Electron-Nucleon Scattering Cross Section Including All Polarization Correlations*

JAMES H. SCOFIELD
(Received 20 August 1965)

The first-order Born-approximation cross section for the electromagnetic scattering of two nonidentical spin one-half particles is given, including the dependence on arbitrary initial and final spin states of both particles. Both particles are treated relativistically with charge and magnetic-moment interactions.

FROM measurements of polarization correlations in electron-nucleon scattering, the relative sign of the electric and magnetic moments of the nucleon can be experimentally determined. Also, a check on the one-photon-exchange picture of the scattering is given, since the form factors can be determined for a fixed square of the four-momentum and fixed center-of-mass energy.

First-order Born-approximation cross sections for electron-nucleon scattering, including the correlations between various pairs of spin states, have previously been given.[1–8] The present calculation includes the dependence on all four spin states and thus applies to all possible spin-correlation experiments. Specifically, there are correlations between all pairs of spin states, and also correlations dependent on the polarization of all four states. The latter type correlation would be very difficult to observe; they are given here for the sake of completeness. The correlation between the two final spin states has not previously been given, and that between the two nucleon spin states has previously been given only in the high-electron-energy limit. General expressions and the high-energy limits for the correlations between the other pairs of spin states are contained in the previous works. The present calculation is distinct from these works in that we use the brickwall coordinate system to define the particle currents and coordinate systems used for the specification of the components of the polarization vectors. This technique is presented by Yennie, Lévy, and Ravenhall[9] in the calculation of the cross section for unpolarized particles. It makes the calculation and resulting expressions relatively simple. The technique does not single out the Dirac magnetic moment. Thus, arbitrary magnetic moments are included for both particles. This generality over the previous works does not appear to have experimental applications.

Throughout the calculation, reference is made only to the spin states of the particles in their rest frames. These states are specified by transforming without spatial rotations to the rest frames of the particles. For a spin one-half particle, the polarization vector $\mathbf{N}$, defined as the expectation value of the Pauli spin $\boldsymbol{\sigma}$, specifies the density operator as

$$\rho=\tfrac{1}{2}(1+\mathbf{N}\cdot\boldsymbol{\sigma})\,.$$

The magnitude of $\mathbf{N}$ assumes its maximum value of one for a pure state and is equal to zero for an unpolarized particle.

In the brickwall coordinate system, i.e., the coordinate system in which the initial and final moments of a particle are equal and opposite, the matrix element of the nucleon current (at the origin) between the momentum states is taken as

$$J_4=ieG_E\,,$$
$$\mathbf{J}=ieG_MK\boldsymbol{\sigma}\times\hat{q}\,,$$

with momentum states normalized to

$$\langle\mathbf{P}_1|\mathbf{P}\rangle=\delta(\mathbf{P}_1-\mathbf{P})E/M\,.$$

The magnitude of the momentum transfer has been absorbed into the magnetic moment μ_N such that $K=(q^2)^{1/2}\mu_N/e$; the analogous quantity for the electron is denoted by k. The momentum transfer is $q=P'-P=p-p'$. The symbols M, $\mathbf{P}$, E, and $\mathbf{N}$ denote respectively the nucleon mass, initial momentum, energy, and polarization vector; m, $\mathbf{p}$, ϵ, and $\mathbf{n}$ are the analogous quantities for the electron. Final quantities are indicated by a prime.

In terms of the covariant expression for the current

$$J_\mu=ie\bar{u}'[F_1\gamma_\mu+\kappa F_2q_\nu\sigma_{\nu\mu}]u\,;$$

the form factors G_E and G_M are given by

$$G_E=F_1-\kappa F_2q^2/4M^2\,,$$
$$G_M\mu_N=e(F_1+\kappa F_2)/2M\,.$$

The G's are not explicitly included in the expression for the cross section.

The trace over the spin states, $\mathrm{Tr}(\rho_fJ_\mu\rho_iJ_\nu{}^\dagger)$, was carried out in the particles' brickwall coordinate-

* Work performed under the auspices of the U. S. Atomic Energy Commission.

