad (19)$$

where

$$A_{ab\alpha\beta}(\mathbf{k}_0'')\equiv\sum(-1)^{l_b-m_b+l_\beta-m_\beta}\langle l_al_b, m_a-m_b|J_nM_n\rangle\langle l_al_\beta, m_\alpha-m_\beta|J_nM_n\rangle\times\int\int R_{nal_a}(r_1)R_{nbl_b}(r_1)Y_{l_a}{}^{m_a}(\omega_1)Y_{l_b}{}^{m_b*}(\omega_1)V(|\mathbf{r}_{01}|)e^{i(\mathbf{k}_0''-\mathbf{k}_0')\cdot\mathbf{r}_0}d\mathbf{r}_0d\mathbf{r}_1\times\int\int R_{n\alpha l_\alpha}(r_1')R_{n\beta l_\beta}(r_1')Y_{l_\alpha}{}^{m_\alpha*}(\omega_1')Y_{l_\beta}{}^{m_\beta}(\omega_1')V(|\mathbf{r}_{01}'|)e^{i(\mathbf{k}_0-\mathbf{k}_0'')\cdot\mathbf{r}_0'}d\mathbf{r}_0'd\mathbf{r}_1' \quad (20)$$

and $\hat{a}\equiv(2a+1)^{1/2}$.

In the case of a Wigner force, the summation over intermediate nuclear states is restricted to $T_n=0$ states.

Both the double integrals of Eq. (20) can be factorized into an integral over $\mathbf{r}_0-\mathbf{r}_1$ (or $\mathbf{r}_0'-\mathbf{r}_1'$), which is the Fourier transform of the potential, and an integral over $\mathbf{r}_1$ (or $\mathbf{r}_1'$). Then $A_{ab\alpha\beta}$ is obtained as

$$A_{ab\alpha\beta}=\sum_{m_a,m_b,m_\alpha,m_\beta}(-1)^{l_b-m_b+l_\beta-m_\beta}\mathcal{A}_{ab\alpha\beta},$$

where

$$\mathcal{A}_{ab\alpha\beta}\equiv\langle l_al_b, m_a-m_b|J_nM_n\rangle\langle l_al_\beta, m_\alpha-m_\beta|J_nM_n\rangle V(\mathbf{q})V(\mathbf{q}')\int R_{nal_a}(r_1)R_{nbl_b}(r_1)Y_{l_a}{}^{m_a}(\omega_1)Y_{l_b}{}^{m_b*}(\omega_1)\times e^{i(\mathbf{k}_0''-\mathbf{k}_0')\cdot\mathbf{r}_1}d\mathbf{r}_1\int R_{n\alpha l_\alpha}(r_1')R_{n\beta l_\beta}(r_1')Y_{l_\alpha}{}^{m_\alpha*}(\omega_1')Y_{l_\beta}{}^{m_\beta}(\omega_1')e^{i(\mathbf{k}_0-\mathbf{k}_0'')\cdot\mathbf{r}_1'}d\mathbf{r}_1', \quad (21)$$

where $\mathbf{q}=\mathbf{k}_0''-\mathbf{k}_0$, $\mathbf{q}'=\mathbf{k}_0'-\mathbf{k}_0''$, $V(\mathbf{q})=\int e^{i\mathbf{q}\cdot\boldsymbol{\rho}'}V(\boldsymbol{\rho}')d\boldsymbol{\rho}'$.

Both integrals, over $\mathbf{r}_1$ and $\mathbf{r}_1'$, involve $\mathbf{k}_0''$ and the last integration over $\mathbf{k}_0''$ is cumbersome. Thus we should like to transform (21) in such a way that the dependence on $\mathbf{k}_0''$ could be put in only one integral. The most convenient way of doing it is to replace $\mathbf{r}_1$ and $\mathbf{r}_1'$ by $\boldsymbol{\rho}$ and $\mathbf{R}$ (a Talmi transformation)

$$\boldsymbol{\rho}=(\mathbf{r}_1-\mathbf{r}_1')/\sqrt{2}\equiv(\rho,\omega),\qquad \mathbf{R}=(\mathbf{r}_1+\mathbf{r}_1')/\sqrt{2}\equiv(R,\Omega). \quad (22)$$

The Talmi transformation of harmonic-oscillator wave functions has been given by Moshinsky and Brody[25] in the form

$$R_{nal_a}(r_1)R_{n\alpha l_\alpha}(r_1')Y_{l_a}{}^{m_a}(\omega_1)Y_{l_\alpha}{}^{-m_\alpha}(\omega_1')=\sum_{\substack{nlNL\\ \lambda\mu mM}}\langle l_al_\alpha, m_a-m_\alpha|\lambda\mu\rangle\langle lLmM|\lambda\mu\rangle R_{nl}(\rho)R_{NL}(R)\times Y_l{}^m(\omega)Y_L{}^M(\Omega)\langle nlNL,\lambda|n_al_an_\alpha l_\alpha,\lambda\rangle \quad (23)$$

with a similar transformation for the (b,β) pair. The

$$\langle nlNL,\lambda|n_al_an_\alpha l_\alpha,\lambda\rangle\text{'s}$$

are the Moshinsky-Brody "brackets."

An $A_{ab\alpha\beta}$ can be expressed now as a product of two integrals, I_R and I_ρ

$$I_R=\int d\mathbf{R}e^{i\mathbf{K}\cdot\mathbf{R}}R_{NL}(R)R_{N'L'}(R)Y_L{}^M(\Omega)Y_{L'}{}^{M'}(\Omega),\qquad I_\rho=\int d\boldsymbol{\rho}e^{i\mathbf{Q}'\cdot\boldsymbol{\rho}}R_{nl}(\rho)R_{n'l'}(\rho)Y_l{}^m(\omega)Y_{l'}{}^{m'}(\omega) \quad (24)$$

with

$$\mathbf{K}=(\mathbf{k}_0-\mathbf{k}_0')/\sqrt{2},\qquad \mathbf{Q}'=\mathbf{Q}-\sqrt{2}\mathbf{k}_0''=(\mathbf{k}_0+\mathbf{k}_0')/\sqrt{2}-\sqrt{2}\mathbf{k}_0''. \quad (24'1)$$

The radial wave function of a particle in an harmonic-

[25] T. A. Brody and M. Moshinsky, *Tables of Transformation Brackets* (Monografias del Instituto de Fisica, Mexico, 1960).

oscillator well is given by

$$R_{nl}(r)=\alpha_{nl}e^{-r^2/(2b_0^2)}(r/b_0)^l\sum_{i=0}^{n}a_i^{(n,l+1/2)}(r/b_0)^{2i}\ (n=0,1,\cdots),$$

$$\alpha_{nl}=[2n!/\Gamma(n+l+\tfrac{3}{2})]^{1/2}, \tag{25}$$

$$a_i^{(n,l+1/2)}=(-1)^i\frac{1}{i!}\frac{\Gamma(n+l+\tfrac{3}{2})}{\Gamma(n-i+1)\Gamma(i+l+\tfrac{3}{2})}.$$

