Production and Absorption of S-Wave Pions at Low Energy by Two Nucleons*

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The cross sections for the reactions $p+p \rightarrow d+\pi^+$ and $p+p \rightarrow p+p+\pi^0$ are calculated for s-wave pions at threshold. The pion is produced by either nucleon and emitted directly or rescattered in an s wave by the second nucleon, through a modified static interaction. The two nucleons interact through a phenomenological potential. For the Hamada-Johnston potential both calculated cross sections agree with experimental results. Application of the method of calculation to the nuclear absorption of s-wave pions is discussed.

I. INTRODUCTION

THE reactions in which a pion is produced or absorbed at low energy by a two-nucleon system have been studied for some time. The early interest was in establishing the nature of the pion (spin, parity) and the qualitative features of its interaction with nucleons, using the known symmetries of the two-nucleon system.[1] As the understanding of the meson-nucleon interaction at low energy became more quantitative, attempts were made to apply the theory to calculations of the two-nucleon production or absorption processes; in particular, the reaction

$$p+p \rightarrow d+\pi^+$$

has been studied for energies near the pion threshold.[2]

If an adequate theoretical treatment of the mesonic interactions existed, the theory of the absorption process could be reversed, and used as a tool for the study of the nuclear system. In particular, both theoretical and experimental interest in the nuclear absorption of pions at rest, or at low energy, leading to the ejection of two nucleons, has developed recently.[3] Certainly, a theory for such a process depends in part on an understanding of the absorption of the meson directly by two nucleons. It is for this reason that we have investigated the production and absorption of a pion by a free two-nucleon system. We have calculated, in particular, the cross sections for the reactions

$$p+p \rightarrow d+\pi^+, \tag{1a}$$

$$p+p \rightarrow p+p+\pi^0, \tag{1b}$$

with the pion emerging at low energy and in an s state, for which processes experimental information is available.[4,5] We have also calculated the transition rate for nonradiative absorption by a deuteron of a π^- in an atomic s state, for which no direct measurement is yet available.

We follow the theoretical approach of Woodruff,[6] who calculated the amplitudes for reaction (1a) leading to s- and p-wave pions, but did not get good agreement with experiment for the s-wave reaction. The meson-nucleon interaction is taken as a modified static-model interaction, consistent with low-energy pion-nucleon s- and p-wave scattering. The two-nucleon system is treated nonrelativistically, with the interaction given by a phenomenological potential. The meson processes considered include direct production by either nucleon, and production by one nucleon, with the meson rescattered by the second nucleon.[7] Our approximate treatment of the rescattering process differs from that of Woodruff in some details, involving the relative phase of forward and backward propagation of the virtual meson. These changes alter the analytic expressions and the numerical results, and lead to better agreement with experiment. We have also used the newer phenomenological potential of Hamada and Johnston[8] as well as that of Gammel and Thaler[9] used by Woodruff. Our result for the cross section for the reaction (1a) agrees with the data[4] within the (10 percent) experimental uncertainty.

Using the same approach, we obtain a cross section for the reaction (1b) which is consistent with the measured cross section near threshold, which is, however, not well determined.[5]

We present the theory in Sec. II, the results of numerical calculations in Sec. III, and return to a discussion of the nuclear absorption of pions in Sec. IV.

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[3] T. Ericson, in *Proceedings of the International Conference on High-Energy Physics and Nuclear Structure, CERN, 1963* (CERN, Geneva, 1963), p. 47; M. Jean, Nuovo Cimento, Suppl. **2**, 400 (1964); see also Refs. 19 and 21.

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[5] R. A. Stallwood, R. B. Sutton, T. H. Fields, J. G. Fox, and J. A. Kane, Phys. Rev. **109**, 1716 (1958); A. F. Dunaĭtsev and Yu. D. Prokoshkin, Zh. Eksperim. i Teor. Fiz. **36**, 1656 (1959) [English transl.: Soviet Phys.—JETP **9**, 1179 (1959)].

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[7] D. B. Lichtenberg, Phys. Rev. **105**, 1084 (1957); A. Aitken, H. Mahmoud, E. M. Henley, M. A. Ruderman, and K. M. Watson, *ibid* **93**, 1349 (1954); B. Durney, Proc. Phys. Soc. (London) **71**, 654 (1958).

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[9] J. L. Gammel and R. M. Thaler, Phys. Rev. **107**, 291 (1957). We used the potential given by Eqs. (9), (9.1), and (10) of this reference.

