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Modulations of Generically Related Galactic Cosmic Rays

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A method of analysis of the spectra of generically related cosmic-ray species is formulated. The effects of various spatial modulations between the cosmic-ray sources and the earth are separated from the effect of fragmentation in interstellar space. Predictions are made on how the ratios of the intensities of related cosmic-ray species, for both rigidity and energy/nucleon intervals, would be expected to vary with geomagnetic latitude, solar activity, and energy. These predictions are compared with the experimental results. Variations in the reported abundance ratios of helium isotopes and the light ($3\leq Z\leq 5$) to medium ($6\leq Z\leq 9$) elements can be explained in terms of solar and geomagnetic modulation, rather than by invoking the hypothesis of longer interstellar path lengths for low-energy cosmic rays. At a geomagnetic latitude of 55°, on 21 April 1961, the relative transmission of He^3 to He^4 through interplanetary and geomagnetic fields in the energy/nucleon interval 250–350 MeV/nucleon is found to be 0.38 ± 0.2. Statistically limited data favor a rigidity-dependent rather than velocity-dependent solar modulation above a rigidity of 1.2 BV. The local interstellar intensity of cosmic-ray helium (i.e., before solar modulation) at 1.7 BV is 0.6 ± 0.4 of that at 1.3 BV. Assuming the differential rigidity (R) spectrum of local interstellar helium to vary as $1/R^q$, the exponent q is found to be 1.8 ± 2 in the rigidity interval 1.2–1.7 BV. The result suggests that considerable solar modulation of the intensity of low-energy cosmic rays may occur even at solar minimum. Based upon the limited available data, the mean interstellar path length for cosmic rays in the region 200–400 MeV/nucleon is found to be 5 ± 2 g/cm^2. The values of the mean path length of cosmic rays and the exponent of the local interstellar helium spectrum are consistent with the values obtained for higher energy cosmic rays. The effects of modulations occurring in the cosmic-ray source regions are also explored.

I. INTRODUCTION

WHEN a cosmic-ray nucleus with charge $Z\geq 2$ interacts with interstellar matter it frequently fragments into two or more daughter species, establishing a generic relationship between the intensity spectra of parent and daughter species. The observed spectra of cosmic-ray species, having local interstellar intensity spectra which are generically related in this manner, are analyzed in terms of the effects of additional spatial modulations between local interstellar space and earth. In particular, the energy spectrum of He^3 is related to that of He^4 on the basis of interstellar fragmentation of He^4 and the spectrum of the fragmented light elements $L(3\leq Z\leq 5)$ is similarly related to the spectra of the medium elements $M(6\leq Z\leq 9)$ and heavy elements $H(Z\geq 10)$. The utility of the generic relationship arises from the fact that the daughter products retain approximately the same velocity as their parents upon fragmentation (hence the same energy per nucleon), but undergo a considerable systematic change in the mass-to-charge ratio and thus a systematic change in rigidity. Differences in the observed energy/nucleon spectra of parent and daughter particles can be interpreted in terms of differences in particle transmission through the fields traversed between local interstellar space and earth. In the case of He^4 fragmenting into He^3 the reduction in rigidity is 25%, and in the case of M fragmenting into L the rigidity on the average increases approximately 10%.

In the low-rigidity region emphasized herein (1–2 BV), the relative abundances of cosmic-ray species observed at the earth are affected not only by fragmentation in the interstellar space but also by transmission of particles through interstellar, interplanetary, and geomagnetic fields. The degree of transmission of daughter products and parent species through the geomagnetic field will differ since the transmission is rigidity dependent. Similarly, should the transmission through interplanetary fields have a significant rigidity dependence then the parent and daughter species having the same value of energy/nucleon will undergo different modifications of intensity. Previous determinations of the mean interstellar path length from generically related low-energy cosmic rays have not considered the effects of transmission.[1–8] The increase in the intensity of low-rigidity particles with increasing geomagnetic latitude is well established.[9–11] The details of the change in the intensity of low-rigidity particles with the 11-year variation in solar activity are in the study stage. The transmission of cosmic rays is considered to be affected by a change in interplanetary fields associated with the 11-year variation in solar

