n-n and *n-p* Scattering Lengths and Charge Independence*

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The degree of charge-independence violation for the low-energy nucleon-nucleon system is studied on the basis of a semiphenomenological meson theory. The difference between the singlet *n-p* and *n-n* scattering lengths induced by the $\pi^{\pm}$-π^0 and $\rho^{\pm}$-ρ^0 (Δm) electromagnetic mass splittings is investigated for the one-pion-, two-pion-, and ρ-exchange potentials. A charge-independent boundary condition is used to replace the short-range behavior. It is found that approximately 60% of the measured difference between the scattering lengths can be accounted for by the one-pion and two-pion potentials with charge-independent pseudoscalar coupling. The calculated results are sensitive to parameters of the ρ-exchange potential, such as Δm and coupling constants, which are not well known. The dependence of the scattering lengths on these parameters is found. Finally, we note that a negative Δm, or a change of the order of 2% in the boundary condition or in the ratio of charged to neutral coupling constants, can account for the discrepancy that remains after pion electromagnetic mass splittings are included.

I. INTRODUCTION

RECENT measurements of the *n-n* scattering length[1] indicate that charge symmetry, but not exact charge independence, is valid for the low-energy two-nucleon system. The "best" values for the scattering lengths a and effective ranges r of this system are[1–4]

$$a_{nn}=-16.4\pm1.9\text{ F}, \qquad (1a)$$

$$a_{np}{}^{(\text{singlet})}=-23.679\pm0.028\text{ F}, \qquad (1b)$$

$$a_{pp}=-7.815\pm0.008\text{ F} \quad (\approx-17\text{ F for nuclear part}), \qquad (1c)$$

$$r_{np}{}^{(\text{singlet})}=2.51\pm0.11\text{ F}, \qquad (1d)$$

$$r_{pp}=2.795\pm0.025\text{ F } (\approx r_{nn}). \qquad (1e)$$

The difference in the *n-n* and *n-p* scattering lengths in the singlet spin state appears to be quite large. However, it only requires a small difference in the nuclear potentials to account for this difference because the scattering lengths are all large compared to the range of nuclear forces. Although no direct Coulomb forces are present in a comparison of the *n-n* and *n-p* systems, other (indirect) electromagnetic effects do influence the nuclear S matrix or potential.[5] Among these are the difference in the neutron and proton electromagnetic form factors,[6] but a primary one is the mass difference between the charged and neutral pions and other mesons responsible for the nucleon-nucleon force.

In this paper, we examine how much of the difference between the *n-n* and *n-p* singlet state scattering lengths can be understood on the basis of the electromagnetic mass differences of the pions and ρ mesons. The effect of the mass difference on the one-pion and one-ρ-exchange potentials were investigated by Heller, Signell, and Yoder, and others.[7,8] Heller *et al.* also made an estimate of the electromagnetic mass splitting effect for two pion exchange, but no detailed investigation was carried out by them. More detailed considerations of the $\pi^{\pm}$-π^0 mass difference for the two-pion-exchange potential were carried out by Lin.[9] However, he did not assume charge symmetry and used a different procedure than we do to evaluate the effect of the pion-mass splitting on the fourth-order potential. We study the effect of the $\pi^{\pm}$-π^0 mass difference on the two-pion-exchange potential in detail, and also consider the effects of single-pion and ρ exchange.

If the interaction Hamiltonian [e.g., $H_{\pi\text{-}N}=(4\pi)^{1/2}\times g\int\bar{\psi}\gamma_5\boldsymbol{\tau}\cdot\boldsymbol{\phi}\psi d^3r$] is assumed to be charge-independent, then the only difference between the *n-n* and *n-p* scattering lengths occurs from direct (e.g., M_n-M_p) and indirect electromagnetic effects. In the latter category, we expect the inequality of the physical masses of the neutral and charged mesons to be most important. In this paper we consider the potentials which arise from one-pion, two-pion and ρ exchange. The boundary condition model of Feshbach, Lomon, and Tubis[10] (F.L.T.) is employed to specify the two pion exchange potential and to replace the unknown short-range [$r\lesssim(2m_\pi)^{-1}$] behavior. We assume charge symmetry since it appears to be valid experimentally.[1,7] Although our formulation allows for an inequality of the coupling constants of neutral g_0 and charged g_+ mesons to nucleons (the actual coupling constant for charged mesons to nucleons is then $2^{1/2}g_+$), our main concern is the mass effect. For this purpose we first assume $g_+=g_0$ and a charge-independent