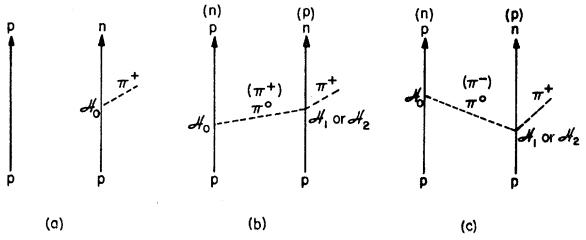

FIG. 1. Diagrams for the processes taken into account for the reaction $p+p\rightarrow d+\pi^+$. Similar diagrams apply to the reaction $p+p\rightarrow p+p+\pi^0$, except that $\mathcal{H}_2$ does not appear.

II. THEORY

We shall compute the cross sections for the reactions

$$p+p\rightarrow d+\pi^+, \tag{1a}$$

$$p+p\rightarrow p+p+\pi^0, \tag{1b}$$

near threshold, with the pion emerging in an s state with respect to the two-nucleon center of mass. For low enough pion momentum, this is also an s state with respect to either nucleon. The calculation of (1a) also leads to an expression for the absorption rate for the reaction

$$\pi^-+d\rightarrow n+n$$

from an atomic s state of the pion.

The pion-nucleon interaction density $\mathcal{H}(j)$ is taken to be of the form used by Woodruff.[6] We write

$$\mathcal{H}(j)=\mathcal{H}_0(j)+\mathcal{H}_1(j)+\mathcal{H}_2(j), \tag{2}$$

where, for the nucleon labeled by j, at position x_j,

$$\mathcal{H}_0=(4\pi)^{1/2}(f/\mu)i\boldsymbol{\sigma}\cdot\{\boldsymbol{\nabla}_\pi[\tau\cdot\phi(x)] +(2M)^{-1}[\mathbf{p}\tau\cdot\pi(x)+\tau\cdot\pi(x)\mathbf{p}]\}, \tag{2a}$$

$$\mathcal{H}_1=4\pi\lambda_1\mu^{-1}\phi^2(x), \tag{2b}$$

$$\mathcal{H}_2=4\pi\lambda_2\mu^{-2}\tau\cdot\phi(x)\times\pi(x). \tag{2c}$$

The operators $\boldsymbol{\sigma}$ and τ are the usual nucleon spin and isospin operators, while $\mathbf{p}$ is the nucleon momentum operator. The mass of the pion is μ, and that of the nucleon, M. The gradient $\boldsymbol{\nabla}_\pi$ operates on the pion field, given by the operator $\phi(x)$, while $\pi(x)$ is the conjugate pion field operator. Both field operators are treated as vectors in isospin. The matrix elements of these operators between a pion state of momentum q, charge $c(c=1, 0, -1)$, and the vacuum state, are given by

$$\begin{aligned}\langle 0|\phi(\mathbf{x})|\mathbf{q},c\rangle&=(2\omega_q)^{-1/2}\exp(i\mathbf{q}\cdot\mathbf{x}),\\ \langle 0|\pi(\mathbf{x})|\mathbf{q},c\rangle&=-i(\tfrac{1}{2}\omega_q)^{1/2}\exp(i\mathbf{q}\cdot\mathbf{x}),\end{aligned} \tag{3}$$

where the pion energy $\omega_q=(q^2+\mu^2)^{1/2}$. We use $\hbar=1$.

The first term in Eq. (2a) is the usual static p-wave pion-nucleon interaction density, with the coupling constant $f^2=0.088$. The second term is included to make $\mathcal{H}_0$ Galilean-invariant, and corresponds to nucleons interacting with *s-wave pions*. The terms $\mathcal{H}_1$ and $\mathcal{H}_2$ are direct and charge exchange interactions which have been introduced[10,11] to explain the low-energy s-wave scattering of pions by nucleons. These terms are formally of second order in the pion field; however, the coefficients are not calculated from meson theory, but fixed phenomenologically.

Since the pion is to be produced in an s state, the lowest-order process is given by the s-wave (second) term of $\mathcal{H}_0$ (see Fig. 1a). The next correction involves a meson produced at one nucleon through either term of $\mathcal{H}_0$, rescattering in the s wave at the second nucleon, through $\mathcal{H}_1$ or $\mathcal{H}_2$ [see Fig. 1(b), (c)]. This virtual meson may propagate forward or backward in time. This correction competes with the lowest order term, in part because of the factor (μ/M) in the s-wave part of $\mathcal{H}_0$. Further corrections are not calculated.