Expanding the exponentials of the formulas (24) in partial waves and going through the standard Racah algebra, we obtain $A_{ab\alpha\beta}$ in the form

$$A_{ab\alpha\beta}=V(\mathbf{q})V(\mathbf{q}')\delta_{M_n,0}(-1)^{l_b+l_\beta}\hat{J}_n^2\sum_{\substack{\lambda\lambda' ll' LL'\\ \mathfrak{L}\mathfrak{L}' nNn'N'}}(-i)^{\mathfrak{L}+\mathfrak{L}'}(2\mathfrak{L}+1)(2\mathfrak{L}'+1)$$
$$\times\hat{l}\hat{l}'\hat{L}\hat{L}'\hat{\lambda}^2\hat{\lambda}'^2\langle nlNL,\lambda|n_al_an_al_\alpha,\lambda\rangle\langle n'l'N'L',\lambda'|n_bl_bn_\beta l_\beta,\lambda'\rangle\begin{pmatrix}l&l'&\mathfrak{L}'\\0&0&0\end{pmatrix}\begin{pmatrix}L&L'&\mathfrak{L}\\0&0&0\end{pmatrix}$$
$$\times\sum_\kappa\hat{\kappa}^2\begin{pmatrix}\mathfrak{L}'&\mathfrak{L}&\kappa\\0&0&0\end{pmatrix}\begin{pmatrix}J_n&J_n&\kappa\\0&0&0\end{pmatrix}\begin{Bmatrix}l&L&\lambda\\l'&L'&\lambda'\\\mathfrak{L}'&\mathfrak{L}&\kappa\end{Bmatrix}\begin{Bmatrix}l_a&l_b&J_n\\l_\alpha&l_\beta&J_n\\\lambda&\lambda'&\kappa\end{Bmatrix}I_{\mathfrak{L}}(K)I_{\mathfrak{L}'}(Q'), \tag{26}$$

where

$$I_{\mathfrak{L}}(K)=\left(\frac{\pi}{2Kb_0}\right)^{1/2}\alpha_{NL}\alpha_{N'L'}\sum_{i=0}^{N}\sum_{j=0}^{N'}a_i^{(N,L+1/2)}a_j^{(N',L'+1/2)}$$
$$\times\left(\frac{Kb_0}{2}\right)^{\mathfrak{L}+1/2}\frac{1}{2\Gamma(\mathfrak{L}+\frac{3}{2})}\Gamma\left(\frac{\mathfrak{L}+L+L'+2i+2j+3}{2}\right){}_1F_1\left(\frac{\mathfrak{L}+L+L'+2i+2j+3}{2};\mathfrak{L}+\tfrac{3}{2};-\frac{K^2b_0^2}{4}\right) \tag{27}$$

and a similar expression for $I_{\mathfrak{L}'}(Q')$.

We are now left with a last integral over $\mathbf{k}_0''$ before obtaining $\mathcal{U}_0$. $A_{ab\alpha\beta}$ depends on $\mathbf{k}_0''$ through $I_{\mathfrak{L}'}(Q')$ and both Fourier transforms of the potential.

For fixed large k_0'', and for large Q', the function $I_{\mathfrak{L}'}(Q')$ behaves as $Q'^n e^{-Q'^2/4b^2}$ and goes to zero very rapidly. The integration over k_0'' will be in fact an integration from zero to a certain finite value $k_{\max}$. We make the assumption that between 0 and $k_{\max}$ the Fourier transforms $V(\mathbf{q})$ and $V(\mathbf{q}')$ can be replaced by their average values $V_{\mathrm{av}}(q)$. The integration over angles is now easy since only $I_{\mathfrak{L}'}(Q')$ depends on $\mathbf{k}_0''$. Expanding the hypergeometric function in series one gets

$$\int_0^{2\pi}d\varphi_0''\int_0^{\pi}\sin\theta_0''d\theta_0''I_{\mathfrak{L}'}(Q')=\frac{\pi^{3/2}}{\sqrt{2}Qk_0''b_0^2}\alpha_{nl}\alpha_{n'l'}\sum_{i=0}^{n}\sum_{j=0}^{n'}a_i^{(n,l+1/2)}a_j^{(n',l'+1/2)}\sum_{x=0}^{\infty}(-1)^x\frac{2}{\mathfrak{L}'+2x+2}\frac{1}{x!}$$
$$\times\frac{\Gamma(\frac{1}{2}(\mathfrak{L}'+l+l'+2i+2j+3)+x)}{\Gamma(\mathfrak{L}'+x+\frac{3}{2})}[(\tfrac{1}{2}(Qb_0+\sqrt{2}k_0''b_0))^{\mathfrak{L}'+2x+2}-(\tfrac{1}{2}(Qb_0-\sqrt{2}k_0''b_0))^{\mathfrak{L}'+2x+2}]\equiv J_{\mathfrak{L}'}(Q,k_0''). \tag{28}$$

Collecting the previous results, $\mathcal{U}_{0n}^{(2)}$ can be written schematically

$$\mathcal{U}_{0n}^{(2)}=\int_0^\infty\frac{F(k_0'')}{Z_n^2-k_0''^2+i\delta}k_0''^2dk_0''.$$

$F(k_0'')$ was defined in (19) and is given by (26), (27), and (28).

$$\mathcal{U}_{0n}^{(2)}=P\int_0^\infty\frac{F(k_0'')}{Z_n^2-k_0''^2}k_0''^2dk_0''-\tfrac{1}{2}i\pi|Z_n|F(|Z_n|), \tag{29}$$

where P denotes the principal value.

We are interested only in the imaginary part of the optical potential, which is easier to calculate than the real

part:

$$\mathrm{Im}\mathcal{U}_0 = W^{(0)} = -\tfrac{1}{2}\pi \sum_{n,\Delta E_n \le \omega_0} \delta_{T_n,0} W_n{}^{(0)},$$

where the sum over n states is strictly limited to those states with excitation energies ΔE_n below the energy of the incident nucleon, ω_0.

$$W_n{}^{(0)} = -\frac{m}{(2\pi)^5\hbar^2} V_{\mathrm{av}}(q)^2 |Z_n| \sum_{ab\alpha\beta} \chi_n{}^*(ab)\chi_n(\alpha\beta)\hat{j}_a\hat{j}_b\hat{j}_\alpha\hat{j}_\beta \hat{J}_n{}^2 \begin{Bmatrix} l_b & j_b & \frac{1}{2} \\ j_a & l_a & J_n \end{Bmatrix} \begin{Bmatrix} l_\beta & j_\beta & \frac{1}{2} \\ j_\alpha & l_\alpha & J_n \end{Bmatrix}$$

$$\times \sum_{\substack{\mathfrak{L}\mathfrak{L}'\mathfrak{l}\mathfrak{l}' \\ LL'nNn'N'}} (2\mathfrak{L}+1)(2\mathfrak{L}'+1)\hat{\mathfrak{l}}\hat{\mathfrak{l}}'\hat{L}\hat{L}'\hat{\lambda}^2\hat{\lambda}'^2(-i)^{\mathfrak{L}+\mathfrak{L}'}\langle nlNL,\lambda|n_a l_a n_\alpha l_\alpha,\lambda\rangle\langle n'l'N'L',\lambda'|n_b l_b n_\beta l_\beta,\lambda'\rangle$$

$$\times \begin{pmatrix} l & l' & \mathfrak{L}' \\ 0 & 0 & 0 \end{pmatrix}\begin{pmatrix} L & L' & \mathfrak{L} \\ 0 & 0 & 0 \end{pmatrix}\sum_\kappa \hat{\kappa}^2 \begin{pmatrix} \mathfrak{L}' & \mathfrak{L} & \kappa \\ 0 & 0 & 0 \end{pmatrix}\begin{pmatrix} J_n & J_n & \kappa \\ 0 & 0 & 0 \end{pmatrix} \begin{Bmatrix} l & L & \lambda \\ l' & L' & \lambda' \\ \mathfrak{L}' & \mathfrak{L} & \kappa \end{Bmatrix} \begin{Bmatrix} l_a & l_b & J_n \\ l_\alpha & l_\beta & J_n \\ \lambda & \lambda' & \kappa \end{Bmatrix} I_{\mathfrak{L}}(K) J_{\mathfrak{L}'}(Q,|Z_n|), \quad (30)$$

where $I_{\mathfrak{L}}(K)$ is given by (27) and $J_{\mathfrak{L}'}(Q,|Z_n|)$ by (28).