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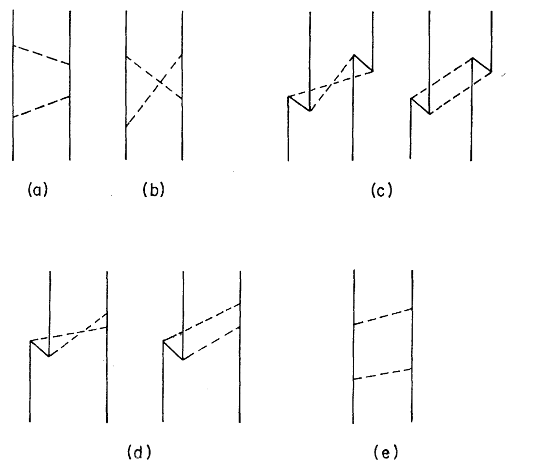

FIG. 1. Feynman-graph contributions to the two-pion exchange potential, TPEP. The various figures are: (a), (b) standard graphs; (c) two-pair type terms; (d) typical one-pair terms; (e) ladder diagram.

boundary condition. The latter is adjusted to give the *n-n* scattering length; the *n-p* scattering length is then computed with the same boundary condition. Later, we investigate how much of the additional violation of charge independence is required at the boundary or for the coupling constants to bring the *n-p* singlet scattering length in complete agreement with measurement. Effects of varying the parameters of the ρ potential (i.e., $m_{\rho^+}-m_{\rho^0}$ and coupling constant) are also considered.

II. THEORY

The one-pion exchange potential (OPEP) is well known. If μ_+, $2^{1/2}g_+$ and μ_0, g_0 are the masses and pseudoscalar coupling constants of the charged (positive or negative) and neutral pions, respectively, then the nonrelativistic second-order potentials in the ${}^1S_0$ state are (see also Refs. 5 and 7)

$$V_{nn}^{(2)}=V_{pp}^{(2)}=-g_0^2(\mu_0/2M)^2e^{-\mu_0r}/r\,,$$

$$V_{np}^{(2)}=-g_0^2\left(\frac{\mu_0}{2M}\right)^2\left[-\frac{e^{-\mu_0r}}{r}+2\left(\frac{g_+^2}{g_0^2}\right)\left(\frac{\mu_+}{\mu_0}\right)^2\frac{e^{-\mu_+r}}{r}\right].\quad(2)$$

In Eq. (2), M is the mass of the neutron or proton; their mass difference is neglected because its effect on the scattering lengths is much smaller than the one we are investigating.

The fourth-order nucleon-nucleon potential (TPEP) has contributions from the Feynman diagrams shown in Fig. 1.

In a nonrelativistic approximation we obtain for the contributions of Figs. 1(a) and 1(b)

$$\begin{aligned}V^{(4)}(\mathbf{r})=(1/2i\pi^5\mu_0^4)\tau_j^a\tau_i^a(\boldsymbol{\sigma}_a\cdot\boldsymbol{\nabla})(\boldsymbol{\sigma}_a\cdot\boldsymbol{\nabla}')&\\ \times[\tau_j^b\tau_i^b(\boldsymbol{\sigma}_b\cdot\boldsymbol{\nabla})(\boldsymbol{\sigma}_b\cdot\boldsymbol{\nabla}')&\\ -\tau_i^b\tau_j^b(\boldsymbol{\sigma}_b\cdot\boldsymbol{\nabla}')(\boldsymbol{\sigma}_b\cdot\boldsymbol{\nabla})]I_{ij}\,,&\quad(3\text{a})\end{aligned}$$

where i and j are summed from 1 to 3, and

$$I_{ij}=G^2(i)G^2(j)\left(\frac{\mu_0}{2M}\right)^4\int\frac{d\omega d^3kd^3k'e^{i(\mathbf{k}\cdot\mathbf{r}+\mathbf{k}'\cdot\mathbf{r}')}}{\omega^2(\omega^2-\mathbf{k}^2-\mu_j^2)(\omega^2-\mathbf{k}'^2-\mu_i^2)}.\quad(3\text{b})$$

