$T=\frac{1}{2}, \frac{3}{2}$ P- and S-Wave $K\pi$ Phase Shifts

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(Received 1 June 1965; revised manuscript received 20 September 1965)

Using the N/D technique, $T=\frac{1}{2}, \frac{3}{2}$ P- and S-wave phase shifts are calculated. The $T=\frac{1}{2}$ P- and S- wave phase shifts are also calculated in the Khuri-Regge representation. For the purpose of comparison with experiments, we use our phase shifts to calculate the angular distributions in the reaction $K^-p \to \bar{K}^0 p\pi^-$.

1. INTRODUCTION

DURING recent years, a large number of dynamic calculations for strong-interaction phase shifts have been done. For this purpose one usually employs the well-known N/D formalism; where the N and D functions have the usual left- and right-hand cuts, respectively. The left-hand discontinuity (which is directly related to the forces responsible for scattering) is assumed to arise mainly due to one-particle exchange diagrams. The phase shifts can then be easily calculated from the resulting unitarized partial-wave amplitude.

In the present work, we have calculated the S- and P-wave $T=\frac{1}{2}$ and $\frac{3}{2}$ $K\pi$ phase shifts. In Sec. 2, the N/D equations are solved in the well-known Balázs[1] two-pole approximation. For purposes of matching, we have evaluated the fixed-energy dispersion integrals, using K^* and ρ exchange diagrams along with the K^* pole in the direct channel.

In Sec. 3, we have attempted an alternative calculation of the S- and P-wave $T=\frac{1}{2}$ $K\pi$ phase shifts, assuming K^* to be a Regge pole à la Khuri.[2] Since no experimental information exists on $K\pi$ scattering, one can only hope to check these results indirectly. In Sec. 4, we assume a simple peripheral model[3] for the reaction $K^-p \to \bar{K}^0 p\pi^-$ and use our phase shifts to calculate the angular distribution. A brief comparison of our results with experiments is also given.

2. N/D CALCULATION

The possibility of a self-bootstrap of K^* in $K\pi$ scattering has been investigated by various authors.[4] In this section we perform a dynamical calculation for S- and P-wave $K\pi$ phase shifts in $T=\frac{1}{2}$ and $\frac{3}{2}$ states. We consider only the exchange of K^* and ρ and use the strip approximation to estimate the high-energy contribution from crossed channels by effectively replacing them with the K^* pole in the direct channel (see Fig. 1). Since these approximations are well known we sketch the calculations briefly (see Fig. 2).

From the structure of the singularities of the partial-wave amplitude[5] one finds that it is convenient to work in the complex S plane. We define the partial-wave amplitude as

$$g_l(s)=(s^{1/2}/k^{2l+1})e^{i\delta_l(s)}\sin\delta_l(s). \qquad (1)$$

Expressing this amplitude in the well-known N/D form

$$g_l(s)=N_l(s)/D_l(s), \qquad (2)$$

we approximate the N function by two poles

$$N_l(s)=\sum_{i=1}^{2} R_{il}/s-s_i. \qquad (3)$$

The D function then is given by

$$D_l(s)=1-\frac{s-s_0}{\pi}\int_{(m+1)^2}^{\infty}\frac{ds'(k')^{2l+1}N_l(s')}{(s'-s)(s'-s_0)(s')^{1/2}}, \qquad (4)$$

where s_0 is the subtraction point and m is the mass of the K meson (the mass of the pion is taken to be unity). The parameters R_{il} are determined by matching the ampli-

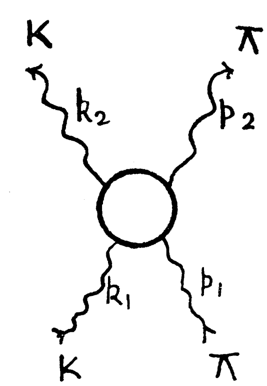

FIG. 1. $K\pi$-scattering diagram.

[1] L. A. P. Balázs, Phys. Rev. **128**, 1935 (1962).
[2] N. N. Khuri, Phys. Rev. **130**, 429 (1963).
[3] G. F. Chew and F. E. Low, Phys. Rev. **113**, 1640 (1959).
[4] R. H. Capps, Phys. Rev. **131**, 1307 (1963); J. L. Gervais, *ibid.* **138**, B1457 (1965); B. Diu, J. L. Gervais, and H. R. Rubinstein, Nuovo Cimento **31**, 27 (1964).
[5] B. W. Lee, Phys. Rev. **120**, 325 (1960).

