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Vector-Vector–Pseudoscalar Interactions

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Vector-vector–pseudoscalar (V-V–PS) intereactions are interpreted exclusively in terms of baryon loops, and application is made to radiative decay of bosons. The model appears to give a highly satisfactory account of A-forbiddenness; it also agrees with at least one decay that is A-allowed, suggesting that intrinsic V-V–PS couplings are of secondary importance.

1. INTRODUCTION

In this paper we calculate effective two-vector–one-pseudoscalar vertices on the basis of intermediate baryon loops. Applications are restricted to the case where one vector is a photon. There are two reasons for employing baryon loops:

(i) A-forbiddenness is complete with boson loops; we want to compute finite results and hence need baryons.

(ii) There is some evidence, cited below, that even A-allowed transitions with boson loops are of secondary magnitude compared with baryon loops.

Strong interactions of elementary particles have been classified according to various groups[1] under charge transformation. A model has also been proposed[2] with the extension of 8×8 Dirac matrices to include all the groups[3–10] so far considered. All these groups are treated as subgroups of R_8, which determines the eightfold structure of particle families, with the conservation of isospin and hypercharge. Real and charge-space statistics are also assumed to be correlated. This latter assumption leads to the following column matrices:

$$\phi=(\phi_a)\,,\quad \text{with}\quad \phi_0=\eta\,,\quad (\phi_1\mp i\phi_2)/\sqrt{2}=\pi^\pm\,,\quad \phi_3=\pi^0\,,$$
$$(\phi_4\mp i\phi_5)/\sqrt{2}=K^\pm\,,\quad (\phi_6\pm i\phi_7)/\sqrt{2}=(K^0,\bar{K}^0)\,;$$

and

$$\psi=\begin{pmatrix}V\\W\end{pmatrix},\quad \text{corresponding to } \rho_3=\pm1\,,$$

$$V=\begin{pmatrix}Y\\Z\end{pmatrix}\quad\text{and}\quad W=\begin{pmatrix}N\\\Xi\end{pmatrix},\quad \text{corresponding to } \sigma_3=\pm1\,,$$

and

$$\sqrt{2}Y=\begin{pmatrix}\sqrt{2}\Sigma^+\\\Sigma^0+\Lambda\end{pmatrix},\quad \sqrt{2}Z=\begin{pmatrix}\Sigma^0-\Lambda\\\sqrt{2}\Sigma^-\end{pmatrix};$$

$$N=\begin{pmatrix}p\\n\end{pmatrix},\quad \Xi=\begin{pmatrix}\Xi^0\\\Xi^-\end{pmatrix},\quad \text{corresponding to } \tau_3=\pm1\,.$$

Here $\boldsymbol{\rho}$, $\boldsymbol{\sigma}$ and $\boldsymbol{\tau}$ are three independent Pauli spin operators, from which a set of 8×8 Dirac matrices are constructed. It is to be emphasized, however, that this formalism is used only for convenience, since it gives a complete description.

The operator A is defined[6] by

$$A\psi=(-1)^{2J}\exp(i\pi Q_2)C\psi\,,$$

where J is the particle spin and C is the real space-charge conjugation operator. The component Q_2 of the charge operator $\mathbf{Q}$ figures in "antiparticulation." $\mathbf{Q}$ is given by

$$\mathbf{Q}=\mathbf{S}+\mathbf{T}+\mathbf{U}\,,$$

with $2\mathbf{S}=\frac{1}{2}(1-\rho_3)\boldsymbol{\sigma}$, $2\mathbf{T}=\boldsymbol{\tau}$, and $2\mathbf{U}=\frac{1}{2}(1+\rho_3)\boldsymbol{\sigma}$.

A has the property that $A^2=1$ and

$$Ap=-(\Xi^-)^C,\quad A\Sigma^+=-(\Sigma^-)^C,$$
$$An=(\Xi^0)^C,\quad A\Sigma^{0+}=(\Sigma^{0-})^C,$$
$$A\phi=-\phi\,,$$
$$A\gamma=\gamma\,,$$

where ϕ is any pseudoscalar meson field and γ is the photon. For vector mesons $A\Phi=+\Phi$ except for the ω meson, where $A\omega=-\omega$.