We have used a standard trick to aid in the calculation of the integral. If we let $(\mathbf{k}+\mathbf{k}')\cdot\mathbf{r}$ go to $\mathbf{k}\cdot\mathbf{r}+\mathbf{k}'\cdot\mathbf{r}'$ in the following expression,

$$\int(\boldsymbol{\sigma}\cdot\mathbf{k})(\boldsymbol{\sigma}\cdot\mathbf{k}')e^{i(\mathbf{k}+\mathbf{k}')\cdot\mathbf{r}}f(\mathbf{k},\mathbf{k}')d^3kd^3k'\,,$$

we can transform it into operator form:

$$-(\boldsymbol{\sigma}\cdot\boldsymbol{\nabla})(\boldsymbol{\sigma}\cdot\boldsymbol{\nabla}')\int e^{i(\mathbf{k}\cdot\mathbf{r}+\mathbf{k}'\cdot\mathbf{r}')}f(\mathbf{k},\mathbf{k}')d^3kd^3k'\,,$$

where it is understood that we must let $\mathbf{r}'$ go to $\mathbf{r}$, the distance between the nucleons, after the calculation. Furthermore $G(1)=G(2)\equiv G_+$ is the fourth-order pseudoscalar coupling constant relevant for $\pi^{\pm}$-N while $G(3)\equiv G_0$ is the fourth-order pseudoscalar coupling constant of π^0-N. That is, in order to be consistent with the work of F.L.T., we allow for a small difference in the pion-nucleon coupling constants which appear in the second-order and fourth-order potentials. This has almost no effect on our results.

Making the approximation that $|\mu_i^2-\mu_j^2|\lesssim 14\text{ MeV}^2$ is small compared to the important contribution of k^2 ($\approx\mu^2$), we get

$$\begin{aligned}I_{ij}=i\pi^4G^2(i)G^2(j)(\mu_0/2M)^4(r+r'/rr')&\\ \times\{K_0[\mu_j(r+r')]+K_0[\mu_i(r+r')]\}\,,&\end{aligned}$$

where $K_\nu(\mu r)$ is a modified Bessel function of order ν.[11] If O_D and O_C are given by

$$O_D=(\boldsymbol{\sigma}_a\cdot\boldsymbol{\nabla})(\boldsymbol{\sigma}_a\cdot\boldsymbol{\nabla}')(\boldsymbol{\sigma}_b\cdot\boldsymbol{\nabla})(\boldsymbol{\sigma}_b\cdot\boldsymbol{\nabla}')\,,$$
$$O_C=-(\boldsymbol{\sigma}_a\cdot\boldsymbol{\nabla})(\boldsymbol{\sigma}_a\cdot\boldsymbol{\nabla}')(\boldsymbol{\sigma}_b\cdot\boldsymbol{\nabla}')(\boldsymbol{\sigma}_b\cdot\boldsymbol{\nabla}),$$

then the terms represented by Figs. 1(a) and 1(b) are

$$\begin{aligned}V_{pp}^{(4)}&=V_{nn}^{(4)}=O_DI_{00}+O_C(I_{00}+4I_{++}),\\ V_{np}^{(4)}&=O_D(I_{00}-4I_{+0}+4I_{++})+O_C(I_{00}+4I_{+0})\,.\quad(4)\end{aligned}$$

Carrying out the prescribed operations, we obtain

$$\begin{aligned}V_{nn}^{(4)}=V_{pp}^{(4)}=G_0^4(\mu_0/2M)^4\mu_0\{\tfrac{1}{3}[S_{ab}U_T(\mu_0r)+\boldsymbol{\sigma}_a\cdot\boldsymbol{\sigma}_bU_\sigma(\mu_0r)]&\\ +(G_+/G_0)^4(\mu_+/\mu_0)^5[U_\tau(\mu_+r)+\tfrac{2}{3}S_{ab}U_T(\mu_+r)+\tfrac{2}{3}\boldsymbol{\sigma}_a\cdot\boldsymbol{\sigma}_bU_\sigma(\mu_+r)]\}\,,&\quad(5\text{a})\end{aligned}$$

