# Nucleon Form Factors and Their Interpretation*

L. H. Chan, K. W. Chen,† J. R. Dunning, Jr., N. F. Ramsey, J. K. Walker, and Richard Wilson
*Department of Physics, Harvard University, Cambridge, Massachusetts*
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All available elastic electron-proton and quasielastic electron-neutron cross sections are analyzed assuming one-photon exchange. The model in which the form factors are caused entirely by vector mesons is discussed. A good fit cannot be obtained assuming $\rho$, $\omega$, and $\phi$ mesons alone. An acceptable fit with an additional $\rho'$ meson is discussed. Possible connections with elastic $pp$ scattering are also discussed.

## INTRODUCTION

TWO preceding papers[1,2] (hereafter called I and II) have discussed the measurements of elastic electron-proton and electron-neutron cross sections. It is the purpose of this paper to discuss these results, together with the results of others in this field, in the light of the various proposed theoretical models. There have been several recent reviews[3–5] but here we propose to discuss primarily the extent to which the preceding two papers extend our knowledge.

We will base our discussion on the use of the electric and magnetic form factors[3] $G_E$ and $G_M$ which are most convenient to describe experimental results.

## SPATIAL DISTRIBUTIONS

In one reference frame, the Breit or brick-wall frame defined by equating the energy transfer in the scattering to zero, the current operator separates into two terms proportional to $G_E$ and $G_M$ which interact with the external electric and magnetic fields, respectively. This may easily be seen by the form-factor separation:

$$J(q^2)=ieG_E(q^2)+(ie/2M)\boldsymbol{\sigma}\times\mathbf{q}G_M(q^2). \quad (1)$$

However, the transition from the laboratory frame to the Breit frame depends upon the momentum transfer and there is therefore no single well-defined Breit frame[6] which can be used independent of momentum transfer. However, the simple form of the fit of Refs. 1 and 2 to the data,

$$G_{Ep}(q^2)=\frac{G_{Mp}(q^2)}{\mu_p}=\frac{G_{Mn}(q^2)}{\mu_N}=\frac{4M^2}{q^2}\frac{G_{En}(q^2)}{\mu_N}=\left(\frac{1}{1+q^2/(18.1)^2}\right)^2,\quad q^2\text{ in F}^{-2}=\left(\frac{1}{1+q^2/(0.71)}\right)^2,\quad q^2\text{ in (BeV}/c)^2 \quad (2)$$

invites a Fourier transformation. With all the appropriate reservations therefore, we present the Fourier transform

$$\begin{aligned}\rho(r)&=2.68\exp[(-4.26\pm0.06)r]\\&\qquad(\text{electron charges})\times\text{F}^{-2},\\ \mu(r)&=7.45\exp[(-4.26\pm0.06)r]\\&\qquad(\text{nuclear magnetons})\times\text{F}^{-2},\end{aligned} \quad (3)$$

with an rms radius $(0.813\pm0.01)\times10^{-13}$ cm. Gourdin[7] has suggested that the form factors be multiplied by $(1+q^2/4M^2)$ before taking the Fourier transform. In view of the vagueness of the concepts, we have not done this.

## THE VECTOR-MESON MODEL

If there exist mesons with spin one and negative parity they will couple to the electromagnetic field. It is also assumed that they couple to the nucleons. It is then immediately possible to write down their contribution to the nucleon form factors. The vector-meson model is then such that nucleon form factors are governed entirely by these vector mesons. Clearly the model is only useful if there are a limited number of such vector mesons, and therefore, a limited number of adjustable constants.

A gamma ray does not conserve isotopic spin ($I$) but the vector mesons have definite isotopic spin (i.e., $I=0$ or 1). It is therefore often convenient to take the isotopic scalar and vector combinations of the proton and neutron form factors to discuss these resonances.

$$\begin{aligned}2G_{MV}&=G_{Mp}-G_{Mn},\\2G_{MS}&=G_{Mp}+G_{Mn}.\end{aligned} \quad (4)$$

Similar equations define the electric-vector and electric-scalar form factors.

* Supported by the Atomic Energy Commission.
† Present address: Palmer Physical Laboratory, Princeton University and Princeton-Pennsylvania Accelerator, Princeton, New Jersey.
[1] K. W. Chen *et al.*, paper I, this issue Phys. Rev. **141**, 1267 (1966).
[2] J. R. Dunning *et al.*, paper II, this issue Phys. Rev. **141**, 1286 (1966).
[3] L. N. Hand, D. G. Miller, and Richard Wilson, Rev. Mod. Phys. **35**, 335 (1963).
[4] J. Levinger and R. R. Wilson, Ann. Rev. Nucl. Sci. **14**, 164 (1964).
[5] T. A. Griffy and L. I. Schiff, in *High Energy Physics* (to be published).
[6] G. Breit, *XII International Conference on High Energy Physics, Dubna, 1964* (Atomizdat, Moscow, 1965).
[7] M. Gourdin, Nuovo Cimento **36**, 129 (1965).

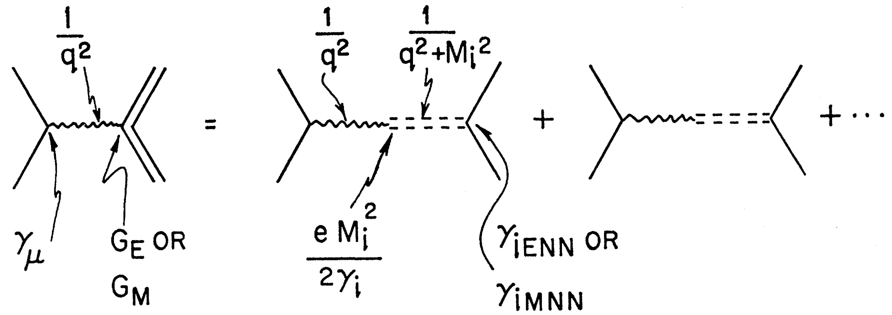


FIG. 1. Diagrams showing how electron scattering could proceed by intermediate vector-meson states. The vertex and propagator functions are shown.

We can express the form factors by the Eq. (5) following Fig. 1.

$$G_{EV}=\sum\frac{M_i^2}{M_i^2+q^2}\frac{\gamma_{iENN}}{2\gamma_i}=\sum\frac{\alpha_i}{1+q^2/M_i^2},$$
$$G_{MV}=\sum\frac{M_i^2}{M_i^2+q^2}\frac{\gamma_{iMNN}}{2\gamma_i}=\sum\frac{\beta_i}{1+q^2/M_i^2},\tag{5}$$

where the summation is taken only over $I=1$ (isovector) mesons $M_i$. Similar relations hold for $G_{ES}$ and $G_{MS}$ where the summation is taken over $I=0$ (isoscalar) mesons $M_i$.

