Helicon Propagation in Metals near the Cyclotron Edge*

A. W. OVERHAUSER
Scientific Laboratory, Ford Motor Company, Dearborn, Michigan
AND
SERGIO RODRIGUEZ
Department of Physics, Purdue University, Lafayette, Indiana
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The theory of Doppler-shifted cyclotron resonance with helicon waves is elaborated by calculating the magnetic-field dependence of the surface impedance near the cyclotron absorption (Kjeldaas) edge. It is found that the location of the edge, defined as that field for which the field derivative of the surface reactance is a minimum, deviates significantly from the threshold field defined by the Doppler-shifted frequency criterion. The deviations are large when $\omega_c\tau<50$. The theory is compared with the experimental work of Taylor on Na and K. The data agree well with the free-electron model for Na, but there is a large discrepancy (770 G for 10-Mc/sec helicons) in K. The surface impedance is also calculated for a metal with a spin-density-wave ground state, in order to test the hypothesis of such a ground state, previously made to explain other anomalies in K. The edge field derived for a spin-density-wave state appropriate to K is 730 G less than that required by the free-electron model, and agrees with the published data.

I. INTRODUCTION

HELICONS[1] are circularly polarized electromagnetic waves propagating in an electron gas in the presence of a magnetic field. A helicon has low damping if the cyclotron period of the electrons in the magnetic field is short compared to the average time between two successive collisions of an electron, i.e., if $\omega_c\tau\gg1$ where ω_c is the electron cyclotron frequency and τ the electron relaxation time. Now, if the frequency of the helicon ω is such that $\omega\ll\omega_c\ll\omega_p^2/\omega$ where ω_p is the plasma frequency of the electron gas, the complex wave number q of the helicon is given by

$$q^2=(q_1+iq_2)^2=(4\pi\omega ne/Bc)f_\pm. \tag{1}$$

In Eq. (1), B is the applied magnetic field, n the concentration of electrons, and $-e$ the charge on the electron. We have assumed here that the wave propagates parallel to the direction of the magnetic field **B** which we take parallel to the z axis of a Cartesian coordinate system. The factor $f_\pm$ is equal to ±1 in the limit of long wavelengths $[|q|\ll(\omega_c/v_F)]$, but differs from unity if $|q|\gtrsim(\omega_c/v_F)$. The analytical expression for $f_\pm$ is[2]

$$f_\pm=\frac{3}{4}\int_0^\pi d\theta\,\sin^3\theta[(qv_F/\omega_c)\cos\theta\pm1+(i/\omega_c\tau)]^{-1}. \tag{2}$$

In the relations given above the upper and lower signs correspond to left-circularly-polarized and right-circularly-polarized waves, respectively. Of these, of course, the right-circularly-polarized wave has a large attenuation and need not be considered further.[3] Typical experimental conditions are, for example, $\nu=(\omega/2\pi)=10$ Mc/sec, $B=10$ kG, and $n=10^{22}$ cm^{-3}. Then $|q|\approx4\times10^3$ cm^{-1}. In this case, the phase velocity of the helicon is approximately $(\omega/|q|)\approx10^4$ cm/sec. This velocity is several orders of magnitude smaller than the Fermi velocity v_F of the electrons. Thus, an electron whose component of velocity parallel to the direction of **B** is v_z experiences a periodic electric field of frequency

$$\omega_a=\omega\pm q_1v_z\approx\pm q_1v_z. \tag{3}$$

Clearly, a resonant absorption of energy occurs when $\omega_c=\omega_a$, i.e., if electrons exist having a component v_z of the velocity satisfying the condition,

$$\omega_c=q_1v_z. \tag{4}$$

If the radiofrequency ω is kept fixed and the magnetic field is decreased from a value such that $\omega_c>q_1v_F$ to values which reverse the inequality, then one expects an absorption edge[4] (called the Kjeldaas edge) at $\omega_c=q_1v_F$. This resonance is also called the Doppler-shifted cyclotron resonance.[5] The Kjeldaas resonance has been observed by Kirsh[6] in bismuth using conditions appropriate for propagation of Alfvén waves[7] and by Taylor *et al.*,[8] and Taylor[9] using helicon waves in sodium, potassium, and indium.