$$\begin{aligned}V_{np}^{(4)}=G_0^4(\mu_0/2M)^4\mu_0\{\tfrac{1}{3}[1-2(G_+/G_0)^2][S_{ab}U_T(\mu_0r)+\boldsymbol{\sigma}_a\cdot\boldsymbol{\sigma}_bU_\sigma(\mu_0r)]&\\ +(G_+/G_0)^2[U_\tau(\mu_0r)+\tfrac{2}{3}S_{ab}U_T(\mu_0r)+\tfrac{2}{3}\boldsymbol{\sigma}_a\cdot\boldsymbol{\sigma}_bU_\sigma(\mu_0r)]&\\ +\tfrac{2}{3}(\mu_+/\mu_0)^5[2(G_+/G_0)^4-(G_+/G_0)^2][S_{ab}U_T(\mu_+r)+\boldsymbol{\sigma}_a\cdot\boldsymbol{\sigma}_bU_\sigma(\mu_+r)]&\\ -(\mu_+/\mu_0)^5[(G_+/G_0)^4-(G_+/G_0)^2][U_\tau(\mu_+r)+\tfrac{2}{3}S_{ab}U_T(\mu_+r)+\tfrac{2}{3}\boldsymbol{\sigma}_a\cdot\boldsymbol{\sigma}_bU_\sigma(\mu_+r)]\}\,,&\quad(5\text{b})\end{aligned}$$

[11] G. N. Watson, *Theory of Bessel Functions* (Cambridge University Press, Cambridge, 1944), pp. 78, 172.

where the U's are the ones derived by Taketani, Machida, and Ohnuma[12] (T.M.O.):

$$U_\tau(x)=-\frac{8}{\pi}\left[\left(\frac{23}{4x^4}+\frac{3}{x^2}\right)K_1(2x)+\left(\frac{23}{4x^3}+\frac{1}{x}\right)K_0(2x)\right],$$

$$U_T(x)=-\frac{8}{\pi}\left[\left(\frac{15}{4x^4}+\frac{1}{x^2}\right)K_1(2x)+\frac{3}{x^3}K_0(2x)\right], \quad (6)$$

$$U_\sigma(x)=\frac{8}{\pi}\left[\left(\frac{3}{x^4}+\frac{2}{x^2}\right)K_1(2x)+\frac{3}{x^3}K_0(2x)\right],$$

and $S_{ab}=3(\boldsymbol{\sigma}_a\cdot\mathbf{r})(\boldsymbol{\sigma}_b\cdot\mathbf{r})/r^2-(\boldsymbol{\sigma}_a\cdot\boldsymbol{\sigma}_b)$.

The same potential for nn, np and pp is found from the one and two pair terms [see Figs. 1(c) and 1(d)].

$$\begin{pmatrix}V_{2\,\text{pair}}\\ V_{1\,\text{pair}}\end{pmatrix}=(\tau_i^a\tau_j^a\tau_i^b\tau_j^b+\tau_i^a\tau_j^a\tau_j^b\tau_i^b)\times\begin{pmatrix}C^{(2)} & V_{ij}^{(2)}\\ C^{(1)} & V_{ij}^{(1)}\end{pmatrix}G^2(i)G^2(j)\left(\frac{\mu_0}{2M}\right)^4,$$

where

$$V_{ij}^{(2)}=\int\frac{d^3k_id^3k_je^{i(\mathbf{k}_i+\mathbf{k}_j)\cdot\mathbf{r}}}{\omega_i\omega_j(\omega_i+\omega_j)}=\frac{(2\pi)^3}{2r^2}[\mu_iK_1(2\mu_ir)+\mu_jK_1(2\mu_jr)],$$

$$C^{(2)}=-\lambda^2M^2/2\pi^4\mu_0^4,\quad C^{(1)}=-\lambda M/4\pi^4\mu_0^4,$$

$$\omega_i^2=k_i^2+\mu_i^2,$$

$$V_{ij}^{(1)}=\int d^3k_id^3k_je^{i(\mathbf{k}_i+\mathbf{k}_j)\cdot\mathbf{r}}\frac{\mathbf{k}_i\cdot\mathbf{k}_j}{\omega_i\omega_j}\times\left[\frac{1}{\omega_i(\omega_i+\omega_j)}+\frac{1}{\omega_j(\omega_i+\omega_j)}+\frac{1}{\omega_i\omega_j}\right]=-2I_iI_j(4\pi/r)^2,$$