In Eqs. (5) and Fig. 1, $(eM_i^2/2\gamma_i)$ is the coupling of the vector meson to the virtual photon and $M_i$ is the mass of the meson. The coupling of the vector meson to the nucleon is $\gamma_{iMNN}$ if helicity is transferred to the proton and $\gamma_{iENN}$ if no helicity is transferred.

It is perhaps useful to remember that Eq. (5) may be derived as a special case of a general dispersion-theory treatment of nucleon form factors. Analyticity is assumed, except over a branch cut *from* $t$ $(=-q^2)=4m_\pi^2$ to $\infty$. This assumption *may not be* valid.[8] If we assume it is, we find a dispersion relation[9]

$$G(q^2)=\frac{1}{\pi}\int_{-4m_\pi^2}^{-\infty}\frac{\mathrm{Im}G(q')^2}{q^2-(q')^2}d(q')^2\tag{6}$$

if no subtractions are needed. This reduces to Eq. (5) if we replace $\mathrm{Im}G[(q')^2]$ by $\mathrm{Im}G[(q')^2]=\alpha\pi\delta[(q')^2-M^2]$.

There is no specific recipe for taking account of the width of a resonance without a detailed model of that resonance. We estimate the effect by replacing Eq. (5) by a simple expression with the correct analytic behavior and a resonance at $q^2=-M_V^2$ with half-width $\Gamma$.

$$F(q^2)=\frac{1}{M_V^2+q^2+\Gamma^2+2\Gamma(q^2+4m_\pi^2)^{1/2}}.\tag{7}$$

For $q^2\sim M_V^2$, Eq. (7) gives a shift in the resonant mass equal to $\Gamma$ which is not large.

## VECTOR MESONS

The vector mesons now known are the $\rho$ mesons, mass $M_\rho=760$ MeV, width $\Gamma_\rho=100$ MeV, isotopic spin $I=1$; the $\omega$ meson, $M_\omega=782$ MeV, $I=0$, width $\Gamma_\omega=10$ MeV; the $\phi$ meson, $M_\phi=1020$, $\Gamma_\phi=3$ MeV, $I=0$; no others are known. They have been found in strong interactions, principally $\pi p$ scattering and are believed to contribute to nucleon-nucleon forces. Their couplings to $\gamma$ rays have not as yet been measured. Attempts to measure the decay into leptons of the $\omega$ meson result in an estimate of $10^{-4}$ of other decays,[10] but they are unable to separate $\rho$ and $\omega$ decays. The width of the $\omega$ meson is about 10 MeV, yielding a partial width for leptonic decay of about 1 keV. This partial width at once tells us $1/\gamma_\omega<\frac{1}{2}$.

Because of the absence of definitive evidence for leptonic decays we cannot say for certain that any particular vector meson contributes to nucleon form factors. In particular, Low[11] would have the coupling $1/\gamma_\omega$ anomalously reduced by the violation of the $A$ quantum number.

Recently, Lanzerotti *et al.*[12] have found that pairs of $\pi$ mesons are copiously photoproduced in the forward direction from hydrogen and coherently from nuclei. The invariant mass of the system of $\pi$-meson pairs gives a peak at the $\rho$ mass with about the right width. One interpretation is that $\rho$ mesons do indeed couple to $\gamma$ rays directly and are diffraction-scattered. The $\rho$ mesons seem to be coherently produced from nuclei in agreement with this picture. All $I=1$ mesons should exhibit a $2\pi$ decay but no other peaks were found with a mass below 1500 MeV.

Other places where these vector mesons are believed to contribute and where in principle the coupling

[8] This has been especially pointed out to one of the authors by G. Källén.

[9] S. D. Drell and F. Zachariasen, *Electromagnetic Structure of the Nucleon* (Oxford University Press, Oxford, 1961).

[10] R. A. Zdanis, L. Madansky, R. W. Kraemer, S. Hertzbach, and D. E. Strand, Phys. Rev. Letters **14**, 721 (1965). This estimate has recently been confirmed by Binnie *et al.*, Phys. Letters **18**, 348 (1965).

[11] J. B. Bronzan and F. E. Low, Phys. Rev. Letters **12**, 522 (1965).

[12] L. J. Lanzerotti *et al.*, Bull. Am. Phys. Soc. **10**, 445 (1965).

constants could be derived are

(1) $\pi^0$ decay,[13]
(2) nucleon-nucleon scattering,[14]
(3) pion-nucleon scattering.[15]

However, the values obtained from these experiments are uncertain and different authors disagree by factors of 2 or more. They seem to agree in believing that the current most certain source of information is the study of the nucleon-electromagnetic form factors—which, as we shall see, is not good.

Notwithstanding these uncertainties, we thought it worthwhile to attempt to fit the nucleon form factors by the known vector mesons plus any others that the fit might show to be necessary. We did not consider it probable that vector mesons with masses less than the $\omega$ or $\rho$ could have been missed in the various experiments, so we have assumed that any added mesons must be of higher mass.

We fit directly to the measured cross sections or ratios of cross sections since the errors on the derived form factors are correlated to each other.

## RESTRICTIONS ON FORM FACTORS

We have said that we will take the dispersion relations for $G_{E,M}(q^2)$ with no subtraction (core term).

It has long been argued,[16] that one less subtraction is needed in $F_2$ than in $F_1$. Until recently, it was suspected on experimental grounds, that a subtraction was necessary for $F_1$ and one sometimes has been used also for $F_2$.[17] When the alternative form factors $G_E$ and $G_M$ were suggested for general use,[16] it was pointed out that the subtraction properties were in fact different for the $F_1$, $F_2$ combination than for the $G_E$, $G_M$ combination.

It follows at once from the definitions

$$\begin{aligned} G_E &= F_1 - \tau K F_2\,, \\ G_M &= F_1 + K F_2\,, \\ \tau &= q^2/4M^2\,, \end{aligned} \tag{8}$$

that if $F_2$ tends to a constant as $q^2 \to \infty$, $G_E \to \infty$, and *two* subtractions are necessary. Indeed, Sachs[18] has further stressed that if ($G_E \to 0$ as $q^2 \to \infty$) then $q^2 F_2 \to 0$ as $q^2 \to \infty$. Thus the statements that $G_E \to 0$ and $G_M \to 0$ as $q^2 \to \infty$ are *more restrictive* than the corresponding statements for $F_1$ and $F_2$.