In this approximation the transition amplitude is given in terms of the matrix elements of the operator

$$\begin{aligned}T&=T_0+T_1+T_2,\\ T_0&=\sum_{i=1,2}\mathcal{H}_0(i),\\ T_{1,2}&=\sum_{i\neq j}\mathcal{H}_{1,2}(i)(E-H+i\epsilon)^{-1}\mathcal{H}_0(j),\end{aligned} \tag{4}$$

where E is the initial center-of-mass energy of the two protons, and H is the Hamiltonian for the two-nucleon system plus a free pion.

Taking matrix elements of T between the final meson state $\mathbf{q}$, c, and the meson vacuum, we obtain, using Eqs. (2a) and (3),

$$\langle\mathbf{q},c|T_0|0\rangle=-(4\pi)^{1/2}fM^{-1}(2\mu)^{-1/2} \times[\boldsymbol{\sigma}(1)\tau_{-c}(1)-\boldsymbol{\sigma}(2)\tau_{-c}(2)]\cdot\mathbf{p}_i+\text{terms in }\boldsymbol{\sigma}\cdot\mathbf{q}, \tag{5}$$

where we have used $\omega_q\simeq\mu$, and

$$\mathbf{p}_i=\tfrac{1}{2}(\mathbf{p}_i(1)-\mathbf{p}_i(2)),$$

where i and f refer to initial and final momenta. For the nucleon producing the pion,

$$\mathbf{p}_f(j)=\mathbf{p}_i(j)-\mathbf{q}.$$

The terms in $\boldsymbol{\sigma}\cdot\mathbf{q}$ do not contribute to the s-wave production. Note that

$$\tau_{\pm1}=\mp(2)^{-1/2}(\tau_x\pm i\tau_y).$$

We may calculate the matrix elements of T, by introducing intermediate meson states $\mathbf{k}$, c, $(\pm)$, where the $(\pm)$ refers to forward (backward) propagation in time. Neglecting the neutron-proton mass difference, binding energy of the deuteron, and the kinetic energy of the emitted pion, we may take

$$\langle\mathbf{k},c,(\pm)|(E-H+i\epsilon)^{-1}|\mathbf{k},c,(\pm)\rangle=(\pm\tfrac{1}{2}\mu-\omega_k)^{-1}. \tag{6}$$

[10] S. D. Drell, M. H. Friedman, and F. Zachariasen, Phys. Rev. **104**, 236 (1956).
[11] A. Klein, Phys. Rev. **99**, 998 (1955).

Then

$$\langle \mathbf{q},c|T_1|0\rangle = -\lambda_1(4\pi)^{3/2}f\mu^{-2}(2\mu)^{-1/2}[\boldsymbol{\sigma}(1)\tau_{-c}(1)-\boldsymbol{\sigma}(2)\tau_{-c}(2)]$$
$$\cdot\sum_{\pm}\int [d^3k/(2\pi)^3][\mathbf{k}(1\pm\omega_k/2M)\pm(\omega_k/M)\mathbf{p}]\omega_k^{-1}(\pm\tfrac{1}{2}\mu-\omega_k)^{-1}\exp[i\mathbf{k}\cdot(\mathbf{x}_1-\mathbf{x}_2)], \quad (7)$$

and similarly,

$$\langle \mathbf{q},c|T_2|0\rangle = \lambda_2(4\pi)^{3/2}f\mu^{-3}(2\mu)^{-1/2}i[\boldsymbol{\tau}(1)\times\boldsymbol{\tau}(2)]_{-c}[\boldsymbol{\sigma}(1)+\boldsymbol{\sigma}(2)]$$
$$\cdot\sum_{\pm}\int \frac{d^3k}{(2\pi)^3}\frac{1}{2\omega_k}\frac{\pm\omega_k+\mu}{\pm\frac{1}{2}\mu-\omega_k}\left[\mathbf{k}\left(1\pm\frac{\omega_k}{2M}\right)\pm\frac{\omega_k}{M}\mathbf{p}\right]\exp[i\mathbf{k}\cdot(\mathbf{x}_1-\mathbf{x}_2)]. \quad (8)$$