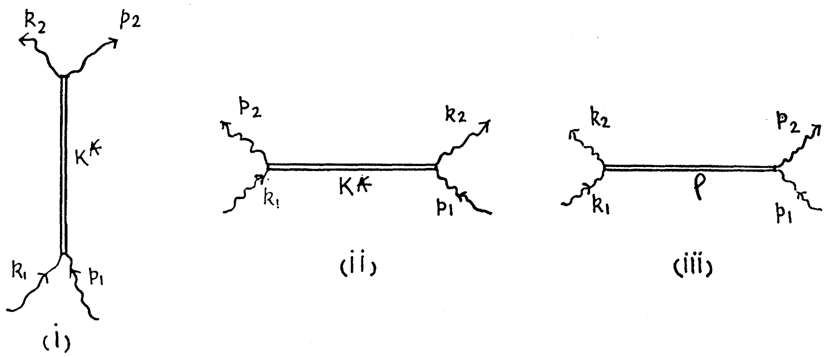

Fig. 2. Input forces.

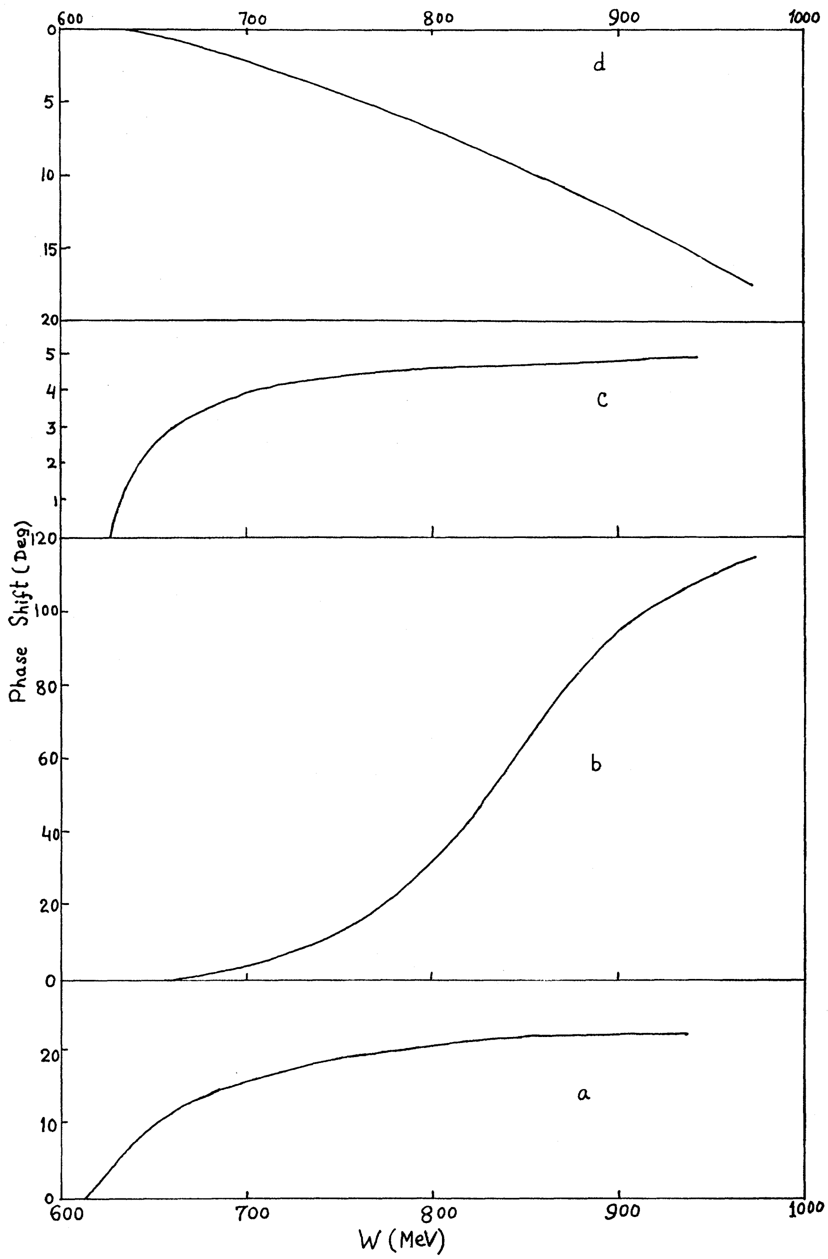

Fig. 3. $K\pi$-scattering phase shifts in (a) $T=\frac{1}{2}$ S wave, (b) $T=\frac{1}{2}$ P wave, (c) $T=\frac{3}{2}$ S wave, and (d) $T=\frac{3}{2}$ P wave.

tude (2) with

$$
\begin{aligned}
g_l^{(\frac{1}{2},\frac{3}{2})}(s)=\left(\frac{1}{8\pi}\right)\frac{1}{k^{2l}}\Bigg[&(3,0)\frac{g_{K^*K\pi}^2}{s-m_{K^*}^2}\left\{\frac{\delta_{l0}}{2l+1}\left(s-2-2m^2+\frac{(m^2-1)^2}{m_{K^*}^2}\right)-4k^2\frac{\delta_{l0}}{2l+1}+4k^2\frac{\delta_{l1}}{2l+1}\right\}\\
&+(-1,2)g_{K^*K\pi}^2\left\{\frac{\delta_{l0}}{2l+1}+\frac{1}{2k^2}\left(-2s+\frac{(m^2-1)^2}{m_{K^*}^2}-m_{K^*}^2+2m^2+2\right)Q_l\left(1-\frac{m_{K^*}^2-2m^2-2+s}{2k^2}\right)\right\}\\
&-(-2,1)g_{\rho KK}g_{\rho\pi\pi}\left\{\frac{\delta_{l0}}{2l+1}+\frac{1}{2k^2}(m_\rho^2-2m^2-2+2s)Q_l\left(1+\frac{m_\rho^2}{2k^2}\right)\right\}\Bigg]
\end{aligned}
\quad (5)
$$

obtained by partial-wave projection of the Born terms. The coupling constants involved in Eq. (5) are expressed in terms of the $\rho\pi\pi$ coupling constant using $SU(3)$ symmetry.[6]

Choosing $s_1=-m^2$, $s_2=-16m^2$, and $s_0=3.5$, we fix our matching points by requiring a zero in ReD at the experimental K^* mass. The S- and P-wave phase shifts for $T=\frac{1}{2}$ and $\frac{3}{2}$ states are plotted in Fig. 3. The general qualitative features of our results are in agreement with those found by earlier authors,[4] and in particular the output width of K^* turns out to be too large.

3. K^* REGGE-POLE CALCULATION

Here, we calculate the $T=\frac{1}{2}$, S- and P-wave phase shifts for $K\pi$ scattering on the assumption that the K^* resonance lies on a Regge trajectory in the 1^-, $T=\frac{1}{2}$ state. Using the Khuri representation,[2] the contribution