Since the choice of form factors $F_{1,2}$ or $G_{E,M}$ is just an alternative description of the data, the description in terms of $G_{E,M}$ is in some other sense *less* restrictive than that in $F_{1,2}$. Again, this is seen most simply from the defining Eq. (8). If we solve these equations for $F_{1,2}$ we find

$$\begin{aligned} F_1 &= (G_E + \tau G_M)/(1+\tau)\,, \\ KF_2 &= (G_M - G_E)/(1+\tau)\,, \end{aligned} \tag{9}$$

which show that $F_1$ and $F_2$ become infinite at $\tau = -1$ unless $G_M = G_E$. The question then arises: Should we enforce this auxiliary condition which we hereafter call the annihilation threshold condition, when we fit the resonance model to the form factors $G_{E,M}$, or can we allow $F_1$ and $F_2$ to become infinite? Other papers discuss this in detail.[19,20]

We may consider the electron-positron production of proton-antiproton pairs at threshold. Then the cross section becomes

$$[G_E^2 \sin^2\theta + \tau G_M^2 (1+\cos^2\theta)]\,. \tag{10}$$

The only states of the system compatible with the quantum numbers of the gamma ray ($1^-$) are $^3S$ and $^3D$. The former will give an isotropic angular distribution and a finite matrix element: The $^3D_1$ gives distribution with terms in $\cos^2\theta$ but the cross section varies as the square of the c.m. momentum or $-(q^2 + 4M^2)$ at threshold. The $^3S_1$ state is *not* an eigenfunction of the helicity operator ($G_E$ and $G_M$ are) but give $G_E = G_M$.

We note in passing that for electron-positron production of $\pi$ mesons, the form factor $G_M$, which corresponds to helicity change $\pm 1$, is absent. The cross section must, therefore, vary as $\sin^2\theta$ at threshold, *but* the matrix varies as the c.m. momentum. Thus $G_E = 0$ at $q^2 = -4m_\pi^2$. Therefore, as for the nucleon form factor case, $G_E = G_M$ since both are zero.

Gourdin's choice of form factors[7] $(1+q^2/4M^2)G_E(q^2)$ and $(1+q^2/4M^2)G_M(q^2)$ give still different behavior as $q^2 \to \infty$ and will be 0 at the annihilation threshold.

We therefore now believe that we *should* make the restriction $G_E = G_M$ at $q^2 = -4M^2$, but we also try to fit data *without* this restriction to see its effect. One result we may at once see: Since our data show that as $q^2 \to \infty$ $G_{Ep}(q^2)$ and $G_{En}(q^2) \to 0$, then $q^2 F_{2p}(q^2)$ and $q^2 F_{2n}(q^2) \to 0$. An observation of Eq. (5) shows at once that at least *two* resonances are needed for each of the isotopic scalar and vector form factors. If we remove the restriction that $F_2$ be finite at $q^2 - 4M^2$, one of these resonances may become the "kinematic" one at this value. Previous fits with one isovector and isoscalar resonance[17] have included hard cores.

## INPUT DATA

We have included in our fits to data, all the results of the preceding two papers (I and II) plus those of the following experiments.

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[13] M. Gell-Mann, D. Sharp, and W. G. Wagner, Phys. Rev. Letters **8**, 261 (1962).
[14] A. Scotti and D. Y. Wong, Phys. Rev. Letters **10**, 142 (1963).
[15] D. G. Miller, Phys. Rev. **127**, 1365 (1962).
[16] L. N. Hand, D. G. Miller, and Richard Wilson, Phys. Rev. Letters **8**, 110 (1962).
[17] S. Bergia, A. Stanghellini, S. Fubini, and C. Villi, Phys. Rev. Letters **6**, 367 (1961).
[18] R. G. Sachs, Phys. Rev. Letters **12**, 231 (1964).
[19] S. Bergia and L. Brown, *Nucleon Structure, Proceedings of the International Conference at Stanford University, 1963*, edited by R. Hofstadter and L. I. Schiff (Stanford University Press, Stanford, California, 1964).
[20] V. Barger and R. Carhart, Phys. Rev. **136**, B281 (1964).

### Stanford Results

(1) Ninety-three electron-proton scattering cross sections measured by Janssens[21] from $q^2=0.16$ to 1.2 $(\mathrm{BeV}/c)^2$.

(2) Thirty-three ratios of neutron-to-proton scattering cross sections measured by Hughes *et al.*,[22] from $q^2=0.29$ to 1.36 $(\mathrm{BeV}/c)^2$. The measurements at $q^2=0.18$ $(\mathrm{BeV}/c)^2$ and below were omitted because of theoretical uncertainties.

### Stanford and Harvard Results

(3) Eight electron-proton scattering cross sections from $q^2=0.012$ to 0.09 $(\mathrm{BeV}/c)^2$ by Drickey and Hand.[23]

### Cornell Results

(4) Twenty-one electron-proton scattering cross sections of Berkelman *et al.*[24] The values of these cross sections have been increased by 3% to allow for differences in calibration between the Cornell and Harvard quantameters. All the errors were multiplied by 1.5 to compensate for the fact that the average $\chi^2$ per point was otherwise twice that of the other data.

(5) Ten ratios of neutron-to-proton scattering cross sections measured by Akerlof *et al.*,[25] from $q^2-0.43$ to 1.36 $(\mathrm{BeV}/c)^2$. Errors were multiplied by 1.5.

### Orsay Results

(6) Twenty-three *e-p* scattering cross sections measured by Lehmann *et al.*,[26] from $q^2=0.06$ to 0.4 $(\mathrm{BeV}/c)^2$.

### Columbia and Argonne Results

(7) Measurements of the electron-neutron interaction near $q^2=0$. We have used a new accurate value for this measured at Argonne by Krohn and Ringo.[27] This value ($dG_{En}/dq^2=+0.0178\pm0.0009$ $F^2$ at $q^2=0$) is 20% smaller than the average result given by Melkonian *et al.*[28] This change affects our conclusions only slightly.

### Harvard Results

(8) Seven electron-proton cross sections from an earlier paper[29] as amended in I.

[21] T. Janssens, thesis, Stanford University, 1964 (unpublished).
[22] E. B. Hughes, T. A. Griffy, M. R. Yearian, and R. Hofstadter, Phys. Rev. **139**, B458 (1965).
[23] D. Drickey and L. N. Hand, Phys. Rev. Letters **9**, 521 (1962).
[24] K. Berkelman, M. Feldman, R. M. Littauer, G. Rouse, and R. R. Wilson, Phys. Rev. **130**, 2061 (1963).
[25] C. W. Akerlof, K. Berkelman, G. Rouse, and M. Tigner, Phys. Rev. **135**, B810 (1964).
[26] P. Lehmann, R. Taylor, and Richard Wilson, Phys. Rev. **126**, 1183 (1962). P. Lehmann, *XII International Conference on High Energy Physics, Dubna, 1964* (Atomizdat, Moscow, 1965). The work of B. Dudelzak, G. Sauvage, and P. Lehmann, Nuovo Cimento **28**, 18 (1963) was omitted by accident. Our results are consistent with their form factors.
[27] V. E. Krohn and G. R. Ringo, Phys. Letters **18**, 297 (1965).
[28] E. Melkonian, B. M. Rustad, and W. W. Havens, Phys. Rev. **114**, 1571 (1959).
[29] J. R. Dunning, K. W. Chen, N. F. Ramsey, J. R. Rees, W. Shlaer, J. K. Walker, and Richard Wilson, Phys. Rev. Letters **10**, 500 (1963).

