Dynamical Inelasticity and High-Energy p-p Scattering*

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A recently proposed method for calculating partial-wave inelasticity dynamically is applied to the high-energy p-p scattering. The two nucleons are considered spinless. Their left-hand-cut contribution is calculated by making a δ-function approximation of the absorptive part in the crossed channel. Partial waves with $l\leq l_a$ are taken to be completely imaginary and those with $l>l_a$ are considered to be given by the Born term. Certain interesting features of the present model are discussed.

I. INTRODUCTION

A METHOD for calculating partial-wave inelasticity $\eta_l=e^{-2\,\mathrm{Im}\delta_l}$, where δ_l is the phase shift, has has been proposed recently.[1] The basic assumption of the method is that above the inelastic threshold a large number of reaction channels open so that the partial-wave amplitude is essentially imaginary in the inelastic region. Correspondingly, $\mathrm{Re}\delta_l\approx n\pi$, where n is an integer, above the inelastic threshold. In this paper, we apply the above method to high-energy p-p scattering.[2–4] The problem of explaining p-p scattering at high energies has been considered by many authors.[5–23] We assume that the two protons are spinless and that their left-hand-cut contribution is given by a δ-function approximation of the absorptive part in the $N\bar{N}$ channel. Partial waves with orbital angular momentum $l\leq l_a$ are taken to be imaginary (i.e., they are diffraction scattered), and those with $l>l_a$ are considered to be given by the Born term and therefore are real.

In Sec. II the present model is formulated. In Sec. III the results of calculation are given. In Sec. IV certain features of the model are discussed and compared with other models.

II. FORMULATION OF THE MODEL

We first recall here some of the basic equations which are needed for calculating the inelasticity η_l dynamically.[1] The physical partial-wave amplitude is given by

$$A_l=(e^{2i\delta_l}-1)/2i\rho=(\eta_l e^{2i\,\mathrm{Re}\delta_l}-1)/2i\rho\,, \qquad (1)$$

where $\rho=[\nu/(\nu+1)]^{1/2}$ and ν is the square of the c.m. energy.[24] A function θ_l is defined in the following way:

$$\theta_l=\frac{\nu^{l+1/2}}{\pi}\int_{\nu_i}^{\infty}\frac{\mathrm{Im}\delta_l(\nu')d\nu'}{\nu'^{l+1/2}(\nu'-\nu)}\,, \qquad (2)$$

where ν_i is the inelastic threshold. If now a new partial-wave amplitude a_l is constructed by writing

$$1+2i\rho a_l=e^{-2i\theta_l}[1+2i\rho A_l]\,, \qquad (3)$$

then a_l can be expressed as

$$a_l=(e^{2i\alpha_l}-1)/2i\rho\,,\quad \text{where}\quad \alpha_l=\delta_l-\theta_l\,. \qquad (4)$$

Since $\mathrm{Im}\delta_l=\mathrm{Im}\theta_l$, α_l is real throughout the physical region ($\nu>0$) and a_l, therefore, always obeys elastic unitarity. For $\nu>\nu_i$, we can write $\theta_l=\Delta_l+i\,\mathrm{Im}\delta_l$, so that from Eq. (2),

$$\frac{\Delta_l(\nu)}{\nu^{l+1/2}}=\frac{1}{\pi}P\int_{\nu_i}^{\infty}\frac{\mathrm{Im}\delta_l(\nu')d\nu'}{\nu'^{l+1/2}(\nu'-\nu)}\,. \qquad (5)$$

Equation (5) can be inverted by using the following boundary conditions[1]:

(i) $\mathrm{Im}\delta_l(\nu)=0$ at $\nu=\nu_i$,
(ii) $\mathrm{Im}\delta_l(\nu)/\nu^{l+1/2}$ vanishes when $\nu\to\infty$,
(iii) $\theta_l(\nu)/\nu^{l+1/2}$ behaves as a constant when $\nu\to 0$.