$$I_i=-\tfrac{1}{2}\pi\mu_i(1+1/\mu_ir)e^{-\mu_ir}.$$

Then for nn, np, and pp we have

$$V_{2\,\text{pair}}=-G_0^4\left(\frac{\mu_0}{2M}\right)^2\mu_0\frac{2\lambda^2}{\pi}\times\left[\frac{K_1(2\mu_0r)}{(\mu_0r)^2}+2\left(\frac{\mu_+}{\mu_0}\right)^3\left(\frac{G_+}{G_0}\right)^4\frac{K_1(2\mu_+r)}{(\mu_+r)^2}\right], \quad (7)$$

where λ is the "pair suppression" parameter,[10] and

$$V_{1\,\text{pair}}=G_0^4\left(\frac{\mu_0}{2M}\right)^3 2\mu_0\lambda\left[\left(\frac{1+\mu_0r}{(\mu_0r)^2}\right)^2e^{-2\mu_0r}+2\left(\frac{G_+}{G_0}\right)^4\left(\frac{\mu_+}{\mu_0}\right)^4\left(\frac{1+\mu_+r}{(\mu_+r)^2}\right)^2e^{-2\mu_+r}\right]. \quad (8)$$

[12] M. Taketani, S. Machida, and S. Ohnuma, Progr. Theoret. Phys. (Kyoto) **6**, 638 (1951); **7**, 45 (1952).

TABLE I. Parameters used in the boundary-condition model.

$g_0^2=14.400$,	$\xi=0.0555$,
$G_0^2=17.255$,	$\mu_0=135.01$ MeV,
$r_{\text{in}}=0.7011$ F,	$\mu_+=139.59$ MeV,
$\lambda=0.89148$,	$M=939$ MeV.

The ladder term, Fig. 1(e), being smaller by an order of magnitude than $V_{np}^{(4)}$ or $V_{nn}^{(4)}$ was taken to be that given by F.L.T.[10] Mass-difference effects are neglected for this term and we obtain for the $^1S_0$ state

$$V_{\text{ladder}}=G_0^4\left(\frac{\mu_0}{2M}\right)^4\frac{2\xi}{\pi}\left[\frac{K_1(\mu_0r)}{(\mu_0r)^2}-\frac{K_0(\mu_0r)}{(\mu_0r)}\right]e^{-\mu_0r}, \quad (9)$$

where ξ is the "ladder parameter."

The various parameters involved in the derived potentials (for $g_+=g_0$, $G_+=G_0$) have been determined by F.L.T. by fitting scattering data up to sizeable energies ($\sim$350 MeV). They used a boundary condition (logarithmic derivative of the wave function) at a radius r_{in} to treat short-range effects. The mass difference between neutral and charged pions was taken into account in an "average" manner by using μ_0 for the p-p and μ_+ for the n-p potential. We have used the parameters of F.L.T., except for charge-dependent effects. The parameters for our calculation are summarized in Table I. If we let $g_+=g_0$, $G_+=G_0$ and $\mu_+=\mu_0$, the above potentials reduce to those of F.L.T. If, in addition, we also let $\lambda=\xi=0$, we obtain the T.M.O. potential.[12]

Finally we have the charge-dependent potential produced by the exchange of the vector ρ meson which in the $^1S_0$ state is given by[13]

$$V_\rho^{nn}=-\tfrac{3}{2}\eta_0^2\left[-A_0+\frac{(1+2g_v)^2}{2}\left(\frac{m_0}{M}\right)^2\right]\frac{e^{-m_0r}}{r},$$

$$V_\rho^{np}=-\tfrac{3}{2}\eta_0^2\left\{\left[A_0-\frac{(1+2g_v)^2}{2}\left(\frac{m_0}{M}\right)^2\right]\frac{e^{-m_0r}}{r}-2\left(\frac{\eta_+}{\eta_0}\right)^2\left[A_+-\frac{(1+2g_v)^2}{2}\left(\frac{m_+}{M}\right)^2\right]\frac{e^{-m_+r}}{r}\right\}, \quad (10a)$$

where, to zeroth order in $(m/M)^2$ and $(p/m)^2$ [p is the momentum of either nucleon] we have in agreement with McKean[14] and Cottingham and Vinh Mau[14]

$$A_+=A_0=1. \quad (10b)$$

[13] This potential differs from that used by the authors of Refs. 7 and 8. Blin-Stoyle and Yalgin apparently have the opposite sign for the quantity $(1+2g_v)^2$ which arises from a spin-spin interaction. Heller, Signell, and Yoder have found an error in their ρ-exchange potential (private communication from P. Signell).