TABLE I. Limiting values of the form factors.

| $q^2$ | Form factor | Normalized value | Reference |
|---|---|---|---|
| 0 | $G_{Ep}$ | 1 | |
| 0 | $G_{Mp}$ | 2.792 | |
| 0 | $G_{En}$ | 0 | |
| 0 | $G_{Mn}$ | 1.914 | |
| 0 | $dG_{Ep}/dq^2$ | $-2.95$ $(\mathrm{BeV}/c)^{-2}$ | 21, 23, 26 |
| 0 | $dG_{Mp}/dq^2$ | $-7.98$ $(\mathrm{BeV}/c)^{-2}$ | 21, 23, 26 |
| 0 | $dG_{En}/dq^2$ | $+0.454$ $(\mathrm{BeV}/c)^{-2}$ | 27, 28 |
| 0 | $dG_{Mn}/dq^2$ | $+5.61$ $(\mathrm{BeV}/c)^{-2}$ | 22 |
| $\infty$ | $G_{Ep}$ | 0 | 1 |
| $\infty$ | $G_{Mp}$ | 0 | 1 |
| $\infty$ | $G_{En}$ | 0 | 2 |
| $\infty$ | $G_{Mn}$ | 0 | 2 |
| $-4M^2$ | $G_{Ep}$ | $G_{Mp}$ | 19, 20 (theory) |
| $-4M^2$ | $G_{En}$ | $G_{Mn}$ | 19, 20 (theory) |
| $-7M^2$ | $G_{Ep}$ | $<1$ | 30 |
| $-7M^2$ | $G_{Mp}$ | $<1$ | 30 |

An upper limit on the form factors in the time-like region could be derived from an experiment on

$$p+p\to e^+ + e^-,$$

or

$$p+p\to \mu^+ + \mu^-. \tag{11}$$

The experiments[30] are not definitive on this, but provide upper limits for the form factors

$$G_{Ep}(q^2)<1\,,\quad G_{Mp}(q^2)<1\,, \tag{12}$$

at $q^2=-7M^2$.

We do not include these data for our computer adjustments, but our final fits satisfy these inequalities. Other published data are considered to be either superseded or too inaccurate to contribute usefully.

We include all these points as separate independent data points. Of course, there are errors systematic for a set of points from one laboratory. It is hard to include these. In particular the systematic errors of normalization of Janssens' cross sections are *not* reduced by a factor of $\sqrt{(93)}\simeq 9.4$ by including 93 points. Thus our $\chi^2$ values are higher than they should be because we do not take correct account of the differences between laboratories. We see the effect on the final fit by varying the normalization of, for example, the Janssens'[21] data and readjusting the fit. This modification changes the over-all $\chi^2$ by 12%, but the parameters at the fit shift less than this. We have not done this in the final analysis.

In particular, we note the following important assumptions already mentioned in papers I and II. We assume that no two-photon exchange terms appear in the cross section in agreement with rough theoretical ideas, but neglect, as barely significant, a disquieting

[30] A. Zichichi (private communication). More recent experiments from Zichichi (CERN) and Brookhaven both suggest $G_{Ep}<\frac{1}{10}$, $G_{Mp}<\frac{1}{10}$. This is in agreement with our four pole fits.

TABLE II. Summary of fits.

| Fit | Annihilation threshold constraint | Core terms | $\chi_p^2$ per point | $\chi_p^2$ per point | $\chi^2$ total per point | Remarks |
|---|---|---|---|---|---|---|
| (1) Eq. (2) | No | No | 2.6 | 1.66 | 2.35 | One-parameter model |
| (2) three-pole | No | No | 20.6 | 8.6 | 11.2 | $M_\rho=510$ MeV |
| (3) three-pole | Yes | Yes | 2.28 | 1.63 | 2.09 | $M_\rho=540$ MeV |
| (4) four-pole | Yes | No | 1.69 | 1.80 | 1.70 | $M_{\rho'}=875$ MeV |
| (5) four-pole | No | No | 1.67 | 1.39 | 1.60 | $M_{\rho'}=875$ MeV |

Parameters of these fits

| Fit No. | $I=1$ $\alpha_1$ | $\alpha_2$ | $I=0$ $\alpha_3$ | $\alpha_4$ | $I=1$ $B_1$ | $\beta_2$ | $I=0$ $\beta_3$ | $\beta_4$ | $I=1$ $M_1$ | $M_2$ | $I=0$ $M_3$ | $M_4$ |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 2 | 0.5 | 0.0 | 1.477 | −0.977 | 2.353 | 0.0 | 1.612 | −1.172 | 510 | ⋯ | 782 | 1020 |
| 3 | 0.525 | 0.0 | 1.090 | −0.555 | 2.471 | 0.0 | 1.060 | −0.594 | 540 | ⋯ | 782 | 1020 |
| (cores) | −0.025 | | −0.035 | | −0.118 | | −0.031 | | | | | |
| 4 | 2.347 | −1.847 | 1.214 | −0.714 | 8.268 | −5.915 | 1.093 | −0.653 | 760 | 875 | 782 | 1020 |
| 5 | 2.303 | −1.803 | 1.184 | −0.684 | 8.164 | −5.811 | 1.18 | −0.741 | 760 | 875 | 782 | 1020 |

result from Stanford,[31] which gives the ratio of electron-to-positron scattering as $0.93\pm0.03$ at 0.73 $(\text{BeV}/c)^2$ and 90°. However, if we were to take this data at its face value, and assume also that the deviation from first Born approximation varies as $q^2$, we could still conclude that the data cannot be understood using $\rho$, $\omega$, and $\varphi$ resonances alone.