Condition (i) follows from the fact that at the inelastic threshold only the elastic channel is open and no

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[24] The proton mass is taken as unity. Also, $c=\hbar=1$.

reaction occurs. Conditions (ii) and (iii) are essentially contained in Eq. (2). In this equation, we have assumed that

$$\int_{\nu_i}^{\infty} \frac{\mathrm{Im}\delta_l(\nu')d\nu'}{\nu'^{l+1/2}(\nu'-\nu)}$$

is a well-defined integral and this implies that $\mathrm{Im}\delta_l(\nu)/\nu^{l+1/2}$ should vanish for $\nu \to \infty$. Condition (iii) indicates the threshold behavior of the function $\theta_l(\nu)$. This behavior in turn is imposed so that the partial wave amplitude $a_l(\nu)$ has the same threshold behavior as the physical partial-wave amplitude $A_l(\nu)$. After inverting Eq. (5), we obtain

$$\begin{aligned}\frac{\mathrm{Im}\delta_l(\nu)}{\nu^{l+1/2}} &= -\frac{(\nu-\nu_i)^{1/2}}{\pi} P\int_{\nu_i}^{\infty} \frac{\Delta_l(\nu')d\nu'}{\nu'^{l+1/2}(\nu'-\nu_i)^{1/2}(\nu'-\nu)} \\ &= -\frac{(\nu-\nu_i)^{1/2}}{\pi} P\int_{\nu_i}^{\infty} \frac{\mathrm{Re}\delta_l(\nu')d\nu'}{\nu'^{l+1/2}(\nu'-\nu_i)^{1/2}(\nu'-\nu)} \\ &\quad + \frac{(\nu-\nu_i)^{1/2}}{\pi} P\int_{\nu_i}^{\infty} \frac{\alpha_l(\nu')d\nu'}{\nu'^{l+1/2}(\nu'-\nu_i)^{1/2}(\nu'-\nu)} \qquad (6)\end{aligned}$$

(using $\alpha_l = \delta_l - \theta_l = \mathrm{Re}\delta_l - \Delta_l$).

Equation (6) is exact. In the model proposed to calculate inelasticity, ν_i is assumed to be large so that $\mathrm{Re}\delta_l(\nu')$ in Eq. (6) can be approximated by its asymptotic value $n\pi$; the phase shift $\alpha_l(\nu')$, on the other hand, is obtained from a given force (or left-hand-cut contribution) and the elastic unitarity, using N/D or inverse technique. Equation (6) thus furnishes a dynamical method for calculating inelasticity.

However, Eq. (6) as it stands is not suitable for calculating $\mathrm{Im}\delta_l(\nu)$ when ν is large and l is large. This is because of the factor $\nu^{l+1/2}$ which will multiply the right-hand side of Eq. (6) in the evaluation of $\mathrm{Im}\delta_l$. The factor $\nu^{l+1/2}$ occurs in the theory because in defining θ_l by Eq. (2) we imposed the threshold behavior for this function. What is needed now is a suitable asymptotic behavior for θ_l. Let us require that for $|\nu| \to \infty$, $|\theta_l(\nu)/\nu^{1/2}| = O(1/|\nu|^{\epsilon})$ where $\epsilon > 0$. An important consequence of this asymptotic behavior is that the definition of θ_l used by us [Eq. (2)] will be the same as that given by Froissart.[25] The equation for $\theta_l(\nu)/\nu^{l+1/2}$ corresponding to Eq. (6) is

$$\begin{aligned}\frac{\theta_l(\nu)}{(\nu-\nu_i)^{1/2}\nu^{l+1/2}} &= \frac{1}{\pi i}\int_{\nu_i}^{\infty} \frac{\mathrm{Re}\delta_l(\nu')d\nu'}{\nu'^{l+1/2}(\nu'-\nu_i)^{1/2}(\nu'-\nu)} \\ &\quad - \frac{1}{\pi i}\int_{\nu_i}^{\infty} \frac{\alpha_l(\nu')d\nu'}{\nu'^{l+1/2}(\nu'-\nu_i)^{1/2}(\nu'-\nu)}. \qquad (7)\end{aligned}$$

We expand $(\nu'-\nu)^{-1}$ in the following way:

$$\frac{1}{\nu'-\nu} = -\frac{1}{\nu} - \frac{\nu'}{\nu^2} \cdots - \frac{\nu'^{l-1}}{\nu^l} + \frac{\nu'^l}{\nu^l(\nu'-\nu)}. \qquad (8)$$