Invariance rules under A are valid only to the extent that the $\Xi-N$ mass difference 2Δ can be neglected. Thus A-forbiddenness generally means reducing the matrix element of a process by (Δ/m_0), where m_0 is the average baryon mass ≈ 1.1 BeV.

The lighter loops are all bosons. Elementary boson-boson interactions are A-invariant, so that for A-forbidden transitions one must go to baryon loops. One of the main features of A is that $A\gamma=\gamma$, while γ is not an eigenfunction of G $[C\exp(i\pi I_2)]$. Thus it is possible to compute all two-vector–one-pseudoscalar interactions with baryon loops, regardless of the number of photons among the vectors.

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TABLE I. Matrix elements for photon-vector–pseudoscalar interactions.

0^- \ 1^-	γ	ρ	ϕ	ω
π	$(e^2/m_0)(\Delta/m_0)g_\pi(2+\frac{2}{3}\mu)$	$(eg_\rho/m_0)(\Delta/m_0)g_\pi(2+\frac{2}{3}\mu)$	$(eg_\phi/m_0)(\Delta/m_0)g_\pi(2+\frac{2}{3}\mu)$	$(eg_\omega/m_0)g_\pi(2+\frac{2}{3}\mu)$
η	$(e^2/m_0)(\Delta/m_0)g_\eta(2+\frac{2}{3}\mu+\frac{1}{3}\mu^2)$	$(eg_\rho/m_0)(\Delta/m_0)g_\eta(2+\frac{2}{3}\mu+\frac{1}{3}\mu^2)$	$(eg_\phi/m_0)(\Delta/m_0)g_\eta(2+\frac{2}{3}\mu+\frac{1}{3}\mu^2)$	$(eg_\omega/m_0)g_\eta(2+\frac{2}{3}\mu+\frac{1}{3}\mu^2)$
X	$(e^2/m_0)g_X(2+\frac{2}{3}\mu+\frac{1}{6}\mu^2)$	$(eg_\rho/m_0)g_X(2+\frac{2}{3}\mu+\frac{1}{6}\mu^2)$	$(eg_\phi/m_0)g_X(2+\frac{2}{3}\mu+\frac{1}{6}\mu^2)$	$(eg_\omega/m_0)(\Delta/m_0)g_X(2+\frac{2}{3}\mu+\frac{1}{6}\mu^2)$

2. METHOD OF CALCULATION

The matrix element corresponding to the process considered in Fig. 1 is given by

$$M=\int \mathrm{Tr}\left[\gamma_5 g(p+k'-m)^{-1}\left(G\gamma_\mu+C\frac{F}{2im}\sigma_{\mu\rho}k_\rho'\right)\times(p-m)^{-1}(G'\gamma_\nu+C(F'/2im)\sigma_{\nu\sigma}k_\sigma'')\times(p-k''-m)^{-1}\right]d^4pA_\mu'\Phi_\nu'',$$

where C is a convergence factor and m is the baryon mass. For this factor we take $C=m^2(p^2-m^2)^{-1}$, corresponding to the notion that a ρ or ϕ meson is a baryon-antibaryon resonance that will transmit energy-momentum transfers to about this extent. A_μ' and Φ_ν'' are the photon and vector-meson wave functions, respectively.