B. Exchange Forces

In this section we look for the imaginary part of the optical potential when the Bartlett, Majorana, and Heisenberg exchange forces are included in V_{01}. Let us consider

$$V_{01} = [a_0 + a_\sigma \boldsymbol{\sigma}_0\cdot\boldsymbol{\sigma}_1 + a_\tau \boldsymbol{\tau}_0\cdot\boldsymbol{\tau}_1 + a_{\sigma\tau}(\boldsymbol{\sigma}_0\cdot\boldsymbol{\sigma}_1)(\boldsymbol{\tau}_0\cdot\boldsymbol{\tau}_1)]V(r_{01}). \quad (31)$$

In the calculation of the matrix element (18) there appear now four different terms corresponding to the four terms of V_{01}. Then, when $\mathcal{U}_{0n}{}^{(2)}$ is calculated, one has 16 terms. It is easy to see that any "cross term" with one spin- (or isotopic-spin-) independent part of V_{01} in one matrix element and one spin- (or isotopic-spin-) dependent part of V_{01} in the second matrix element gives no contribution to $\mathcal{U}_{0n}{}^{(2)}$. Consequently, we are left with only four types of terms to calculate, one of which has been calculated in the first section and corresponds to $a_0 V(r_{01})$.

Let us consider first the term corresponding to $a_\tau \boldsymbol{\tau}_0\cdot\boldsymbol{\tau}_1 V(r_{01})$, which contains the product

$$\langle \mathbf{k}_0'\Psi_0|V(r_{01})\boldsymbol{\tau}_0\cdot\boldsymbol{\tau}_1|\mathbf{k}_0''\psi_n\rangle\langle\mathbf{k}_0''\psi_n|V(r_{01})\boldsymbol{\tau}_0\cdot\boldsymbol{\tau}_1|\mathbf{k}_0\Psi_0\rangle.$$

The isotopic-spin part can be factorized and calculated independently of the spin and space part. After summation and averages over isospin, the contribution of this term is found as

$$\sum_n \delta_{T_n,1}\mathcal{U}_{0n}{}^{(2)}, \quad (32)$$

where $\mathcal{U}_{0n}{}^{(2)}$ has been calculated in Sec. 1.

Let us define

$$\mathcal{U}_\sigma{}^{(2)} = \frac{2m}{\hbar^2}\sum_n \sum_{\substack{\sigma_{0z}',\sigma_{0z}'' \\ \tau_{0z}',\tau_{0z}''}} \mathcal{S}_{\sigma_{0z},\tau_{0z}} \int d\mathbf{k}_0'' \frac{\langle \mathbf{k}_0'\psi_0|V(r_{01})\boldsymbol{\sigma}_0\cdot\boldsymbol{\sigma}_1|\mathbf{k}_0''\psi_n\rangle\langle\mathbf{k}_0''\psi_n|V(r_{01})\boldsymbol{\sigma}_0\cdot\boldsymbol{\sigma}_1|\mathbf{k}_0\psi_0\rangle}{Z_n{}^2 - k_0''{}^2 + i\delta}. \quad (33)$$

For the calculation of $\mathcal{U}_\sigma{}^{(2)}$, the same method is used as in Sec. 1. Its contribution to the imaginary part of the optical potential is obtained

$$\mathrm{Im}\mathcal{U}_\sigma = W^{(\sigma)} = \sum_{n,\Delta E_n \le \omega_0} \delta_{T_n,0} W_n{}^{(\sigma)}, \quad (34)$$

where

$$W_n{}^{(\sigma)} = -\frac{m}{(2\pi)^5\hbar^2} V_{\mathrm{av}}(q)^2 |Z_n| \sum_{ab\alpha\beta} \chi_n{}^*(ab)\chi_n(\alpha\beta)(-1)^{j_b+j_\beta+1+l_b+l_\beta}\hat{j}_a\hat{j}_b\hat{j}_\alpha\hat{j}_\beta \sum_\Lambda \hat{\Lambda}^2 \begin{Bmatrix} l_a & \frac{1}{2} & j_a \\ l_b & \frac{1}{2} & j_b \\ \Lambda & 1 & J_n \end{Bmatrix} \begin{Bmatrix} l_\alpha & \frac{1}{2} & j_\alpha \\ l_\beta & \frac{1}{2} & j_\beta \\ \Lambda & 1 & J_n \end{Bmatrix}$$

$$\times \sum_{\substack{\lambda\lambda'\mathfrak{l}\mathfrak{l}'LL'\mathfrak{L}\mathfrak{L}' \\ nNn'N'}} (-i)^{\mathfrak{L}+\mathfrak{L}'}\hat{\lambda}^2\hat{\lambda}'^2(2\mathfrak{L}+1)(2\mathfrak{L}'+1)\hat{\mathfrak{l}}\hat{\mathfrak{l}}'\hat{L}\hat{L}'\langle nlNL,\lambda|n_a l_a n_\alpha l_\alpha,\lambda\rangle\langle n'l'N'L',\lambda'|n_b l_b n_\beta l_\beta,\lambda'\rangle$$

$$\times \begin{pmatrix} l & l' & \mathfrak{L}' \\ 0 & 0 & 0 \end{pmatrix}\begin{pmatrix} L & L' & \mathfrak{L} \\ 0 & 0 & 0 \end{pmatrix}\sum_\kappa \hat{\kappa}^2 \begin{pmatrix} \mathfrak{L}' & \mathfrak{L} & \kappa \\ 0 & 0 & 0 \end{pmatrix}\begin{pmatrix} \Lambda & \Lambda & \kappa \\ 0 & 0 & 0 \end{pmatrix} \begin{Bmatrix} l & L & \lambda \\ l' & L' & \lambda' \\ \mathfrak{L}' & \mathfrak{L} & \kappa \end{Bmatrix} \begin{Bmatrix} l_a & l_b & \Lambda \\ l_\alpha & l_\beta & \Lambda \\ \lambda & \lambda' & \kappa \end{Bmatrix} I_{\mathfrak{L}}(K) J_{\mathfrak{L}'}(Q,|Z_n|). \quad (35)$$

The fourth term of interaction with $(\boldsymbol{\sigma}_0\cdot\boldsymbol{\sigma}_1)(\boldsymbol{\tau}_0\cdot\boldsymbol{\tau}_1)\times V(r_{01})$ gives the following contribution

$$\mathrm{Im}\mathcal{U}_{\sigma\tau}=W^{(\sigma\tau)}=\sum_{n,\Delta E_n\leq\omega_0}\delta_{T_n,1}W_n{}^{(\sigma)}. \quad (36)$$

Collecting the previous results, the nonantisymmetrized imaginary optical potential obtained from the interaction (31) is

$$W(\mathbf{k}_0,\mathbf{k}_0')=\sum_{n,\Delta E_n\leq\omega_0}[(a_0{}^2W_n{}^{(0)}+a_\sigma{}^2W_n{}^{(\sigma)})\delta_{T_n,}+(a_\tau{}^2W_n{}^{(0)}+a_{\sigma\tau}{}^2W_n{}^{(\sigma)})\delta_{T_n,1}], \quad (37)$$

where $W_n{}^{(0)}$ and $W_n{}^{(\sigma)}$ are given, respectively, by formulas (30) and (35).