Performing the summations and integrations in Eqs. (7) and (8), we obtain

$$\langle q,c|T_1|0\rangle = \lambda_1(4\pi)^{1/2}f\mu^{-2}(2\mu)^{-1/2}[\boldsymbol{\sigma}(1)\tau_{-c}(1)-\boldsymbol{\sigma}(2)\tau_{-c}(2)]\cdot[(2+\mu/2M)(-i\hat{r})f'(r)+(\mu/M)\mathbf{p}f(r)], \quad (7')$$

$$\langle q,c|T_2|0\rangle = -\tfrac{3}{2}\lambda_2(4\pi)^{1/2}\mu^{-2}f(2\mu)^{-1/2}i[\boldsymbol{\tau}(1)\times\boldsymbol{\tau}(2)]_{-c}[\boldsymbol{\sigma}(1)+\boldsymbol{\sigma}(2)]\cdot[(1+\mu/4M)(-i\hat{r})f'(r)+(\mu/2M)\mathbf{p}f(r)], \quad (8')$$

where $\hat{r}$ is a unit vector and

$$f(r)\equiv r^{-1}\exp(-\mu' r),\quad f'(r)=df(r)/dr,\quad (\mu')^2=\tfrac{3}{4}\mu^2. \quad (9)$$

In passing from Eq. (8) to Eq. (8′) we drop a term proportional to $\delta(\mathbf{x})$ which does not contribute to the nuclear matrix elements in the presence of a repulsive core.

For the reaction (1a) the initial and final nuclear states are, respectively,

$$\psi_i(\mathbf{r}) = \mp(\sqrt{2}/pr)iu_{1,1}(r)e^{i\delta_{1,1}}(2\pi\cdot 3)^{1/2}|^{3,3}P_1\rangle,\quad M_J=\pm 1,$$
$$\psi_f(\mathbf{r}) = r^{-1}[u(r)|^{1,3}S_1\rangle+w(r)|^{1,3}D_1\rangle], \quad (10)$$

where we normalize

$$\int_0^\infty dr(u^2+w^2)=N^2\mu^{-1}.$$

For the reaction (1b) we have

$$\psi_i(\mathbf{r}) = (\sqrt{2}/pr)iu_{1,0}(r)e^{i\delta_{1,0}}(4\pi)^{1/2}|^{3,3}P_0\rangle,$$
$$\psi_f(\mathbf{r}) = (1/p'r)u_0(r)e^{i\delta_{0,0}}(4\pi)^{1/2}|^{3,1}S_0\rangle, \quad (11)$$

where the isospin and angular momenta are given in the form $^{2T+1,2S+1}L_J$; $\delta_{L,J}$ is the phase shift,

$$u_{L,J}(r)\longrightarrow \sin(pr-\tfrac{1}{2}\pi L+\delta_{L,J}),\quad r\to\infty,$$

and p, p' are the initial and final relative momenta of the protons. We take the nuclear matrix element of $\langle q,+1|T|0\rangle$, using the states (10), sum the absolute square over initial and final spin states, and obtain (some details of the calculation are given in the Appendix)

$$\sum|T_a|^2 = 8(4\pi)^2f^2M^{-2}\mu^{-2}\Big|\sum_{i=1}^{6}I_i\Big|^2N^{-2}, \quad (12)$$

where

$$I_1=\frac{-\mu}{p}\int_0^\infty dr\, r^2\frac{u}{r}\left(\frac{d}{dr}+\frac{2}{r}\right)\frac{u_{1,1}}{r},$$

$$I_2=-(2)^{-1/2}\frac{\mu}{p}\int_0^\infty dr\, r^2\frac{w}{r}\left(\frac{d}{dr}-\frac{1}{r}\right)\frac{u_{1,1}}{r},$$

$$I_3=(\lambda_1+\tfrac{3}{2}\lambda_2)\frac{1}{p}\int_0^\infty dr\, r^2\frac{u}{r}f(r)\left(\frac{d}{dr}+\frac{2}{r}\right)\frac{u_{1,1}}{r},$$

$$I_4=(2)^{-1/2}(\lambda_1+\tfrac{3}{2}\lambda_2)\frac{1}{p}\int_0^\infty dr\, r^2\frac{w}{r}f(r)\left(\frac{d}{dr}-\frac{1}{r}\right)\frac{u_{1,1}}{r}, \quad (13)$$

$$I_5=\frac{M}{\mu}\left(2+\frac{\mu}{2M}\right)(\lambda_1+\tfrac{3}{2}\lambda_2)\frac{1}{p}\int_0^\infty dr\, uf'(r)u_{1,1},$$

$$I_6=(2)^{-1/2}\frac{M}{\mu}\left(2+\frac{\mu}{2M}\right)(\lambda_1+\tfrac{3}{2}\lambda_2)\frac{1}{p}\int_0^\infty dr\, wf'(r)u_{1,1}.$$