With the convergence factors we can integrate over all momenta p for the baryon loops, obtaining

$$M=M_0\epsilon_{\mu\nu\rho\sigma}k_\rho'k_\sigma''A_\mu'\Phi_\nu'', \quad (1)$$

where

$$M_0=i(2\pi)^2[m^{-1}g\{GG'+\tfrac{1}{6}(GF'+G'F)+\tfrac{1}{12}FF'\}].$$

For the interaction in charge space we use the σ_3, τ_3, and ρ_3 formalism discussed in Sec. 1. Accordingly the charge coupling for a photon is

$$\gamma: e[\tfrac{1}{2}(\sigma_3+\tau_3)+\mu\{\tau_3+\tfrac{1}{4}(\sigma_3-\tau_3)(1+\rho_3)\}], \quad (2)$$

with $\mu=1.5$, an average value at the photon energies involved and

$$m^{-1}=m_0^{-1}[1+(\Delta/m_0)\tfrac{1}{2}(1-\rho_3)\sigma_3]. \quad (3)$$

Thus we get for $\gamma\gamma$

$$[GG'+\tfrac{1}{6}(GF'+G'F)+\tfrac{1}{12}FF'] = e^2\{\tfrac{1}{2}(1+\sigma_3\tau_3)+\tfrac{1}{6}\mu(1+\sigma_3\tau_3)+\tfrac{1}{12}\mu^2[1-\tfrac{1}{4}(1-\sigma_3\tau_3)(1+\rho_3)]\}. \quad (4)$$

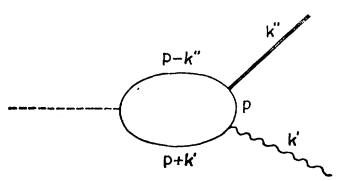

FIG. 1. Diagram showing photon-vector–pseudoscalar interactions with intermediate baryon loops. Wavy line—photon, double line—vector meson, and dashed line—pseudoscalar meson.

Now when the pseudoscalar meson is a π^0,

$$g=g_\pi[\tau_3-\tfrac{1}{2}(1+\rho_3)\sigma_3] \quad (5)$$

and when an η meson

$$g=g_\eta[\tfrac{1}{2}(1-\rho_3)\sigma_3]. \quad (6)$$

Hence, using Eqs. (2)–(6) and taking the trace, we find

$$M(\pi\to 2\gamma)=i(2\pi)^2(e^2/m_0)(\Delta/m_0)\times g_\pi(2+\tfrac{2}{3}\mu)\epsilon_{\mu\nu\rho\sigma}k_\rho'k_\sigma''A_\mu'A_\nu'' \quad (7)$$

and

$$M(\eta\to 2\gamma)=i(2\pi)^2(e^2/m_0)(\Delta/m_0)\times g_\eta(2+\tfrac{2}{3}\mu+\tfrac{1}{3}\mu^2)\epsilon_{\mu\nu\rho\sigma}k_\rho'k_\sigma''A_\mu'A_\nu''. \quad (8)$$

It is worth mentioning at this stage that any coupling scheme can be expressed along this line. However, we try to avoid very specific coupling schemes in the face of our current ignorance by keeping to general average values. Accordingly we use the following assumptions:

(i) $M_0(\gamma 0^-\rho)+M_0(\gamma 0^-\phi)\approx 2M_0(\gamma 0^-\gamma)$,

(ii) $M_0(\gamma 0^-\rho)\approx M_0(\gamma 0^-\phi)$,

(iii) $M_0(\gamma 0^-\omega)\approx M_0(\gamma 0^-\phi)$ except for (Δ/m_0) factor. ω is just like ϕ in J^P and I^G, the only difference being $A=-1$ for ω, where $A=+1$ for ϕ; therefore $(\gamma 0^-\omega)$ is A-allowed where $(\gamma 0^-\phi)$ was A-forbidden.

These assumptions are based on the following theoretical and experimental considerations. Theoretically we have ϕ, ρ, and γ coupling proportional to $g_\phi\frac{1}{2}(1-\rho_3)\sigma_3$, $g_\rho[\tau_3+\frac{1}{2}(1+\rho_3)\sigma_3]$ and $e(\tau_3+\sigma_3)$, respectively. Thus we get ϕ and ρ coupling together proportional to (g_v/e) (γ-coupling), when we use $g_\rho=g_\phi=g_v$. Also from the experimental study of the electromagnetic properties of the nucleon, one asserts that ρ and ϕ enter on the same footing.