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[1] P. Aigrain, *Proceedings of the International Conference on Semiconductor Physics, Prague, 1960* (Czechoslovak Academy of Sciences, Prague, 1961), p. 224. R. Bowers, C. Legendy, and F. Rose, Phys. Rev. Letters **7**, 339 (1961).

[2] See for example, J. J. Quinn and S. Rodriguez, Phys. Rev. **133**, A1589 (1964).

[3] This is true for the model considered in the text, namely, a free-electron gas embedded in a uniform background of positive charge of the same density. In a metal in which the conduction is due to holes the right-circularly-polarized wave is undamped.

[4] See T. Kjeldaas, Jr., Phys. Rev. **113**, 1473 (1959) for a discussion of the edge in connection with the attenuation of ultrasonic shear waves propagating parallel to an applied magnetic field. For helicons see E. A. Stern, Phys. Rev. Letters **10**, 91 (1963).

[5] P. B. Miller and R. R. Haering, Phys. Rev. **128**, 126 (1962).

[6] J. Kirsh, Phys. Rev. **133**, A1390 (1964).

[7] S. J. Buchsbaum and J. K. Galt, Phys. Fluids **4**, 1514 (1961).

[8] M. T. Taylor, J. R. Merrill, and R. Bowers, Phys. Letters **6**, 159 (1964).

[9] M. T. Taylor, Phys. Rev. Letters **12**, 497 (1964); Phys. Rev. **137**, A1145 (1965).

It is of the utmost importance to distinguish the theoretical and experimental definitions of the edge. The "theoretical" edge B_e is

$$B_e \equiv \text{that field for which } \omega_c = q_1 v_{\max}, \quad (5)$$

where $v_{\max}$ is the maximum value of the z component of electron velocity on the Fermi surface. The experimental definition of the edge is arbitrary. Because the most salient feature of Taylor's data[9] is the sharp minimum in dX/dB, where X is the reactive part of the surface impedance Z, we define the experimental edge B_X to be

$$B_X \equiv \text{that field for which } dX/dB \text{ is a minimum}. \quad (6)$$

Taylor made the conjecture that $B_X = B_e$, an assumption which we shall demonstrate to be generally invalid.

For the free-electron model B_e is easily calculated. Equation (1) together with $v_{\max} = v_F$ yield

$$B_e = (8\hbar^2 c/3e)^{1/3} k_F^{5/3} \nu^{1/3} f_e^{1/3}, \quad (7)$$

where k_F is the Fermi wave number, and

$$f_e \equiv \{\mathrm{Re}[f_+(B_e)]^{1/2}\}^2. \quad (8)$$

Taylor has evaluated approximately the dependence of f_e on the collision time. $f_e \approx 1.5$ for $\omega_c\tau > 50$, and decreases monotonically from this asymptotic value with decreasing $\omega_c\tau$. We have verified this behavior by a more precise calculation. For potassium (taking the lattice constant[10] $a = 5.225$ Å) we find, for $\nu = 10$ Mc/sec,

$$B_e(\omega_c\tau = \infty) = 18.58 \text{ kG}. \quad (9)$$

The calculation of the "experimental" edge B_X is considerably more difficult than that of B_e. The details are given in Sec. II. The conclusions may be summarized here:

(a) $B_X \approx B_e$ for large $\omega_c(18)\tau$ (in excess of about 40).[11]

(b) B_X *increases* monotonically with *decreasing* τ. This behavior is the reverse of that found for B_e.

(c) For the case of potassium at 10 Mc/sec, and $\omega_c(18)\tau \sim 25$, we find B_X to be about 18.73 kG. This is to be compared with the edge observed by Taylor in material of comparable collision time, which was 17.96 kG. There is therefore a discrepancy of 770 G, which seems significant. In Table I of Taylor's paper he found what appeared to be agreement between experiment and theory for the free-electron model. It is clear that this apparent agreement was a spurious consequence of the conjecture which identified B_X with B_e. It should be appreciated that both of these quantities are independent of the electron effective mass.

The existence of this large discrepancy has been verified independently by T. Kushida (private communication). The magnetic field near the Kjeldaas edge was accurately calibrated by Cs NMR. Furthermore, the shape of the Cs NMR was used to precisely determine the pure resistance and reactance modes; and this was verified by an independent experiment using (instead of an impedance bridge) a Pound box, which automatically selects the pure resistance mode.