Similarly, the summed absolute square of the nuclear matrix elements of $\langle q,0|T|0\rangle$, for the states (11), is

$$\sum|T_b|^2 = 4(4\pi)^3f^2M^{-2}\mu^{-5}\Big|\sum_{i=1}^{3}J_i\Big|^2, \quad (14)$$

where

$$J_1=\frac{-\mu^2}{pp'}\int_0^\infty dr\, r^2\frac{u_0}{r}\left(\frac{d}{dr}+\frac{2}{r}\right)\frac{u_{1,0}}{r},$$

$$J_2=\frac{\lambda_1\mu}{pp'}\int_0^\infty dr\, r^2\frac{u_0}{r}f(r)\left(\frac{d}{dr}+\frac{2}{r}\right)\frac{u_{1,0}}{r}, \quad (15)$$

$$J_3=\frac{\lambda_1 M}{pp'}\left(2\pm\frac{\mu}{2M}\right)\int_0^\infty dr\, u_0f'(r)u_{1,0}.$$

TABLE I. The cross section σ_a for the reaction $p+p\rightarrow d+\pi^+$ with s-wave pions, and the rate R for the reaction $\pi^-+d\rightarrow n+n$ with pions in the $1s$ orbit. The integrals K_i are defined in Eqs. (13) and (23), and the quantity η is the pion momentum in units of μc. The p-p or n-n wave functions have been obtained by using the potentials of Hamada and Johnston (Ref. 8) (H-J) and Gammel and Thaler (Ref. 9) (G-T), and the deuteron wave functions are those of Hamada and Johnston (Ref. 8) (H-J) and Gartenhaus (Refs. 13, 14) (G).

p-p or n-n state	Deuteron state	K_1	K_2	K_3	K_4	K_5	K_6	$\Sigma_i K_i$	σ_a/η (in μb)	R (in 10^{15} sec^{-1})
H-J	H-J	−0.080	0.083	0.002	−0.004	−0.126	−0.035	−0.160	146	0.84
H-J	G	−0.087	0.087	0.003	−0.004	−0.129	−0.033	−0.163	151	0.87
G-T	H-J	−0.079	0.086	0.001	−0.004	−0.129	−0.036	−0.161	147	0.85
G-T	G	−0.089	0.090	0.003	−0.004	−0.134	−0.034	−0.168	160	0.93

The cross section for reaction (1a), $p+p\rightarrow d+\pi^+$, averaged over initial spin states, may be obtained directly from Eq. (12),

$$\sigma_a=(2\pi/v)\tfrac{1}{4}\sum|T_a|^2\rho(E_f)$$
$$=16\pi f^2(\mu/M)^{3/2}\mu^{-2}\eta|\sum I_i|^2N^{-2} \quad (16)$$

for a pion of momentum $q=\mu\eta$. We use $p^2=M\mu$ at threshold, and $v=2p/M$.

The expression for the rate of absorption of a π^- by a deuteron is

$$R=2\pi\times\tfrac{1}{3}\sum|T_a|^2|\phi_\pi(0)|^2\times\tfrac{1}{2}Mp(2\pi)^{-2}N^{-2}, \quad (17)$$

where the pion wave function at the origin, for an atomic ns orbit is

$$\phi_{ns}(0)=(4\pi)^{-1/2}2(\mu e^2)^{3/2}n^{-3}.$$

The cross section per unit energy E_p in the center-of-mass system of the outgoing protons, for the reaction (1b), $p+p\rightarrow p+p+\pi^0$, averaged over initial spin states, may be similarly obtained,

$$d\sigma/dE_p=(2\pi/v)\tfrac{1}{4}\sum|T_b|^2[d\rho(E_f)/dE_p], \quad (18)$$

where

$$d\rho/dE_p=(2\pi)^{-4}(2\mu M)^{3/2}(E_f-E_p)^{1/2}E_p^{1/2},$$
$$E_f=E_p+q^2/2\mu. \quad (19)$$