On the basis of similar reasoning we assume

(iv) $M_0(\gamma X1^-)\approx M_0(\gamma\eta 1^-)\approx\frac{1}{2}[M_0(\gamma\pi 1^-)+M_0(\gamma\eta 1^-)]$ except for (Δ/m_0) factor. Here again the X and η are assumed identical in J^P and I^G but opposite in A, by exact analogy with the ω and ϕ; thus $X\to 2\gamma$ is A-allowed because $A=+1$ for the X. The relation of this particular assignment to experiment on the X has been discussed separately.[11] Thus, we need only to compute $\pi\to 2\gamma$ and $\eta\to 2\gamma$ matrix elements and all the rest

[11] S. K. Kundu and D. C. Peaslee, Nuovo Cimento 36, 277 (1965).

follow. The results for all $-i(2\pi)^{-2}M_0(\gamma 0^- 1^-)$ are shown in Table I.

3. COMPARISON WITH EXPERIMENT

The decay width for the process $0^- \to 1^- + \gamma$ is given by

$$\Gamma(0^- \to 1^- + \gamma) = \pi k^3 |M_0(0^- \to 1^- + \gamma)/(2\pi)^5|^2, \quad (9)$$

where $k = (m_0^2 - m_1^2)/2m_0$.

Similarly

$$\Gamma(1^- \to 0^- + \gamma) = \tfrac{1}{3}\pi k^3 |M_0(1^- \to 0^- + \gamma)/(2\pi)^5|^2, \quad (10)$$

where $k = (m_1^2 - m_0^2)/2m_1$.

In case $0^- \to 2\gamma$, the decay width is given by Eq. (9) with $k = \frac{1}{2}m_0$, i.e.,

$$\Gamma(0^- \to 2\gamma) = \pi k^3 |M_0(0^- \to 2\gamma)/(2\pi)^5|^2. \quad (11)$$

Using the values for pseudoscalar and vector meson couplings (assumed roughly independent of charge type in agreement with recent fits to N-N scattering)

$$g_{ps}^2/4\pi \approx 14 \quad \text{and} \quad g_v^2/4\pi \approx 1-2,$$

we get from Table I and Eqs. (9)–(11)

$$\begin{aligned}
(\pi \to 2\gamma) &= 5.3 \text{ eV},\\
(\eta \to 2\gamma) &= 0.6 \text{ keV},\\
(X \to 2\gamma) &= 0.1 \text{ MeV},\\
(X \to \gamma + \rho) &= 0.6(g_\rho^2/4\pi) \text{ MeV} \approx 1 \text{ MeV}, \quad (12)\\
(\omega \to \gamma + \pi) &= 1.4(g_\omega^2/4\pi) \text{ MeV} \approx 2 \text{ MeV},\\
(\omega \to \gamma + \eta) &= 0.3(g_\omega^2/4\pi) \text{ MeV} \approx 0.4 \text{ MeV},\\
(\phi \to \gamma + \pi) &= 0.1(g_\phi^2/4\pi) \text{ MeV} \approx 0.1 \text{ MeV},\\
(\phi \to \gamma + \eta) &= 0.05(g_\phi^2/4\pi) \text{ MeV} \approx 0.1 \text{ MeV}.
\end{aligned}$$

The first entry in Eq. (12) is in good agreement with observation and is calculated without any adjustable parameters; it includes an A-forbiddenness factor[12] of $(\Delta/m_0)^2 \approx 1/36$.[12a] The second entry indicates a total η width of order 2 keV. The calculated $\omega \to \gamma + \pi^0$ is in fair agreement with observation[13]; this is an A-allowed process, and the agreement suggests that no boson loops or hence intrinsic V-V–PS couplings are needed. Since such couplings must be symmetric in the charge coordinates, while antisymmetry is the fundamental expression of any charge group, there is some *a priori* argument against boson loops in this case. The last three entries of Eq. (12) do not yet have suitable data for comparison.