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emission.[12–20] The effect of the interplanetary medium on the ratio of daughter-to-parent species has been considered by Appa Rao.[21] We do not consider cosmic-ray variations occuring in short time intervals such as those due to magnetic storms, Forbush decreases, and the 27-day effects. The nature of interstellar fields, and their effects on cosmic rays are still largely speculative.[22–24]

Recently the NASA[25–27] and Minnesota[28,29] groups have reported on the differences in the low-rigidity spectra between the hydrogen and helium components of primary cosmic rays. From a study of these spectra alone, it is difficult to establish to what extent these differences in the spectra are due to the effects of interplanetary fields or differences in the local interstellar spectra of these elements.

A method of analysis is given in Sec. II which separates the effects of production and modulation in cosmic-ray source regions,[30–38] fragmentation,[30–34,39–44] and transmission. The differences in the abundance ratios of related cosmic-ray species are interpreted in terms of transmission through geomagnetic, interplanetary, and interstellar fields. Calculations of the variations of the relative abundance ratios are made on the basis of observed modulations of the parent spectra. The method is applied to existing experimental data in Sec. III and yields qualitative and quantitative information on geomagnetic and solar modulation, on mean interstellar path length, and on the local interstellar spectrum of He^4. The conclusions are summarized in Sec. IV.

II. ANALYSIS

A. Notation

ϵ = Energy/nucleon (velocity parameter).

R = Rigidity (momentum/charge).

p = Parent species.

s = Secondary species (i.e., daughter species).

R_p = Rigidity of parent species.

R_s = Rigidity of daughter species, whose parent species had rigidity R_p.

$K_{sp}=R_s/R_p$.

$\tilde{S}_i(\epsilon)$ = Intensity per unit energy/nucleon interval of species i, at an energy/nucleon ϵ, in local interstellar space.

$S_i(R)$ = Intensity per unit rigidity interval of species i, at a rigidity R, in local interstellar space.

$\tilde{O}_i(\epsilon)$ = Intensity per unit energy/nucleon interval of species i at an energy/nucleon ϵ, observable at the top of the earth's atmosphere.

$O_i(R)$ = Intensity per unit rigidity interval of species i, at a rigidity R, observable at the top of the earth's atmosphere.

$T_i(R)=O_i(R)/S_i(R)$ = The transmission fraction of species i between local interstellar space and earth, i.e., the fraction of particles of species i that are transmitted through the intervening fields.

$T^\lambda(R)$ = The transmission fraction through the geomagnetic field.

$T_i^I(R)$ = The transmission fraction of species i through the interplanetary field.

$\rho_{sp}(\bar{x},\epsilon)$ = The ratio of the energy/nucleon differential intensities of daughter species to parent species in local interstellar space upon traversal of mean thickness of interstellar matter $\bar{x}$, considering fragmentation only and neglecting other interstellar modulations.

$\rho_{sp}(\bar{x},\epsilon)f_{sp}(R_p)$ = The ratio of the energy/nucleon differential intensities of daughter species to parent species, in local interstellar space which

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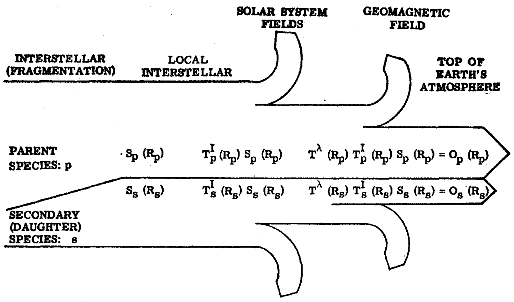

FIG. 1. Flow diagram to illustrate the modulations of the local interstellar differential intensity S_i of the daughter ($i=s$) and parent ($i=p$) species. In interstellar space the daughter intensity grows largely due to fragmentation of the parent species. $T_i^I(R_i)$ represents the fractional reduction of species i through interplanetary fields at a value of rigidity R_i. The product $T_i^I(R_i) \times S_i^I(R_i)$ represents the intensity prior to entry into the effective geomagnetic field. $T^\lambda(R_i)$ represents the fractional reduction of the intensity at rigidity R_i upon transmission through the geomagnetic field at geomagnetic latitude λ, and $O_i(R_i)$ represents the observable differential intensity above the earth's atmosphere.

considers both fragmentation and other modulations in interstellar space and in the source regions.