C. Antisymmetrization

The optical potential has been calculated replacing V_{01} by $\bar{V}_{01}$. In this section, we should like to calculate the antisymmetrized optical potential using $\bar{V}_{01}=V_{01}(1-P_{01})$ instead of V_{01}. The P_{01} antisymmetrization operator is

$$P_{01}=P^rP^\sigma P^\tau,$$

P^r, P^σ, P^τ being, respectively, the exchange operator of space, spin, and isospin variables; P^σ and P^τ can be expressed in terms of $\boldsymbol{\sigma}_0\cdot\boldsymbol{\sigma}_1$ and $\boldsymbol{\tau}_0\cdot\boldsymbol{\tau}_1$, respectively, and the introduction of such operators will not change the structure of the results. But the introduction of P^r, the operator which exchanges space variables, will complicate calculations. We avoid these difficulties by working now with a zero-range potential. With this type of potential, P^r does not affect our determination of the optical potential.

From now on we work with the potential

$$V_{01}=-V_0(a_0+a_\sigma\boldsymbol{\sigma}_0\cdot\boldsymbol{\sigma}_1)\delta(\mathbf{r}_0-\mathbf{r}_1). \quad (38)$$

It is important to note that our choice of the 0-nucleon-1-nucleon interaction must be the same as the nucleon-nucleon interaction involved in the determination of nuclear wave functions. Now it has been shown that a zero-range interaction can be determined such as to reproduce the same nuclear properties as a Rosenfeld mixture force. If we take for the parameters V_0, a_0, and a_σ the values which were deduced in this way, we do not destroy the coherence of our treatment, and will obtain the optical potential to a good approximation. Furthermore, such a potential has a Fourier transform equal to $-V_0$, and the previous expressions for $W_n{}^{(0)}$ and $W_n{}^{(\sigma)}$ are still valid, when we replace $V_{\mathrm{av}}(q)^2$ by $V_0{}^2$. The imaginary part of the nonantisymmetrized optical potential should be

$$W=\sum_{n,\Delta E_n\leq\omega_0}(a_0{}^2W_n{}^{(0)}+a_\sigma{}^2W_n{}^{(\sigma)})\delta_{T_n,0}.$$

The antisymmetrized optical potential is now easy to calculate if we replace V_{01} by

$$\begin{aligned}V_{01}(1-P_{01})&=V_{01}(1-P^\sigma P^\tau)\\&=-V_0[A_0+A_\sigma\boldsymbol{\sigma}_0\cdot\boldsymbol{\sigma}_1+A_\tau\boldsymbol{\tau}_0\cdot\boldsymbol{\tau}_1\\&\quad+A_{\sigma\tau}(\boldsymbol{\sigma}_0\cdot\boldsymbol{\sigma}_1)(\boldsymbol{\tau}_0\cdot\boldsymbol{\tau}_1)]\delta(\mathbf{r}_0-\mathbf{r}_1),\end{aligned} \quad (39)$$

where

$$\begin{aligned}A_0&=\tfrac{3}{4}(a_0-a_\sigma),\\A_\sigma&=\tfrac{1}{4}(5a_\sigma-a_0),\\A_\tau&=-\tfrac{1}{4}(a_0+3a_\sigma),\\A_{\sigma\tau}&=-\tfrac{1}{4}(a_0-a_\sigma).\end{aligned} \quad (40)$$

This operator has the same structure as the most general V_{01} given by (31) and we write directly the corresponding imaginary part of the antisymmetrized optical potential

$$W(\mathbf{k}_0,\mathbf{k}_0')=\sum_{n,\Delta E_n\leq\omega_0}[(A_0{}^2W_n{}^{(0)}+A_\sigma{}^2W_n{}^{(\sigma)})\delta_{T_n,0}+(A_\tau{}^2W_n{}^{(0)}+A_{\sigma\tau}{}^2W_n{}^{(\sigma)})\delta_{T_n,1}], \quad (41)$$

where A_0, A_σ, A_τ, and $A_{\sigma\tau}$ are related to the parameters of the force by Eq. (40), and where $W_n{}^{(0)}$ and $W_n{}^{(\sigma)}$ are given by Eqs. (31) and (35) with

$$V_{\mathrm{av}}(q)^2=V_0{}^2.$$

IV. NUMERICAL RESULTS AND DISCUSSION

We have calculated the imaginary part of the optical potential as its Fourier component $W(\mathbf{k}_0,\mathbf{k}_0')$ from Eq. (41) for the case of ^{12}C and for several choices of the TD and RPA eigenvector sets and several mixtures of exchange forces. For the incident nucleon energy we choose $\omega_0=20$ MeV. At this energy the target-exchange-type terms and other higher order corrections to our approximation to the antisymmetrization should still be rather small, while the TD and RPA approximations are valid for most of the nuclear levels involved. For higher incident energies ω_0, modes involving heavy weights of two-particle–two-hole components should be quite important for many nuclear levels involved, and treatments of them are not available in the literature. In addition, the number of levels involved increases greatly with increasing ω_0, thus rendering computations extremely tedious. All our numerical analyses were performed on the Univac computer of the Faculté des Sciences at Orsay.

The RPA eigenvectors of all but one of the nuclear levels involved have been taken from Vinh Mau[22b] as corresponding to the nucleon-nucleon potential of the Gaussian radial dependence: $-V_0e^{-(r_{12}/\mu)^2}$ with $V_0=40$ MeV, $\mu=1.7$ F, and the exchange force mixture $W=0.37$, $m=0.38$, $b=-0.15$, $h=0.40$. Unfortunately, the lowest lying level with $T=0$, the $2+$ state observed at 4.43 MeV, could not be reproduced well by the above calculation, probably because it should involve an appreciable admixture of two-particle–two-hole components ignored in the simple TD or RPA (the lowest

TABLE I. Values of the parameters a_0 and a_σ for the zero-range nucleon-nucleon potential $V_{01}=V_0\delta(\mathbf{r}_{01})(a_0+a_\sigma\boldsymbol{\sigma}_0\cdot\boldsymbol{\sigma}_1)$, and corresponding values of the A parameters, defined in Eq. (40), for different exchange-force mixtures.

Forces	a_0	a_σ	A_0	A_σ	A_τ	$A_{\sigma\tau}$
Gillet (a)	0.9	0.1	0.6	−0.1	−0.3	−0.2
Vinh-Mau (b)	0.862	0.125	0.553	−0.059	−0.309	−0.184
Soper (c)	0.865	0.135	0.5475	−0.0475	−0.3175	−0.1825
Rosenfeld (d)	0.9	0.1	0.6	−0.1	−0.3	−0.2
Ferrell-Visscher (e)	0.9085	0.0915	0.613	−0.113	−0.296	−0.204
Serber (f)	1	0	0.75	−0.25	−0.25	−0.25

TD $T=0$ 2+ state obtained with the above nuclear force lies at 9.16 MeV, while the corresponding RPA eigenvalue turns out to be imaginary). The harmonic-oscillator wave-function parameter b_0 is fixed so that $\mu/b_0=1.02$. The (most important) 12 RPA levels involved taken from Ref. 22(b) are

$T=0$: 1^-, 4.62 MeV; 1^-, 10.4 MeV; 1^+, 13.2 MeV; 2^-, 14.0 MeV; 2^-, 15.2 MeV; 2^+, 16.7 MeV;

$T=1$: 2^+, 16.1 MeV; 1^+, 17.4 MeV; 1^-, 18.1 MeV; 2^-, 18.6 MeV; 3^-, 18.8 MeV; 2^-, 19.8 MeV.