Secondly, we assume that the electron-deuteron elastic-scattering cross section,[23,32] combined with its analysis using the nonrelativistic deuteron wave function derived from nucleon-nucleon scattering experiments, is incorrect in spite of the accuracy of both sets of experiments. These experiments, if taken at their face value, suggest that $G_{En}=0$ out to $q^2=0.25$ $(\text{BeV}/c)^2$, in contrast to the precise value from the neutron-electron interaction[27,28]

$$dG_{En}/dq^2=0.0178\pm0.0009 \text{ F}^2. \tag{13}$$

Unless we assume that $d^2G_{En}/(dq^2)^2$ is very large near the origin, we must reject one set of data and we choose

TABLE III. Individual contributions to $\chi^2$ from the four best fits.

| Fit No. / Author | Number of points | 1 | 3 | 4 | 5 |
|---|---|---|---|---|---|
| | | $\chi^2$ per point | | | |
| Proton data: | | | | | |
| Janssens | (93) | 3.03 | 1.53 | 1.57 | 1.61 |
| Dunning | (7) | 1.74 | 1.35 | 3.52 | 3.16 |
| Hand | (8) | 1.91 | 4.64 | 3.11 | 2.71 |
| Chen | (19) | 2.43 | 5.9 | 1.52 | 1.35 |
| Lehmann | (23) | 3.2 | 1.70 | 1.18 | 1.27 |
| Berkelman | (21) | 2.1 | 2.26 | 1.77 | 1.74 |
| Neutron data: | | | | | |
| Dunning | (10) | 2.0 | 2.93 | 2.81 | 2.06 |
| Hughes | (33) | 1.7 | 1.37 | 1.76 | 1.43 |
| Akerlof | (10) | 0.8 | 0.85 | 0.72 | 0.62 |

[31] A. Browman, F. Liu, and C. Schaerf, Phys. Rev. Letters 12, 183 (1964).

[32] D. Benakas, D. Drickey, and D. Frèrejacque, Phys. Rev. Letters 13, 824 (1964).

to reject the elastic electron-deuteron scattering on the ground that the theory may be inadequate. Halpern,[33] in a recent paper, has thrown doubt on the interpretation of the neutron-electron interaction experiments. The difference in any case makes only a small effect on the conclusions of this paper.

It is possible to summarize the results of these experiments in several different ways. Equation (1) forms a useful summary. Another is to tabulate, as shown in Table I, the limiting values and slopes at $q^2=0$ and $\infty$. These values together with the assumption, supported in general by experiment, that the form factors vary smoothly with four-momentum transfer, enable us to obtain the parameters of any reasonable phenomenological fit.

The values at $q^2=\infty$ allow us to neglect a "core" term in the resonance model. Neglecting the last two upper limits,[30] we then have ten additional constraints. In practice, only the value of $dG_{En}/dq^2$ at $q^2=0$ was used as input data. Thus our fitting procedure imposes seven constraints. For each resonance we have two variables (one each for $G_E$ and $G_M$) if we assume the mass fixed and another if we allow the mass to vary. The four-meson fit discussed below with one variable mass (the $\rho'$ meson) gives nine variables. It is noteworthy that we manage a moderate fit.

## INCONSISTENCIES AMONG THE DATA

The quality of a particular fit depends to a degree on the input data used. For example, Hughes *et al.*[22] manage to fit theirs and Janssens' *et al.*[21] data with three mesons plus cores with an average $\chi^2$ per point of 1.1. We confirm their arithmetic. However, when the set of data listed above is included, the average $\chi^2$ rises to 1.7 and part of this $\chi^2$ is provided by the Janssens' and Hughes' points. The parameters of the fit are changed only slightly (see Table II, fit No. 3). It is clear, therefore, that either the data of different

[33] O. Halpern, Phys. Rev. 133, B579 (1964).

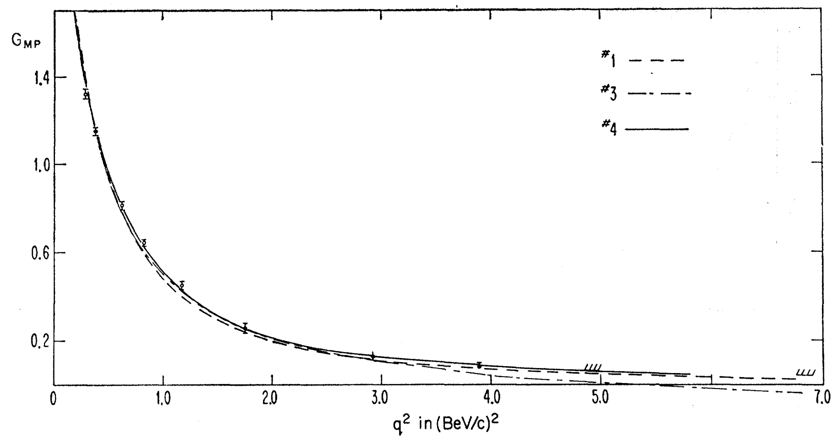


FIG. 2. The form factors $G_{Mp}$ separated as in I compared with fits to the data. The numbers refer to Table II.

laboratories are inconsistent, or that form factors vary more erratically than we have assumed. The same phenomenon was observed with other analytic fits discussed below.

An examination of the individual $\chi^2$ values tells us the following inconsistencies with either three or four pole fits:

(1) Janssens' *e-p* data[21] are consistently low at backward angles compared with the fit; yet, Lehmann's data[26] fit well and Hand's data[23] are high. This is a systematic difference in angular distributions.

(2) The early Dunning[26] data are high compared with the fit for very small scattering angles. This is not a region which overlaps other data and may be a real (2-photon exchange) effect.

(3) Berkelman's[24] 90° points are high and 145° points are low. They contribute significantly to the $\chi^2$ even after increasing the errors as discussed earlier.

(4) Hughes' neutron data[22] contribute significantly when we insist (as we insist in II) that electron-neutron scattering has a low cross section at high $q^2$.

## FITS TO THE RESONANCE MODEL

Table II gives the parameters of the various fits we have tried, using the notation of Eq. (5). Table III lists the $\chi^2$ values and the contributions from the various data groups. The fits are dominated by the proton data at momentum transfers below 0.5 $(\text{BeV}/c)^2$ which are the most numerous and most precise.

Figures 2, 3, 4, and 5 show how well these parameters fit the form factors, separated as in paper I. At low momentum transfers more points exist, but these are omitted to avoid confusion. In observing this, it should be remembered that the computer minimized $\chi^2$ for a fit to the cross sections, so that the errors in form factor separation do not appear in the $\chi^2$ plot. Figures 6 and 7 show the fit to the cross sections at 31° for *e-p* and *e-n* scattering.

Fit 1 is that of Eq. (2) which has the attractive merit of simplicity. It corresponds approximately to two resonances (poles) very near to each other with opposite signs of coupling. We call it the "dipole" fit.

Fit 2 is a three-meson fit without cores and without the annihilation constraint. This also is not good. Figure 8 shows the variation of $\chi^2$ with the isovector mass.

