PHYSICAL REVIEW VOLUME 141, NUMBER 4 JANUARY 1966

Meson Exchange Effects on the Electromagnetic Structure of the Deuteron

R. J. ADLER
University of Washington, Seattle, Washington
(Received 21 July 1965)

In a recent paper, Adler and Drell reported on a calculated effect of the $\rho\pi$ mesonic exchange current on the electromagnetic interactions of the deuteron. The present paper describes more fully the method of calculation used and discusses the confrontation of the results with more recent large-angle elastic electron-deuteron scattering experiments. The most striking aspect is that at q^2 values of 6, 7, and 8 (fermi)$^{-2}$ the recent data are in excellent agreement with the present theory, but the magnetic scattering does not agree with the predictions of the older and well-studied impulse approximation.

INTRODUCTION

THERE are a number of reasons for studying the electromagnetic structure of the deuteron in detail. The two that we are concerned with in this paper are:

(1) The static charge, quadrupole moment, and magnetic moment are both calculable and experimentally measurable, which allows us to test rather directly our theoretical understanding of the deuteron.[1]

(2) Given a model of the deuteron, such as the nonrelativistic wave functions we will use in this paper, one can calculate expressions for elastic electron-deuteron (*e-d*) scattering in terms of the nucleon form factors and the deuteron structure. The neutron form factor then can be extracted from elastic *e-d* scattering measurements.[2] Since the simplest target containing neutrons is deuterium, this is therefore probably the most direct way to investigate neutron structure. (See Refs. 3 and 4 however for a discussion of inelastic *e-d* scattering.)

We will show here that mesonic exchange currents appear to be capable of explaining a small but disturbing 2% discrepancy between experimental and previous theoretical values of the deuteron magnetic moment,[5] and can remove a rather large discrepancy between recent experimental data and theory on elastic *e-d* magnetic or back-scattering.[6] The effect on forward or charge scattering and on the analysis of neutron structure appears to be quite small. Thus the over-all effect of including the exchange currents we discuss is to ameliorate several difficulties connected with the deuteron magnetic structure while introducing no new ones.

Lastly, we would like to emphasize that the mesonic exchange effects discussed here constitute one particularly simple type of relativistic effect. Additional relativistic corrections to the current operator[7–9] can also be expected to play an interesting role, especially in connection with the neutron charge structure; purely kinematical corrections are believed to be small.[7]

I. A REVIEW OF THE IMPULSE APPROXIMATION

A number of authors have studied the impulse approximation for the elastic scattering of relativistic electrons from deuterons using a nonrelativistic wave function to describe the deuteron structure.[2,7,10] The basis of the impulse approximation is that one assumes that the electron interacts with the individual nucleons in the deuteron, and that the interaction can be accurately described using free nucleon form factors. The theoretical treatments are in substantial agreement: the only points not totally agreed upon are rather small kinematical relativistic corrections[7] and the most appropriate frame in which to describe the deuteron by a nonrelativistic wave function—the laboratory frame of the target deuteron or the Breit frame. However, since a nonrelativistic wave function is used to describe the deuteron structure, we do not believe that agreement on these minor points is in principle important and we will therefore consider the impulse approximation, in terms of a nonrelativistic deuteron wave function, as basically understood.

Recent attempts to treat the deuteron using a relativistic wave function or dispersion relations with approximate unitarity[11,12] constitute an important and very interesting alternative approach to the problem. Unfortunately quantitative understanding of the dispersion relation approach is still somewhat lacking as a result of the large number of relevant diagrams. Thus, little is really known about intrinsically relativistic "corrections," if indeed the difference between relativistic results and nonrelativistic results can be considered as corrections. Thus, in this paper we will limit ourselves to a description of the deuteron in terms of a

[1] J. Blatt and V. Weisskopf, *Theoretical Nuclear Physics* (John Wiley & Sons, New York, 1952).
[2] N. Glendenning and G. Kramer, Phys. Rev. **126**, 2159 (1962).
[3] S. Drell and F. Zachariasen, *Electromagnetic Structure of Nucleons* (Oxford University Press, New York, 1961).
[4] L. N. Hand, D. G. Miller, and R. Wilson, Rev. Mod. Phys. **35**, 335 (1963).
[5] R. J. Adler and S. Drell, Phys. Rev. Letters **13**, 349 (1964).
[6] C. D. Buchanan and M. R. Yearian, Phys. Rev. Letters **15**, 303 (1965).
[7] R. J. Adler, thesis, Stanford University, 1965 (unpublished).
[8] H. F. Jones, Nuovo Cimento **26**, 4622 (1962).
[9] R. J. Adler and E. F. Erickson, Nuovo Cimento (to be published).
[10] V. Z. Jankus, Phys. Rev. **102**, 1586 (1956).
[11] J. Nuttal, Nuovo Cimento **29**, 841 (1963).
[12] F. Gross, Phys. Rev. **134**, B405 (1964).

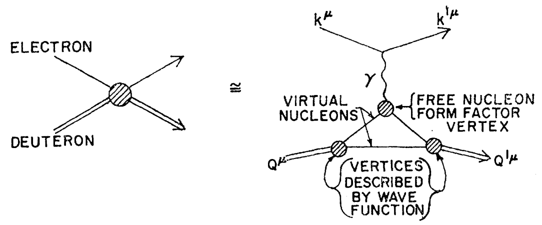

FIG. 1. Impulse approximation graph.

wave function. In another paper in preparation[9] we discuss, following Cutkosky[13] and Jones,[8] a plausible phenomenological modification of the standard nonrelativistic wave function results to include relativistic effects.

The exchange current that we discuss in the next section can therefore be considered as one of a number of relativistic effects which should be understood before the over-all elastic e-d scattering problem can be considered understood.

The impulse approximation (i.a.) involves the study of a γ-ray interacting with the individual nucleons in the deuteron, and we can picture the situation diagrammatically as in Fig. 1. We have used a calculational method analogous to the familiar Feynman graphic technique of quantum electrodynamics with the following assumptions and approximations:

(1) The effect of binding in the deuteron is taken account of by using a deuteron bound-state wave function for the incoming and outgoing neutron and proton in place of Dirac plane-wave spinors. The same will be done when we consider exchange currents in the next section. Incidentally, one may treat the deuteron center-of-mass coordinates completely relativistically and the nonrelativistic nature of the wave function only enters when the relative neutron-proton coordinates are considered.

(2) As a result of using a nonrelativistic deuteron wave function we have chosen to use Pauli spinors.[2]

(3) The bound-nucleon electromagnetic form factors are assumed to be the same as the free-nucleon form factors.

(4) The target deuteron is assumed to be at rest in the lab frame, where it is described by a nonrelativistic wave function. (As noted above, it is difficult to say whether the lab frame or the Breit frame is more appropriate.) We also assume that the dueteron recoil momentum is quite small compared to its mass, i.e., low momentum transfer. Only second-order terms in the momentum transfer q are retained in the cross section.