(d) For the case of sodium the agreement found by Taylor between experiment and the free-electron model is pertinent because the specimen collision times, $\omega_c(25)\tau \sim 100$, were sufficiently long to justify the use of conclusion (a), above.

In order to explore a possible reason for the large discrepancy between experiment and theory for the case of potassium, we have computed B_X, assuming a spin-density-wave ground state.[12] The details are given in Sec. III. The calculated value of B_X at 10 Mc/sec is 18.0 kG, assuming, as above, $\omega_c(18)\tau = 25$. The spin-density-wave energy gap was taken to be 0.62 eV, the value required[12] to explain the optical anomalies[13] in K. The agreement between this calculated value and Taylor's experimentally observed edge is an interesting coincidence.

II. THE SURFACE IMPEDANCE

In this section we calculate the surface impedance of a metal in the presence of a magnetic field $\mathbf{B}$ perpendicular to the surface and of a radiofrequency electromagnetic field of frequency $\omega \ll \omega_c$. The calculation is carried out for the free-electron model. The numerical applications that we give are for the case of potassium with the parameters given in Sec. I. The surface impedance of the material for a left-circularly-polarized wave, assuming that the electrons are reflected specularly from the surface of the metal, is given by the relation

$$Z = R - iX = (8i\omega/c^2)\int_0^\infty dq\left[q^2 + \frac{4\pi i\omega}{c^2}(\sigma_{xx} - i\sigma_{xy})\right]^{-1}, \quad (10)$$

In Eq. (10) σ_{xx} and σ_{xy} are components of the magnetoconductivity tensor with respect to a Cartesian coordinate system whose z axis is parallel to $\mathbf{B}$.

We now show how to calculate $\sigma_{ij}(i,j = x,y,z)$ for an electron gas with an arbitrary dispersion formula $E(\mathbf{k})$. This calculation is considerably more general than is required for our purposes. However we give it here to provide a basis for the analysis of Sec. III and because of its intrinsic interest. The procedure we use is that due to Eckstein.[14] We consider the electrons within a metal in the presence of the dc magnetic field $\mathbf{B}$ and of an alternating electric field $\mathcal{E}$ that varies as

[10] C. S. Barrett, Acta Cryst. **9**, 671 (1956).

[11] We have defined for convenience the parameter $\omega_c(18)\tau$ to mean the value of $\omega_c\tau$ when the applied magnetic field is 18 kG, the approximate position of the Kjeldaas edge in potassium. Thus $\omega_c(18)\tau$ is merely a measure of τ and is independent of B.

[12] A. W. Overhauser, Phys. Rev. Letters **13**, 190 (1964).

[13] M. H. El Naby, Z. Physik **174**, 269 (1963); H. Mayer and M. H. El Naby, *ibid.* **174**, 280, 289 (1963).

[14] S. G. Eckstein, Bull. Am. Phys. Soc. **9**, 550 (1964); and private communication.

$\exp(i\omega t - i\mathbf{q}\cdot\mathbf{r})$. The Boltzmann transport equation for the electron distribution function $f=f(\mathbf{k},\mathbf{r},t)$ is

$$\frac{\partial f}{\partial t}+\mathbf{v}\cdot\nabla f-\frac{e}{\hbar}\left(\mathcal{E}+\frac{1}{c}\mathbf{v}\times\mathbf{B}\right)\cdot\nabla_k f=-(f-f_0)/\tau, \quad (11)$$

where $\mathbf{v}$ is the velocity of the electron having wave vector $\mathbf{k}$ and $f_0=f_0(E(\mathbf{k}))$ is the distribution function in thermal equilibrium. Equation (11) is linearized in $\mathcal{E}$ the usual way and its solution is found to be

$$f=f_0+f_1, \quad (12)$$

where

$$f_1=\int_{-\infty}^{u} du'\, e\mathcal{E}\cdot\mathbf{v}'\frac{df_0}{dE}\exp\left[\left(\frac{1}{\tau}+i\omega\right)\times(u'-u)+i\mathbf{q}\cdot(\mathbf{R}-\mathbf{R}')\right]. \quad (13)$$