Inserting Eq. (8) into Eq. (7), we get

$$\begin{aligned}\frac{\theta_l(\nu)}{(\nu-\nu_i)^{1/2}\nu^{l+1/2}} &= -\sum_{n=0}^{l-1} \frac{M_n}{\nu^{n+1}} \\ &\quad + \frac{1}{\nu^l \pi i}\int_{\nu_i}^{\infty} \frac{\mathrm{Re}\delta_l(\nu')d\nu'}{\nu'^{1/2}(\nu'-\nu_i)^{1/2}(\nu'-\nu)} \\ &\quad - \frac{1}{\nu^l \pi i}\int_{\nu_i}^{\infty} \frac{\alpha_l(\nu')d\nu'}{\nu'^{1/2}(\nu'-\nu_i)^{1/2}(\nu'-\nu)}, \qquad (9)\end{aligned}$$

where

$$\begin{aligned}M_n &= \frac{1}{\pi i}\int_{\nu_i}^{\infty} \frac{\mathrm{Re}\delta_l(\nu')d\nu'}{\nu'^{l-n+1/2}(\nu'-\nu_i)^{1/2}} \\ &\quad - \frac{1}{\pi i}\int_{\nu_i}^{\infty} \frac{\alpha_l(\nu')d\nu'}{\nu'^{l-n+1/2}(\nu'-\nu_i)^{1/2}}. \qquad (10)\end{aligned}$$

Now, the assumed asymptotic behavior for $\theta_l/\nu^{1/2}$ requires

$$\left|\frac{\theta_l(\nu)}{(\nu-\nu_i)^{1/2}\nu^{l+1/2}}\right| = O\left(\frac{1}{|\nu|^{l+1/2+\epsilon}}\right) \quad \text{when} \quad |\nu| \to \infty. \qquad (11)$$

From Eqs. (9) and (11) we therefore get

$$M_n = 0, \quad n = 0, 1, \cdots, l-1 \qquad (12)$$

and

$$\begin{aligned}\frac{\theta_l(\nu)}{(\nu-\nu_i)^{1/2}\nu^{1/2}} &= \frac{1}{\pi i}\int_{\nu_i}^{\infty} \frac{\mathrm{Re}\delta_l(\nu')d\nu'}{\nu'^{1/2}(\nu'-\nu_i)^{1/2}(\nu'-\nu)} \\ &\quad - \frac{1}{\pi i}\int_{\nu_i}^{\infty} \frac{\alpha_l(\nu')d\nu'}{\nu'^{1/2}(\nu'-\nu_i)^{1/2}(\nu'-\nu)}. \qquad (13)\end{aligned}$$

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[25] This can be easily shown in the following way: Using the identity

$$\frac{1}{\nu'-\nu} = -\sum_{n=0}^{l-1} \frac{\nu'^n}{\nu^{n+1}} + \frac{\nu'^l}{\nu^l(\nu'-\nu)}$$

in Eq. (2), we have

$$\frac{\theta_l}{\nu^{l+1/2}} = -\sum_{n=0}^{l-1} \frac{\mathfrak{M}_n}{\nu^{n+1}} + \frac{1}{\nu^l \pi}\int_{\nu_i}^{\infty} \frac{\mathrm{Im}\delta_l(\nu')d\nu'}{\nu'^{1/2}(\nu'-\nu)},$$

where

$$\mathfrak{M}_n = \frac{1}{\pi}\int_{\nu_i}^{\infty} \frac{\mathrm{Im}\delta_l(\nu')d\nu'}{\nu'^{l-n+1/2}}.$$

Now, for $|\nu| \to \infty$, $|\theta_l/\nu^{l+1/2}| = O(1/|\nu|^{l+\epsilon})$. Therefore, from the above equation it follows that $\mathfrak{M}_n = 0$ and

$$\theta_l = \frac{\nu^{1/2}}{\pi}\int_{\nu_i}^{\infty} \frac{\mathrm{Im}\delta_l(\nu')d\nu'}{\nu'^{1/2}(\nu'-\nu)},$$

which is the definition used by M. Froissart, Nuovo Cimento **22**, 191 (1961).