Fit 3 has been discussed by Hughes[22] who found it was a good fit to his and Janssens'[21] data. It is a fit with isoscalar poles at the $\omega$ and $\varphi$ masses and a single isovector pole at an adjustable $\rho$ mass with hard core terms (poles at $M=\infty$). Hughes found an average $\chi^2$

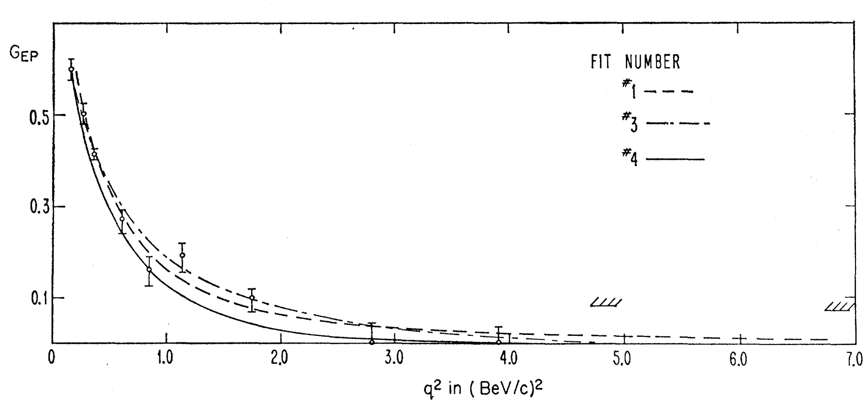


FIG. 3. The form factors $G_{Ep}$ separated in I compared with fits to the data.

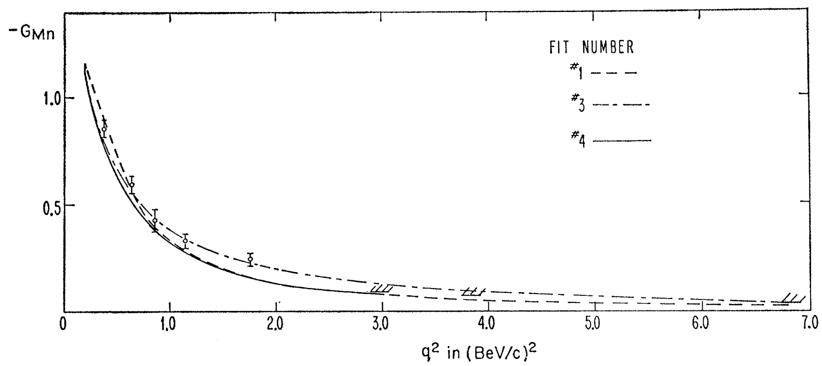


FIG. 4. The form factors $G_{Mn}$ separated in II compared with fits to the data.

of 1.1 per point. When we try to fit the complete set of data, however, the $\chi^2$ rises because of the inconsistencies between laboratories noted above, and also by a systematic failure to fit the high momentum transfer points. In appraising this failure it must be noted that the core terms, although small, are necessary to the fit. The difference between fits No. 2 and No. 3 show this. One cannot reduce one core to zero without incurring correspondingly large changes in the remaining cores. The presence of these cores causes the proton magnetic form factor to go through zero at some four-momentum transfer. For fit No. 3 this occurs near our point at $q^2=4.86$ $(\mathrm{BeV}/c)^2$. Here the fit is an order of magnitude too low (see Fig. 6). This discrepancy persists to $q^2=6.89$ $(\mathrm{BeV}/c)^2$ where the fit is a factor of 3 too low. Further, the fit is correspondingly a factor of 2 high for the $q^2=3.89$ and 2.75 $(\mathrm{BeV}/c)^2$ neutron data (Fig. 7). The fitting procedure uses linearized errors. Thus the $\chi^2$ associated with such large discrepancies do not appear as large as they otherwise would.

Fit 4 is a four-meson fit without cores and including the annihilation threshold constraint. Three meson masses are held at the $\omega$, $\varphi$, and $\rho$ masses and the mass of the second isovector meson ($\rho'$) is allowed to vary. Figure 9 shows how $\chi^2$ varies. Releasing the annihilation threshold constraint (fit No. 5) reduces $\chi^2$ somewhat but still does not give a good fit.

We present as our "best" fit the four-pole resonance fit without core terms. The data of Papers I and II tend to exclude core terms and therefore this is the minimum acceptable fit. The improvement of fit by the addition of extra parameters by including cores or releasing the annihilation threshold constraint is slight and we cannot regard their necessity as established. The relationship $G_{Ep}=G_{Mp}/\mu_p$ is not particularly well satisfied by this fit at momentum transfers above 0.5 $(\mathrm{BeV}/c)^2$. Below this $q^2$ the relationship is satisfied with increasing precision.

The four-pole fit is very analogous to the dipole fit (fit No. 1) for each of the isoscalar ($I=0$) and isovector ($J=1$) form factors; there are two resonances (poles) with opposite signs of coupling, with masses on either side of the "dipole" mass $\sqrt{0.71}$ $\mathrm{BeV}^2=0.84$ BeV. Clearly if we were to choose a lower value than the "correct" one, 760 MeV, for the $\rho$ mass, we could find a higher value than 890 MeV for the $\rho'$ mass. As seen from Fig. 9, the mass of the $\rho'$ meson is not well determined; Hughes[22] finds a higher value (1100 MeV). We believe that this is due to his adjustment to a limited set of data, particularly, his exclusion of the Orsay data.[26]

It is clear that the nucleon form factors cannot be understood on the simple resonance model, with only $\omega$, $\varphi$, and $\rho$ resonances. In order to obtain even an

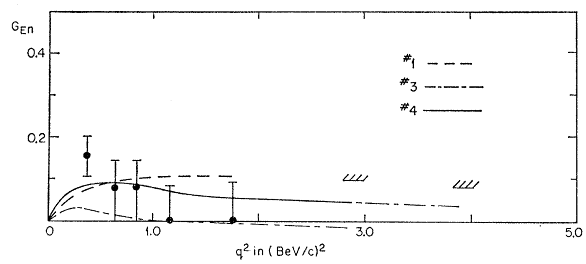


FIG. 5. The form factors $G_{En}$ separated in II compared with fits to the data. Note that the positive sign has been assumed near $q^2=0$.

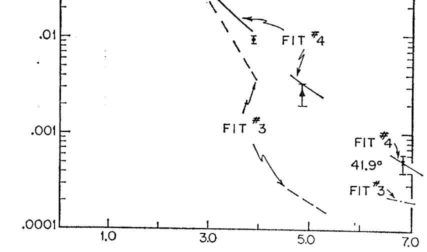


FIG. 6. The $e$-$p$ cross section at 31° compared with the various fits to the data.

approximate fit, we must postulate the existence of a $\rho'$ meson, at a mass where it is hard to see why it has not been found. In particular it should probably be photoproduced as easily as the $\rho$ meson, if it couples to the virtual $\gamma$ ray and is strongly interacting. Recent data on photoproduction[12] at 6 BeV show no sign of it. Moreover, Low's $A$ quantum number[11] forbids $\varphi\gamma$ coupling, which is necessary for our scheme.