The result of our impulse approximation analysis[7] is an S-matrix element of the form,

$$S_{\text{i.a.}}=-ie^2(m^2M_d/k_0k_0'Q_0')^{1/2}(2\pi)^4 \times\delta^4(Q'+k'-Q-k)\Lambda^\mu j_\mu(1/q^2)\,, \tag{1.1}$$

where the factor Λ^μ can be identified as the deuteron four-vector electromagnetic current. This current contains 3 form factors and is given to order q^2 by

$$\begin{aligned}\Lambda^0&=\chi_f^\dagger[(G_{Ed}-G_{Qd}(S_{12}/\sqrt{8}))(1+q^2/4M_d^2)]\chi_i\\ \Lambda^i&=-\chi_f^\dagger[(i/2M_d)\epsilon^{imn}q^m\sigma_{(S)}{}^nG_{Md}]\chi_i\,,\end{aligned} \tag{1.2}$$

where the notation follows Refs. (5) and (7): the χ are pairs of Pauli spinors in the triplet state, S_{12} is the tensor operator familiar from nuclear physics, and $\sigma_{(S)}{}^n$ is the isoscalar Pauli spin matrix, $\sigma_n{}^n+\sigma_p{}^n$, and repeated indices are summed over. Since the deuteron has spin 1, it can only have 3 independent form factors so the current (1.2) must actually be general if the functions G_{Ed}, G_{Qd}, and G_{Md} are considered arbitrary.[14] The cross section which results from the current (1.2) is

$$\frac{d\sigma}{d\Omega}=\left(\frac{d\sigma}{d\Omega}\right)_{\text{Mott}}\left[(G_{Ed}^2+G_{Qd}^2)\left(1+\frac{q^2}{2M_d^2}\right)-\frac{2}{3}\frac{q^2}{M_d^2}[2\tan^2(\theta/2)+1]G_{Md}^2\right]. \tag{1.3}$$

In the special case of the impulse approximation, the deuteron G functions appearing in (1.2) can be explicitly written in terms of the two nucleon isoscalar form factors G_{ES} and G_{MS} and integrals over the deuteron S and D radial wave functions, u and w, as[2,7] follows,

$$\begin{aligned}G_{Ed}&=G_{ES}\int_0^\infty (u^2+\omega^2)j_0(qy/2)dy\,,\\ G_{Qd}&=G_{ES}\int_0^\infty 2w(u-w/\sqrt{8})j_2(qy/2)dy\,,\\ G_{Md}&=G_{MS}\int_0^\infty [(u^2-\tfrac{1}{2}w^2)j_0(qy/2)\\ &\quad+w/\sqrt{2}(u^2+w^2/\sqrt{2})j_2(qy/2)]dy\\ &\quad+\tfrac{3}{4}G_{ES}\int_0^\infty w^2[j_0(qy/2)+j_2(qy/2)]dy\,.\end{aligned} \tag{1.4}$$

These expressions contain q which means $|\mathbf{q}|$ and should be considered reliable only for $q\ll M_d$; in particular, small corrections to these quantities such as obtained in Ref. (7) are not considered sufficiently reliable to include here.

Let us lastly note that in the limit of zero momentum transfer, the deuteron form factors in (1.3) correspond to the static electric and magnetic moments of the deuteron as follows:

$$\begin{aligned}&G_{Ed}(0)=1\,,\quad \text{(charge)}\\ &G_{Md}(0)=\mu_d=(\mu_p+\mu_n)-\tfrac{3}{2}P_D(\mu_p+\mu_n-\tfrac{1}{2})\,,\quad \text{(magnetic moment)}\\ &\lim_{q^2\to 0}\left[\frac{6G_{QD}(q^2)}{\sqrt{2}\mathbf{q}^2}\right]=Q\,.\quad \text{(quadrupole moment)}\end{aligned} \tag{1.5}$$

[13] R. E. Cutkosky, in *1960 Annual Conference on High Energy Physics* (Interscience Publishers, Inc., New York, 1960).

[14] V. Glazer and B. Jaksic, Nuovo Cimento **5**, 1197 (1957).

The theoretical expression for the magnetic moment is derived from (1.4) and is model-independent except for P_D, the percent of D wave in the deuteron, which we take to be[15]

$$P_D \equiv \int_0^\infty w^2 dy = 0.07.$$

With this value of P_D, the value of μ_d is 0.840 whereas the experimental value is 0.857. This small discrepancy is nevertheless considered rather serious since one must reduce P_D to 0.04 to remove it, in conflict with much analysis of n-p scattering data.[2] We will show in the next section that it may be explainable in terms of mesonic exchange effects, and that this explanation is consistent with recent large angle e-d scattering experiments at large q^2 values.

II. THE $\rho\pi$ EXCHANGE CURRENT

The results of the impulse approximation which we discussed in Sec. I have been found to be reasonably consistent and useful for the purpose of extracting the neutron form factors from elastic e-d scattering measurements at low q^2 values; however several difficulties do occur. Specifically these are:

(1) The scattering data indicate that the neutron electric form factor G_{En} is very small and consistent with zero up to $q^2 \simeq 10$ (fermi)$^{-1}$,[16] whereas the well-established neutron-electron interaction (as discussed by Foldy[17]) indicates that G_{En} has a positive slope of about 0.02 (fermi)$^{+2}$ at $q^2=0$. Taken together these results imply a surprisingly rapid curvature of G_{En} at small q^2.[9]

(2) The theoretical value of the deuteron magnetic moment as discussed in section I (0.840 nm) differs from the experimental value (0.857 nm) by about 2%.[1,2] This seemingly small discrepancy is disturbing because the magnetic moment is a static quantity and is model-independent except for P_D, the amount of D wave admixture.

(3) Recent large-angle (or magnetic) e-d scattering experiments seem to be in rather poor agreement with the predictions of the impulse approximation. In fact at q^2 above 6 (fermi)$^{-2}$ there appears to be a factor of two discrepancy in the preliminary results of Buchanan *et al.*[6] We will discuss these difficulties in detail in the next section: they constitute a motivation to study non-impulse contributions to elastic e-d scattering.

Let us limit ourselves at the start to the simplest and lightest "exchange currents." By exchange currents we will mean, in this paper, those processes which do not involve the interaction of the γ ray with the individual

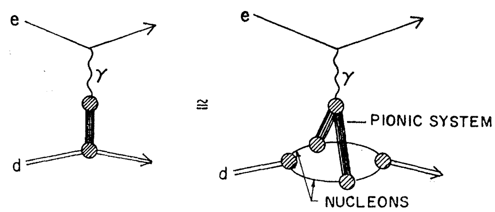

FIG. 2. General exchange current graph.

dressed neutron or proton, but with the two-nucleon system. For simplicity we will consider only single pions or pionic resonances being exchanged by the neutron and proton as pictured on the right of Fig. 2.

It has already been pointed out by several authors, recently by Gourdin,[18] that the lightest pionic system which can occur in Fig. 2 is not 2 but 3 pions. Indeed no even number of pions can occur. The argument leading to this conclusion is simple and for completeness we will repeat it here. In Fig. 2 we see that the pionic system must have isospin $I=0$ since the deuteron enters and emerges from the interaction with $I=0$. Thus the G parity of the pionic system is

$$G = e^{i\pi I_y} C = C. \tag{2.1}$$

Any pionic system is an eigenstate of G parity,[19] in general $G=(-1)^n$, and it must join to the γ ray which has $C=-1$. Thus from (2.1)

$$G = C = (-1)^n = -1, \tag{2.2}$$

so n must be odd and therefore at least 3. One thus arrives at the result that the lightest and simplest exchange diagram is that of Fig. 3, as also discussed by Gourdin.

To estimate the contribution of Fig. 3 we have assumed that the 2-pion system which lands on one nucleon can be approximated by the ρ-resonance.[20] Thus we investigate the last diagram in Fig. 3.