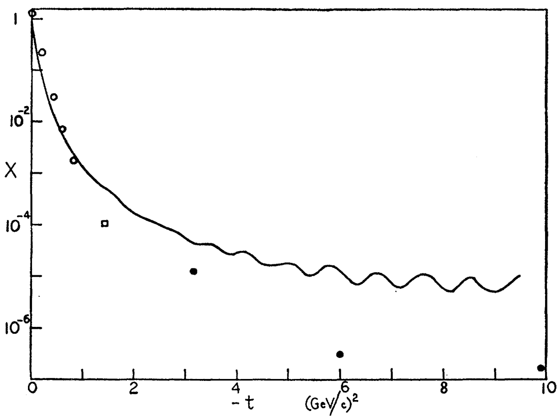

FIG. 1. The normalized differential elastic cross section X is plotted against the invariant momentum transfer $-t$ for $p_0=11$ GeV/c. The experimental points are: ○ Foley *et al.*,[3] □ Diddens *et al.*,[2] ● Cocconi *et al.*[4]

From Eq. (13) we have

$$\mathrm{Im}\delta_l(\nu)=-\frac{(\nu-\nu_i)^{1/2}\nu^{1/2}}{\pi}P\int_{\nu_i}^{\infty}\frac{\mathrm{Re}\delta_l(\nu')d\nu'}{\nu'^{1/2}(\nu'-\nu_i)^{1/2}(\nu'-\nu)}+\frac{(\nu-\nu_i)^{1/2}\nu^{1/2}}{\pi}P\int_{\nu_i}^{\infty}\frac{\alpha_l(\nu')d\nu'}{\nu'^{1/2}(\nu'-\nu_i)^{1/2}(\nu'-\nu)}. \quad (14)$$

Instead of Eq. (6), Eq. (14) can now be used to calculate $\mathrm{Im}\delta_l(\nu)$.

To obtain $\alpha_l(\nu)$, let us first consider a fixed energy dispersion relation:

$$A(s,\cos\theta)=\frac{1}{\pi}\int_{4\mu^2}^{\infty}\frac{A_t(s,t')dt'}{t'-t}+\frac{1}{\pi}\int_{4\mu^2}^{\infty}\frac{A_u(s,u')du'}{u'-u}. \quad (15)$$

Here, $s=4(\nu+1)=$ square of c.m. energy, $t=-2\nu\times(1-\cos\theta)$, $u=-2\nu(1+\cos\theta)$, and $\theta=$ c.m. scattering angle; 2μ is the mass of the lowest intermediate state in t and u channels. In Eq. (15), $A_t(s,t')$ is related through crossing symmetry to the absorptive part in the t channel, where t' is the square of c.m. energy and s is the momentum transfer. Similarly, $A_u(s,u')$ is related to the absorptive part in the u channel, where u' is the square of c.m. energy and s is the momentum transfer. Since the amplitude $A(s,\cos\theta)$ will be an even function of $\cos\theta$ for identical spinless particles, therefore, by interchanging t and u in Eq. (15), we get $A_t(s,t')=A_u(s,t')$. The invariant partial-wave amplitude which we obtain now from Eq. (15) is

$$A_l(\nu)=\frac{1}{2}\int_{-1}^{1}d\cos\theta P_l(\cos\theta)A(s,\cos\theta)=\frac{[1+(-1)^l]}{2\pi\nu}\int_{4\mu^2}^{\infty}A_t(s,t')Q_l\left(1+\frac{t'}{2\nu}\right)dt'. \quad (16)$$

If a simple δ-function approximation for $A_t(s,t')$ is made, say,

$$A_t(s,t')=\pi\gamma\delta(t'-m_0^2), \quad (17)$$

then Eq. (16) gives the following Born amplitude

$$A_l^B(\nu)=(\gamma/\nu)Q_l(1+m_0^2/2\nu) \quad l \text{ even},$$
$$=0 \quad l \text{ odd}. \quad (18)$$

We can take, as an approximation, Eq. (18) to be also the Born term of the purely elastic amplitude a_l; that is, we are approximating the left-hand-cut contribution of a_l by that of A_l.

