Three-Nucleon Interaction*

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A contribution to the long-range part of the three-nucleon potential arising from the interaction of virtual pions with the cloud of *s*-wave, two-pion resonances around a nucleon is investigated. Crude estimates are made for the effect of this potential in nuclear matter and in the triton. In nuclear matter the corresponding effective two-nucleon interaction at small distances is repulsive and strong enough to dominate the one-pion-exchange potential. The contribution to the triton binding energy is about 1.1 MeV.

INTRODUCTION

ALTHOUGH calculations in nuclear physics generally are made using only two-particle interactions, it has long been realized that the meson theory of nuclear forces predicts multinucleon interactions.[1–11] The calculation of these effects is, unfortunately, extremely difficult; at the present time there is little hope of going much beyond a rough estimate of the long-range part of the three-nucleon interaction.

Fairly recent calculations with this aim have been carried out by Fujita and Miyazawa,[7] and by Smith and Sharp.[9] Using static-source meson theory (but rather different methods of calculation), these authors have estimated the strength of the three-nucleon interaction corresponding to Fig. 1. Since their work was done, however, it has become increasingly clear that mesons can interact strongly amongst themselves and thus produce a different type of long-range multinucleon interaction. In this paper we wish to consider the simplest of these: the interaction corresponding to Fig. 2.

To simplify the calculation we shall treat the pair of interacting pions as a $T=0$, $J=0$, even parity particle of definite mass m. Although a two-pion resonance with these quantum numbers is perhaps not as well established as some of the other meson resonances, there is considerable evidence[12–17] for its existence, and it appears to give the most important contribution of the type depicted in Fig. 2.

THE POTENTIAL

To define our coupling constants unambiguously we use an effective interaction density

$$\mathcal{H}_I = ig\bar{\psi}\gamma_5\boldsymbol{\tau}\psi\cdot\boldsymbol{\phi} + g_s\bar{\psi}\psi\phi_s + \tfrac{1}{2}f\boldsymbol{\phi}\cdot\boldsymbol{\phi}\phi_s\,, \qquad (1)$$

where ψ, ϕ, and ϕ_s are the nucleon, pion, and scalar meson (i.e., the two-pion resonance) fields, respectively. With this definition $(4\pi)^{-1}g^2\approx 14$ and $(4\pi)^{-1}g_s^2\approx 10$.[13] If we assume the two-pion resonance to have a mass $m=4\,\mu$ and width $\Gamma=\frac{1}{2}\,\mu$, then $(4\pi)^{-1}f^2\approx 6\,\mu^2$, where μ is the pion mass. Furthermore, the analysis of the two-pion exchange contribution to pion-nucleon scattering[18] seems to indicate that $fg_s>0$.

In the nonrelativistic limit the contribution to the transition amplitude corresponding to Fig. 2 is

$$\frac{1}{2}\left(\frac{g}{2M}\right)^2 fg_s\frac{\boldsymbol{\sigma}^{(1)}\cdot\mathbf{p}_1\boldsymbol{\sigma}^{(2)}\cdot\mathbf{p}_2\boldsymbol{\tau}^{(1)}\cdot\boldsymbol{\tau}^{(2)}}{(p_1^2+\mu^2)(p_2^2+\mu^2)(p_3^2+m^2)}, \qquad (2)$$

where $\mathbf{p}_i=\mathbf{q}_i'-\mathbf{q}_i$ and M is the nucleon mass. After symmetrization the corresponding contribution to the three-nucleon potential is[19]

$$V(1,2,3)=v(1,2;3)+v(3,1;2)+v(2,3;1)\,, \qquad (3)$$

where $v(1,2;3)$ is symmetric in the coordinates of nucleons 1 and 2, and is given by

$$v(1,2;3)=\int d^3x\,\hat{v}(\mathbf{x}_1-\mathbf{x},\,\mathbf{x}_2-\mathbf{x};\,\mathbf{x}_3-\mathbf{x})\,, \qquad (4)$$

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[1] H. Primakoff and T. Holstein, Phys. Rev. **55**, 1218 (1938).
[2] L. Janossy, Proc. Cambridge Phil. Soc. **35**, 616 (1939).
[3] A. Klein, Phys. Rev. **90**, 1101 (1953).
[4] S. D. Drell and K. Huang, Phys. Rev. **91**, 1527 (1953).
[5] G. Wentzel, Phys. Rev. **91**, 1573 (1953).
[6] E. M. Gelbard, Phys. Rev. **100**, 1530 (1955).
[7] J. Fujita and H. Miyazawa, Progr. Theoret. Phys. (Kyoto) **17**, 360 (1957).
[8] S. Pandya and J. French, Ann. Phys. (N. Y.) **2**, 166 (1957).
[9] R. C. Smith and R. T. Sharp, Can. J. Phys. **38**, 1154 (1960).
[10] J. Fujita, M. Kawai, and M. Tanifuji, Nucl. Phys. **29**, 252 (1962).
[11] B. Gruber, Phys. Letters **5**, 278 (1963).
[12] L. M. Brown and P. Singer, Phys. Rev. **133**, B812 (1964); H. Faier and L. M. Brown, Bull. Am. Phys. Soc. **10**, 467 (1965).
[13] R. A. Bryant and B. L. Scott, Phys. Rev. **135**, B434 (1964).
[14] L. Durand, III, and Y. T. Chiu, Phys. Rev. Letters **14**, 329, 680 (1965).
[15] P. G. Thurnauer, Phys. Rev. Letters **14**, 985 (1965).
[16] V. Hagopian, W. Selove, J. Alitti, J. B. Baton, and M. Neven-Rene, Phys. Rev. Letters **14**, 1077 (1965).
[17] J. Halpern, Bull. Am. Phys. Soc. **10**, 585 (1965).
[18] J. Hamilton, T. D. Spearman, and W. S. Woolcock, Ann. Phys. (N. Y.) **17**, 1 (1962).
[19] In the limit $m\to\infty$, $g_sfM\to g^2m^2$, this potential is equivalent to V_{3a} of Ref. 4, Appendix A.