This exchange current has the attractive and important virtue that the $\rho\pi\gamma$ coupling can be directly observed experimentally in high-energy 2-pion peripheral photoproduction as well as in the direct measurement of the decay width for $\rho \to \pi+\gamma$. The decay width of ρ into $\pi+\gamma$ is at present provisionally estimated to be about 0.5 MeV.[21] This value is inferred from the energy

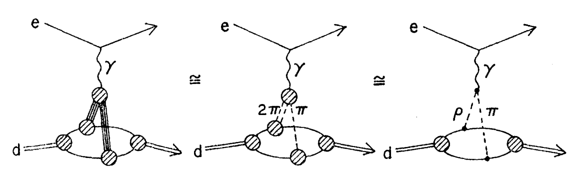

FIG. 3. $\rho\pi$ exchange current graph.

[15] R. Wilson, *The Nucleon-Nucleon Interaction* (Oxford University Press, New York, 1964).

[16] D. Benaksas, D. J. Drickey, and D. Frèrejacque, Phys. Rev. Letters 13, 353 (1964).

[17] L. L. Foldy, Rev. Mod. Phys. 30, 471 (1958).

[18] M. Gourdin, Nuovo Cimento 28, 2097 (1963).

[19] J. J. Sakurai, *Invariance Principles and Elementary Particles* (Princeton University Press, Princeton, New Jersey, 1964).

[20] This assumption is also made by Y. Fujii and M. Kawagachi, Progr. Theoret. Phys. (Kyoto) 26, 519 (1961).

[21] S. Berman and S. Drell, Phys. Rev. 133, B791 (1964).

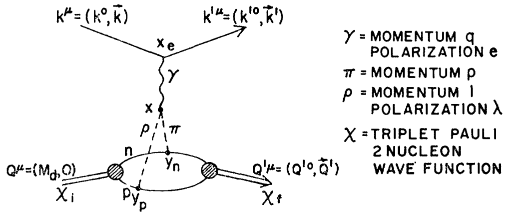

FIG. 4. Feynman diagram for $\rho\pi$ exchange current.

dependence of forward ρ° photoproduction from hydrogen in the 2- to 3-BeV energy region, which indicates that diffraction production is not the whole story. With neglect of final-state absorption corrections (which would increase the coupling strength) this estimate is arrived at by assigning the energy-dependent part of the ρ° photoproduction amplitude to a one-pion exchange graph. Thus we do not invent a new process. We are invoking one which is already believed to exist,[21,22] and computing its role in the e-d cross section.

The $\rho\pi$-d over-all vertex in Fig. 3 will be the product of a π-N and a ρ-N vertex. For the π-N vertex we use the usual nonrelativistic reduction of the pseudoscalar meson-nucleon vertex,[15]

$$(iG/2M)\chi_f^\dagger(\boldsymbol{\sigma}\cdot\mathbf{p})\chi_i+O(p^3)\,;\quad G^2/4\pi\cong 14\,,\quad(2.3)$$

where $\mathbf{p}$ is the 3-momentum transfer to the nucleon[15] and the χ are nucleon Pauli spinors. The restriction to small momentum transfers explicit in (2.3) will be satisfied in the case of the deuteron since, as can be seen intuitively, the momentum-space wave function has a width of the order of $\frac{1}{2}$ (fermi)$^{-1}\ll M$, and we will restrict our analysis to external momentum transfers of $|q|\ll M$.

The ρ-N vertex will be assumed to have the same form as the general γ-N vertex but with "point" coupling constants $a=G_{EV}(0)=\frac{1}{2}$, $b=G_{MV}(0)=2.35$. This is equivalent to assuming that the ρ-resonance propagator nearly dominates the q^2 dependence of the nucleon isovector form factor. The nonrelativistic limit of this interaction vertex is then a four vector "current" with components given by[3]

$$\chi_f^\dagger\Gamma^0\chi_i=a\chi_f^\dagger\chi_i\,,\quad\text{(0th component)}$$
$$\chi_f^\dagger\Gamma^i\chi_i=\chi_f^\dagger[a(P^i/2M)-b(i/2M)\epsilon^{ijk}l^j\sigma^k]\chi_i\,,\quad\text{(space component)}\quad(2.4)$$

where P^i is the sum of the nucleon three-momenta before and after interaction, and the l^j is the three-momentum transfer to the nucleon. The constants a and b in (2.4) are simply related to the static values of the nucleon isovector form factors by[3]

$$a\cong F_1{}^V(0)=0.5\,,\quad b/a\cong G_{MV}(0)/F_1{}^V(0)=4.7\,.\quad(2.5)$$

If the q^2 dependence is considered to occur from a 2-pole fit, the values of a and b tend to be about twice the above values[4]:

$$a\cong 1\,,\quad b/a\cong 4.7\,.\quad\text{(2-pole fit)}.\quad(2.6)$$

The remaining vertex in Fig. 3, the $\rho\pi\gamma$ vertex itself, has the general unique and gauge invariant form

$$\mathfrak{L}_{\mathrm{I}}=(g_{\rho\pi\gamma}/2m_\rho)\epsilon_{\alpha\beta\gamma\delta}F^{\alpha\beta}\rho_\gamma(\partial\pi/\partial x^\delta)\,;\quad g_{\rho\pi\gamma}{}^2/4\pi\cong 1/50\,,\quad(2.7)$$

where the absolute value of the dimensionless constant $g_{\rho\pi\gamma}$ corresponds to a decay width of $\frac{1}{2}$ MeV as estimated in Ref. 21, $\epsilon_{\alpha\beta\gamma\delta}$ is the antisymmetric pseudotensor of rank 4, and ρ and π are the rho and pion field amplitudes. In momentum space, the vertex (2.7) takes the simple form (see Fig. 4)

$$(g_{\rho\pi\gamma}/m_\rho)\epsilon_{\alpha\beta\gamma\delta}e^\alpha q^\beta\lambda^\gamma p^\delta\,,\quad(2.8)$$

with a possible momentum-dependent form factor, which we neglect.

Before proceeding to write the Feynman amplitude let us consider the effect of isotopic spin on our amplitude. The ρ and π systems have $I=1$ and are exchanged between nucleons in an isotopic singlet state, $I=0$. If the $\rho\pi\gamma$ vertex is assumed isospin invariant, a sum over the possible isospin states therefore gives a factor of -3. Otherwise one obtains a factor of $-(2\lambda+1)$, where λ is a dimensionless parameter which expresses the altered coupling of *charged* mesons at the vertex. If we use $SU(3)$ symmetry with the electromagnetic current considered as a member of the octet we obtain $\lambda=1$, which corresponds to the above value of $SU(2)$ symmetric coupling and is the value we will adopt.

It is now possible to calculate the S-matrix element for the $\rho\pi$ exchange picture in Fig. 4. In complete analogy with quantum electrodynamics[23] we obtain

$$\begin{aligned}S_{\rho\pi}=ie\int d^4y_pd^4y_n\chi_f{}^\dagger(y_p,y_n)\\ \times\Bigg\{\frac{iG}{2M}p^i(\sigma^i\colon\Gamma^\mu)\lambda_\mu\left[\frac{d^4p}{(2\pi)^4}\frac{e^{-ip\cdot(y_n-x)}}{p^2-m_\pi{}^2}\right]\\ \times\left[\frac{d^4l}{(2\pi)^4}\frac{e^{-il\cdot(y_p-x)}}{l^2-m_\rho{}^2}\right]\left[\frac{g_{\rho\pi\gamma}}{m_\rho}\epsilon^{\alpha\beta\gamma\delta}e_\alpha\lambda_\beta p_\gamma q_\delta\right]\\ \times\left[\frac{d^4q}{(2\pi)^4}\frac{e^{-iq\cdot(x-x_e)}}{q^2}\right]e^\nu[\bar{u}(k')\gamma_\nu u(k)]\\ \times(m^2M_D/k_0k_0'Q_0')^{1/2}e^{-i(k-k')\cdot x_e}\Bigg\}\\ \times\chi_i(y_p,y_n)d^4xd^4x_e+\text{(diagram with}\\ \text{neutron and proton reversed)}\,,\quad(2.9)\end{aligned}$$

[22] J. J. Szymanski *et al.*, Bull. Am. Phys. Soc. **9**, 408 (1964).