For the most important first $T=0$, 2^+ state, we have considered three other theoretical models available.[26] The one most consistent with the above is that of Gillet[22a] which corresponds to the same radial well shape as the one above, the same μ/b_0 ratio, $w=m=h=0.4$, and $b=-0.2$. The energy of this level is then found[22a] at 4.80 MeV. Goswami and Pal[27] have assumed for this level its experimental value of 4.43 MeV and found the corresponding TD eigenvector on adjusting the most important radial matrix elements of the V involved. In the third model considered in our calculations the same authors[28] found the 4.43-MeV-state RPA eigenvector for the nucleon-nucleon potential with the Yukawa radial well shape $-V_0e^{-r/\alpha}/(r/\alpha)$, $V_0=37$ MeV, $\alpha=1.36$ F, and the exchange-force parameters of Soper: $w=0.3$, $m=0.43$, $b=0.27$, and $h=0$.

In general, our RPA is to be interpreted as an improved TD in the sense that we obviously throw away the unphysical negative-energy RPA states as discussed above. On the other hand, the ground-state correlations involved in the RPA and the at least partial elimination of the center-of-mass spuriousness by the same method render it generally much superior relative to TD.

In calculating $W(\mathbf{k}_0,\mathbf{k}_0')$ of Eq. (51) with our zero-range force, we have considered several exchange-force mixtures:

(a) Gillet[22a] as mentioned above: $w=m=h=0.4$, $b=-0.2$;

(b) Vinh Mau[22b] as mentioned above: $w=0.37$, $m=0.38$, $b=-0.15$, $h=0$;

(c) Soper as mentioned above[27]: $w=0.3$, $m=0.43$, $b=0.27$, $h=0$;

(d) Rosenfeld: $w=-0.13$, $m=0.93$, $b=0.46$, $h=-0.26$;

(e) Ferrell and Visscher[29]: $w=0.317$, $m=0.5$, $b=0$, $h=0.183$; and

(f) Serber: $w=m=0.5$, $b=h=0$.

It is remarkable that our parameters A_0, A_σ, A_τ, and $A_{\sigma\tau}$ of Eq. (40) differ very little among themselves in spite of large differences in the corresponding parameters w, m, b, and h. These constants of Eq. (40) are found for the mentioned cases in Table I.

In view of this insensitivity of our $W(\mathbf{k}_0,\mathbf{k}_0')$ to exchange-force mixtures[30] we may confine ourselves to the cases (a) and (b) [the latter case is practically equivalent to that of the Soper mixture (c)].

In Table II we present our results for the imaginary part of the optical-model potential $W(\mathbf{k}_0,\mathbf{k}_0')$ of Eq. (51) for $\omega_0=20$ MeV; the zero-range nucleon-nucleon potential with $V_0/(4\pi b_0^3)=-10.2$ MeV, $b_0=1.61$ F; the exchange-force mixtures (a) and (b); and the energies and wave functions (RPA eigenvalues and eigenvectors) corresponding to Ref. 22(b), with the exception of the lowest lying $T=0$, 2_1^+ level, for which we $T=0$, 2_1^+ level, for which we take the results of Ref. 22(a) or Ref. 28.

In order to exhibit the effect of antisymmetrization, we have also calculated our $W(\mathbf{k}_0,\mathbf{k}_0')$ with nonantisymmetrized V-matrix elements.

We observe that the differences between the cases corresponding to the 2_1^+ state chosen as in Ref. 22(a) and in Ref. 28, for the same exchange-force mixture, are quite small, and that these two cases are practically equivalent. The respective differences in $W(\mathbf{k}_0,\mathbf{k}_0')$ between the exchange-force mixtures (a) and (b) for the same model of the 2_1^+ state are at most of the order of 15 MeV F^3.

If we had chosen the RPA 2_1^+-state eigenvector of Ref. 22(b) and replaced its imaginary eigenvalue by the experimental energy of 4.43 MeV, we would have found,

[26] The objection to the TD (RPA) 2_1^+ state as calculated in Ref. 22b, may be applied to these models as well (and to some other even-parity states). However, we may view the TD (RPA) configuration mixing of these other models corresponding to a good 2_1^+ state energy as "reasonably adjusted."

[27] A. Goswami and M. K. Pal, Nucl. Phys. **35**, 544 (1962).

[28] A. Goswami and M. K. Pal, Nucl. Phys. **44**, 294 (1963).

[29] R. A. Ferrell and W. M. Visscher, Phys. Rev. **102**, 450 (1956).

[30] It should be noted, however, that this property, valid for our zero-range nucleon-nucleon potential, may be invalid for finite-range potentials.

TABLE II. The imaginary part of the optical $W(\mathbf{k}_0,\mathbf{k}_0')$ for different choices of force mixtures and wave functions (in units MeV F^3), and for $\omega_0=20$ MeV.

	Force mixture	Wave function	18°	36°	54°	72°	90°	108°	126°	162°	180°
I	(a) (nonanti-symmetrized)	State 2_1^+ of Ref. 22a	−167.8	−166.3	−162.6	−155.7	−144.1	−126.6	−102.7	−28.2	−0.2
II	(a)	State 2_1^+ of Ref. 22a	−70.0	−69.4	−68.0	−65.2	−60.5	−53.3	−43.3	−11.6	−0.1
III	(b)	State 2_1^+ of Ref. 22a	−57.3	−56.8	−55.7	−53.4	−49.6	−43.7	−35.5	−9.5	−0.1
IV	(a)	State 2_1^+ of Ref. 28	−71.5	−71.2	−70.0	−67.3	−62.3	−54.5	−44.0	−11.5	+0.1
V	(b)	State 2_1^+ of Ref. 28	−58.0	−57.8	−56.9	−54.8	−50.7	−44.5	−35.9	−9.4	+0.1

for the Soper exchange-force mixture, $W(\mathbf{k}_0,\mathbf{k}_0')$ at $\theta=18°$, the value of the order of −200 MeV F^3, which is much too large; the TD 2_1^+-state eigenvector of Ref. 27 would have given 4.43 MeV for this state; the same exchange-force mixture and θ yields $W(\mathbf{k}_0,\mathbf{k}_0')$ of the order −17.0 MeV F^3 which is much too little as compared with our results of Table II.

From the results of the first line of Table II, we see that the use of nonantisymmetrized V-matrix elements (which would have followed from a direct application of the Feshbach[5] formula) leads to values of $W(\mathbf{k}_0,\mathbf{k}_0')$ that are much too large (by a factor of 2–3).