[1] A. I. Akhiezer, L. N. Rozentsveig, and I. M. Shumushkevich, Zh. Eksperim. i Teor. Fiz. 33, 765 (1957) [English transl.: Soviet Phys.—JETP 6, 588 (1958)].
[2] A. M. Bincer, Phys. Rev. 107, 1467 (1957).
[3] G. V. Frolov, Zh. Eksperim. i Teor. Fiz. 34, 764 (1958); 40, 296 (1961) [English transls.: Soviet Phys.—JETP 7, 525 (1958); 13, 200 (1961)].
[4] R. G. Newton, Phys. Rev. 109, 2213 (1958); 110, 1483 (E) (1958).
[5] J. H. Scofield, Phys. Rev. 113, 1599 (1959).
[6] L. J. Weigert and M. E. Rose, Nucl. Phys. 51, 529 (1964).
[7] C. K. Iddings, G. L. Shaw, and Y. S. Tsai, Phys. Rev. 135, B1388 (1964).
[8] G. Ramachandran and R. K. Umerjee, Phys. Rev. 137, B978 (1965).
[9] D. R. Yennie, M. M. Lévy, and D. G. Ravenhall, Rev. Mod. Phys. 29, 144 (1957); the electron-nucleon cross section for unpolarized particles is given originally by M. N. Rosenbluth, Phys. Rev. 79, 615 (1950).

systems. The explicit Lorentz transformation of velocity $\mathbf{v}$ was used to transform the electron current to the brickwall system of the nucleon, and the scalar products of the two currents subsequently formed. In the brickwall system of the nucleon, $\mathbf{v}$ is given by $\mathbf{v}=(\mathbf{p}+\mathbf{p}')/2\epsilon$ and is perpendicular to $\mathbf{q}$. The zeroth component of the four-velocity is given by

$$\gamma=(1-v^2)^{-1/2}=-(4m^2+q^2)^{-1/2}(4M^2+q^2)^{-1/2}\times(P+P')(p+p').$$

The procedure gives the cross section with the components of the polarization vectors specified as those with respect to the coordinate system used in the particles' brickwall systems. In the brickwall systems, we use the $\mathbf{q}$, $\mathbf{v}$, and $\mathbf{q}\times\mathbf{v}$ directions for the 3, 1, and 2 directions, respectively. In terms of the momenta in a general coordinate system, the 3 and 1 basis vectors for the initial-electron polarization are

$$\begin{aligned}\mathbf{e}^{(3)}(\mathbf{p})&=(q^2)^{-1/2}(1+q^2/4m^2)^{-1/2}\times[\hat{p}(\epsilon'|\mathbf{p}|-\epsilon\hat{p}\cdot\mathbf{p}')/m-\hat{p}\times(\mathbf{p}'\times\hat{p})]\\&=(q^2)^{-1/2}(1+q^2/4m^2)^{-1/2}\times[\mathbf{p}(\epsilon'-p\cdot p'/m)/(\epsilon+m)-\mathbf{p}'],\end{aligned}$$

$$\begin{aligned}\mathbf{e}^{(1)}(\mathbf{p})=v^{-1}\{&\mathbf{p}[(4M^2+q^2)^{-1/2}\gamma^{-1}(E+E')\\&+(4m^2+q^2)^{-1/2}(m-\epsilon')]/(m+\epsilon)\\&+\mathbf{p}'(4m^2+q^2)^{-1/2}-(\mathbf{P}+\mathbf{P}')(4M^2+q^2)^{-1/2}\gamma^{-1}\}.\end{aligned}$$

Substituting the corresponding momenta, etc. for the other states yields the 3-basis-vector for the final nucleon and its negative for the initial nucleon and final electron, and the 1-basis-vector for the final electron and its negative for the nucleons. For the usual situation with all the momenta coplanar, the 2 axis for all the polarizations is normal to the plane and is given by $\mathbf{p}\times\mathbf{p}'$ in the rest frame of either nucleon. In this case, the 2 and 3 axes give the simplier specification of the coordinate systems. The first form given for $\mathbf{e}^{(3)}$ gives its decomposition into longitudinal and transverse components.