[23] J. Bjorken and S. Drell, *Relativistic Quantum Mechanics* (McGraw-Hill Book Company, Inc., New York, 1964).

where we introduce the symbol $(\sigma^j:\Gamma^\mu)$ which indicates that σ^j acts on the neutron spinor and Γ^μ on the proton spinor. As usual in diagrammatic calculations we sum over the polarization vectors of the virtual γ and ρ to obtain the covariant recipe

$$e_\alpha e_\nu \longrightarrow -g_{\alpha\nu}, \quad \lambda_\mu\lambda_\beta \longrightarrow -g_{\mu\beta}, \tag{2.10}$$

as prescribed by current conservation at the ρ-nucleon and γ-electron vertices. To put the deuteron wave functions in manageable form we next introduce 4-dimensional relative and center-of-mass coordinates for the deuteron

$$\begin{aligned} Y&=\tfrac{1}{2}(y_p+y_n), & y&=y_p-y_n, \\ y_p&=Y+\tfrac{1}{2}y, & y_n&=Y-\tfrac{1}{2}y, \end{aligned} \tag{2.11}$$

and assume

$$\begin{aligned} \chi_f^\dagger&=e^{iQ'\cdot Y}\varphi_f^\dagger(\mathbf{y}), \\ \chi_i&=e^{-iQ\cdot Y}\varphi_i(\mathbf{y}). \end{aligned} \tag{2.12}$$

That is, we assume that the deuteron as a whole is described by a plane wave and the structure by the time-independent Pauli triplet wave functions φ which are obtained by phenomenological analyses of nucleon-nucleon interactions as discussed in Ref. (2). With the above assumptions and substitutions, (2.9) can be integrated over x, x_e, Y, and y_0.

$$\begin{aligned} S_{\rho\pi}=2ie(2\pi)^4\delta^4(Q'-Q-q)\int d^3y\ \varphi_f^\dagger(y) \\ \times\left\{\frac{iG}{2M}p^j(\sigma^j:\Gamma_\beta)\left[\frac{e^{-ip\cdot y/2}}{p^2-m_\pi^2}\right]\left[\frac{e^{il\cdot y/2}}{l^2-m_\rho^2}\right]\frac{g_{\rho\pi\gamma}}{m_\rho}\right. \\ \left.\times\epsilon^{\alpha\beta\gamma\delta}p_\gamma q_\delta j_\alpha(1/q^2)(m^2M_d/k_0k_0'Q_0')^{1/2}\right\} \\ \times\varphi_i(\mathbf{y})2\pi\delta^0(p_0-l_0)d^4p/(2\pi)^4 \\ +(\text{diagram with neutron and proton} \\ \text{reversed}):\quad j_\alpha=\bar{u}(k')\gamma_\alpha u(k). \end{aligned} \tag{2.13}$$

Note the occurrence of the factor $(2\pi)\delta^0(p_0-l_0)$. This merely means that "no" energy can be transferred between the nucleons; it is a result of using the nonrelativistic time-independent wave functions $\varphi_f^\dagger$ and φ_i to describe the deuteron structure in a non-covariant way and in reality only limits our analysis to "low" momentum transfer. To further simplify the exchange S matrix, we transform to a new four-vector τ which is the difference between p and l

$$\begin{aligned} \tau&=p-l, & q&=p+l, \\ p&=\tfrac{1}{2}(q+\tau), & l&=\tfrac{1}{2}(q-\tau). \end{aligned} \tag{2.14}$$

The dp_0 or, equivalently, the $d\tau_0$ integral in (2.13) is then easily performed:

$$\begin{aligned} S_{\rho\pi}=ie(2\pi)^4\delta^4(Q'-Q-q)\int d^3y\frac{d^3\tau}{8(2\pi)^3} \\ \times\left\{\varphi_f^\dagger(\mathbf{y})(\sigma^j:\Gamma_\beta)\varphi_i(\mathbf{y})e^{i\tau\cdot\frac{1}{2}\mathbf{y}}\frac{iG}{2M}p^j\right. \\ \times[(p^2-m_\pi^2)(l^2-m_\rho^2)]^{-1}(g_{\rho\pi\gamma}/m_\rho) \\ \left.\times\epsilon^{\alpha\beta\gamma\delta}p_\gamma q_\delta j_\alpha q^{-2}(m^2M_d/k_0k_0'Q_0')^{1/2}\right\} \\ +(\text{diagram with neutron and} \\ \text{proton reversed}). \end{aligned} \tag{2.15}$$

Now only $\mathbf{y}$ and $\boldsymbol{\tau}$ remain as three-dimensional integration variables. Using (2.15) we can proceed to calculate the value of the mesonic exchange current.

To extract an exchange *current* from the S matrix (2.15) we recall that the impulse approximation S matrix (1.1) could be written as

$$\begin{aligned} S_{\text{i.a.}}&=-i\mathfrak{D}\Lambda_{\text{i.a.}}{}^\mu j_\mu(1/q^2) \\ \mathfrak{D}&=e^2(m^2M_d/k_0k_0'Q_0')^{1/2}(2\pi)^4\delta^4(Q'-Q-q), \end{aligned} \tag{2.16}$$

where $\Lambda_{\text{i.a.}}{}^\mu$ was identified as the electromagnetic current and was written explicitly in (1.2). Likewise (2.15) can be written as

$$\begin{aligned} S_{\rho\pi}&=i\mathcal{E}W^\mu j_\mu(1/q^2), \\ \mathcal{E}&=\left(\frac{Gg_{\rho\pi\gamma}e}{64Mm_\rho}\right)\left(\frac{m^2M_d}{k_0k_0'Q_0'}\right)^{1/2}(2\pi)^4\delta^4(Q'-Q-q), \\ W^\mu&=4i\int d^3y\frac{d^3\tau}{(2\pi)^3}\left\{\varphi_f^\dagger(\mathbf{y})(\sigma^j:\Gamma_\beta)\varphi_i(\mathbf{y})\right. \\ &\quad\left.\times\left[\frac{e^{i\tau\cdot\frac{1}{2}\mathbf{y}}}{(p^2-m_\pi^2)(l^2-m_\rho^2)}\right]\epsilon^{\alpha\beta\gamma\delta}p^jp_\gamma q_\delta\right\} \\ &\quad+(\text{diagram with neutron and} \\ &\quad\text{proton reversed}). \end{aligned} \tag{2.17}$$

It thus becomes obvious that the $\rho\pi$ S matrix (2.15) leads to an exchange current

$$\Lambda_{\rho\pi}{}^\mu=-(\mathcal{E}/\mathfrak{D})W^\mu, \quad \mathcal{E}/\mathfrak{D}=-Gg_{\rho\pi\gamma}/64Mm_\rho e, \tag{2.18}$$

and a total current and S matrix

$$\Lambda_{\text{total}}{}^\mu=\Lambda_{\text{i.a.}}{}^\mu+\Lambda_{\rho\pi}{}^\mu, \quad S_{\text{total}}=-i\mathfrak{D}\Lambda_{\text{total}}{}^\mu j_\mu(q^2). \tag{2.19}$$

Moreover the form of current in (1.2) is *general*, as noted, so $\Lambda_{\rho\pi}{}^\mu$ must have this same form and can thus only alter the three form factors from the impulse approximation values by amounts ΔG_{Ed}, ΔG_{Qd}, and ΔG_{Md}. Thus our remaining task is to put $\Lambda_{\rho\pi}{}^\mu$ into the form (1.2), identify the form-factor corrections, and then calculate or estimate their magnitude. We therefore must perform a bit of algebraic manipulation with the W^μ expression in (2.16) in the next section.