In writing Eq. (13) we found it convenient to express $\mathbf{k}$ in terms of the new variables E, k_z, and u where u is a quantity having the dimension of a time and which describes the position of the electron on the orbit defined by the intersection of the surface $E(\mathbf{k})=E$ and the surface k_z equals a constant when there is no radio-frequency field present. Thus u is the time parameter in the equation $\hbar(d\mathbf{k}/du)=-(e/c)\mathbf{v}\times\mathbf{B}$ which governs the motion of the electron in $\mathbf{k}$ space in the presence of $\mathbf{B}$ alone. The function $\mathbf{R}(u)$ is the position in real space of an electron of energy E and component k_z of the wave

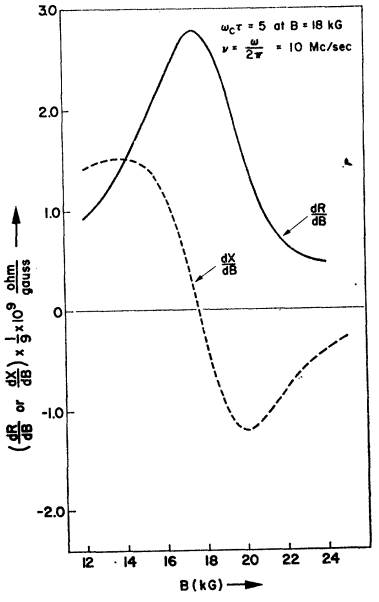

FIG. 1. Derivatives of the surface resistance and surface reactance with respect to an applied magnetic field B for a potassium slab with the field B perpendicular to the surface of the slab. The model used is the free-electron gas for a frequency of 10 Mc/sec and $\omega_c(18)\tau=5$.

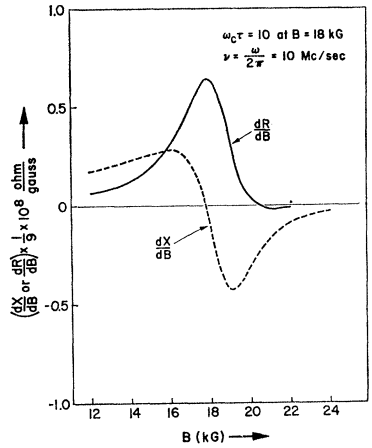

FIG. 2. dR/dB and dX/dB for potassium with $\omega_c(18)\tau=10$.

vector at the time u under these conditions. Thus, with a suitable choice of origin $\mathbf{k}=-(e/\hbar c)\mathbf{R}(u)\times\mathbf{B}$, so that $\mathbf{R}(u)$ describes an orbit in space which is identical to that described by $\mathbf{k}$ but rotated by 90° in the clockwise direction about an axis parallel to $\mathbf{B}$ and amplified by the factor $\hbar c/eB$. Of course, the component of $\mathbf{R}$ parallel to $\mathbf{B}$ is not determined by these considerations. In Eq. (13) $\mathbf{R}$ is the function $\mathbf{R}(E,k_z,u)$ while $\mathbf{R}'$ is the same function evaluated for the value u' of u, i.e., $\mathbf{R}'=\mathbf{R}(E,k_z,u')$. In the same manner $\mathbf{v}'=\mathbf{v}(E,k_z,u')$. We can always separate $\mathbf{R}$ into two terms $\mathbf{R}_s$ and $\mathbf{R}_p$ so that $\mathbf{R}=\mathbf{R}_s+\mathbf{R}_p$. Here $\mathbf{R}_p$ is a periodic function of $\mathbf{k}$ while $\mathbf{R}_s=\mathbf{v}_s(E,k_z)u$ increases linearly with the time u. Now $\mathbf{v}_s(E,k_z)$ is the average velocity of an electron on the orbit defined by E and k_z. In the free-electron model $\mathbf{v}_s$ is the component of the velocity parallel to the applied magnetic field but in the more general case, to which our discussion is applicable, $\mathbf{v}_s$ can have another component when the orbit defined by E and k_z in $\mathbf{k}$ space corresponds to an open orbit. Making use of the facts that the volume element in $\mathbf{k}$ space $d\mathbf{k}=(eB/\hbar^2c)dE\,dk_z\,du$ and that the electrical current density is $j=[(-2e)/(2\pi)^3]\int\mathbf{v}f_1 d\mathbf{k}$, we obtain