$r_{sp}^{\Delta\epsilon}$ = The ratio of daughter to parent species in the energy-per-nucleon interval $\Delta\epsilon$ observable at the top of the earth's atmosphere.

$r_{sp}^{\Delta R}$ = The ratio of daughter to parent species in the rigidity interval ΔR observable at the top of the earth's atmosphere.

$n_{HM}^{\Delta\epsilon}$ = The ratio of heavy nuclei (with charge $Z \geq 10$) to medium nuclei (with $6 \leq Z \leq 9$) in the energy/nucleon interval $\Delta\epsilon$ observable at the top of the earth's atmosphere.

ψ_{sp} = The ratio of $r_{sp}^{\Delta R}$ to $r_{sp}^{\Delta\epsilon}$.

ψ_{sp}' = The ratio of $r_{sp}^{\Delta\epsilon}$ to $r_{sp}^{\Delta R}$ for the case in which the $\Delta\epsilon$ and ΔR intervals of the daughter species coincide.

B. Relationships between Spectra

The one-dimensional diffusion equations are frequently used[30–34,39–43,1] to determine the ratio of the intensities of parent and daughter species resulting from fragmentation processes occurring in a mean interstellar path $\bar{x}$ (see Appendix). The intensity ratio of daughter to parent species $\rho(\bar{x},\epsilon)$ is a function of energy per nucleon because the partial cross sections of the interaction (interaction mean free paths and fragmentation probabilities) depend on energy per nucleon. The possibility that $\bar{x}$ itself depends significantly on rigidity or energy[1,45,46] is an open question and is to be decided by experiment. $\tilde{S}_s(\epsilon)/\tilde{S}_p(\epsilon)$, the ratio of the intensities per unit energy/nucleon interval of daughter and parent species in local interstellar space, at an energy/nucleon ϵ, would be identical with $\rho(\bar{x},\epsilon)$ under the following assumptions: (a) The energy per nucleon is conserved on the average in the fragmentation process upon collision of the parents with interstellar matter. (b) No daughter species is injected at the source. (c) Acceleration and transmission in interstellar space is primarily velocity-dependent. (d) Differences in the energy loss rates in interstellar matter between parent and daughter species are negligible.

To permit the possibility that one or more of the above conditions are not satisfied, the ratio of $\tilde{S}_s(\epsilon)$ and $\tilde{S}_p(\epsilon)$ is assumed to be of the form

$$\tilde{S}_s(\epsilon)/\tilde{S}_p(\epsilon) \equiv \rho_{sp}(\bar{x},\epsilon) f_{sp}(R_p). \tag{1}$$

Thus any departure of the intensity ratio in local interstellar space from $\rho_{sp}(\bar{x},\epsilon)$ due to violation of the above assumptions is taken up by the unknown function $f_{sp}(R_p)$, where R_p is the rigidity of the parent species.

In order to account for differences in transmission through the solar system and geomagnetic fields, the transmission fraction between local interstellar space and earth, evaluated at a rigidity R, is defined as

$$T_i(R) \equiv O_i(R)/S_i(R). \tag{2}$$

$O_i(R)$ represents the intensity per unit rigidity of the cosmic-ray species i at a rigidity R, at the top of the earth's atmosphere. $S_i(R)$ is the intensity per unit rigidity interval of cosmic-ray species i at a rigidity R in local interstellar space. $T_i(R)$ may be represented as the product of two transmission fractions

$$T_i(R) = T^\lambda(R) T_i^I(R), \tag{3}$$

where $T^\lambda(R)$ is the transmission fraction due to the earth's field at a geomagnetic latitude λ. $T^\lambda(R)$ is a function of rigidity and the direction of arrival. $T_i^I(R)$ is the transmission fraction, evaluated at a rigidity R, of the cosmic-ray species i due to the interplanetary fields. For the case in which the interplanetary transmission has some velocity dependence as well as rigidity dependence, as suggested by Parker,[17] $T_i^I(R)$ will differ for parent ($i=p$) and daughter ($i=s$) species. For the case of an interplanetary transmission function which is purely rigidity-dependent, $T_i^I(R)$ and hence $T_i(R)$ are independent of the species i.