III. EVALUATION OF THE MESONIC FORM-FACTOR CORRECTIONS

To compare our calculation with experiment we wish to put it in a form where the mesonic form-factor corrections (ΔG's) become manifest. Since the coupling of the ρ to the nucleon is predominantly due to the a term (of order 1) in Γ_0 (2.4) and not the b term (of order p), we first investigate the effect of Γ_0. We accordingly write, very symbolically, the indexed quantities which appear in (2.17) as

$$\begin{aligned} W^\mu &\sim \epsilon^{\mu\beta\gamma\delta}\Gamma_\beta p_\gamma q_\delta \\ &\sim \epsilon^{\mu\beta\gamma\delta}\Gamma_\beta \tau_\gamma q_\delta. \end{aligned} \tag{3.1}$$

The last line follows from the definition of τ, $p=\frac{1}{2}(q+\tau)$, in (2.14). Since τ_0 is very small as discussed in Sec. II and since the energy transfer is second order, $q_0\cong|\mathbf{q}|^2/4m$, we see that the W^μ terms for $\mu\neq 0$ should dominate the exchange current on the basis of order of magnitude considerations. Thus we will first consider

$$W^i\sim\epsilon^{i0kl}\Gamma_0\tau_k q_l\,,\quad \epsilon^{i0kl}=-\epsilon^{ikl}\,, \tag{3.2}$$

where we now adopt the convention that Latin indices run from 1 to 3. The separation—Γ_0 corresponds to space terms or magnetic effects—is however another low q^2 approximation.

Using the above we will calculate only Γ_0 and magnetic current effects in this section. That is we will deal only with the magnetic form factor correction ΔG_{Md}. There do exist ΔG_{Ed} and ΔG_{Qd} corrections but we have found that these are small, as expected, and will return to them later in Sec. IV.

Since Γ_0 is simply aI, the calculation of W^i is rather simple. For $\mu=i$ the expression (2.17) becomes

$$W^i=-4ia\int d^3y\frac{d^3\tau}{(2\pi)^3}[\varphi_f^\dagger\sigma_{(s)}{}^i\varphi_i] \times\frac{e^{i\tau\cdot\frac{1}{2}\mathbf{y}}}{(p^2-m_\pi{}^2)(l^2-m_\rho{}^2)}p^jp^kq^l\epsilon^{ikl}, \tag{3.3}$$

where $\boldsymbol{\sigma}_{(s)}=(\boldsymbol{\sigma}:I)+(I:\boldsymbol{\sigma})$ is isoscalar Pauli spin matrix. In terms of the radial u and w functions the deuteron wave function may be written[1]

$$\varphi=\frac{1}{(4\pi)^{1/2}}\left(\frac{u}{y}+\frac{S_{12}w}{(\sqrt{8})y}\right)\chi=\left(\psi_s+\frac{\psi_D}{\sqrt{8}}S_{12}\right)\chi, \tag{3.4}$$

where S_{12} is the familiar tensor operator and χ is a *pair* of Pauli spinors in the triplet state.

The integral over y in (3.3) then becomes

$$\begin{aligned} &\int d^3y[\varphi_f^\dagger\sigma_{(s)}{}^i\varphi_i]e^{i\tau\cdot\frac{1}{2}\mathbf{y}} \\ &=\int\psi_s{}^2e^{i\tau\cdot\frac{1}{2}\mathbf{y}}d^3y(\chi_f^\dagger\sigma_{(s)}{}^i\chi_i) \\ &\quad+\chi_f^\dagger\left(\int\frac{\psi_s\psi_D}{\sqrt{8}}\{S_{12}(\hat{y}),\sigma_{(s)}{}^i\}e^{i\tau\cdot\frac{1}{2}\mathbf{y}}d^3y\right)\chi_i \\ &\quad+\chi_f^\dagger\left(\int\frac{\psi_D{}^2}{8}S_{12}(\hat{y})\sigma_{(s)}{}^iS_{12}(\hat{y})e^{i\tau\cdot\frac{1}{2}\mathbf{y}}d^3y\right)\chi_i{}^k. \end{aligned} \tag{3.5}$$

To explicitly evaluate these we define a tensor T^{lm} by

$$\begin{aligned} S_{12}(\hat{y})&=3\hat{y}^l\hat{y}^m(\sigma^l:\sigma^m)-(\sigma^l:\sigma^l) \\ &\equiv T^{lm}(\hat{y})(\sigma^l:\sigma^m)\,, \\ T^{lm}(\hat{y})&=3\hat{y}^l\hat{y}^m-\delta^{lm}\,, \end{aligned} \tag{3.6}$$

and use the easily proven theorems[7]

$$\begin{aligned} &\{S_{12}(\hat{y}),\sigma_{(s)}{}^j\}=2T^{jm}(\hat{y})\sigma_{(s)}{}^m\,, \\ &S_{12}(\hat{y})\sigma_{(s)}{}^jS_{12}(\hat{y})=4[T^{jm}(\hat{y})-\delta^{jm}]\sigma_{(s)}{}^m\,, \\ &\int d\Omega\; T^{jm}(\hat{y})e^{i\tau\cdot\frac{1}{2}\mathbf{y}}=4\pi j_2(\tau y/2)T^{jm}(\hat{\tau})\,, \\ &\int d\Omega\;\delta^{jm}e^{i\tau\cdot\frac{1}{2}\mathbf{y}}=4\pi j_0(\tau y/2)\delta^{jm}. \end{aligned} \tag{3.7}$$

Equation (3.5) then becomes

$$\begin{aligned} &\int d^3y[\varphi_f^\dagger\sigma_{(s)}{}^i\varphi_i]e^{i\tau\cdot\frac{1}{2}\mathbf{y}} \\ &=(\chi_f^\dagger\sigma_{(s)}{}^m\chi_i)\left[\int_0^\infty(u^2-\tfrac{1}{2}w^2)j_2(\tau y/2)dy\;\delta^{im}\right. \\ &\qquad\left.-\int_0^\infty\frac{2w}{\sqrt{8}}\left(u+\frac{w}{\sqrt{2}}\right)j_2(\tau y/2)dy\;T^{im}(\hat{\tau})\right] \\ &\equiv(\chi_f^\dagger\sigma_{(s)}{}^m\chi_i)[f_1(\tau)\delta^{im}-f_2(\tau)T^{im}(\hat{\tau})]\,. \end{aligned} \tag{3.8}$$

It remains now to integrate this over τ as indicated in (3.3). To do this we orient the z axis along $\mathbf{q}$. The evaluation is then straightforward and yields

$$W^i=-ia\epsilon^{ikl}q^l(\chi_f^\dagger\sigma_{(s)}{}^k\chi_i) \times\int\frac{d^3\tau(\tau_z)^2[f_1(\tau)-f_2(\tau)]}{(2\pi)^3(p^2-m_\pi{}^2)(l^2-m_\rho{}^2)}. \tag{3.9}$$

Comparison of this expression with the expressions

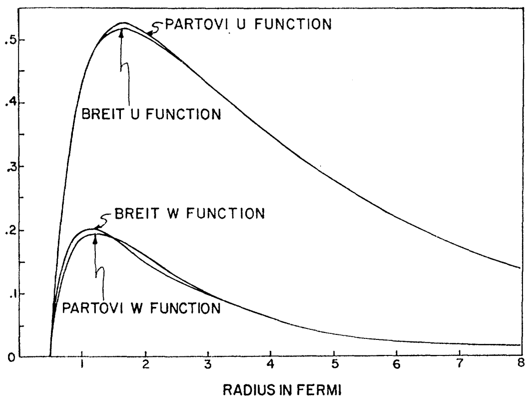

FIG. 5. Deuteron wave functions.