$$\mathbf{j}=\boldsymbol{\sigma}\cdot\mathcal{E}, \quad (14)$$

where

$$\boldsymbol{\sigma}=\frac{2e^3B}{(2\pi)^3\hbar^2c}\int dE\left(-\frac{df_0}{dE}\right)\int dk_z T(E,k_z)\sum_{n=-\infty}^{\infty}\mathbf{V}_n\mathbf{V}_n^*\times\left[\frac{1}{\tau}+i\left(\omega-\frac{2\pi n}{T(E,k_z)}-\mathbf{q}\cdot\mathbf{v}_s\right)\right]^{-1}. \quad (15)$$

In this expression $T(E,k_z)$ is the period of the motion in $\mathbf{k}$ space of an electron having energy E and a component of its wave vector equal to k_z along the direction of $\mathbf{B}$. The quantities $\mathbf{V}_n$ are the Fourier components of $\mathbf{v}(E,k_z,u)\exp[i\mathbf{q}\cdot\mathbf{R}_p(E,k_z,u)]$ considered as periodic

functions of u, i.e., we have

$$\mathbf{v}(E,k_z,u)\exp[i\mathbf{q}\cdot\mathbf{R}_p(E,k_z,u)] = \sum_{n=-\infty}^{\infty}\mathbf{V}_n(E,k_z)\exp\left[\frac{2\pi i n u}{T(E,k_z)}\right], \quad (16)$$

so that

$$\mathbf{V}_n(E,k_z)=\frac{1}{T}\int_0^T du\,\mathbf{v}(E,k_z,u) \times\exp[i\mathbf{q}\cdot\mathbf{R}_p(E,k_z,u)-(2\pi i n u/T)]. \quad (17)$$

The application of these results to the case of interest to us is particularly simple. We wish to consider the components of the conductivity tensor σ in the particular situation in which the Fermi surface possesses an axis of rotational symmetry about $\mathbf{B}$. In such a case we obtain $V_{nx}\pm iV_{ny}=v_\perp\delta_{n,\,n\pm1}$, where $v_\perp$ is the component of the electron velocity on a plane perpendicular to $\mathbf{B}$. Then we obtain

$$\sigma_{xx}-i\sigma_{xy}=\frac{e^2}{4\pi^2\hbar^2}\int dk_z\frac{m\tau v_\perp^2}{1+i(\omega\tau-\omega_c\tau-qv_z\tau)}, \quad (18)$$

where m is the cyclotron effective mass. For the free-electron model we use $E(\mathbf{k})=\hbar^2k^2/2m$, where m is the mass of the electron and assume that τ is a constant over the Fermi surface. The latter assumption is not really necessary as long as $\omega_c\bar{\tau}\gg1$, where $\bar{\tau}$ is an appropriate average collision time but we make it for the sake of simplicity. In this case $\sigma_+=\sigma_{xx}-i\sigma_{xy}$ can be evaluated in terms of elementary functions. The calculation of Z has been carried out numerically and the results

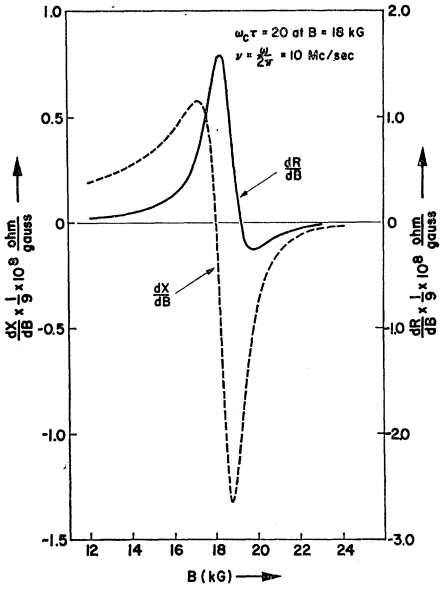

FIG. 3. dR/dB and dX/dB for potassium with $\omega_c(18)\tau=20$.