When a parent species, of rigidity R_p, fragments into a daughter species with rigidity R_d, the average of the ratio $R_s/R_p \equiv K_{sp}$ is assumed to be constant. Because the analysis compares the local interstellar intensities and the transmission of two cosmic-ray species at values of rigidity which differ by the factor K_{sp}, subscripts p and s have also been assigned to the value of rigidity R at which the functions $T_i(R)$ and $S_i(R)$ are evaluated. This bookkeeping procedure also helps in understanding the influences of transmission and the local interstellar spectra upon the observable intensity ratios of related cosmic-ray species.

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The intensities of the daughter and parent species at the top of the earth's atmosphere are related to the intensity of the parent species in local interstellar space by Eqs. (5) and (6), which follow from Eqs. (1), (2), and (4):

$$\tilde{S}_i(\epsilon)d\epsilon = S_i(R_i)dR_i, \tag{4}$$

$$O_s(R_s)dR_s = T_s(R_s)\rho_{sp}(\bar{x},\epsilon)f_{sp}(R_p)S_p(R_p)dR_p, \tag{5}$$

$$O_p(R_p)dR_p = T_p(R_p)S_p(R_p)dR_p. \tag{6}$$

Various modulations which occur after fragmentation are illustrated in Fig. 1 in terms of the intensity per unit rigidity interval.

The ratios of the abundance of daughter-to-parent species at the top of the earth's atsmophere are defined for the energy/nucleon interval $\Delta\epsilon = \epsilon_b - \epsilon_a$ and the rigidity interval $\Delta R = R_f - R_e$, respectively,

$$r_{sp}{}^{\Delta\epsilon} \equiv \int_{\epsilon_a}^{\epsilon_b} \tilde{O}_s(\epsilon)d\epsilon \bigg/ \int_{\epsilon_a}^{\epsilon_b} \tilde{O}_p(\epsilon)d\epsilon, \tag{7}$$

$$r_{sp}{}^{\Delta R} \equiv \int_{R_e}^{R_f} O_s(R_s)dR_s \bigg/ \int_{R_e}^{R_f} O_p(R_p)dR_p. \tag{8}$$

More information can be extracted from cosmic-ray spectra by utilizing these equations in differential form. However, because of the limited statistics of presently available experimental data the results have been expressed in integral form. Using Eqs. (1), (2), (5), and (6), the Eqs. (7) and (8) become

$$r_{sp}{}^{\Delta\epsilon} \equiv \int_{R_a}^{R_b} \frac{T_s(R_s)\rho_{sp}(\bar{x},\epsilon)f_{sp}(R_p)O_p(R_p)}{T_p(R_p)} dR_p \bigg/ \int_{R_a}^{R_b} O_p(R_p)dR_p, \tag{9}$$

$$r_{sp}{}^{\Delta R} \equiv \int_{R_e}^{R_f} \frac{T_s(R_s)\rho_{sp}(\bar{x},\epsilon)f_{sp}(R_p)S_p(R_p)O_p(R_s)}{K_{sp}T_p(R_s)S_p(R_d)} dR_s \bigg/ \int_{R_e}^{R_f} O_p(R_p)dR_p. \tag{10}$$

Upon averaging over the intensity $O_p(R_p)$, the abundance ratios become

$$r_{sp}{}^{\Delta\epsilon} = \langle \rho_{sp}(\bar{x},\epsilon)f_{sp}(R_p)[T_s(R_s)/T_p(R_p)]\rangle_{\mathrm{av}}, \tag{11}$$

$$r_{sp}{}^{\Delta R} = \left(\frac{\rho_{sp}(\bar{x},\epsilon)f_{sp}(R_p)}{K_{sp}} \frac{T_s(R_s)}{T_p(R_s)} \frac{S_p(R_p)}{S_p(R_s)}\right)_{\mathrm{av}}. \tag{12}$$