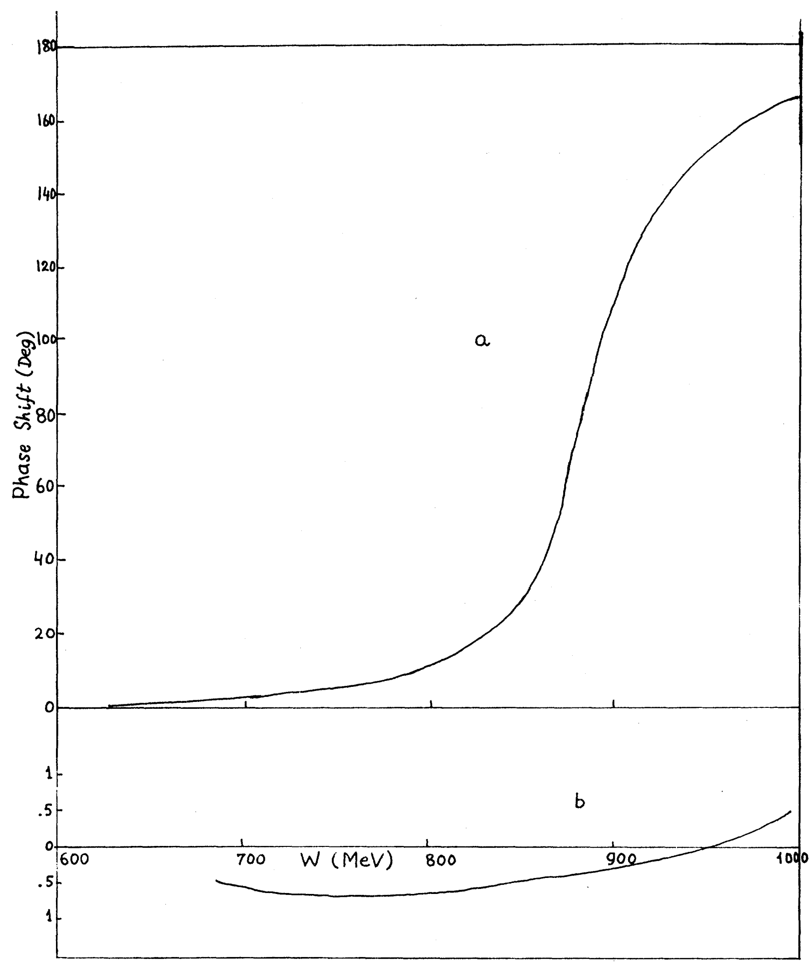

FIG. 4. $K\pi$-scattering phase shifts in (a) $T=\frac{1}{2}$ P wave and (b) $T=\frac{1}{2}$ S wave.

[6] J. J. de Swart, Rev. Mod. Phys. **135, 916** (1963).

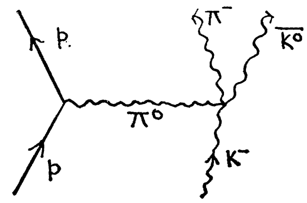

FIG. 5. One-pion-exchange diagram for the reaction $K^-p \to \bar{K}^0\pi^-p$.

of the K^* Regge pole to the $K\pi$ S- and P-wave amplitude is given by

$$g_l(w) = -\frac{1}{2}\frac{w}{q^{2l}}\frac{\beta_l(w)}{[\alpha(w)-l]}[e^{(\alpha-l)\xi_1}+e^{(\alpha-l)\xi_2}], \tag{6}$$

where w is the total center-of-mass energy ($w=\sqrt{s}$), $\alpha(w)$ is the trajectory of the K^* Regge pole, and $\beta_l(w)$ is the corresponding residue. ξ_1 and ξ_2 are the two thresholds given by

$$\cosh\xi_1 = 1+2/k^2, \tag{7}$$

$$\cosh\xi_2 = ((m+1)^2-2-2m^2+w^2)/2k^2-1.$$

From the threshold behavior of $\beta_l(k^2)$, it easily follows that the product of $\beta_l e^{\alpha\xi_1}$ is slowly varying and essentially real and constant near threshold. We approximate it by a real constant C_l for the entire range under consideration. Thus,

$$\beta_l \exp(\alpha\xi_1) = C_l. \tag{8}$$

We assume the Regge trajectory of K^* to be a straight line,[7]

$$\mathrm{Re}\alpha(w) = 1+\epsilon(w-m_{K^*}), \tag{9}$$

where ϵ is the slope of the trajectory $\alpha(w)$, and that

$$\mathrm{Im}\alpha(w) = C_1[w-(m+1)]^{\alpha_0+1/2}, \tag{10}$$

where $\alpha_0 = 1+\epsilon(m+1-m_{K^*})$. Equation (10) satisfies two requirements: (i) $\alpha(w)$ must be purely real below threshold, and (ii) $\mathrm{Im}\alpha(w) \simeq k^{2\alpha_0+1}$ as $k^2 \to 0$. Using the relation

$$\Gamma = \frac{1}{m_{K^*}}\left[\mathrm{Im}\alpha(w)\Big/\frac{d}{dw^2}\mathrm{Re}\alpha(w)\right]_{w=m_{k^*}} \tag{11}$$

we get

$$C_1 = \tfrac{1}{2}\Gamma\epsilon/[m_{K^*}-(m+1)]^{\alpha_0+1/2}. \tag{12}$$

Combining Eqs. (9) to (12), the full trajectory $\alpha(w)$ is obtained as

$$\alpha(w) = 1+\epsilon(w-m_{K^*})+\frac{i}{2}\Gamma\epsilon\left(\frac{w-(m+1)}{m_{K^*}-(m+1)}\right)^{\alpha_0+1/2}. \tag{13}$$