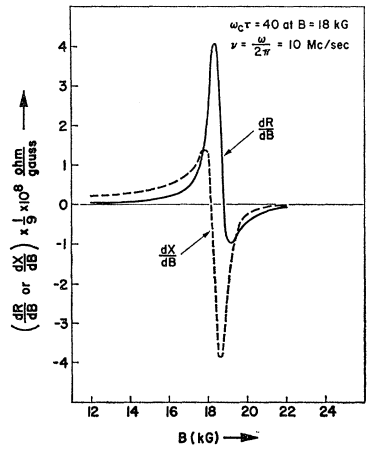

FIG. 4. dR/dB and dX/dB for potassium with $\omega_c(18)\tau=40$.

are displayed in Figs. 1–5. We notice that B_X increases monotonically with decreasing τ. We can understand this physically as follows. For the case in which $\omega_c\tau$ is not very much larger than unity the helicon is attenuated and is not a single sinusoidal wave in space. If a Fourier analysis of the wave is made one would find a Lorentzian distribution of wave numbers q and therefore the condition for the absorption edge is reached at higher magnetic fields for lower collision times.

In Fig. 6 we have displayed the variation of B_X and of $(\Delta B)^{-1}$ with τ. ΔB is defined as

$$\Delta B\equiv\text{Difference between the values of } B \text{ for which a minimum and a maximum of } dR/dB \text{ occur.} \quad (19)$$

ΔB is a measure of the width of the resonance line and

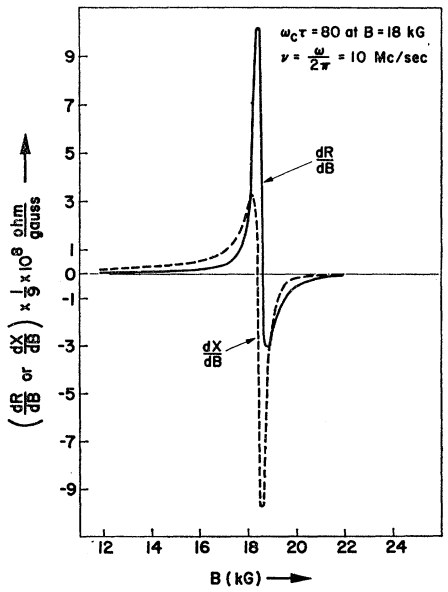

FIG. 5. dR/dB and dX/dB for potassium with $\omega_c(18)\tau=80$.

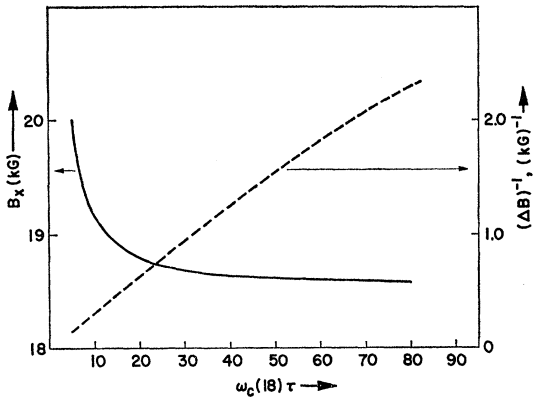

FIG. 6. Graph of B_X and $(\Delta B)^{-1}$ versus $\omega_c(18)\tau$ corresponding to a frequency of 10 Mc/sec for a free-electron gas with a density of electrons equal to that of potassium.

we see that it is approximately proportional to τ^{-1} as one would expect. For large values of τ, $(\Delta B)^{-1}$ deviates from a linear relation as might be anticipated, since even for $\tau=\infty$ the line should have a finite width. It would be desirable to carry out experiments to verify the τ dependence of B_X.

III. HELICON PROPAGATION IN A SPIN-DENSITY WAVE METAL

The anomalous optical reflectivity of potassium[13] gives support to the hypothesis[12] that the electronic ground state of potassium possesses a spin-density wave[15] (SDW). In the SDW model the one-electron states can be labeled by a wave vector $\mathbf{k}$ and have an energy dispersion relation of the form

$$E(\mathbf{k})=\epsilon(\mathbf{k})+\mu(\tfrac{1}{2}Q-k_z)-[\mu^2(\tfrac{1}{2}Q-k_z)^2+\tfrac{1}{4}G^2]^{1/2}, \quad (20)$$

which is valid for $k_z>0$ in the vicinity of $k_z=Q/2$. In this relation k_z is the component of $\mathbf{k}$ along the direction of the wave vector $\mathbf{Q}$ of the SDW. The quantity G is the