ψ_{sp}, a quantity useful for the interpretation of the effects of geomagnetic latitude and solar activity on these abundance ratios, is defined as follows:

$$\psi_{sp} \equiv r_{sp}{}^{\Delta\epsilon}/r_{sp}{}^{\Delta R}. \tag{13}$$

For the special case in which the limits of integration R_e and R_f of Eq. (10) are the values of rigidity of the fragmented daughter corresponding to the energy/nucleon limits ϵ_a and ϵ_b of Eq. (7), respectively, the numerators of Eqs. (9) and (10) cancel, and

$$\psi_{sp}' = \int_{K_{sp}R_a}^{K_{sp}R_b} O_p(R_p)dR_p \bigg/ \int_{R_a}^{R_b} O_p(R_p)dR_p. \tag{14}$$

ψ_{sp}' is a function of the observable parent spectrum only. Thus the left and right sides of Eq. (13) may be independently determined from experimental results.

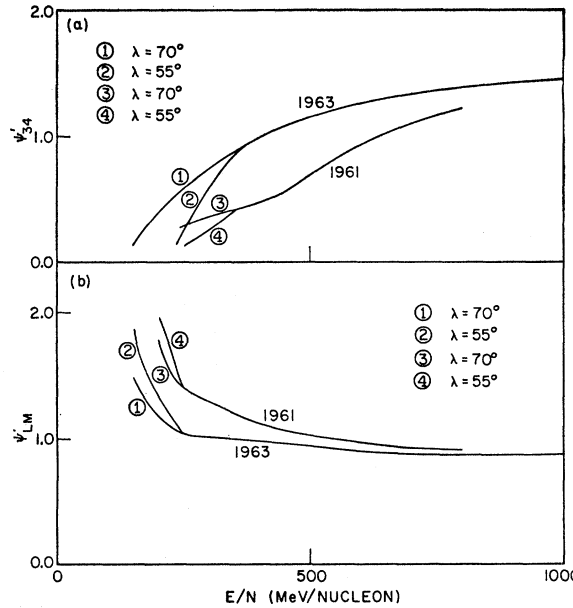

FIG. 2. (a) shows the dependence of $\psi_{34}' = r_{34}{}^{\Delta\epsilon}/r_{34}{}^{\Delta R}$ on energy/nucleon at different times of the solar cycle and different geomagnetic latitudes. Here $r_{34}{}^{\Delta\epsilon}$ and $r_{34}{}^{\Delta R}$ are the ratios of He^3 to He^4 in energy/nucleon intervals and rigidity intervals, respectively. The $\Delta\epsilon$ and ΔR intervals of He^3 are taken to be identical, hence ψ_{34}' depends on the He^4 spectrum alone. $\Delta\epsilon = 100$ MeV/nucleon. (b) shows the corresponding data for ψ_{LM}', for the light and medium nuclei which have nuclear charges of $3 \leq Z \leq 5$ and $6 \leq Z \leq 9$, respectively. The curves for the year 1963, based on the spectra of Ormes and Webber (Ref. 28), give the values of ψ_{sp}' at a time near solar minimum, at geomagnetic latitudes $\lambda = 70°$ and 55°, respectively. The curves for 1961, based on the spectra obtained by Fichtel *et al.* (Ref. 25) and Shapiro *et al.* (Ref. 48), give the values of ψ_{sp}' at a more active part of the solar cycle, at $\lambda = 70°$ and 55°, respectively.

TABLE I. Variation of abundance ratios with solar modulation and geomagnetic latitude at fixed intervals of energy/nucleon or rigidity.

Case	Example	Ratio	Increasing geomagnetic cutoff	Increasing rigidity-dependent solar modulation	Increasing velocity-dependent solar modulation
$R_s < R_p$	He^3/He^4	$r_{sp}{}^{\Delta\epsilon}$	decreases	decreases	constant
		$r_{sp}{}^{\Delta R}$	constant	constant	increases
$R_p < R_s$	L/M	$r_{sp}{}^{\Delta\epsilon}$	increases	increases	constant
		$r_{sp}{}^{\Delta R}$	constant	constant	decreases

TABLE II. Variation of abundance ratios with decreasing rigidity at solar minimum.