At high energy (ν large) the amplitude a_l, which obeys elastic unitarity throughout the physical region, can be considered to approach its Born term,[26] $a_l^B=(\gamma/\nu)Q_l(1+m_0^2/2\nu)$. Again, for large ν, the phase shift α_l can be considered small so that

$$a_l=(e^{2i\alpha_l}-1)/2i\rho\approx\alpha_l/\rho. \quad (19)$$

Equating Eq. (19) with a_l^B, we thus obtain

$$\alpha_l(\nu)=[\gamma/\nu^{1/2}(\nu+1)^{1/2}]Q_l(1+m_0^2/2\nu). \quad (20)$$

Equation (20) is essentially the asymptotic value of α_l.

Inserting Eq. (20) into Eq. (14) and making the approximation that $\mathrm{Re}\delta_l\approx n\pi$ and $n=0$ (i.e., the real parts of the phase shifts vanish asymptotically), we obtain

$$\mathrm{Im}\delta_l(\nu)\approx\frac{(\nu-\nu_i)^{1/2}\nu^{1/2}}{\pi}\gamma\times P\int_{\nu_i}^{\infty}\frac{Q_l(1+m_0^2/2\nu')d\nu'}{\nu'(\nu'+1)^{1/2}(\nu'-\nu_i)^{1/2}(\nu'-\nu)}. \quad (21)$$

To calculate the differential cross section, we have assumed that all partial waves up to $l\leq l_a$ are completely imaginary, i.e.,

$$A_l=i(1-\eta_l)/2\rho \quad \text{for} \quad l\leq l_a. \quad (22)$$

For $l>l_a$, the partial waves are approximated by the corresponding Born term, i.e.,

$$A_l\approx A_l^B=(\gamma/\nu)Q_l(1+m_0^2/2\nu) \quad \text{for } l>l_a \text{ and } l \text{ even}. \quad (23)$$

If we write the differential cross section as

$$d\sigma/d\Omega=|f(\theta)|^2, \quad (24)$$

then

$$f(\theta)=\frac{2}{(\nu+1)^{1/2}}\sum_{l=0,2,\cdots}^{\infty}(2l+1)P_l(\cos\theta)A_l(\nu)=\frac{4}{\sqrt{s}}\left\{i\sum_{l=0,2,\cdots}^{l_a}(2l+1)P_l(\cos\theta)\frac{(1-\eta_l)}{2\rho}+\sum_{l=l_a+2,\cdots}^{\infty}(2l+1)P_l(\cos\theta)A_l^B\right\}. \quad (25)$$

[26] In potential scattering, this is known to be true; see, for example, R. Blankenbecler and M. L. Goldberger, Phys. Rev. **126**, 766 (1962).

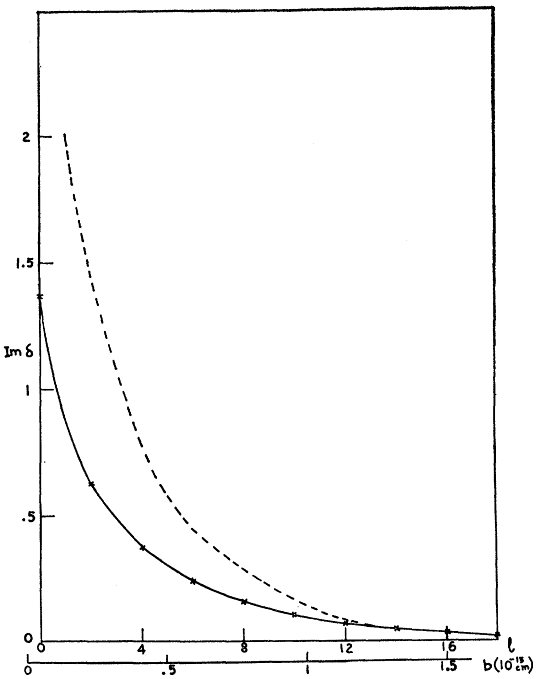

FIG. 2. Im δ_l as a function of l and the impact parameter b. The crosses indicate the calculated values. The dashed curve represents Im$\delta(b)$ as given by Serber (Ref. 5).