with

$$\bar{v}(\mathbf{y}_1,\mathbf{y}_2;\mathbf{y}_3) = -(g/2M)^2 g_s f \boldsymbol{\tau}^{(1)}\cdot\boldsymbol{\tau}^{(2)} \times \boldsymbol{\sigma}^{(1)}\cdot\nabla_1 \boldsymbol{\sigma}^{(2)}\cdot\nabla_2 \exp(-\mu y_1)/4\pi y_1 \times[\exp(-\mu y_2)/4\pi y_2]\exp(-m y_3)/4\pi y_3. \quad (5)$$

The term $v(1,2;3)$ may be looked upon as due to the scattering, by nucleon 3, of a virtual pion being exchanged between nucleons 1 and 2. The reasons for choosing a light, scalar two-pion resonance are clear: The lighter the resonance, the more extended the target presented by nucleon 3, and, for kinematic reasons, a scalar meson scatters low-momentum pions more effectively than, say, a vector meson.

Having obtained an expression for the three-nucleon potential, the obvious question is: How important is it? The answer is not trivially found, since it depends upon three-nucleon correlations in complex nuclei. We shall here be satisfied with rough estimates in two extreme cases: nuclear matter and the triton.

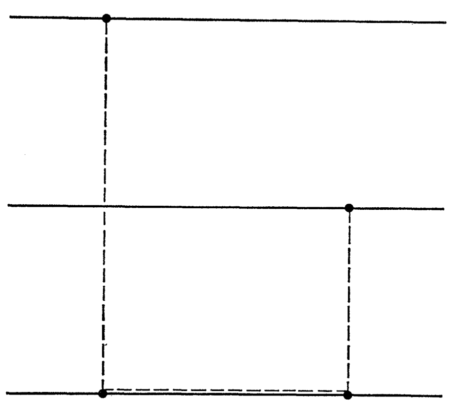

FIG. 1. The diagram considered in Refs. 7 and 9. The double line represents a "3-3" resonance.

NUCLEAR MATTER

We shall use the Fermi-gas model for nuclear matter, thus ignoring any correlations beyond those required by antisymmetry. If we further ignore terms in which all three nucleons are exchanged, our potential can be reduced to an effective two-nucleon potential merely by averaging over the coordinates of the third particle. Assuming equal numbers of "up" and "down" spins and isospins, we obtain

$$V_3(\mathbf{r}) = \rho\int d^3x_3 v(1,2;3) = (2\mu m^2)^{-1} g_s f \rho V_Y'(\mathbf{r}), \quad (6)$$

where $\rho \approx 0.18\ \mathrm{F}^{-3}$ is the density of nucleons in nuclear matter, and

$$V_Y'(\mathbf{r}) = (1/4\pi)(g/2M)^2 \boldsymbol{\sigma}^{(1)}\cdot\nabla\boldsymbol{\sigma}^{(2)}\cdot\nabla\boldsymbol{\tau}^{(1)}\cdot\boldsymbol{\tau}^{(2)} \times\exp(-\mu r) \quad (7)$$

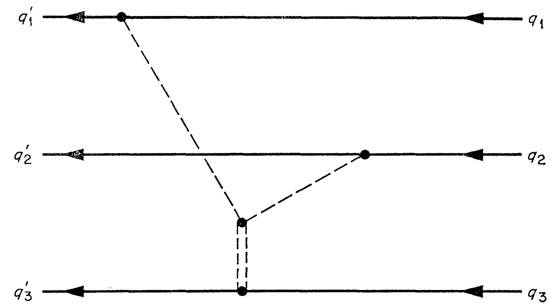

FIG. 2. The diagram considered in this paper. The double line represents two interacting pions, treated as a neutral scalar particle.

is the usual one-pion-exchange (OPE) potential with a factor of r^{-1} removed.