In order to understand the relative importance of the contributions of the various types of nuclear states involved, we have analyzed them, as an example, for the case of the second line in Table II. For this case, we find the total net contribution of all the $T=1$ states very small (+4.5 MeV F^3 at $\theta=18°$). All the $T=0$ states excluding the 2_1^+ state (4.43 MeV) contribute +66.4 MeV F^3 at the same $\theta=18°$. The most important predominant contribution of a negative sign comes then from the 2_1^+ state (it is equal to −141.0 MeV F^3 for $\theta=18°$). The θ angular dependence of all these respective contributions is like that of their sum $[W(\mathbf{k}_0,\mathbf{k}_0')]$.

In Fig. 1, we compare our $W(\mathbf{k}_0,\mathbf{k}_0')$ of the second and the third lines of Table II, with several phenomenological potentials appropriate to ^{12}C and to our energy ω_0. Of the available phenomenological imaginary potentials we have chosen the following ones

(1) the "surface absorption" Gaussian:

$$-W_1\exp[-(r-R)^2/\gamma^2]$$

of Nodvik *et al.*[31] with (a) $W_1=23.2$ MeV, $R=1.25A^{1/3}$ F, $\gamma=0.25$ F and (b) $W_1=12.4$ MeV, $R=1.20A^{1/3}$ F, $\gamma=0.5$ F;

(2) the "surface absorption" of the form of the derivative of a Saxon-Woods well[31]

$$-4aW_1(d/dr)[1+\exp\{(r-R)/a\}]^{-1},$$

with (a) $W_1=23.2$ MeV, $R=1.25A^{1/3}$ F, $a=0.49$ F, (b) $W_1=12.4$ MeV, $R=1.20A^{1/3}$ F, $a=0.53$ F;

(3) the "volume plus surface" absorption of the Saxon-Woods well[31]

$$-Wf(r)-4_aW_1(df/dr),\quad f(r)=[1+\exp\{(r-R)/a\}]^{-1}$$

with $W=2.0$ MeV, $W_1=20.7$ MeV, $R=1.20A^{1/3}$ F, $a=0.53$ F;

(4) the nonlocal form of Perey and Buck[32]

$$W(\mathbf{r},\mathbf{r}')=-H(|\mathbf{r}-\mathbf{r}'|)U(\tfrac{1}{2}|\mathbf{r}+\mathbf{r}'|),$$

where

$$H(x)=\pi^{-3/2}\beta^{-3}\exp(-x^2/\beta^2),$$
$$U(y)=W_D\times 4e^{-(y-R)/a_D}[1+e^{(y-R)/a_D}]^{-2},$$

with $\beta=1.0$ F, $W_D=7.0$ MeV, $R=1.25A^{1/3}$ F, $a_D=0.65$ F. Of these, the potentials (1)–(3) are appropriate to $\omega_0=19.4$ MeV and to ^{12}C; the nonlocal potential (4) is appropriate rather to higher elements, and at lower

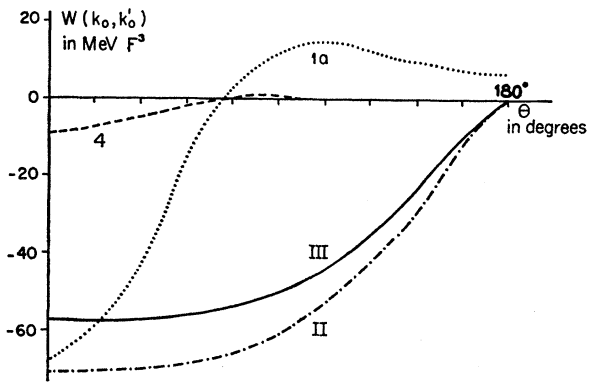

FIG. 1. $\mathrm{Im}v_{\mathrm{opt}}\equiv W(\mathbf{k}_0,\mathbf{k}_0')$ (in MeV F^3) as a function of $\theta=\sphericalangle(\mathbf{k}_0,\mathbf{k}_0')$, nucleon energy $\omega_0=20$ MeV, for the following cases: II: the exchange-force mixture of Gillet (Ref. 22a); III: the exchange-force mixture of Vinh Mau (Ref. 22b); the particle-hole eigenvectors are as from Vinh Mau (Ref. 22b) except for the state 2_1^+ which is taken as from Gillet (Ref. 22a). The corresponding phenomenological $\mathrm{Im}v_{\mathrm{opt}}$ are: (1a) the "surface absorption" Gaussian of Nodvick *et al.* (Ref. 31); (4) the nonlocal form of Perey and Buck (Ref. 32).

[31] J. S. Nodvik, C. B. Duke, and M. A. Melkanoff, Phys. Rev. **125**, 975 (1962).

[32] F. Perey and B. Buck, Nucl. Phys. **32**, 353 (1962).

energies for ^{12}C the corresponding well depth should be considerably increased, according to Wilenzick *et al.*[33]

In Fig. 1, we present the Fourier transforms $(\mathbf{k}_0,\mathbf{k}_0')$ of the phenomenological potentials (1a) and (4). In fact, those of the potentials (1b)–(3) are rather quite similar to that of the potential (1a). Our theoretical $W(\mathbf{k}_0,\mathbf{k}_0')$ are of approximately the same well depth in the forward direction ($\theta \lesssim 20°$), but are much flatter for larger θ than the Fourier transform of the phenomenological potential (1a). The θ angular distribution for the potential (4) is more similar to that of our $W(\mathbf{k}_0,\mathbf{k}_0')$, only the depth of the potential (4) appears somehow to be too shallow, in agreement with Ref. 33. It appears that a phenomenological local $\mathrm{Im}\mathcal{U}_{\mathrm{opt}}$ which could correspond to our $W(\mathbf{k}_0,\mathbf{k}_0')$ should have a volume absorption radial shape of the Saxon-Woods type with a rather small value of the parameter R.

The "flatness" of the θ dependence of our $W(\mathbf{k}_0,\mathbf{k}_0')$ [the disagreement with the corresponding phenomenological $W(\mathbf{k}_0,\mathbf{k}_0')$ at large θ as in Fig. 1] is probably due to the combined effect of the zero range of our $V(0,1)$ and the bad asymptotic (large r) behavior of the radial wave functions of the harmonic-oscillator potential.

Finally, we should like to remark that it would be extremely interesting to perform a similar calculation of $W(\mathbf{k}_0,\mathbf{k}_0')$ with a corresponding RPA set based on Nilsson-type wave functions of ^{12}C, with a negative deformation (cf., e.g., Nilsson *et al.*[22] for the $T=1$, 1^- states). Another basis of nuclear states which could be considered would correspond to recent Hartree-Fock calculations with deformed orbitals.[34]

ACKNOWLEDGMENTS

We would like to thank Dr. C. Marty for many helpful discussions and suggestions. One of us (J.S.) expresses his thanks to Professor M. Jean for the kind hospitality extended to him at the Laboratoire de Physique Théorique d'Orsay (groupe I).

APPENDIX I

A. The Self-Energy Operator $\Sigma(0,0')$ in the Green's Function Formalism

We define the n-particle Green's function as zero-temperature ($T=0$) limits of grand canonical ensemble averages (cf., e.g., Ref. 13), and we use the definition:

$$G_n(12\cdots n;1'2'\cdots n') \equiv (-i)^n \times \langle N0|\,T\{\psi(1)\cdots\psi(n)\psi^\dagger(n')\cdots\psi^\dagger(1')\}\,|N0\rangle, \quad \text{(A1)}$$

where N is the total number of particles ($=A+1$ in our case), $|N0\rangle$ represents the $T=0$ limit state of this system (as a grand canonical ensemble).