Because of the averaging processes used to obtain Table I, a few of the entries in Eq. (12) may contain specific errors, but the general order of magnitude is already in good enough agreement to suggest that the simple baryon loop model is valid for V-V–PS couplings, and in particular as a specific dynamical model to provide examples of the A-selection rule on radiative decay of bosons.

It is interesting to note that the best experimental evidence for $A_X = +1$ comes from a V-V–PS interaction of the type considered here. The decay $X \to \gamma + \omega$ has never been reported, while $X \to \gamma + \rho^0$ is a major mode.[14,15] Since the ρ^0 and ω would be identical in these decay processes except for A value, it seems clear that $A_X = A_\gamma A_\rho = +1$ as opposed to $A_\gamma A_\omega = -1$. The estimates of Brown and Faier[16] for the $X \to 2\pi\eta$ mode must be reduced by $\sim 1/36$ because of A violation; but with extreme allowance for the σ resonance, this could still leave $\Gamma(2\pi\eta) \sim 0.2G^2$ MeV. Using the formula from Eq. (12) yields

$$(X \to 2\pi\eta)/(X \to \gamma\rho^0) \approx (2G/g_\rho)^2. \quad (13)$$

The effective values of G and g_ρ are not known to high precision, one can only guess that they are comparable; but this is in good accord with the latest measurements[17] indicating a value around 2 for the left side of Eq. (13). This observation has prompted the independent remark[17] that $X \to 2\pi\eta$ should be A-forbidden in order to make the electromagnetic decay competitive.

4. CONCLUSIONS

A dynamical theory involving baryon loops is proposed to account for two-vector–one-pseudoscalar interactions where one vector is a photon. The theory yields the observed $\pi \to 2\gamma$ decay rate and an A-forbiddenness factor as suggested by Bronzan and Low.[12] In general the agreement with experimental results appears promising. It is important to note that the averaging pro-

[12] J. B. Bronzan and F. E. Low, Phys. Rev. Letters **12**, 522 (1964).

[12a] *Note added in proof.* This value is an order of magnitude larger than usually quoted, the discrepancy being composed of the following factors: a relatively high (×2) value assumed for $\pi^0 \to 2\gamma$; neglect of the usual SU_3 factor of $\frac{1}{3}$ in our assumption that $g_\eta^2 \approx g_\pi^2 \approx g_{ps}^2$; an extra factor of 1.5 from the μ-dependent factors in Table I. This discrepancy illustrates the extremes of latitude in such crude estimates and has nothing to do with the factor of 40 found by F. A. Berends and P. Singer, Phys. Letters **19**, 249 (1965), who have simply neglected A selection rules in comparing the A-forbidden $\eta \to \gamma + \rho^0$ with the A-allowed $\omega \to \gamma + \pi^0$. Their factor of 40 just confirms Bronzan and Low's original estimate (Ref. 12).

[13] M. Gell-Mann, D. Sharp, and W. G. Wagner, Phys. Rev. Letters **8**, 261 (1962), neglected A-forbiddenness in $\omega \to \gamma$ but instead inserted a factor 1/40 in the $\rho\pi\omega$ vertex (where it should not be) by determining $g_{\rho\pi\omega}$ from a phenomenological fit to the observed $\pi^0 \to 2\gamma$ decay rate. They also calculated $\omega \to \pi^0 + \gamma$ without inconsistency.

[14] G. R. Kalbfleisch, O. I. Dahl, and A. Rittenberg, Phys. Rev. Letters **13**, 349 (1964).

[15] P. M. Dauber, W. E. Slater, L. T. Smith, D. H. Stork, and H. K. Ticho, Phys. Rev. Letters **13**, 449 (1964).

[16] L. M. Brown and H. Faier, Phys. Rev. Letters **13**, 73 (1964).