[14] W. M. Cottingham and R. Vinh Mau, Phys. Rev. **130**, 735 (1963); R. S. McKean, Jr., *ibid.* **125**, 1399 (1962). Our potential differs from that of McKean, whose anomalous magnetic moment term has an error in sign.

It is possible to include higher order effects in $(m/M)^2$ in the static potential, although the uniqueness of the results can be questioned. In fact, the expansion parameter is quite large (≈ 0.65). The most reasonable procedure is to consider the S matrix obtained from one ρ exchange as an analytic function of the variables s and t (squares of the total energy and momentum transfer, respectively). In the limit $s \to 0$, and finite t one then obtains to order $(m/M)^2$

$$A_{+,0}=1+m_{+,0}^2/2M^2+g_v(m_{+,0}^2/M^2). \qquad (10c)$$

This agrees with the potential obtained by Bryan, Dismukes, and Ramsay[15] and by Wong.[15]

In Eqs. (10) g_v is the nucleon isovector gyromagnetic ratio, η_0 (η_+) is the N-ρ_0 (N-$\rho_\pm$) coupling constant, m_0 (m_+) is the mass of the ρ_0 ($\rho_\pm$). Since the mass difference between the ρ_0 and the $\rho_\pm$ is not known experimentally, we allow this to vary somewhat. The SU_6 prediction, with parameters that fit the π^+-π^0 mass difference is m_{ρ^+}-$m_{\rho^0} \approx 1$ MeV. The coupling constant g_v is obtained from the anomalous magnetic moment of the nucleon; it and the ρ^0 mass are taken to be[14,16,17] $g_v=1.85$ and $m_0=5.4\mu_0$, where μ_0 is the mass of the neutral pion. An accurate determination of the ρ-N coupling constant η has not yet occurred. The generally accepted values based on internal symmetry arguments[18] are $\eta_0^2=\eta_+^2\approx 1.3-2$. In the remainder of this work we always take $\eta_0^2=\eta_+^2\equiv\eta^2$ and most of our calculations are performed for $\eta^2=1.668$.

III. RESULTS

The results of the previous calculation are summarized in Tables II and III and in Figs. 2 and 3. The values of the n-p singlet state scattering lengths listed in Table II were obtained as follows. The dimensionless logarithmic derivative B,

$$B=-\frac{r}{u}\frac{du}{dr}\bigg|_{r=r_{\rm in}},$$

where $u(r)$ is the radial wave function, was adjusted at $r_{\rm in}=0.7011$ F to give the experimental n-n scattering length, Eq. (1a). Pseudoscalar coupling was assumed. The n-p scattering length was then computed with the same boundary condition and with $g_+=g_0$, $G_+=G_0$. The second row in the table lists the ratio of $g_+/g_0=G_+/G_0$ still needed to bring a_{np} to the value given in Eq. (1b).

[15] R. H. Bryan, C. R. Dismukes, and W. Ramsay, Nucl. Phys. **45**, 353 (1963); D. Y. Wong, *ibid.* **55**, 212 (1964).

[16] E. Pickup, D. K. Robinson, and E. O. Salant, Phys. Rev. Letters **7**, 192 (1961); D. D. Carmony and R. T. Van de Walle, *ibid.* **8**, 73 (1962).

[17] S. Bergia, A. Stranghellini, S. Fubini, and C. Villi, Phys. Rev. Letters **6**, 367 (1961).

[18] J. J. Sakurai, in *Selected Topics on Elementary Particle Physics*, edited by M. Conversi (Academic Press Inc., New York, 1963), pp. 50–53. *Note added in proof.* The squared coupling constant η^2 should be $\frac{1}{4}$ of that given above. We thank Dr. R. Bryan for pointing this out. Our conclusions are essentially unaltered by this change.