(2.18) and (2.19) then gives the final result

$$\Delta G_{Md}(q)\cong\Gamma\int\frac{d^3\tau(\tau_x)^2}{(2\pi)^3(p^2-m_\pi^2)(l^2-m_\rho^2)}\times\int_0^\infty dy[(u^2-\tfrac{1}{2}w^2)j_0(\tau y/2)-\sqrt{2}w(u+w/\sqrt{2})j_2(\tau y/2)],\quad(3.10)$$

$$\Gamma\equiv Gg_{\rho\pi\gamma}a/16m_\rho e.$$

From (3.10) it is evident that the dependence of ΔG_{Md} on q^2 is not especially great until $q^2\gtrsim 4m_\pi^2$, since $p^2\cong -\frac{1}{2}(\mathbf{q}+\boldsymbol{\tau})^2$ and $l^2\cong -\frac{1}{2}(\mathbf{q}-\boldsymbol{\tau})^2$. For this reason and because $\Delta G_{Md}(0)$ is the static magnetic moment we will now set $q=0$ in (3.10) and simplify that expression. Indeed for the value $q=0$ the angular integral is simple and the τ integral can be easily performed by contour integration. The result is

$$\Delta G_{Md}(0)=\frac{8\Gamma}{3\pi(m_\rho^2-m_\pi^2)}\times\int_0^\infty\left[\left(m_\rho^2\frac{e^{-m_\rho y}}{y}-m_\pi^2\frac{e^{-m_\pi y}}{y}\right)(u^2-\tfrac{1}{2}w^2)+\left(m_\rho^2\left[1+\frac{3}{m_\rho y}+\frac{3}{m_\rho^2y^2}\right]\frac{e^{-m_\rho y}}{y}-m_\pi^2\left[1+\frac{3}{m_\pi y}+\frac{3}{m_\pi^2y^2}\right]\frac{e^{-m_\pi y}}{y}\right)\times\sqrt{2}w(u+w/\sqrt{2})\right]dy,\quad(3.11)$$

which, despite its bulky appearance, is in convenient form for numerical evaluation as we shall discuss in the next section.

IV. THE DEUTERON MAGNETIC MOMENT

In this section we will discuss the relation of the theoretical results of the last section to experiment.

Let us begin with $\Delta G_{Md}(0)$, which we can associate with a mesonic contribution to the deuteron's static magnetic moment. In (3.11) we obtain a form for this quantity in terms of a one-dimensional weighted integral over the S and D radial wave functions of the deuteron. To obtain a numerical value we must estimate the coupling constants in (3.11) and perform the integration over the u and w functions. For the constants appearing in (3.11) we have chosen the values

$$\begin{aligned}&G^2/4\pi=14.0,\quad(\text{Ref. }15)\\&g_{\rho\pi\gamma}^2/4\pi=0.18,\quad(\text{Ref. }21)\\&e^2/4\pi=1/137,\\&a=\tfrac{1}{2}\text{ or }1,\quad(\text{fit to nucleon isovector form factor—1 or 2 poles; Ref. 4})\\&m_\rho=750\text{ MeV},\\&m_\pi=137\text{ MeV}.\end{aligned}\quad(4.1)$$

The $g_{\rho\pi\gamma}$ is to be multiplied by -3 in accord with our discussion of isospin in Sec. II.

For the wave functions we have chosen the results of Breit and co-workers,[24] and of Partovi.[25] These two models represent independent analyses and we expect them to give us some idea of the model dependence of our result. The Breit model is obtained from a Yale potential and the Partovi from a Hamada potential: both have hard cores of about $\frac{1}{2}$ fermi. The wave functions bear a very close resemblance as is evident in Fig. 5. Using numerical integration[26] we obtained

$$\begin{aligned}\Delta G_{Md}&=\pm(0.94\text{ to }1.88)\times10^{-2};\quad\text{Breit model}\\&=\pm(0.90\text{ to }1.80)\times10^{-2};\quad\text{Partovi model}\\&=\pm(1\text{ to }2)\times10^{-2};\\&\qquad\sim5\%\text{ model dependence}.\end{aligned}\quad(4.2)$$

Note that our result involves an arbitrary sign and a factor of about 2 uncertainty due to the 1 or 2 pole fit to the nucleon isovector form factor.

Several points should be noted about our numerical results: (1) the dominant contribution to the numerical value (4.2) comes from the *uw* cross term, so we consider a pure S-wave deuteron model unsuitable for the theoretical analysis of exchange currents. (2) The results are somewhat sensitive to the presence and size of the nucleon hard core of about $\frac{1}{2}$ fermi—which is probably the best phenomenological description we have of very short range strong interaction effects.[2] The agreement to 5% between the different models, however, leads to confidence in the numerical results. (3) The ρ meson is

[24] K. E. Lassilila, M. H. Hull, Jr., H. M. Ruppel, F. A. McDonald, and G. Breit, Phys. Rev. **126**, 881 (1962).
[25] F. Partovi, Ann. Phys. (N. Y.) **27**, 79 (1964).
[26] E. F. Erickson performed the numerical integration on the IBM 7090 at Stanford University.

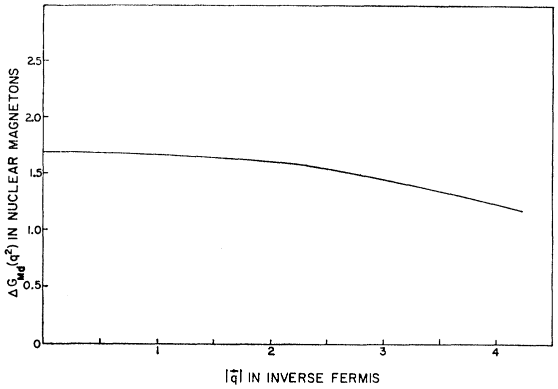

FIG. 6. $\Delta G_{Md}(q^2)$ versus q^2.

not a sharp state; therefore we investigated the effect of using a Breit-Wigner spectral function in its propagator as in Ref. 27 but found only a 5% effect, which cannot be considered significant.

We have already mentioned in Sec. I that the impulse approximation and elementary quantum mechanics give a theoretical value for the deuteron magnetic moment $\mu_d = G_{Md}(0)$ of 0.840 as opposed to the measured value of 0.857. The small but long standing and disturbing discrepancy of +0.017 is consistent with the mesonic contribution (4.2). It is thus an interesting possibility that this discrepancy is due to meson-exchange effects; indeed in the next section we will tentatively assume that this is indeed the case. Better estimates of the constants in (4.1) should determine the feasibility of this assumption. (Needless to say, one cannot ignore the possibility that the magnetic moment discrepancy is at least partially due to relativistic effects.)