It is to be noticed that the amplitude $f(\theta)$ has a real part coming from the partial waves with $l>l_a$. The real part can be exactly calculated by using Eq. (23) and the formula

$$\sum_{l=n+2,n+4,\cdots}^{\infty}(2l+1)P_l(x)Q_l(z)=\frac{(n+1)}{z^2-x^2}[xP_{n+1}(x)Q_n(z)-zP_n(x)Q_{n+1}(z)] \quad (26)$$

for n even. This formula is obtained by using Heine's expansion for $(z-x)^{-1}$ and Christoffel's second summation formula.[27]

III. RESULTS OF CALCULATIONS

There are four parameters in the present model, namely, m_0, ν_i, γ, and l_a. The imaginary parts of the phase shifts have been calculated numerically from Eq. (21) using the Brown University IBM 7070 computer. It has been found that for reasonable values of m_0 and ν_i, γ is negative. This means that the force or the left-hand-cut contribution is repulsive. Calculation of Imδ_l shows that with increasing l it decreases. This is physically expected, since partial waves with large l are not as strongly absorbed as those with small l. We have also found that after a certain value of l, which we call l_a, Imδ_l as calculated from Eq. (21) becomes negative. This implies that for $l>l_a$ the partial waves have to be purely elastic. The results of our calculation for p-p scattering at proton incident momentum $p_0=11$ GeV/c are given in Figs. 1 and 2. The values of the parameters are[24] $m_0=0.2$, $\gamma=-1.25$, $\nu_i=1.5$ and $l_a=30$. In Fig. 1, we have plotted the normalized cross section X against the momentum transfer $-t$. The cross section X is defined by $X=(d\sigma/d\Omega)_{\mathrm{c.m.}}/(k\sigma_T/4\pi)^2$, where σ_T is taken as 40 mb. Some of the experimental results[2–4] are included for the purpose of comparison.[28] It will be seen that the calculated angular distribution has a diffraction peak, a sharp fall with increasing momentum transfer and a flattening of the cross section at large momentum transfer. These are the characteristic features of high-energy p-p scattering for given incident momentum. However, the theoretical cross sections are larger by two orders of magnitude than the experimental cross sections for wide angle scattering.[4] Our calculated values for σ_{el}, σ_{tot}, and $\sigma_{\mathrm{el}}/\sigma_{\mathrm{tot}}$ are respectively 11.26 mb, 36.76 mb, and 0.306; these are to be compared with the corresponding experimental values 11 mb, 40 mb, and 0.28, respectively.[3] Of the total 11.26-mb elastic cross section, the amount contributed by the real part of the amplitude is only 0.08 mb. We find the ratio of the real part to the imaginary part of the amplitude in the forward direction to be -0.250. This is in agreement both in sign and in magnitude with the experimental results.[29–31] The imaginary part of the amplitude is found to be positive throughout the whole momentum-transfer region, while the real part is observed to change sign a number of times. The oscillatory character of the angular distribution at large $-t$ is due to these changes of sign of the real part. In Fig. 2 Imδ_l is plotted against l and also against the impact parameter $b=(l+\frac{1}{2})/k$. For the purpose of comparison, Im$\delta(b)$ as given by Serbert[5] is also plotted. We have also calculated the angular distribution for $p_0=16$ GeV/c. It is found to be very similar in magnitude to that for $p_0=11$ GeV/c. On the other hand, experimentally[4] the wide-angle cross sections for 16 GeV/c are smaller by a factor of 10 than those for 11 GeV/c. This indicates that the present model is inadequate to explain the strong energy dependence of high-energy p-p scattering.

[27] *Higher Transcendental Functions*, edited by A. Eredlyi (McGraw-Hill Book Company, Inc., New York, 1953), Vol. 1, p. 162.

[28] Our value of X in the forward direction is <1, because the experimental total cross section σ_T is larger than the calculated total cross section.

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IV. DISCUSSION

One important point the present model illustrates is that physically meaningful inelasticities can be calculated from the left-hand-cut contribution.[32] It is interesting to compare this with the eikonal approximation where inelasticities are obtained by using purely imaginary potential.[33] The model indicates that the scattering amplitude is not purely imaginary (we recall that $\mathrm{Im}\delta_l$ becomes negative after a certain value of l), but has a real part. The real part can be appreciable for all values of $-t$. It has both the right sign and the right magnitude in the forward direction. The theoretically predicted angular distribution has the qualitative features of the experimentally observed distribution, namely, a forward diffraction peak, a sharp fall, and a levelling off at wide angles. However the present model is very inadequate to explain the strong energy dependence of high-energy p-p scattering.[4] This probably shows that the approximation made to obtain the left-hand cut contribution, viz., replacing the absorptive part in crossed-channel by a δ function, is too simple-minded. It will be interesting to see the effect of improved left-hand-cut contributions. The fact that A_l^B in the present model is the same as the first Born approximation of an energy-dependent repulsive Yukawa potential, may indicate that at high energies the force is perhaps dominantly repulsive.