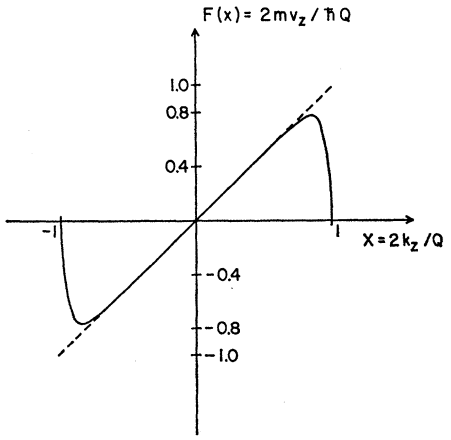

FIG. 7. Schematic diagram showing v_z versus k_z for a SDW model of a metal.

[15] A. W. Overhauser, Phys. Rev. 128, 1437 (1962).

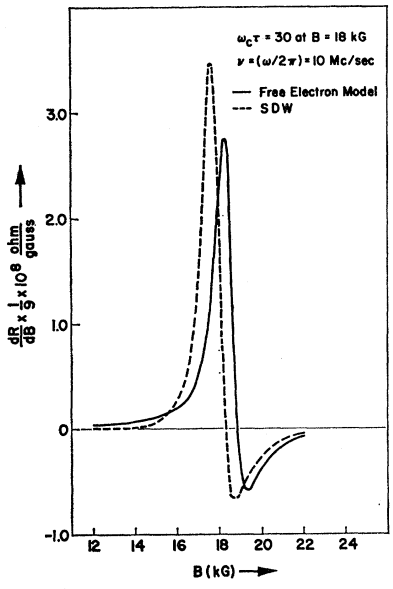

FIG. 8. dR/dB for potassium with $\omega_c(18)\tau=30$ for both the free-electron and SDW models.

energy gap at $k_z=Q/2$, $\mu=\hbar^2Q/2m$, and $\epsilon(\mathbf{k})=\hbar^2k^2/2m$. In virtue of time-inversion symmetry we must have $E(-\mathbf{k})=E(\mathbf{k})$, a relation that defines $E(\mathbf{k})$ for $k_z<0$. On the other hand, de Haas–van Alphen studies by Shoenberg and Stiles[16] and cyclotron resonance experiments by Grimes and Kip[17] have shown that the Fermi surface of potassium is a sphere to within a few parts

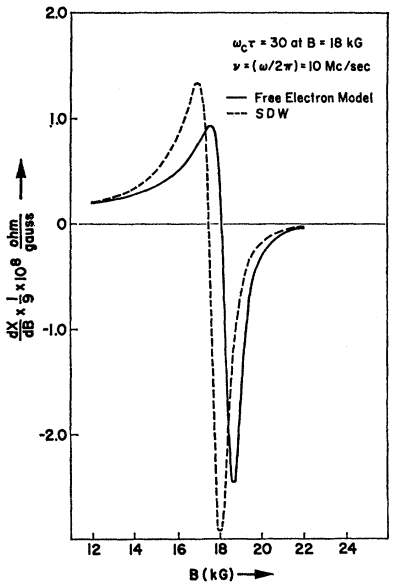

FIG. 9. dX/dB for potassium with $\omega_c(18)\tau=30$ for both the free-electron and SDW models.

[16] D. Shoenberg and P. Stiles, Proc. Roy. Soc. (London) A281, 62 (1964).

[17] C. C. Grimes and A. F. Kip, Phys. Rev. 132, 1991 (1963).

in a thousand. It is possible to reconcile these two results by assuming that the wave vector **Q** of the SDW orients itself parallel to a sufficiently strong dc magnetic field **B**. If this is correct, then the extremal cross sections of the Fermi surface by planes perpendicular to **B** are circles. The area of these circles is, however, slightly less than their values for the free-electron model. This assumption would explain why in a SDW model a rotation of the specimen with respect to the magnetic field would give rise to a configuration in which an extremal cross section of the Fermi surface is always a circle.