[17] J. Badier *et al.*, Phys. Letters **17**, 337 (1965).

cedure adopted in the present calculations may lead to some errors in case of specific decay rates but the general order of magnitude seems quite reasonable.

The foregoing calculation has assumed throughout that failure of A conservation could be entirely represented by the $N-\Xi$ mass asymmetry. This implies that if the asymmetry results from unspecified strong interactions, they must mainly be effective at momentum transfers much higher than those involved here, say $\sim$500 MeV/c. A similar approach appears to be satisfactory in other cases.[18]

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[18] See, for example, E. Johnson and E. R. McCliment, Bull. Am. Phys. Soc. 10, 98 (1965).

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Effective Nonlocal Energy-Independent Potential and the Possibility of a Repulsive Core at Small Distances

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An effective nonlocal potential is constructed by requiring that its Born expansion give the same amplitude as the peripheral-interaction expansion of Cutkosky. The latter is a modified perturbation expansion based on dispersion theory in which each line of a Feynman graph represents not only discrete states, but also all states lying in the continuum which have a limited angular momentum; in higher order diagrams one also has to subtract out certain contributions which are already included in lower order diagrams, so as to avoid double counting. It can then be argued that, at least if we go up to fourth order, there should be a repulsive core in the potential. The Schrödinger equation is thus solvable without cutoff in configuration space. Because of the difficulties of such a program, only a rough momentum-space calculation was carried out for $\pi\pi$ scattering with ρ exchange.

I. INTRODUCTION

IN a previous paper,[1] a method was given for constructing an effective energy-dependent local potential which would reproduce the correct relativistic amplitude. The procedure used was a straightforward generalization of the one introduced by Charap and Fubini[2] to define a nonrelativistic potential. The relativistic amplitude was built up by means of the iterative strip approximation,[3] in which one takes the contribution of a few crossed-channel partial waves in first approximation. In practice, the equations are solved by iteration; such a procedure is equivalent to requiring that, to any given order, the Born expansion for the potential give the same amplitude as the strip approximation. Once the potential has been found, the Schrödinger equation merely serves as a way of imposing unitarity.

Although a local energy-dependent potential is simpler to deal with in a practical calculation, it turns out that a nonlocal energy-independent potential might have certain advantages in principle. The general procedure for constructing such a potential is very similar to the one for the local case. Instead of using the strip approximation, however, we shall use a more elementary and general graphical technique for constructing our relativistic amplitude. Although this technique has been extensively used in special cases, it was first put on a general footing by Cutkosky.[4] In lowest order, it is essentially the Cini-Fubini approximation,[5] and gives expression to the intuitive idea of treating resonances on the same footing as stable particles. The main differences from the usual Feynman graphs arise in higher order graphs, which have to be "renormalized," i.e., the contributions already included in lower order graphs have to be subtracted out to avoid double counting. This has the effect of suppressing the contribution of higher order graphs. Thus the scheme should be meaningful even for strong interactions.

In Sec. II, we discuss briefly the Cutkosky procedure. We do not give the general theory, for which the interested reader should consult Ref. 4, but merely consider the second- and fourth-order graphs, which can be understood intuitively. These graphs are all we will need for the rest of the paper, and they resemble very

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[1] L. A. P. Balázs, Phys. Rev. 137, B1510 (1965).
[2] J. M. Charap and S. P. Fubini, Nuovo Cimento 14, 540 (1959); 15, 73 (1960). See also A. A. Logunov and A. N. Tavkhelidze, Nuovo Cimento 29, 380 (1963) and A. A. Logunov, A. N. Tavkhelidze, I. T. Todorov and O. A. Khrustalev, *ibid.* 30, 134 (1963); these authors considered an effective potential which is energy-dependent and nonlocal in a certain specific way.
[3] G. F. Chew and S. C. Frautschi, Phys. Rev. 123, 1478 (1961).
[4] R. E. Cutkosky, Nucl. Phys. 37, 57 (1962).
[5] M. Cini and S. C. Fubini, Ann. Phys. (N. Y.) 10, 352.