For the slope of the trajectory, we take the Chew-Frautschi[8] value. Using Eqs. (1), (6), and (13), the S- and P-wave phase shifts can be readily calculated. These are plotted in Fig. 4.

[7] N. N. Khuri and B. M. Udgaonkar, Phys. Rev. Letters **10**, 172 (1963).
[8] G. F. Chew and S. C. Frautschi, Phys. Rev. Letters **7**, 394 (1961).

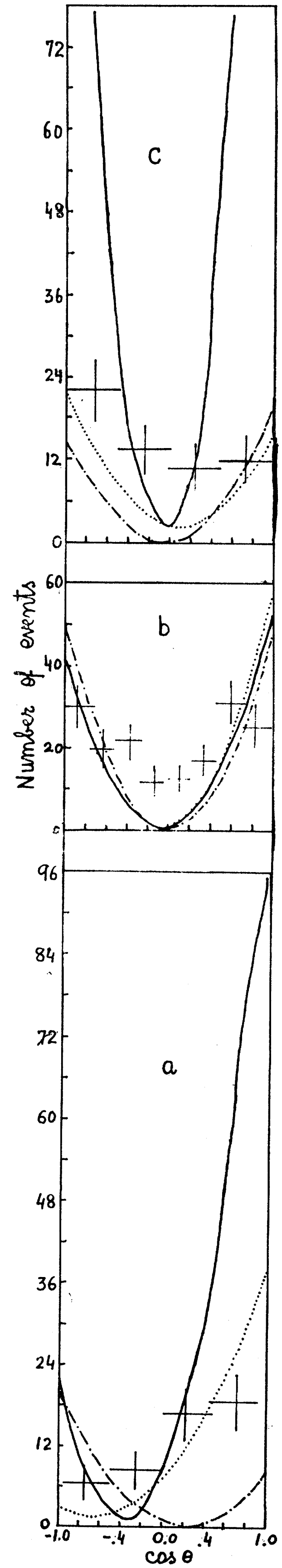

FIG. 6. Distribution in the $K\pi$ scattering angle for events with incident momenta $\sim$1.65 BeV/c. (a) $W=744$ MeV, (b) $W=884$ MeV, and (c) $W=938$ MeV. The full-line curves have been obtained with all the phase shifts from N/D. The broken-line curves have been obtained with $T=\frac{1}{2}$ phase shifts from Regge-Khuri analysis and $T=\frac{3}{2}$ phase shifts from N/D. The dotted curves have been obtained with $T=\frac{1}{2}$ P-wave phase shift from Regge-Khuri analysis and all the rest from N/D. The crosses are the experimental points taken from Ref. 11. (The theoretical curves have been appropriately normalized for comparison with experimental data.)

4. THE PERIPHERAL MODEL FOR $K^-p\rightarrow K^0\pi^-p$

In this section, we consider the production cross section for the reaction $K^-p\rightarrow \bar{K}^0\pi^-p$. The Chew-Low one-pion-exchange model,[3] when applied to Fig. 5, gives

$$\frac{\partial^3\sigma_{K^-p\rightarrow\bar{K}^0\pi^-p}}{\partial w^2\partial\Delta^2\partial\cos\theta}\xrightarrow{\Delta^2\rightarrow -1}\frac{f^2}{2\pi}\frac{\Delta^2}{(\Delta^2+1)^2}\frac{[\frac{1}{4}w^4-\frac{1}{2}w^2(m^2+1)+\frac{1}{4}(m^2-1)^2]}{q_1^2}\frac{\partial\sigma_{K\pi}(w,\cos\theta)}{\partial\cos\theta}, \tag{14}$$

where Δ^2 is the invariant four-momentum transfer squared given to the nucleon, q_1 is the laboratory momentum of the incident K meson, and f^2 is the pion-nucleon coupling constant. We wish to emphasize that here, we use a simple peripheral model involving only a pion exchange and neglect contributions due to absorptive effects and the vector-particle exchange.[9]