One possible, *ad hoc*, way out of the problem of the $\rho'$ meson is to assume that the only isovector meson is the $\rho$ meson, but that the $\rho$ meson couplings to the nucleon $\gamma_{\rho ENN}$ and $\gamma_{\rho MNN}$ are not constants but vary with four-momentum transfer in such a way as to explain the data. While this may be true, it is unsatisfactory because it creates a new pair of functions of momentum transfer which are as arbitrary as the form factors themselves.

The large value of $\chi^2$ (for our best fit) is attributable to many points. We believe that it is primarily due to the inconsistencies of the various data.

## LEVINGER-PEIERLS MODEL

There have been attempts[34–36] to derive by an extrapolation method, the form factors in the timelike region ($t>0$, $q^2<0$) from those in the space-like region ($t<0$, $q^2>0$). These attempts have all suffered from the inadequate precision of the data. They show a large imaginary part of the form factor near or below the $\rho$ or $\omega$ mass reversing in sign above this mass. This is reassuring but does not give details. It is clear that in order to say more about the form factors than can be said from the Levinger-Peierls approach, we must utilize some of the existing information about these vector-meson resonances as initial constraints, which is what is done in this paper.

## RELATION TO $p$-$p$ SCATTERING

The size of the proton as measured by electron scattering is given by the first term in the expansion of the form factor

$$G(q^2)=1-\tfrac{1}{6}\langle r^2\rangle q^2\,, \tag{14}$$

where $\sqrt{\langle r^2\rangle}=0.813$ F, for both electric and magnetic form factors $G_E$ and $G_M$. It is interesting to note that this is close to the size of the proton from $p$-$p$ scattering

$$\sigma(\theta)/\sigma(0)=1-(2/6)\langle r^2\rangle q^2\,. \tag{15}$$

The factor 2 is put on to allow for the mutual interactions of two meson clouds. We see from $p$-$p$ scattering data that

$$\sqrt{\langle r^2\rangle}=0.8-1.0\text{ F}\,. \tag{16}$$

The near equality of the rms radii derived from electron-nucleon and nucleon-nucleon scattering has been observed before.

[34] J. S. Levinger and R. F. Peierls, Phys. Rev. **134**, B1341 (1964).
[35] J. S. Levinger and C. P. Wang, Phys. Rev. **136**, B733 (1964).
[36] R. A. Weiner, Harvard University (unpublished).

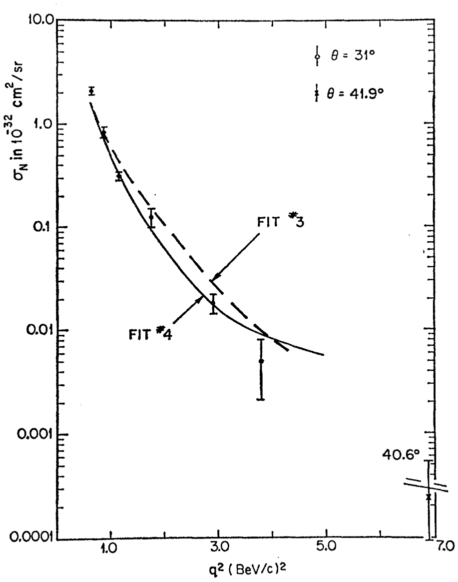


FIG. 7. The $e$-$n$ cross section at 31° compared with the various fits to the data.

We now turn to a conjecture of Wu and Yang.[37] They note that at large momentum transfers $p$-$p$ scattering may be expressed as

$$\sigma(\theta)/\sigma(0)=\exp(-P_\perp/P_0)\,, \tag{17}$$

where $P_\perp$ is the transverse momentum and $P_0=0.15$ BeV/$c$. They guess that for $e$-$p$ scattering a similar relation holds for the form factors

$$G_{Ep}(q^2)=G_{Mp}(q^2)/\mu_p=\exp[-(q^2)^{1/2}/0.6]\,,\quad q^2\text{ in (BeV}/c)^2\,, \tag{18}$$

which is valid at large momentum transfers only.

The fit to the form factors $G_{Ep}$ and $G_{Mp}$ is shown in Fig. 10. The model is not expected to fit near $q^2=0$ and,

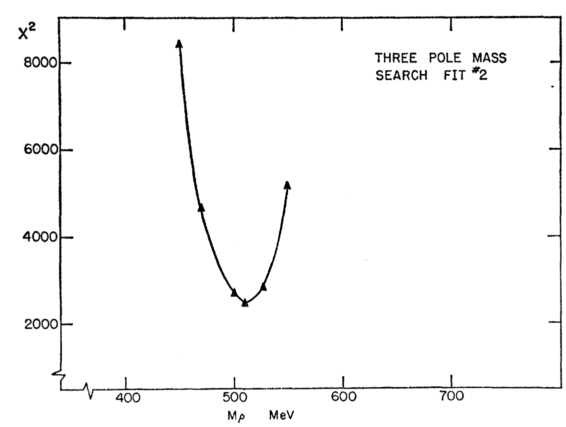


FIG. 8. The variation of $\chi^2$ with the isovector meson ($\rho$) mass for a three-pole fit with cores.

[37] T. T. Wu and C. N. Yang, Phys. Rev. **137**, B708 (1965).

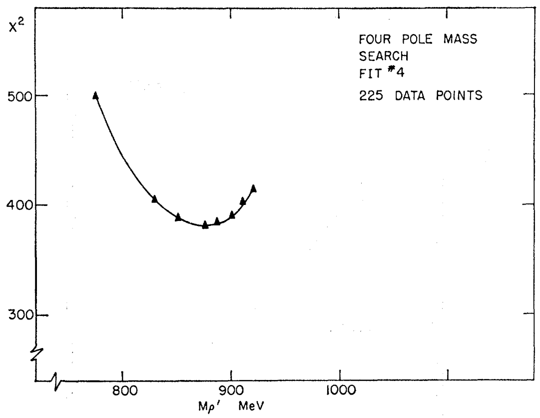


FIG. 9. The variation of $\chi^2$ with the second isovector meson ($\rho'$) mass for a four-pole fit with no cores.

indeed, the analyticity properties are not quite right. $G(q^2)$ should have no imaginary part in the region $-4m_\pi^2<q^2<0$ in contrast to Eq. (18). We could make the analytic properties work out correctly by writing

$$G_{Ep}(q^2)=\exp[-(q^2+4m_\pi^2)^{1/2}/0.6]/\exp(-2m_\pi/0.6). \quad (18a)$$

It fits quite well at higher momentum transfers though there is a tendency for the model to fall *faster* than the data. This is accentuated if we insist that $G_{Mp}=\mu_p G_{Ep}$ and therefore have no error in the form-factor separation.