To evaluate the cross section near threshold, we treat the dependence of $|T_b|^2$ on E_p approximately, following Watson,[12]

$$|T(E_p)|^2\simeq|T(0)|^2/(1+p'^2a^2), \quad (20)$$

where p' is the relative momentum of the protons, and a is the scattering length. For $E_f\gg M^{-1}a^{-2}$ we may approximate Eq. (18) and write

$$\sigma_b=\frac{2\pi}{4v}\frac{1}{Ma^2}\sum|T_b(0)|^2\int_0^{E_f}dE_p\frac{1}{E_p}\frac{d\rho}{dE_p},$$

or

$$\sigma_b=(2)^{1/2}2\pi f^2(\mu/M)(\mu a)^{-2}\mu^{-2}|\textstyle\sum_i J_i|^2\eta^2,$$

where η is the maximum momentum of the emitted pion, in units of μ.

[12] K. M. Watson, Phys. Rev. **88**, 1163 (1952).

III. NUMERICAL RESULTS

The two-nucleon radial wave functions in Eqs. (10) and (11) were obtained by numerical integration of the Schrödinger equation, at the threshold energy for the initial states, and at zero energy for the final state (reaction 1b). For reaction (1a) both Hamada-Johnston[8] and Gammel-Thaler[9] potentials were tried while only the former was taken for reaction (1b). The deuteron wave functions were obtained both from Hamada and Johnston,[8] and from the best Moravcsik fit[13] to the Gartenhaus[14] functions. We used the Hamada-Johnston normalization $N^2=0.9000$, pion mass $\mu=139.4$ MeV, and mass ratio $M/\mu=6.73$, throughout. The squared pion-nucleon coupling constant was taken to be $f^2=0.088$.[15] The s-wave parameters λ_1 and λ_2 are obtained from the s-wave phase shifts δ_1 and δ_3 for pion-nucleon scattering, through the Born-approximation relations[11]

$$\lambda_1=-\tfrac{1}{6}\eta^{-1}(\delta_1+2\delta_3),$$
$$\lambda_2=\tfrac{1}{6}\eta^{-1}(\delta_1-\delta_3). \quad (22)$$

Using the experimental phase shifts[15] $\delta_1=0.17\eta$ and $\delta_3=-0.10\eta$, we find $\lambda_1=0.005$ and $\lambda_2=0.045$.

The radial integrals I_i [Eq. (13)] and J_i [Eq. (15)] were evaluated numerically. For convenience the integrals I_i were transformed by integration by parts:

$$I_1=K_1,$$
$$I_2=K_2,$$
$$I_3=K_3-(2M/\mu-\tfrac{1}{2})^{-1}K_5,$$
$$I_4=K_4-(2M/\mu-\tfrac{1}{2})^{-1}K_6,$$
$$I_5=(2M/\mu+\tfrac{1}{2})(2M/\mu-\tfrac{1}{2})^{-1}K_5,$$
$$I_6=(2M/\mu+\tfrac{1}{2})(2M/\mu-\tfrac{1}{2})^{-1}K_6, \quad (23)$$

where $\sum_i I_i=\sum_i K_i$. The numerical values of the integrals K_i are shown in Table I, for the four choices of potentials for the initial and final states. The integrals J_i are given in Table II.

[13] M. J. Moravcsik, Nucl. Phys. **7**, 113 (1958).
[14] S. Gartenhaus, Phys. Rev. **100**, 900 (1955).
[15] S. W. Barnes, B. Rose, G. Giacomelli, J. Ring, K. Miyake, and K. Kinsey, Phys. Rev. **117**, 226 (1960); S. W. Barnes, H. Winick, K. Miyake, and K. Kinsey, *ibid* **117**, 238 (1960); J. M. McKinley, Rev. Mod. Phys. **35**, 788 (1963).

The calculated cross sections, for the case of the Hamada-Johnston potentials are, for $p+p \rightarrow d+\pi^+$,

$$\sigma_a = 146\eta\ \mu\text{b}, \tag{24}$$

and for $p+p \rightarrow p+p+\pi^0$,[16]

$$\sigma_b = 17\ \eta^2\ \mu\text{b}. \tag{25}$$

The experimental cross section for reaction (1a) is given as[4]

$$\sigma_a = (138\pm 15)\eta\ \mu\text{b}, \tag{26}$$

and for reaction (1b) as[5]

$$\sigma_b = (27\pm 10)\eta^2\ \mu\text{b}, \tag{27}$$

or as[5]

$$\sigma_b = (32\pm 7)\eta^2\ \mu\text{b}. \tag{28}$$

We note that the coefficients of η^2 quoted in Eqs. (27) and (28) are determined by fitting a three-term polynomial in η^2 to the cross section over a large energy range. The coefficient thus determined depends largely on measurements far from threshold, and the limited form of the polynomial. The theoretical result [Eq. (27)] is consistent with measured cross sections near threshold, but not with the fitted curve [Eq. (28)].