For a particle "0" we can write the free one-particle propagator in the form

$$[G_1^{(0)}(0,0')]^{-1}=[i\hbar(\partial/\partial t_0)-h_0(\mathbf{0})]\delta(0-0'), \qquad h_0(\mathbf{0})=(-\hbar^2/2m)\nabla_0^2. \quad \text{(A2)}$$

The equation of motion of the (exact) function G_1 can be written in the form (in the absence of any external forces)

$$[G_1^{(0)}(0,\tilde{0})]^{-1}G_1(\tilde{0},0') = \delta(0-0')-iV(0,1)G_2(01^-,0'1^+). \quad \text{(A3)}$$

Generalization of the present formalism to nonlocal potentials is immediate. If the self-energy operator $\Sigma(0,0')$ is defined by

$$[G_1(0,0')]^{-1}=[G_1^{(0)}(0,0')]^{-1}-\Sigma(0,0'), \quad \text{(A4)}$$

we easily obtain Eq. (1) of the main text.

The lowest order approximation to Σ is usually chosen to follow from the linearization of the equation of motion for G_2, e.g., from a factorization of the respective three-particle Green's function G_3. Following Martin and Schwinger,[10] we can write

$$G_2(01,0'1') = (G_1(0,0')G_1(1,1') - \mathrm{ex}) - iG_1(0,\bar{0})V(\bar{0},2)\{G_3(\bar{0}12,0'1'2) - [G_2(\bar{0}2,0'2)G_1(1,1') - G_2(\bar{0}2,1'2)G_1(1,0')]\}, \quad \text{(A5)}$$

where "ex" stands for the exchange terms. One possible *ansatz* factorization of G_3 is

$$G_3(\bar{0}12,0'1'2)\cong G_2(\bar{0}2,0'2)G_1(1,1') - G_2(\bar{0}2,1'2)G_1(1,0')+G_2(2\bar{0},0'1')G_1(1,2)+\Delta'. \quad \text{(A6)}$$

The higher order terms Δ' can be put in the form

$$\Delta' \equiv G_2(12,1'2)G_1(\bar{0},0') - G_2(12,0'2)G_1(\bar{0},1') + G_2(12,0'1')G_1(\bar{0},2) - G_2(\bar{0}1,0'1')G_1(2,2) + G_2(\bar{0}1,0'2)G_1(2,1') - G_2(\bar{0}1,1'2)G_1(2,0') + \text{other higher order corrections.} \quad \text{(A7)}$$

It is easy to see that in the noninteracting case $\Delta'\equiv 0$.

On substituting Eq. (A6) into (A5) the first two terms cancel out the square-bracket factor in Eq. (A5).

The third term on the right-hand side of Eq. (A6) is the most important one for long-range slowly varying forces and for short-range forces at sufficiently low densities. For this term both the two-particle scattering and correlation refer to the same pair in the final formula for Σ. If we drop Δ' we have

$$G_2(01,0'1')\cong(G_1(0,0')G_1(1,1')-\mathrm{ex}) - iG_1(0,\bar{0})G_1(1,2)V(\bar{0},2)G_2(2\bar{0},0'1'), \quad \text{(A8)}$$

where "ex" stands for the exchange terms. We can now express the solution formally in terms of the Møller Ω matrix which is the Green's function of the G_2 equation, Eq. (4) of the main text.

[33] R. M. Wilenzick, K. K. Seth, P. R. Bevington, and H. W. Lewis, Nucl. Phys. **62**, 511 (1965).

[34] D. Kurath and L. Pičman, Nucl. Phys. **10**, 313 (1959); D. Koltun, Phys. Rev. **124**, 1162 (1961) and thesis, Princeton University, 1961 (unpublished); W. H. Bassichis (private communication).

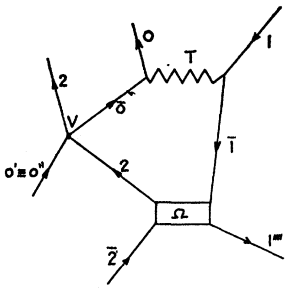

FIG. 2. A diagrammatic representation of $\Delta_{(1)}\Sigma(0,0')$ of Eq. (A10).

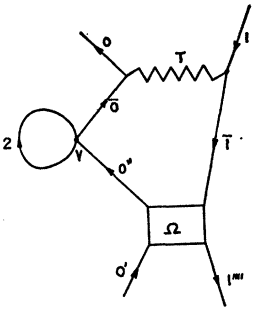

FIG 3. A. diagrammatic representation of $\Delta_{(4)}\Sigma(0,0')$ of Eq. (A11).

Consequently,

$$G_2(01,0'1')=\langle 01|\Omega|\bar{0}\bar{1}\rangle G_1(\bar{0},0')G_1(\bar{1},1'). \quad \text{(A9)}$$

On substituting Eq. (A9) into (1) of the main text, we finally obtain our Eqs. (2), (3) of the same.

The contribution to $\Sigma(0,0')$ of the first term of Δ' of Eq. (A7) can be put in the form

$$\begin{aligned}\Delta_{(1)}\Sigma(0,0')\equiv&-iV(0,x)\Delta_{(1)}G_2(0x,yx)[G_1(y,0')]^{-1}\\ =&-\langle 01|T|\bar{0}\bar{1}\rangle G_1(\bar{0},0'')V(0'',2)\delta(0''-0')\\ &\times\langle\bar{1}2|\Omega|1''''\bar{2}\rangle G_1(1'''',1)G_1(\bar{2},2). \quad \text{(A10)}\end{aligned}$$

This contribution can be represented symbolically as in Fig. 2. The pair correlation matrix Ω we denote by a block diagram, a V interaction by a dot (point), and a T interaction by a wiggly line. We see that we have here a correlation of one pair connected to a V scattering of another one which in turn is coupled to a T scattering of one member of each pair with each other. This is a rather typical term of this group. In the case when, e.g., the lines $0''$, $0'$, $\bar{1}$, and $1''''$ are rewritten as holes, and 2 and $\bar{2}$ as particles, the contribution has a "target exchange" character. Because of this triple "interconnected" character such a $\Delta_{(1)}\Sigma(0,0')$ of Eq. (A10) is a rather small contribution—as are the usual "target exchange" corrections.[16] However, we have not as yet performed any numerical calculations of such corrections for our model.

The contribution of the fourth term of Eq. (A7) is

$$\begin{aligned}\Delta_{(4)}\Sigma(0,0')=&\langle 01|T|\bar{0}\bar{1}\rangle G_1(\bar{0},0'')V(0'',2)\\ &\times G_1(2,2)\langle 0''\bar{1}|\Omega|0'1''''\rangle G_1(1'''',1). \quad \text{(A11)}\end{aligned}$$

$\Delta_{(4)}\Sigma(0,0')$ can be represented graphically as in Fig. 3. This could be the most significant correction term, as it represents a T scattering and an Ω correlation of the same pair with an intermediate interaction of one member of this pair with a third particle 2 in the Fermi sea.

The other terms of Δ' give quite similar contributions. All of them should naturally be expected to be small for a system of not too high a density. An estimate of such terms could be attempted by using a Jastrow-type correlation function replacing the Ω elements (cf., e.g., Ref. 35).

[35] J. Da Providencia and C. M. Shakin, Nucl. Phys. **65**, 54 (1965); **65**, 75 (1965).