We can obtain some feeling for the strength of this potential by comparing it with the OPE potential. After eliminating the spins and isospins one has

$$\bar{V}_3(r) = (g_s f\rho/2m^2\mu^2)(\mu r - 2)\bar{V}_Y(r), \quad (8)$$

where

$$\bar{V}_Y(r) = -\tfrac{3}{4}(1/4\pi)(g/2M)^2\mu^2 \exp(-\mu r)/r \quad (9)$$

is the OPE potential. With our parameters, $(2m^2\mu^2)^{-1} \times g_s f\rho \approx 1.5$, so that $\bar{V}_3$ dominates $\bar{V}_Y$ at small and large distances, but is small near $r = 2\ \mu^{-1}$, as shown in Fig. 3. It is interesting to note that $\bar{V}_3 + \bar{V}_y$ is repulsive at small distances, but in view of the crudeness of our calculation this should not be given too much significance.

THE TRITON

It seems now generally accepted that the triton wave function is predominantly the s-wave, completely spatially symmetric type.[20] If we neglect the presum-

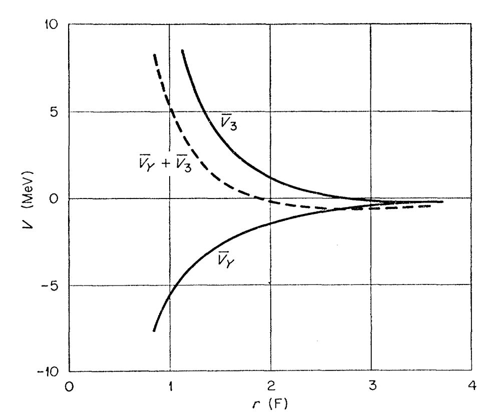

FIG. 3. The effective two-nucleon potential V_3 resulting from our three-nucleon force, compared with the OPE potential V_Y and their sum.

[20] See, for example, L. I. Schiff, Phys. Rev. **133**, B802 (1964), and references given there.

ably small admixtures of the other types, the expectation value of our three-nucleon potential in the triton can be written

$$\langle V_3\rangle_t = -3(g/2M)^2 g_s f \int d^3s_1 d^3s_2 U^2(s_1,s_2,s_3) \times \int \frac{d^3q_1}{(2\pi)^3}\frac{d^3q_2}{(2\pi)^3}\mathbf{q}_1\cdot\mathbf{q}_2 \exp(i\mathbf{q}_1\cdot\mathbf{s}_1+i\mathbf{q}_2\cdot\mathbf{s}_2) \times \frac{1}{q_1^2+\mu^2}\frac{1}{q_2^2+\mu^2}\frac{1}{(\mathbf{q}_1+\mathbf{q}_2)^2+m^2}. \quad (10)$$

Here $\mathbf{s}_1=\mathbf{r}_1-\mathbf{r}_3$, $\mathbf{s}_2=\mathbf{r}_2-\mathbf{r}_3$, $\mathbf{s}_3=\mathbf{s}_1-\mathbf{s}_2$, and $U(s_1,s_2,s_3)$ is the spatial wave function.

Several forms have been suggested for $U(s_1,s_2,s_3)$, none of which has a very convincing theoretical foundation. Mainly to simplify the necessary integration as much as possible, we shall use the Gaussian form

$$U(s_1,s_2,s_3)=A \exp[-\tfrac{1}{2}\alpha^2(s_1^2+s_2^2+s_3^2)] \quad (11)$$

with $A^2=[3^{1/2}\alpha^2/\pi]^3$ and $\alpha\approx 0.38\ \mathrm{F}^{-1}$ as determined by Schiff[20] from fits to the electromagnetic properties of the triton. After a numerical double integration, with the parameters above, we find

$$\langle V_3\rangle_t \approx 1.1\ \mathrm{MeV}, \quad (12)$$

which may be compared to the total binding energy of 8.5 MeV. It should be emphasized that this estimate is quite sensitive to the form of the wave function. It is possible to obtain widely different values for $\langle V_3\rangle_t$ with different wave functions which give equally good fits to the electromagnetic properties of the triton.

CONCLUSION

The exact results of the calculations sketched above should, of course, not be taken too seriously. There is considerable uncertainty in the parameters we have used (especially those relating to the two-pion resonance), and the methods used to estimate the effects of our potential in nuclear matter and the triton are very crude. More fundamentally, we have considered only a single piece of the three-nucleon interaction; there certainly are many other contributions, especially at small distances.

Our main purpose has been to show, through a simple example, that the strong interactions among pions give rise to multinucleon interactions which may be as important as those arising solely from pion-nucleon interactions. For both types, however, the status of multinucleon forces is presently ambiguous, to say the least. There seems to be reason to believe that certain bits and pieces of these forces are relatively strong, but their overall effect will depend upon how much cancellation occurs, and upon the nature of multiparticle correlations in complex nuclei. In view of our inability to make accurate calculations of these effects, it is perhaps most reasonable to go ahead with present programs for calculating the properties of nuclei using only two-body forces. The possibility of important corrections due to multinucleon forces (which may, for example, modify the effective two-nucleon interaction) should, however, always be kept in mind.[21]

[21] Similar conclusions with regard to the $\pi\pi N$ system have been reached in a recent study by R. F. Sawyer, Phys. Rev. **139**, B151 (1965).