We consider now the position of the Kjeldaas edge in the SDW model. Here we expect the edge to occur at considerably lower magnetic fields. In fact the velocity v_z in this case is not linear in k_z but behaves in the fashion displayed schematically in Fig. 7. The position of the Kjeldaas edge is at the value of the magnetic field such that $\omega_c = q_1 v_{\max}$. Now, in this case the dispersion relation (1) is the same as for the free-electron gas provided we set $f_\pm$ equal to

$$f_\pm = \frac{eBm}{4\pi^2 nc\hbar^2 q}\int \frac{v_\perp^2 dk_z}{v_z \pm \omega_c/q + i/q\tau}. \tag{21}$$

In Eq. (21) $v_\perp$ is the magnitude of the component of velocity of an electron perpendicular to **B**. Now

$$v_\perp^2 = \frac{2E_F}{m} - \frac{\hbar^2 k_z^2}{m^2} - \frac{2\mu}{m}\left(\frac{Q}{2} - k_z\right) + \frac{2}{m}\left[\mu^2\left(\frac{Q}{2} - k_z\right)^2 + \frac{G^2}{4}\right]^{1/2}, \tag{22}$$

for $k_z > 0$. In this equation E_F is the Fermi energy. Equation (21) has been obtained using Eq. (18) in Sec. II. The value of $v_{\max}$ is

$$v_{\max} = (\hbar Q/2m)[1-(G/\mu Q)^{2/3}]^{3/2}. \tag{23}$$

For potassium, if we choose $G = 0.62$ eV [and $Q = 1.33(2\pi/a)$ where a is the lattice constant] to obtain agreement with the optical data[13] we obtain $v_{\max} = 0.714\times 10^8$ cm/sec which is to be compared with the Fermi velocity $v_F = 0.864\times 10^8$ cm/sec in the free-electron model. Now, the dielectric constant of the electron gas in a SDW state appropriate to helicon propagation is ϵ_+ where[18]

$$\epsilon_+ = (e^2Q^2/2\pi\omega\hbar q)U_+, \tag{24}$$

and

$$U_+ = \int_{-1}^{1} \frac{(2E_F/\mu Q) - \frac{1}{2}x^2 - (1-|x|) + \{(1-|x|)^2 + (G/\mu Q)^2\}^{1/2}}{F(x) + (\hbar\omega_c/\mu q) + (i\hbar/\mu q\tau)}. \tag{25}$$

In Eq. (25) the function $F(x)$ is the ratio of the velocity v_z to $(\hbar Q/2m)$ as a function of $x = 2k_z/Q$.

The surface impedance of potassium has been calculated using a SDW model in the same way as for the free-electron model. We have displayed the results for the derivative of the real and imaginary parts of the surface impedance $Z = R - iX$ with respect to the applied magnetic field B in Figs. 8 and 9 assuming that $\omega_c\tau = 30$ at $B = 18$ kG. The curves for dR/dB and dX/dB are compared with the results for the free-electron gas for the same value of $\omega_c(18)\tau$. We see that the minimum in (dX/dB) occurs at $B = 18.68$ kG for the free-electron model and at $B = 18.0$ kG for a model with a SDW ground state. We see, then, that if potassium had a spin-density-wave ground state, B_X at $\nu = 10$ Mc/sec would occur at a magnetic field of about 18.0 kG. It is interesting to notice that this agrees with Taylor's observations, especially since the calculation employed no adjustable parameter. Numerical methods that would guarantee at least four figure accuracy were used throughout the calculations.

It is surprising that B_X for the SDW model is only 4% less than B_X for the free-electron model. One might have expected a larger deviation because $v_{\max}$ for the SDW model is 17% less than v_F. In fact, such a large shift is predicted for the location of the Kjeldaas edge of acoustic waves.[19] Evidently the helicon dispersion relation near the Kjeldaas edge is more sensitive to the nature of the electronic model than is the case for acoustic waves.

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[18] The dielectric constant ϵ_+ is related to $\sigma_{xx} - i\sigma_{xy}$ by the relation $\sigma_{xx} - i\sigma_{xy} = (i\omega/4\pi)(\epsilon_+ - 1) \approx (i\omega\epsilon_+/4\pi)$.

[19] R. C. Alig, J. J. Quinn, and S. Rodriguez, Phys. Rev. Letters 14, 981 (1965).