Case	Example	Ratio	No solar[a] modulation at solar minimum	Velocity-dependent[a] modulation at solar minimum	Rigidity-dependent[a] modulation at solar minimum	Cosmic-ray-source model of Kaplon and Skadron,[b] no solar modulation
$R_s<R_p$	He^3/He^4	$r_{sp}^{\Delta\epsilon}$	constant	constant	decreases	...
		$r_{sp}^{\Delta R}$	increases	increases	increases or constant[c]	increases
$R_p<R_s$	L/M	$r_{sp}^{\Delta\epsilon}$	constant	constant	increases	increases
		$r_{sp}^{\Delta R}$	decreases	decreases	decreases or constant[c]	increases

[a] $\bar{x}$, λ, $f_{dp}(R_p)$ are fixed. [b] See Ref. 45. [c] See prediction (d), Sec. IIC.

With the use of Eqs. (2) and (3), we put Eq. (14) into the form

$$\psi_{sp}'=K_{sp}\frac{\langle O_p(R_s)\rangle_{\text{av}}}{\langle O_p(R_p)\rangle_{\text{av}}}=K_{sp}\frac{\langle T^\lambda(R_s)T_p{}^I(R_s)S_p(R_s)\rangle_{\text{av}}}{\langle T^\lambda(R_p)T_p{}^I(R_p)S_p(R_p)\rangle_{\text{av}}}, \quad (15)$$

where the averages have been taken over the rigidity interval $\Delta R=R_b-R_a$.

C. Deduced Variations in Relative Abundances

Equations (11) and (12) indicate that neither of the abundance ratios of daughter-to-parent species $r_{sp}^{\Delta\epsilon}$ and $r_{sp}^{\Delta R}$ are equal to $\rho_{sp}(\bar{x},\epsilon)$ as has been assumed in earlier calculations of the interstellar path length. The effect of solar and geomagnetic modulation is particularly important in the low-rigidity region, below 2 BV. But even at high energies, when the transmission fractions $T(R)$ approach unity, $r_{sp}^{\Delta R}$ differs from $r_{sp}^{\Delta\epsilon}$ by the factor $(1/K_{sp})S_p(R_p)/S_p(R_s)$.

Several qualitative predictions concerning variations in $r_{sp}^{\Delta\epsilon}$, $r_{sp}^{\Delta R}$, and ψ_{sp}' with energy, geomagnetic latitude, and solar activity will be made with the use of Eqs. (11), (12), and (15) plus the conditions i–vii listed below. For the benefit of readers interested in the results rather than in the technicalities of these deductions, all predictions of this section are summarized in Fig. 2 and Tables I and II.

(i) The interstellar quantities $S_i(R)$, K_{sp}, $\rho_{sp}(\bar{x},\epsilon)$, and $f_{sp}(R_p)$ are, by definition, independent of the geomagnetic latitude λ. These quantities are assumed to be constant over short time intervals such as the 11-year solar cycle.

(ii) The quantities $\rho_{sp}(\bar{x},\epsilon)$ and $f_{sp}(R_p)$ are assumed to be constants; (the possibility that these functions vary with energy is considered later).

(iii) The geomagnetic transmission fraction $T^\lambda(R)$ has been experimentally found to increase with increasing rigidity at a given geomagnetic latitude, and approaches unity.[27] Thus the ratio of the fractions transmitted at rigidities R_s and R_p, respectively, $T^\lambda(R_s)/T^\lambda(R_p)$, is less than or equal to one for $R_s<R_p$ and greater than or equal to one for $R_s>R_p$, becoming unity at higher rigidities. Short-term variations produced by solar flares and magnetic storms are not considered and $T^\lambda(R)$ is assumed to be independent of the solar cycle.