The differential cross section for the scattering of the electron through an angle θ from a nucleon initially at rest with the final electron and nucleon in the pure states with polarizations $\mathbf{n}'$ and $\mathbf{N}'$ is given by

$$d\sigma/d\Omega=\{\alpha^2m^2|\mathbf{p}'|\gamma^2/(q^2)^2|\mathbf{p}|[1+(\epsilon-|\mathbf{p}|\cos\theta)/M]\}\times(X_0+X_{NN}+X_{ee}+X_{eN}+X_4),$$

where

$$\begin{aligned}X_0={}&1+(k^2+K^2)v^2+k^2K^2(2-v^2),\\
X_{NN}={}&N_3N_3'[1+(k^2-K^2)v^2-k^2K^2(2-v^2)]+N_1N_1'[1+(k^2-K^2)v^2-k^2K^2v^2]\\&+N_2N_2'[1+(k^2+K^2)v^2+k^2K^2v^2]+2(N_3N_1'-N_1N_3')(1+k^2)Kv,\\
X_{eN}={}&-2(N_2+N_2')(n_2+n_2')kK(1-v^2)-2\gamma^{-1}[(N_1+N_1')(n_1+n_1')kK+(N_1+N_1')(n_3-n_3')k^2Kv\\&+(N_3-N_3')(n_1+n_1')kK^2v+(N_3-N_3')(n_3-n_3')k^2K^2],\\
X_4={}&\mathbf{N}\cdot\mathbf{N}'\mathbf{n}\cdot\mathbf{n}'+2\mathbf{N}\cdot\mathbf{N}'(\mathbf{n}\times\mathbf{n}')_2kv+2(\mathbf{N}\times\mathbf{N}')_2\mathbf{n}\cdot\mathbf{n}'Kv+2(\mathbf{N}\times\mathbf{N}')_2(\mathbf{n}\times\mathbf{n}')_2kK(1+v^2)\\
&+2\gamma^{-1}(\mathbf{N}\times\mathbf{N}')_1(\mathbf{n}\times\mathbf{n}')_1kK+\mathbf{N}\cdot\mathbf{N}'(n_2n_2'-n_1n_1'-n_3n_3')k^2v^2+(N_2N_2'-N_1N_1'-N_3N_3')\mathbf{n}\cdot\mathbf{n}'K^2v^2\\
&+2[(\mathbf{N}\times\mathbf{N}')_2(n_2n_2'-n_1n_1'-n_3n_3')+\gamma^{-1}(\mathbf{N}\times\mathbf{N}')_1(n_1n_2'+n_2n_1')]k^2Kv\\
&+2[(N_2N_2'-N_1N_1'-N_3N_3')(\mathbf{n}\times\mathbf{n}')_2+\gamma^{-1}(N_1N_2'+N_2N_1')(\mathbf{n}\times\mathbf{n}')_1]kK^2v\\
&+[(N_2N_2'-N_1N_1'-N_3N_3')(n_2n_2'-n_1n_1'-n_3n_3')+(N_1N_1'-N_2N_2'-N_3N_3')(n_1n_1'-n_2n_2'-n_3n_3')(1-v^2)\\
&+2\gamma^{-1}(N_2N_1'+N_1N_2')(n_2n_1'+n_1n_2')]k^2K^2.\end{aligned}$$

The expression for X_{ee} is obtained from X_{NN} by replacing $\mathbf{N}$ by $\mathbf{n}$, $\mathbf{N}'$ by $\mathbf{n}'$, and interchanging k and K. The treatment of the scattering in other reference frames requires modifying only the factor giving the density of states divided by the flux.

For the initial nucleon at rest, the 3 axis for both nucleon polarizations is in the direction of the final nucleon momentum. In this reference frame, kinematics gives,

$$\gamma=(1+q^2/4M^2)^{-1/2}(1+q^2/4m^2)^{-1/2}(\epsilon+\epsilon')/2m,$$

and

$$q^2=2M(\epsilon-\epsilon').$$

In the high-electron-energy limit[10] (ϵ and $\epsilon\theta\gg m$), the 3 axes for the initial and final electrons are parallel and antiparallel to their respective momenta. In this high-energy limit, kinematics gives

$$\begin{aligned}\gamma^2&=[1+(1+\eta)\tan^2(\tfrac{1}{2}\theta)]\cot^2(\tfrac{1}{2}\theta)/(1+\eta),\\v^2\gamma^2&=\cot^2(\tfrac{1}{2}\theta)/(1+\eta),\\q^2&=4\epsilon\epsilon'\sin^2(\tfrac{1}{2}\theta),\end{aligned}$$

where

$$\eta=q^2/4M^2.$$

In this limit, the scattering is due predominantly to the electron's magnetic moment; and the terms proportional to k^2 dominate the cross section.

[10] The high-energy limit of the cross section with all the polarization correlations is given in J. H. Scofield, Ph.D. thesis, Indiana University, 1961 (unpublished). A few errors are contained in the expression given for the cross section.