The rate of absorption for a π^- by a deuteron, from an atomic $1s$ state, is calculated to be

$$R = 0.84\times 10^{15}\ \text{sec}^{-1}$$

for the Hamada-Johnston potential.

IV. DISCUSSION

As can be seen from Table I, the calculation of σ_a is not very sensitive to the choice of nuclear potential, since the values of σ_a for the first three combinations of initial and final state potentials agree with the experimental cross section (26) within the quoted uncertainty, while the fourth lies just outside. From the partial cancellation of the deuteron S- and D-state integrals, shown in the table, one would expect the calculation to be sensitive to the D-state probability of the deuteron. This was not tested directly since both wave functions chosen have about the same percentage of D state ($\sim 7\%$). Any attempt to vary the D-state probability would have to be done without changing the deuteron quadrupole moment and the triplet-even phase shifts to which the phenomenological potentials are adjusted. Detailed inspection of the integral K_2 shows that an arbitrary decrease in the D-state probability which is constrained to keep the quadrupole moment fixed, will *tend* to keep the integral K_2 constant as well. This effect is similar to that pointed out by deSwart and Marshak[17] in a calculation of the high-energy photodisintegration of the deuteron, and reduces the sensitivity to the D-state probability. This has not been investigated in detail.

TABLE II. The cross section σ_b for the reaction $p+p \rightarrow p+p+\pi^0$ with s-wave pions. The integrals J_i are defined in Eq. (15), and the quantity η is the maximum pion momentum in units of μc. The initial and final diproton states are obtained from the potentials of Hamada and Johnston (Ref. 8).

J_1	J_2	J_3	$\Sigma_i J_i$	σ_b/η^2 (in μb)
−0.88	0.01	−0.16	−1.03	17

We should like to point out that although the pion-nucleon interaction we have chosen has been adjusted to reproduce the low-energy s- and p-wave pion-nucleon scattering, not every interaction which reproduces these data will yield the same results for the processes we have calculated. Because of the energy dependence of the field operator $\pi(x)$, the propagation of the virtual pion in our Born treatment will differ from that, say, in an impulse-approximation calculation, based on the same pion-nucleon parameters. The largest difference is in the charge-exchange rescattering term T_2, leading to a ratio of about two for the cross section σ_a calculated by impulse, over that by Born approximation. In fact, an impulse-approximation calculation of the reaction $p+p \rightarrow d+\pi^+$ in the pion energy range 20–170 MeV, has been reported by Dosch,[18] which does overestimate σ_a at 20 MeV by a factor ~ 1.5. This could also be due to his omission of the D state in the deuteron wave function.

We also note, that although Coulomb effects are important very close to threshold, they can be (and have been) neglected for the purpose of comparing with experiments,[4,5] for which the pion momentum $\eta > 0.1$. We have also omitted reduced mass effects, which are not consistent with the static approximation, and we have ignored the π^0-π^+ mass difference. We have not calculated higher corrections to the single rescattering process.

Finally, we return to the problem of nuclear absorption of s-wave pions, with two-nucleon ejection. We assume that this process can be treated in terms of the absorption of the pion by a nucleon pair in the target, which pair is emitted from the nucleus, although other processes may have to be taken into account. Several authors[19] have formulated or performed calculations of the absorption rate or the spectrum of the particles emitted, using the linear, Galilean-invariant pion-nucleon interaction given in Eq. (2a). The absorption amplitude is given by nuclear matrix elements of T_0 [Eq. (5)], modified to include center-of-mass motion of the nucleon pair. However, for our deuteron absorption calculation, we found a large contribution from the

[16] Note from Eq. (22) that λ_1 is less well determined than λ_2, for a given uncertainty in the phase shifts. This is reflected in a larger uncertainty in the computed value (25) than in (24).

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rescattering of the meson through the charge-exchange s-wave interaction $\mathcal{H}_2$, given in Eq. (2c) [see Table I]. We would expect similar contributions in the general nuclear case.