(iv) In the latitude-sensitive region of rigidities, the geomagnetic transmission fraction $T^\lambda(R)$ increases with geomagnetic latitude at a given value of rigidity, approaching unity.[9–11,27] Hence $O_i(R_s)/O_i(R_p)$, the ratio of the observable intensities at the top of the atmosphere at rigidities R_s and R_p, respectively, increases (decreases) with geomagnetic latitude for $R_s<R_p$ ($R_s>R_p$) in the rigidity interval where $0<T^\lambda(R)<1$.

(v) The interplanetary transmission fractions $T_i{}^I(R)$ increase with rigidity or energy/nucleon for $R>0.5$ BV, and approach unity at high rigidities.[27,46] Hence the ratio of the fractions transmitted at rigidities R_s and R_p, respectively, $T_i{}^I(R_s)/T_i{}^I(R_p)\leq 1$ for $R_s<R_p$ and $T_i{}^I(R_s)/T_i{}^I(R_p)\geq 1$ for $R_s>R_p$, approaching unity at high values of rigidity.

(vi) At a fixed value of rigidity, or energy/nucleon, the interplanetary transmission fraction $T_i{}^I(R)$ increases with decreasing solar modulation of cosmic rays.[27] Hence, at a fixed value of R_s, the ratio of the observable intensities at the top of the atmosphere $O_i(R_s)/O_i(R_p)$ increases (decreases) with decreasing solar modulation for the case $R_s<R_p$ ($R_s>R_p$) at a fixed value of geomagnetic latitude.

(vii) At a fixed value of geomagnetic latitude and a fixed time in the solar cycle, the ratio of the observable intensities at the top of the atmosphere $O_i(R_s)/O_i(R_p)$ increases (decreases) with rigidity or energy/nucleon[27] for $R_s<R_p$ ($R_s>R_p$), for 1 BV$\leq R\leq$3 BV.

The predictions concerning $r_{sp}^{\Delta\epsilon}$, the ratio of intensities of daughter-to-partent species in an energy/nucleon interval, are

(a) On the basis of conditions ii, iii, and v (regarding the constancy of the relevant interstellar quantities and the increase of the geomagnetic and interplanetary transmission fractions with increasing rigidity) and Eq. (11), we find that $r_{sp}^{\Delta\epsilon}$ increases (decreases) with increasing energy/nucleon for $R_s<R_p$ ($R_s>R_p$) at a given value of the ratio of the geomagnetic transmission fractions $T^\lambda(R_s)/T^\lambda(R_p)$ and at a given time of the solar cycle, if the solar modulation is rigidity-dependent. If the solar modulation is velocity dependent then $r_{sp}^{\Delta\epsilon}$

does not vary with ϵ for a given value of $T^{\lambda}(R_s)/T^{\lambda}(R_p)$ and a fixed time in the solar cycle.

(b) On the basis of conditions i and iv (regarding the constancy of the relevant interstellar quantities and the increase of the geomagnetic transmission fraction with increasing geomagnetic latitude) and Eq. 11, we find that $r_{sp}^{\Delta\epsilon}$ decreases (increases) with an increasing value of cutoff rigidity, i.e., with decreasing geomagnetic latitude, for $R_s<R_p$ ($R_s>R_p$), at a given value of rigidity, and at a given time of the solar cycle.

(c) On the basis of conditions i and vi (regarding the constancy of the relevant interstellar quantities and the increase of the interplanetary transmission fraction with decreasing solar modulation) and Eq. (11), we find that $r_{sp}^{\Delta\epsilon}$ decreases (increases) with increasing solar modulation for $R_s<R_p$ ($R_s>R_p$), at a given rigidity and at a given geomagnetic latitude. If, however, the solar modulation is purely velocity dependent, $r_{sp}^{\Delta\epsilon}$ will be independent of solar modulation.