The formula (18) gives an oscillating behavior for the form factor in the time-like region; this is equivalent to choosing many vector resonances whose coupling constants have the same signs but whose magnitude alternates as a function of $q^2$.

Another approach can be based on the analysis of the $p$-$p$ scattering by Serber.[38] Serber notes that the amplitude is mainly imaginary and that therefore many cancellations can occur. He finds that the major features of the experiment can be summarized by assuming an imaginary interaction

$$V(r)=i\hbar c/r\exp(-\lambda r),\quad \lambda=2.12\ \mathrm{F}^{-1}. \quad (19)$$

We may now guess at the consequences of this approach for $e$-$p$ scattering. We assume that this imaginary potential is caused by a cloud of strongly interacting particles, interacting at short range and that most of these particles have a charge; then we might attribute a charge and magnetic moment distribution equal to the potential.

$$\rho(r)=\exp(-\lambda r)/r, \quad (20)$$

$$G(q^2)=\lambda^2/(q^2+\lambda^2). \quad (21)$$

Again, however, we must allow for the fact that in $p$-$p$ scattering there are two interacting clouds and that the form factor of Eq. (21) is the fold of the two nucleon form factors. We must thus take the square root of Eq. (21) to obtain

$$G_{Mp}(q^2)/\mu_p=G_{Ep}(q^2)=\left(\frac{\lambda^2}{q^2+\lambda^2}\right)^{1/2}=1-\tfrac{1}{2}q^2/\lambda^2+\cdots, \quad (22)$$

which gives a rms radius of 0.67 $\mathrm{F}^{-1}$ which is about right but falls off too slowly at high momentum transfer. This is shown in Fig. 10.

The difference in the behavior at high momentum transfers from the idea of Wu and Yang is that Serber attributes the cancellations giving the small $p$-$p$ cross section at high momentum transfer entirely to the phase which is imaginary; these cancellations do not then occur in $e$-$p$ scattering which has a real amplitude.

It is clear that this connection is still confused and that more precise data may lead to further understanding.

## INTERNAL SYMMETRIES

There have been some interesting recent attempts[38,39,40] to relate the form factors using the internal symmetries. These symmetries seem to predict the relations between the various form factors

$$G_{Mp}(q^2)/\mu_p=G_{Mn}(q^2)/\mu_n=G_{Ep}(q^2),\quad G_{En}(q^2)=0, \quad (23)$$

which we found experimentally to be reasonable approximations to the data.

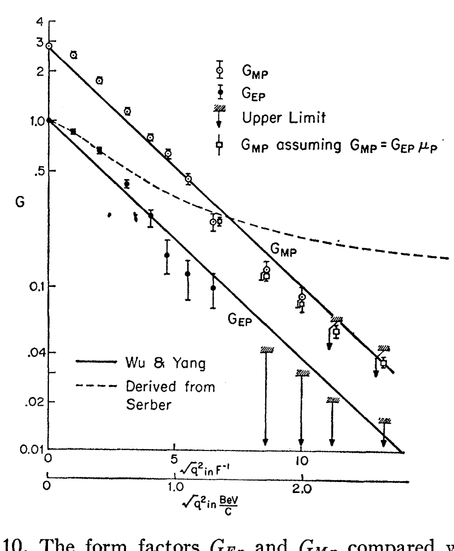


FIG. 10. The form factors $G_{Ep}$ and $G_{Mp}$ compared with the simple model of Wu and Yang [Eq. (18)] and a model derived from Serber's fit to $p$-$p$ scattering [Eq. (22)].

[38] R. Serber, Phys. Rev. Letters **10**, 357 (1963).

[39] K. J. Barnes, P. Carruthers, and F. Hipple, Phys. Rev. Letters **14**, 82 (1965).

[40] N. N. Bogoliubov, Nguyen Van Hieu, D. S. Stoyanov, B. V. Struminsky, A. N. Tavkhelidze, and V. P. Shelest, Dubna Report D2075, 1965 (unpublished).

## FORM FACTORS IN THE TIME-LIKE REGION

The only measurements of form factors in the time-like region give upper limits of $G_{Ep}$ and $G_{Mp}<1$.[30] It is of interest to see what predictions we can safely make in order to guide further experiments. In particular, experiments with colliding beams,

$$e^{+}+e^{-}\rightarrow p+\bar{p}\,, \tag{24}$$

have attracted recent attention. The experiments are likely to be limited by a low counting rate and small form factors will make them impossible.

Firstly, let us assume that the ideas of Wu and Yang are approximately correct. Then the form factors in the time-like region become

$$F(q^2)=\exp[i(-q^2)^{1/2}/0.6] \tag{25}$$

and satisfy $F^{2(0)}=1$ and have oscilllating phase. This then corresponds to many resonances spaced equally with a mass separation of 1900 MeV, with equal magnitude but alternating sign of coupling. Clearly, we have an average counting rate for strong-interaction experiments equal to that for particles with a form factor equal to the value for $q^2=0$. In particular, for recently discussed storage rings for collisions of 3-BeV electrons and positrons with a luminosity $L=$ (counting rate)/(cross section) equal to $10^{30}$ sec$^{-1}$ cm$^{-2}$ we find $GP^2=(2.79)^2$ and a counting rate of one count per minute.

On the other hand we may take our best four-pole fit number 4 of Table II. At $q^2=-860$ F$^{-2}$ (3-BeV storage rings) we find $GP_p=0.022$, $G_{Ep}=0.006$, and a count rate $4\times10^{-4}$ of that of particles with no form factor. This particular fit has cancellations between the terms which lead to a very rapid fall off of the form factor with increasing momentum transfer.

This range, $10^4$, should clearly be narrowed by experiment and one awaits anxiously definite results from experiments of the type of Zichichi *et al.*[26]

In experiments with a storage ring there will also be produced pairs of all known baryons and "mixed" pairs such as

$$e^{+}+e^{-}\rightarrow \bar{N}+N^{*}\,,$$

where the pairs have the same spin and parity but may differ in isotopic spin by 1. In the space-like region we know that the cross section for the reaction

$$e+p\rightarrow e+N^{*}$$

is comparable to the elastic-scattering cross section.[41] Also symmetry theories predict that the form factors of the baryons are equal to each other within small numerical factors. We can then find 200 possible pairs of baryons using known particles. The over-all counting rate for a storage ring will then be 200 times the previous figures.

## ACKNOWLEDGMENTS

We are grateful to other workers in the field, Dr. Hughes, Dr. Janssens, and Dr. Lehmann, for sending us their data and discussing their errors before publication.

[41] A. A. Cone, K. W. Chen, J. R. Dunning, Jr., G. Hartwig, N. F. Ramsey, J. K. Walker, and Richard Wilson, Phys. Rev. Letters **14**, 326 (1965).