TABLE II. Neutron-proton scattering-length predictions for pseudoscalar coupling. The first row lists the predicted n-p scattering lengths for a charge-independent coupling and the second one the ratio of $(g_+/g_0)=(G_+/G_0)$ required to fit the experimental scattering length. We take $\eta_+^2=\eta_0^2=1.668$, $r_{\rm in}=0.7011$, $a_{nn}=-16.4$ F, $\Delta m=2$ MeV, and $A_+=A_0=1$.

	OPEP	OPEP+TPEP	OPEP+TPEP $+V_\rho$
a_{np} (F)	-19.9	-20.80	-18.27
$g_+/g_0=G_+/G_0$	1.0094	0.9953	0.9832

The first column of the table is with a value of B adjusted to fit a_{nn} with OPEP alone,[19] the second column with OPEP and TPEP and the last one includes ρ exchanges as well, with $m_{\rho+}-m_{\rho0}\equiv\Delta m=2$ MeV, $\eta^2=1.668$ and $A_+=A_0=1$. It should be noted that a change of the order of 2% from unity in the ratio of $g_+/g_0=G_+/G_0$ is needed to bring a_{np} to the experimentally determined value. In Table III the same results as Table II are presented for pseudovector coupling with f_+ (or F_+) and f_0 (or F_0) related to g_+ (or G_+) and g_0 (or G_0) by $f_+=(\mu_+/2M)g_+$ and $f_0=(\mu_0/2M)g_0$, but otherwise with the same potentials. A charge-independent coupling $f_+=f_0$, $F_+=F_0$ thus implies that $\mu_0 g_0=\mu_+ g$ and $\mu_0 G_0=\mu_+ G_+$. In this case, the calculated scattering lengths are more sensitive to the potential. For OPEP alone, $a_{np}>a_{nn}$, whereas for OPEP+TPEP or together with the ρ potential, $a_{np}<a_{nn}$. This sensitivity is due to the last term in Eq. (5b), which is zero for a charge-independent pseudoscalar coupling, but yields an appreciable negative contribution to the n-p potential for a charge-independent pseudovector coupling. However, the change $(f_+-f_0)/f_0$ required to bring a_{np} in line with experiment is approximately equal but opposite in sign to that for the pseudoscalar coupling.

Instead of varying g_+/g_0, one may ask how much charge dependence of the shorter range potential ($r<r_{\rm in}$) is necessary to bring agreement between the calculated and measured values of a_{np}. For this purpose we set $g_+/g_0=G_+/G_0=1$ in the pseudoscalar theory and compute the value of B required to fit the n-p scattering length. With the above parameters for the ρ-potential, we find that $B_{np}/B_{nn}\approx 0.98$ (a 2% change of the boundary condition) is required to fit the experimental $s_{np}-a_{nn}$.

TABLE III. Neutron-proton scattering-length predictions for pseudovector coupling. The listing is the same as for Table II, except that f and F replace g and G.

	OPEP	OPEP+TPEP	OPEP+TPEP $+V_\rho$
a_{np} (F)	-12.6	-123	-32.35
$f_+/f_0=F_+/F_0$	1.043	1.029	1.017

[19] This differs from the value given in Ref. 7 because the two-pion exchange potential was included there but without a $\pi^\pm$-π^0 mass difference.

Lastly, we already pointed out that Δm is unknown and that η^2 is poorly determined. The effect of changing Δm is shown in Fig. 2 with $\eta^2=1.668$ and A_+, A_0 given in Eq. (10b) for curve I and in Eq. (10c) for curve II. In agreement with Heller, Signell, and Yoder,[7] we find that Δm must be *negative* ($m_{\rho^0}-m_{\rho^+}=4$ MeV for I, and 5.2 MeV for II) to fit the experimental *n-p* scattering length in the $^1S_0$ state. This result appears to be relatively insensitive to variations in $r_{\rm in}$. For $r_{\rm in}=0.35$ F, $\Delta m=-3.8$ MeV (rather than -4 MeV for $r_{\rm in}\approx 0.7$ F) fits the experimental a_{np} for curve I. The effect of varying η^2 is shown in Fig. 3 for $\Delta m=-2$ MeV. By extrapolation we find that agreement with experiment requires $\eta^2>4$.