The rescattering contribution to the π^0 production, however, is smaller than the direct term, since the charge-exchange interaction $\mathcal{H}_2$ does not enter [see Table II]. This has an interesting consequence if one argues that the relative S state of the absorbing nucleon pair is most important for nuclear absorption. Then the amplitude for absorption by a ^3S pair would be larger than that by a ^1S pair, because of the contribution of the charge-exchange rescattering to the former. If we now assume that the ratio of contributions from these different mechanisms in the nucleus is similar to that displayed in Tables I and II, we obtain a ratio of $\sim 2:1$ for the amplitudes $^3S:^1S$, or 4:1 for probabilities. Since n-p pairs may occupy either ^3S or ^1S states, while p-p pairs must be in ^1S, this ratio enhances the absorption of π^- by n-p pairs relative to p-p pairs, over and above the effect of having a greater number of n-p pairs in a nucleus.

If we apply this somewhat too simple argument to π^- absorption by O^{16}, we find by a counting of magnetic substates which can contribute, that the ratio of n-n pairs to n-p pairs *ejected* (reflecting absorptions by n-p and p-p, respectively) to be $\sim 9:1$. Recent experiments[20] on O^{16} have given this ratio as about 8:1 for emitted pairs at a relative angle of π radians. Without taking this too literally, we find a qualitative explanation for the excess neutron-pair emission, first reported by Ozaki *et al.*[21] An alternate explanation has been discussed by Kohmura,[22] who suggests that the n-p correlation is stronger than the p-p correlation in nuclei.

This proposed effect of the charge-exchange rescattering is in disagreement with the conclusions of Eckstein,[23] who has applied a phenomenological model to the π^- absorption by He^4. Her method of analysis leads to equal absorption by n-p and p-p S-state pairs. However, her transition operator has zero range for the absorbing pair. Since the extraction of the unknown strength parameter from the deuteron does not, then, include the D-state contribution, which we calculate to be large, her conclusion about the effective strengths of the ^3S and ^1S is doubtful. Our calculation produces the equivalent of her unknown parameters g_0^-, g_1^-, but not in a form easily compared, since our transition operator T has terms of finite range.

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APPENDIX

For the computation of the nuclear matrix elements leading to Eqs. (12) and (14), we need the isospin matrix elements

$$\begin{aligned}\langle 0,0|\{\tau_{-1}(1)-\tau_{-1}(2)\}|1,1\rangle&=-2,\\ \langle 0,0|i\{\tau(1)\times\tau(2)\}_{-1}|1,1\rangle&=2,\\ \langle 1,1|\{\tau_0(1)+\tau_0(2)\}|1,1\rangle&=2,\end{aligned}\tag{A1}$$

where the states are labeled by T, T_3. Other relevant matrix elements are zero.

For reaction (1a) we need the spin-angle matrix elements

$$\begin{aligned}\langle {}^3S_1|\mathbf{S}\cdot\hat{r}|{}^3P_1\rangle&=(\tfrac{2}{3})^{1/2},\\ \langle {}^3S_1|\mathbf{S}\cdot\mathbf{p}|{}^3P_1\rangle&=-i(\tfrac{2}{3})^{1/2}(\partial/\partial r+2/r),\\ \langle {}^3D_1|\mathbf{S}\cdot\hat{r}|{}^3P_1\rangle&=(\tfrac{1}{3})^{1/2},\\ \langle {}^3D_1|\mathbf{S}\cdot\mathbf{p}|{}^3P_1\rangle&=-i(\tfrac{1}{3})^{1/2}(\partial/\partial r-1/r),\end{aligned}\tag{A2}$$

where $\mathbf{S}=\frac{1}{2}\{\boldsymbol{\sigma}(1)+\boldsymbol{\sigma}(2)\}$.

For reaction (1b) we use

$$\begin{aligned}\langle {}^1S_0|\boldsymbol{\Sigma}\cdot\hat{r}|{}^3P_0\rangle&=1,\\ \langle {}^1S_0|\boldsymbol{\Sigma}\cdot\mathbf{p}|{}^3P_0\rangle&=-i(\partial/\partial r+2/r),\end{aligned}\tag{A3}$$

where $\boldsymbol{\Sigma}=\frac{1}{2}\{\boldsymbol{\sigma}(1)-\boldsymbol{\sigma}(2)\}$.