V. BACK SCATTERING: COMPARISON WITH EXPERIMENT

In the preceding section we discussed a possible explanation of the disagreement between the measured value of the deuteron magnetic dipole moment and the value calculated by elementary methods. However such a test of a theory involves only one number, which is clearly an undesirable fact. It is therefore interesting to consider $\Delta G_{Md}(q^2)$ for nonzero q^2 and see if large angle scattering conforms to the predictions.

For nonzero q^2, the expression (3.10) is simple but tedious to calculate. However for small q^2 values, one would expect the denominator in (3.10) to be nearly constant and thus that $\Delta G_{Md}(q^2) \cong \Delta G_{Md}(0)$ until a value of q^2 in the region $4m_\pi^2$ or $4m_\rho^2$ is reached. We have verified that this is indeed the case as follows. Define the τ dependence in (3.10) as

$$F(\tau)=\int_0^\infty \left[(u^2-\tfrac{1}{2}w^2)j_0(\tau y/2) -\sqrt{2}w(u+\tfrac{1}{2}w)j_2(\tau y/2)\right]dy. \quad (5.1)$$

Then the angular integral in (3.10) can be explicitly done, yielding

$$\Delta G_{Md}(q^2)=\frac{\Gamma}{2(2\pi)^2}\int_0^\infty \tau^4 F(\tau)M(\tau,\mathbf{q}^2)d\tau$$

$$\begin{aligned} M(\tau,\mathbf{q}^2)=\frac{8}{\tau^2\mathbf{q}^2}+&\frac{1}{|\mathbf{q}|^3\tau^3(\mathbf{q}^2+\tau^2+2m_\rho^2+2m_\pi^2)} \\ &\times\left\{\left[(\mathbf{q}^2+\tau^2+4m_\rho^2)^2-4\mathbf{q}^2\tau^2\right]\right. \\ &\times\ln\left[\frac{\mathbf{q}^2+\tau^2+4m_\rho^2-2|\mathbf{q}|\tau}{\mathbf{q}^2+\tau^2+4m_\rho^2+2|\mathbf{q}|\tau}\right] \\ &+\left[(\mathbf{q}^2+\tau^2+4m_\pi^2)^2-4\mathbf{q}^2\tau^2\right] \\ &\left.\times\ln\left[\frac{\mathbf{q}^2+\tau^2+4m_\pi^2-2|\mathbf{q}|\tau}{\mathbf{q}^2+\tau^2+4m_\pi^2+2|\mathbf{q}|\tau}\right]\right\}. \quad (5.2)\end{aligned}$$

This expression has been integrated numerically and we have found that $\Delta G_{Md}(q^2)$ is indeed slowly varying out to $q \lesssim 3$ fermi as shown in Fig. 6.

We believe the lessons to be learned from the above and from Sec. III are:

(1) To realistic accuracy $\Delta G_{Md}(q^2) \cong \Delta G_{Md}(0)$ for $q \lesssim 3$.

(2) For larger q our analysis fails anyway because of neglect of higher order terms and the use of a nonrelativistic wave function to describe the deuteron.

(3) We normalize to 1.7×10^{-2} at $q^2=0$ for obvious reasons.

(4) The slow variation of $G_{Md}(q^2)$ is actually dependent only on the assumption of a *heavy* exchange current and not especially sensitive to the specific current chosen.

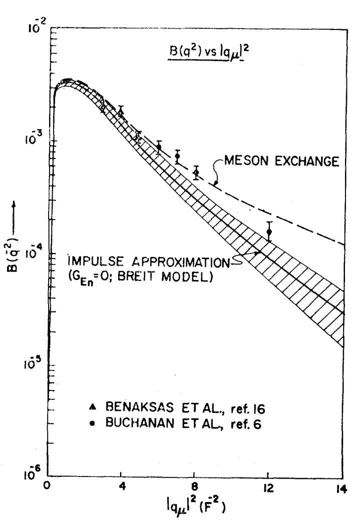

FIG. 7. Predicted backscatter versus. experiment, from Buchanan *et al.*, Ref. 6.

[27] M. W. Kirson, Phys. Rev. **132**, 1249 (1963).

In order to analyze several recent back-scattering experiments we thus need only the assumption that $\Delta G_{Md}(q^2)\cong 1.7\times 10^{-2}$ and some assumption about the isoscalar form factors of the nucleons. Buchanan[6] has empirically fitted the nucleon form-factor data and obtained the theoretical prediction shown in Fig. 7. The function $B(q^2)$ here represents the angle-dependent part of the cross section when written as

$$\frac{d\sigma}{d\Omega}=\left(\frac{d\sigma}{d\Omega}\right)_{\text{Mott}}\left[A(q^2)+B(q^2)\tan^2\tfrac{1}{2}\theta\right]. \qquad (5.3)$$

This function is clearly easy to compute if the nucleon form factors are known. The cross-hatched region represents theoretical uncertainties due to wave function uncertainty, relativistic corrections,[7] and uncertainty in the nucleon form factors.

Three sets of data now exist to test the prediction in Fig. 7. Goldemberg and Schaerf[28] have measured G_{Md} at q^2 values of 0.26 and 0.41 (fermi)$^{-2}$. Their data agree quite well with (5.4), but this must be considered as a consequence of using a correct normalization for $G_{Md}(0)$, not as a critical check of the theory and we have not included their points in Fig. 7. The second set of points at $q^2=3$, 4, and 5 (fermi)$^{-2}$ are those of Benaksas, Dricky, and Frerejacque,[16] which appear to be somewhat noncritical and do not clearly favor either curve. The last and most recent set of points are those of Buchanan *et al.*[6] at $q^2=6$, 7 and 8 (fermi)$^{-2}$. In this region the G_{Md} due to the impulse approximation has dropped to a value of the order of ΔG_{Md}, which is roughly a constant: thus the effect of ΔG_{Md} is large and noticeable. From Fig. 7 it appears that the curve *including* the $\rho\pi$ exchange current is strongly favored, despite the rather large theoretical uncertainty. It should be noted that the point at $q^2=12$ is noticeably lower than the exchange curve. This is to be expected since we do not trust our results too close to $q^2=m_\rho^2\cong 14$ (fermi)$^{-2}$. (In fact one may assume a form factor $(1+q^2/m_\omega^2)$ for $g_{\rho\pi\gamma}$, corresponding to an ω intermediate state in $\gamma\rightarrow\pi+\rho$. The result is to lower the exchange curve precisely to coincide with the part at 12.0. This is however too speculative to take very seriously.)

VI. FORWARD SCATTERING

For low q^2 and small-angle electron scattering, the electric monopole and quadrupole form factors of the deuteron dominate the *e-d* cross section (1.3). Indeed from the cross section (1.3) and the low q^2 limits (1.5), we see that the analysis of the neutron form factor via *e-d* scattering depends almost entirely on G_{Ed}. However we noted in Sec. III that the magnetic correction ΔG_{Md} is associated with the ρ-nucleon coupling Γ_0 while ΔG_{Ed} and ΔG_{Qd} are associated with the space components Γ_i of the ρ-nucleon coupling, which are an order of magnitude smaller than Γ_0. In this section we will discuss our estimates of ΔG_{Ed} and ΔG_{Qd} and their relevance to forward scattering and the analysis of neutron structure. Since we expect the corrections to be small we will ignore terms in w^2, which are expected to be doubly small. This simplifies the algebra considerably and provides what appears to be an adequate answer to our investigation, namely that the exchange has very little relevance to forward scattering at low q^2.