It is instructive to compare the present model with various other models which have been suggested to explain high-energy p-p scattering. In the diffraction scattering models,[5,7–11] the scattering amplitude is considered purely imaginary. The partial-wave inelasticities are calculated by assuming certain potential or by using some other phenomenological description.[34] In the statistical model,[13–15] one only attempts to explain the wide angle scattering by assuming this to be due to the decay of a compound system. The small angle scattering is considered due to peripheral processes and therefore, beyond the scope of this model. In nonlocal field-theoretic model,[16] one simply calculates the scattering amplitude corresponding to some Feynman diagrams and modifies them by suitable form factors. The scattering amplitude is completely real in this model and as such the diffraction region is regarded outside the scope of the model. In Regge pole models,[23] one tries to explain the diffraction region in terms of the dominance of a few Regge poles. In the large momentum transfer region, contributions from many Regge poles and also from Regge cuts are thought to be important and therefore, cannot be described in a simple manner.

In conclusion, we would like to emphasize the dynamical nature of the present model and to point out that in a model of this type, one has to explain in a unified manner the small-angle as well as the wide-angle scattering.

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Note added in proof. The following two remarks are worth making:

(i) In the context of this paper, by ν_i large what has been meant is that, above the value ν_i, we can physically expect a large number of inelastic channels to be open; partial waves with small l can then be thought of as diffraction scattered and should be essentially imaginary (i.e., $\mathrm{Re}\delta_l \approx 0$) above ν_i. The value $\nu_i = 1.5m^2$ (m = nucleon mass) corresponds to $s = 10m^2$ and c.m. energy 3 GeV. This is appreciably high above the p-p elastic threshold and we can expect a large number of inelastic channels open at this energy.

(ii) $\mathrm{Im}\delta_l(\nu)$ as given by Eq. (21) has not been shown to be consistent with condition (12). In fact, it is not. The reason is that $\mathrm{Im}\delta_l(\nu)$ as given by (21) is an approximate result, while (12) will only be satisfied by the exact result. Now, derivation of Eq. (21) is based on the following two approximations:

$$\text{(a)}\quad \mathrm{Re}\delta_l(\nu) \approx 0 \quad \text{for} \quad \nu > \nu_i,$$

$$\text{(b)}\quad \alpha_l(\nu) \text{ is given by Eq. (20).}$$

Arguments for the first approximation are given in our previous remark. As for (b) this approximation is based on two arguments: first, for ν large $\alpha_l(\nu)$ is expected to be small, and second, $\alpha_l(\nu)$ is expected to approach its Born term $\alpha_l^B(\nu)$. Thus, we can consider Eq. (21) to be a reasonable approximation. However, if condition (12) is not satisfied, then the function $\theta_l(\nu)/(\nu-\nu_i)^{1/2}\nu^{1/2}$ corresponding to Eq. (21) does not have the right threshold behavior. What this means is that Eq. (21) cannot be considered reasonable when we are using it near threshold, but it is a sensible approximation when we are at high energy. For $p_0 = 11$ GeV/c, $\nu = 5.38m^2$ and we are in a region where it should be valid. It is worth noting that if we want to calculate $\mathrm{Im}\delta_l(\nu)$ at low energy, we should use Eq. (7) in preference to Eq. (13), since the former manifestly preserves threshold behavior.

[32] In contrast, in Ref. 1 it was found that the usual one-pole and two-pole approximations of the left-hand cut do not give physically acceptable inelasticities at high energy.

[33] R. Omnes, Phys. Rev. **137**, B653 (1965) has shown how an imaginary optical potential can be found which, when used in eikonal approximation, fits a given scattering amplitude at high energy.

[34] Optical-model calculation with a real part of the potential has also been done. See Ref. 6.