The differential cross section for $K\pi$ scattering in S and P waves for $T=\frac{1}{2}$ and $\frac{3}{2}$ states is given by[10]

$$\begin{aligned}\frac{d\sigma_{K\pi}(w,\cos\theta)}{d\cos\theta}=2\pi\lambda\!\!\!^{-2}[&(2/9)\{\sin^2\delta_0{}^1+\sin^2\delta_0{}^3-2\sin\delta_0{}^1\sin\delta_0{}^3\cos(\delta_0{}^1-\delta_0{}^3)\}+\tfrac{4}{3}\{\sin\delta_0{}^1\sin\delta_1{}^1\cos(\delta_0{}^1-\delta_1{}^1)\\&-\sin\delta_0{}^3\sin\delta_1{}^1\cos(\delta_0{}^3-\delta_1{}^1)-\sin\delta_1{}^3\sin\delta_0{}^1\cos(\delta_1{}^3-\delta_0{}^1)+\sin\delta_1{}^3\sin\delta_0{}^3\cos(\delta_1{}^3-\delta_0{}^3)\}\cos\theta\\&+2\{\sin^2\delta_1{}^1+\sin^2\delta_1{}^3-2\sin\delta_1{}^1\sin\delta_1{}^3\cos(\delta_1{}^1-\delta_1{}^3)\}\cos^2\theta].\end{aligned} \tag{15}$$

Here, $\lambda\!\!\!^-$ is the reduced K^- wavelength in the $K\pi$ system.

Equations (14) and (15) and the results of previous sections enable us to obtain the angular distribution curves of the reaction $K^-p\rightarrow\bar{K}^0\pi^-p$. These are shown in Fig. 6. The experimental curves have been taken from Wojcicki.[11] The angular distribution obtained by using the $T=\frac{1}{2}$ and $T=\frac{3}{2}$ S- and P-wave $K\pi$ phase shifts from N/D calculations is in disagreement with the experimental data. In one-channel calculations, this is not surprising, since the width of the K^* resonance using the calculated phase shifts is rather large. On the other hand, the angular-distribution curve plotted by using $T=\frac{1}{2}$ S- and P-wave phase shifts as obtained from the Regge-Khuri analysis, and the corresponding phase shifts for the $T=\frac{3}{2}$ channel taken from N/D calculations is in lesser disagreement with the data. This arises mainly because the $T=\frac{1}{2}$ P-wave phase shifts as obtained by the Regge-Khuri analysis are presumably more reliable since the experimental width of K^* has been fed in. It is of interest to mention that if one takes the $T=\frac{1}{2}$ P-wave phase shifts from the Regge-Khuri analysis and the rest of the information from the N/D calculations, one obtains a much better agreement with the experimental data. The Regge-Khuri K^* pole gives understandably a negligible contribution to the S-wave $T=\frac{1}{2}$ $K\pi$ phase shifts; on the other hand, the N/D calculations yield much larger values in the energy range of interest, in qualitative agreement with the phenomenological analysis of Wojcicki.[11]

Finally, we would like to mention that the $T=\frac{1}{2}$ S-wave $K\pi$ resonance (κ) has not been taken into consideration, since the conclusion of Ref. 11 does not warrant it in the energy range considered.

[9] It has recently been shown that these corrections may not be negligible [see J. D. Jackson, Rev. Mod. Phys. **37**, 484 (1965) and also the literature quoted there]. Inclusion of these effects presumably will alter the results considerably. Such calculations are in progress and will be published elsewhere.

[10] We use the notation $\delta_J{}^{2T}$, where T is the isotopic spin number and J the total angular momentum.

[11] S. G. Wojcicki, Phys. Rev. **135**, B484 (1964).

ACKNOWLEDGMENTS

We are thankful to Professor S. N. Biswas for many helpful discussions. We are also grateful to Professor R. C. Majumdar for his interest in the present work. One of us (K.C.G.) wishes to thank the University Grants Commission, India for a fellowship.