B. Approximation for the T Matrix by Iterations in Terms of $T^{(0)}$

The system of Eqs. (2), (3), and (4) of the main text is, in general, very difficult to solve because of its nonlinearity. It is customary to work rather with $T^{(0)}\equiv V\Omega^{(0)}$ where the matrix $\Omega^{(0)}$ satisfies Eq. (9) with G_1 replaced by $G_1^{(0)}$, the free (unperturbed) propagators.

One practical successive-approximation procedure for the operator Σ could be developed in the following way (in the obvious shorthand notation)

$$\Sigma^{(0)}=-iT^{(0)}G_1^{(0)}, \quad T^{(0)}=V(1-iG_1^{(0)}G_1^{(0)}T^{(0)}). \quad \text{(A12)}$$

Now

$$G_1^{(1)}\equiv G_1^{(0)}-iG_1^{(0)}VG_2^{(0)}, \quad \text{(A13)}$$

where

$$\begin{aligned}G_2^{(0)}=&(G_1^{(0)}G_1^{(0)}-\text{ex})\\ &-iG_1^{(0)}G_1^{(0)}VG_2^{(0)}\equiv\Omega^{(0)}G_1^{(0)}G_1^{(0)}, \quad \text{(A14)}\end{aligned}$$

and $VG_2^{(0)}=T^{(0)}G_1^{(0)}G_1^{(0)}$,
whence

$$G_1^{(1)}=G_1^{(0)}(1-iT^{(0)}G_1^{(0)}G_1^{(0)}). \quad \text{(A15)}$$

We define now $T^{(1)}$ by the equation

$$T^{(1)}=V(1-iG_1^{(1)}G_1^{(1)}T^{(1)}). \quad \text{(A16)}$$

Thus, on eliminating V,

$$\begin{aligned}\Delta T=T^{(1)}-T^{(0)}\cong&-iT^{(0)}(G_1^{(1)}G_1^{(1)}-G_1^{(0)}G_1^{(0)})T^{(1)}\\ \cong&-iT^{(0)}(G_1^{(1)}G_1^{(1)}-G_1^{(0)}G_1^{(0)})T^{(0)}\\ \cong&-T^{(0)}(G_1^{(0)}T^{(0)}G_1^{(0)}G_1^{(0)}G_1^{(0)}\\ &+G_1^{(0)}G_1^{(0)}T^{(0)}G_1^{(0)}G_1^{(0)})T^{(0)}. \quad \text{(A17)}\end{aligned}$$

Thus the lowest order correction is cubic in $T^{(0)}$ and so is

$$\Delta\Sigma^{(1)}=\Sigma^{(1)}-\Sigma^{(0)}=-i(\Delta T)G_1.$$

One can see this result also from the difference $(G_1^{(1)}G_1^{(1)}-G_1^{(0)}G_1^{(0)})$ in $\Delta G_2=G_2^{(1)}-G_2^{(0)}$ and which one uses in

$$\begin{aligned}\Delta\Sigma^{(1)}=&\Delta(VG_2)\cdot(G_1)^{-1}\\ =&V\Omega^{(0)}(\Delta G_2)G_1^{-1}=T^{(0)}(\Delta G_2)G_1^{-1}.\end{aligned}$$

If we employ the formulas of Eqs. (15) and (89)–(91) of Puff,[11] we obtain

$$\Delta\Sigma(1,1')=i\langle 1,2|T^{(0)}|\bar{1}\bar{2}\rangle G_1^{(0)}(\bar{1},1'''')G_1^{(0)}(\bar{2},2'''')\times\{\langle 1''''2'|T^{(0)}|1''2''\rangle\langle 2''''1'''|T^{(0)}|2'''1'\rangle+\langle 1''''2'|T^{(0)}|1'2''\rangle\langle 2''''1'''|T^{(0)}|2'''1''\rangle\}\times G_1^{(0)}(2''',2')G_1^{(0)}(2'',2)G_1^{(0)}(1'',1'''). \quad \text{(A18)}$$

This is just the result of Eq. (A17).

This expansion, with such complicated terms involving products of three $T^{(0)}$ operators as the lowest order corrections, converges rapidly for sufficiently high energies. Here again we have no numerical results. This expansion in successive corrections to the mutual interaction of two particles (1,2)—one pair only—has obviously nothing to do with the Watson multiple-scattering formalism.

APPENDIX II

$\mathcal{U}_{\text{opt}}$ for Infinite Nuclear Matter

Equations (2) and (11) are valid for the case of infinite nuclear matter in which the ground state of the A system is represented by a degenerate (or slightly correlated) Fermi sea of plane waves, and the excited states by a complete set of the RPA states as constructed by Sawada *et al.*[36] for the case of the electron gas. We find

$$\Sigma(\mathbf{k}_0,\mathbf{k}_0';\omega_0)=AV(0)\delta(\mathbf{k}_0-\mathbf{k}_0')-\tfrac{1}{4}\sum_{\kappa<\kappa_F}V(\mathbf{k}_0'-\boldsymbol{\kappa})+\sum_n\sum_{\boldsymbol{\kappa}\boldsymbol{\kappa}'\boldsymbol{\kappa}''\boldsymbol{\kappa}'''}n_\kappa(1-n_{\kappa'})a_n(\boldsymbol{\kappa}\boldsymbol{\kappa}')V(\boldsymbol{\kappa}-\boldsymbol{\kappa}')n_{\kappa''}(1-n_{\kappa'''})\times a_n^*(\boldsymbol{\kappa}''\boldsymbol{\kappa}''')[V(\mathbf{k}_0-\mathbf{k}_0'-\boldsymbol{\kappa}+\boldsymbol{\kappa}')-\tfrac{1}{4}V(\mathbf{k}_0'+\boldsymbol{\kappa}-\boldsymbol{\kappa}'-\boldsymbol{\kappa}'')]\delta(\boldsymbol{\kappa}'''-\mathbf{k}_0-\boldsymbol{\kappa}''+\mathbf{k}_0'-\boldsymbol{\kappa}'+\boldsymbol{\kappa})\times[\tilde{E}-\tilde{E}_n+i\delta]^{-1}, \quad \text{(A19)}$$

where $V(0,1)$ has been taken as a simple Wigner force and the spin and isospin included implicitly. Consistently with the RPA model, $\tilde{E}_n$ should be assumed in the form

$$\tilde{E}_n=E_0+\Delta\omega_n+\hbar^2k_0''^2/2m.$$

According to Sawada

$$a_n(\boldsymbol{\kappa},\boldsymbol{\kappa}'=\boldsymbol{\kappa}+\mathbf{q})=N_{\mathbf{q}}^{(n)}[\Delta\omega_n(\mathbf{q})-\epsilon_{\kappa'}^{(0)}+\epsilon_\kappa^{(0)}]^{-1} \quad \text{(A20)}$$

and

$$|N_{\mathbf{q}}^{(n)}|^{-2}=\Big(\sum_{\substack{\kappa<\kappa_F\\ \kappa'>\kappa_F}}-\sum_{\substack{\kappa>\kappa_F\\ \kappa'<\kappa_F}}\Big)\times[\Delta\omega_n(\mathbf{q})-\epsilon_{\kappa'}^{(0)}+\epsilon_\kappa^{(0)}]^2, \quad \text{(A21)}$$

where

$$\epsilon_\kappa^{(0)}=\hbar^2\kappa^2/2m.$$

In the present paper we present no numerical results for this case.

[36] K. Sawada, K. A. Brueckner, N. Fukuda, and R. Brout, Phys. Rev. **108**, 507 (1957).