IV. CONCLUSIONS

The mass differences between neutral and charged mesons are able to account for a large fraction, or possibly all, of the difference between the measured *n-n* and *n-p* scattering lengths. The electromagnetic mass splittings of the $\pi^\pm$-π^0 yield approximately 60% of the difference if we use an OPEP and TPEP with a boundary condition at 0.7011 F. The addition of a ρ potential with $\Delta m\geq 0$ raises the calculated a_{np}, and thus makes agreement with experiment worse. We find that a negative $\Delta m<-4$ MeV is required to fit experiment if the square of the charge-independent ρ-nucleon coupling constant, η^2, is 1.668. On the other hand with $\Delta m=-2$ MeV, a value of $\eta^2>4$ is necessary to give $a_{np}\approx -23.68$ F if $a_{nn}=-16.4$ F.

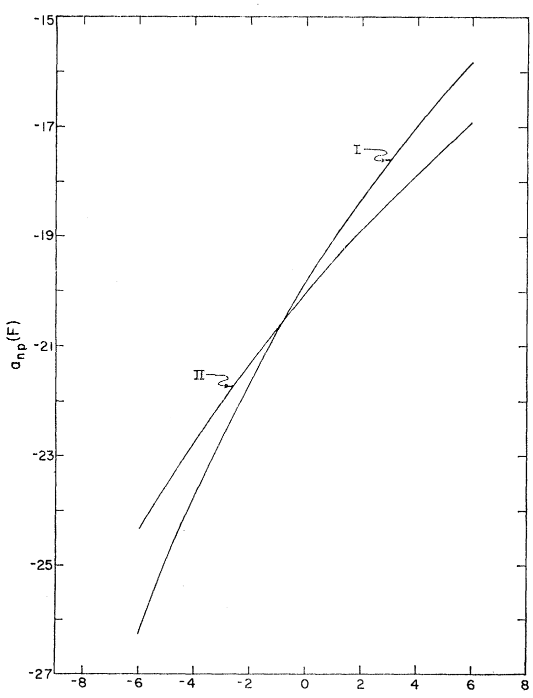

FIG. 2. Effect of the $\rho^\pm$-ρ^0 mass difference Δm on a_{np} with a charge-independent coupling, $\eta^2=1.668$, and $r_{\rm in}=0.7011$ F. Curves I and II correspond to the ρ potential, Eq. (10a), with parameters given by Eqs. (10b) and (10c), respectively.

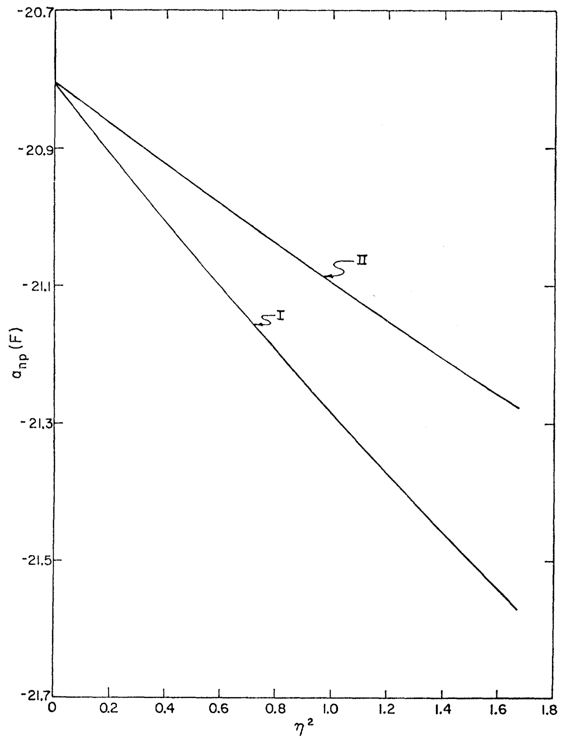

FIG. 3. Variation of the calculated a_{np} with η^2. The other parameters are the same as for Fig. 2, except that $\Delta m=-2$ MeV.

The above conclusions assume charge-independent couplings and boundary conditions. Roughly a 2% change in the ratio of the charged to neutral pion-nucleon coupling constant or in the boundary condition can give agreement with experiment if $\Delta m=+2$ MeV and $\eta^2=1.668$.

In order to make firmer theoretical statements concerning the origin of the discrepancy between a_{nn} and a_{np}, it will be necessary to know the parameters of the theory more accurately and to have a better theoretical framework for low energy scattering.

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