[28] J. Goldemberg and C. Schaerf, Phys. Rev. Letters **12**, 298 (1964).

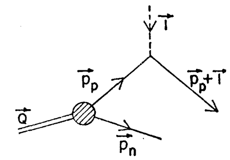

FIG. 8. Kinematics of ρ-nucleon interaction.

To obtain ΔG_{Ed} and ΔG_{Qd} we must evaluate W^0 in (2.17). As noted previously we need consider only the space parts of Γ_μ so we obtain, using (2.14),

$$W^0=i\int d^3y\frac{d^3\tau}{(2\pi)^3}\left\{\varphi_f^\dagger(\sigma^j:\Gamma_i)\varphi_i\frac{e^{i\tau\cdot\frac{1}{2}\mathbf{y}}}{(p^2-m_\pi^2)(l^2-m_\rho^2)}\times\epsilon^{0ikl}(q^j+\tau^j)\tau^k q^l\right\}+(\text{diagram with neutron and proton switched}). \qquad (6.1)$$

The evaluation of this integral is somewhat tedious; it proceeds as follows. We write the operator Γ_i, which appears explicitly in (2.4), as

$$\Gamma_i=a(P_i/2M)+b(i/2M)\epsilon^{ijt}l^j\sigma^t. \qquad (6.1')$$

Then we introduce the deuteron total and relative momenta, both of which we expect to be small, in a configuration space representation

$$\mathbf{Q}=\mathbf{p}_n+\mathbf{p}_p=0,\quad\text{(target at rest)}$$
$$\boldsymbol{\lambda}=\tfrac{1}{2}(\mathbf{p}_p-\mathbf{p}_n), \qquad (6.2)$$

$\lambda_i\rightarrow i(\partial/\partial y^i)=i\partial_i$. (relative momentum operator)

From Fig. 8 and (2.14) we then obtain for the proton term of (6.1)

$$\begin{aligned}\Gamma_i&=(a/2M)(2p_{pi}+l_i)+b(i/2M)\epsilon^{irt}l^r\sigma^t\\&=(a/2M)(2i\partial_i+\tfrac{1}{2}q_i-\tfrac{1}{2}\tau_i)\\&\qquad+b(i/4M)\epsilon^{irt}(q^r-\tau^r)\sigma^t.\end{aligned} \qquad (6.3)$$

But the presence of τ^k, q^l, and $\epsilon^{0ikl}=\epsilon^{ikl}$ in (6.1) allows us to drop the q_j and τ_j terms in (6.3), so we need only consider

$$\Gamma_i=(ia/M)\partial_i+(ib/4M)\epsilon^{irt}(q^r-\tau^r)\sigma^t. \qquad (6.4)$$

The operator (6.4) may now be inserted in (6.1) and the integral evaluated as we did in Sec. III. Deleting w^2 terms we obtain after somewhat lengthy algebra[7]

$$(\mathcal{E}/\mathfrak{D})W^0\cong\chi_f^\dagger(\Delta G_{Ed}-\Delta G_{Qd}(S_{12}/\sqrt{8}))\chi_i, \qquad (6.5)$$

where

$$\Delta G_{Ed}=\hat{\Gamma}\frac{2}{3}\frac{q^2}{M_d^2}\int\frac{d^3\tau(\tau_x)^2}{(2\pi)^3(p^2-m_\pi^2)(l^2-m_\rho^2)}\int_0^\infty[u^2j_0(\tau y/2)-\sqrt{2}uwj_2(\tau y/2)]dy\,,$$

$$\begin{aligned}\Delta G_{Qd}\cong&-\hat{\Gamma}\frac{\sqrt{2}}{3}\frac{q^2}{M_d^2}\int\frac{d^3\tau(\tau_x)^2[u^2j_0(\tau y/2)-\sqrt{2}uwj_2(\tau y/2)]dy}{(2\pi)^3(p^2-m_\pi^2)(l^2-m_\rho^2)}\\&-\hat{\Gamma}\frac{q^2}{M_d^2}\int\frac{d^3\tau(\tau_z/|\mathbf{q}|)[\sqrt{2}u^2j_0(\tau y/2)+uwj_2(\tau y/2)]dy}{(2\pi)^3(p^2-m_\pi^2)(l^2-m_\rho^2)}-\Gamma\frac{q^2}{M_d^2}\int\frac{d^3\tau(\tau_z/|\mathbf{q}|)[24w(u'y-u)j_2(\tau y/2)]dy}{(2\pi)^3(p^2-m_\pi^2)(l^2-m_\rho^2)\tau^2y^2}.\end{aligned}\tag{6.6}$$

Here

$$\Gamma=\frac{Gg_{\rho\pi\gamma}a}{16m_\rho e},\quad \hat{\Gamma}=\frac{Gg_{\rho\pi\gamma}b}{16m_\rho e},\quad u'=\frac{du}{dy}.\tag{6.7}$$

To obtain a useful result from (6.6) we first compare it with (3.10) and obtain

$$\Delta G_{Ed}\cong\tfrac{2}{3}(b/a)(q^2/M_d^2)\Delta G_{Md}\cong(6\times10^{-2})q^2/M_d^2.\tag{6.8}$$

Since we investigated ΔG_{Md} in Sec. III the above expression is all we need to discuss ΔG_{Ed}.

For the evaluation of ΔG_{Qd} we must expand (6.6) in powers of $|\mathbf{q}|$ inside the integral, then perform contour integrals around second-order poles. Even then the results must ultimately be gotten from numerical integration. The details may be found in Ref. (7), but we will here quote only the result

$$\Delta G_{Qd}\cong-(5\times10^{-2})q^2/M_d^2.\tag{6.9}$$

In (6.8) and (6.9) we have the end result of the numerical evaluations necessary to compare theory and experiment. Erickson has investigated the relevance of our estimates to experiment and finds that (6.8) and (6.9) significantly affect forward scattering only for $|\mathbf{q}|\gtrsim3.7$ (fermi)$^{-1}$, but $m_\rho\cong3.7$ (fermi)$^{-1}$ and we clearly cannot trust our low q results in this range.

Lastly let us note that ΔG_{Qd} gives rise to a quadrupole moment correction according to (1.5). Indeed the numerical value is

$$\Delta Q=\lim_{q\to0}[6\Delta G_{Qd}(q^2)/\sqrt{2}\mathbf{q}^2]\cong2\times10^{-3}\text{ fermi}.\tag{6.10}$$

This is about 1% of the measured value and is too small to be relevant to the comparison of theory and experiment at present; the theoretical value of Q is unfortunately more dependent on the inner structure of the deuteron than μ_d.

VII. CONCLUSIONS

The confrontation of our theoretical results with experiment, as discussed in Secs. IV and V, leads to optimism that the $\rho\pi$ exchange current explains both the magnetic moment anomaly in the deuteron and recent *e-d* back-scattering results. Since we found very little effect on forward scattering in Sec. VI, however, the relevance to the standard analysis of the neutron form factor via forward *e-d* scattering appears to be negligible.

ACKNOWLEDGMENTS

The author would like to thank Professor S. D. Drell for numerous invaluable discussions, Dr. E. F. Erickson for computational assistance, and C. Buchanan for discussions on the experimental situation.