The predictions concerning $r_{sp}^{\Delta R}$, the ratio of daughter-to-parent species in a rigidity interval, are

(d) On the basis of conditions i and ii (regarding the constancy of relevant interstellar quantities) and Eqs. (3) and (12), we find that $r_{sp}^{\Delta R}\propto[T_s^I(R_s)S_p(R_p)]/[T_p^I(R_s)S_p(R_s)]$. For a purely rigidity-dependent interplanetary transmission mechanism, $r_{sp}^{\Delta R}$ varies as $S_p(R_p)/S_p(R_s)$, the ratio of the intensities of the parent species at rigidities of R_p and R_s, respectively, in local interstellar space. In this case the variation of $r_{sp}^{\Delta R}$ with increasing rigidity yields information on the variation of the local interstellar spectrum $S_p(R)$ with with rigidity; e.g., if $S_p(R)$ has a negative curvature, then $r_{sp}^{\Delta R}$ increases (decreases) for $R_s<R_p$ ($R_s>R_p$) with decreasing rigidity. If $S(R)\propto 1/R^q$ (q=constant), then $r_{sp}^{\Delta R}$ is independent of rigidity. If, however, a velocity-dependent interplanetary transmission mechanism is assumed, condition v, regarding the increase of the interplanetary transmission fraction $T_i^I(R)$ with increasing energy/nucleon, is used to deduce that $T_s^I(R_s)/T_p^I(R_s)$ decreases (increases) for $R_s<R_p$ ($R_s>R_p$) with increasing rigidity, approaching unity at high rigidities. If $S(R)\propto 1/R^q$ (q=constant), then $S_p(R_p)/S_p(R_s)$ is constant, and the variation in $r_{sp}^{\Delta R}$ with rigidity, at a fixed time of the solar cycle, yields information on the degree of velocity dependence of the interplanetary transmission mechanism.

(e) On the basis of condition i (regarding the constancy of relevant interstellar quantities), considering the geomagnetic transmission fraction to be purely rigidity-dependent, and applying Eqs. (3) and (12), we find that $r_{sp}^{\Delta R}$ is independent of geomagnetic latitude.

(f) On the basis of conditions i and vi (regarding the constancy of relevant interstellar quantities and the increase of the interplanetary transmission fraction with decreasing solar modulation) and Eqs. (3) and (12), we find that $r_{sp}^{\Delta R}$ increases (decreases) for $R_s<R_p$ ($R_s>R_p$) with increasing solar modulation, at a fixed value of rigidity, if the solar modulation is velocity-dependent. If, however, the solar modulation is purely rigidity-dependent, then $r_{sp}^{\Delta R}$ does not vary with the solar cycle at a fixed value of rigidity.

The predictions concerning ψ_{sp}', the ratio of the abundance ratios $r_{sp}^{\Delta\epsilon}$ to $r_{sp}^{\Delta R}$ for the case where the energy/nucleon and rigidity intervals of the daughter species coincide, are:

(g) On the basis of condition vii (regarding the variation of the ratio of the observable intensities at the top of the atmosphere, $O_i(R_s)/O_i(R_p)$, with rigidity), and Eq. (15), we find that ψ_{sp}' increases (decreases) with rigidity for $R_s<R_p$ ($R_s>R_p$) in the interval between 1 and 3 BV, at a fixed value of geomagnetic latitude and a fixed time of the solar cycle.

(h) On the basis of condition iv (regarding the increase of the geomagnetic transmission fraction with increasing geomagnetic latitude) and Eq. (15), we find that ψ_{sp}' increases (decreases) for $R_s<R_p$ ($R_s>R_p$) with increasing geomagnetic latitude, for a fixed rigidity interval and at a fixed time in the solar cycle.

(i) On the basis of condition vi (regarding the increase of the interplanetary transmission fraction with decreasing solar modulation) and Eq. (15), we find that ψ_{sp}' increases (decreases) for $R_s<R_p$ ($R_s>R_p$) with decreasing solar modulation, for a fixed rigidity interval and at a fixed value of geomagnetic latitude.

Since ψ_{sp}' depends only on the observable parent spectra, predictions (g), (h), and (i) can be demonstrated quantitatively. Figure 2(a) indicates the dependence of the ratio $r_{34}^{\Delta\epsilon}$ to $r_{34}^{\Delta R}$ on the energy/nucleon interval at different times of the solar cycle and at different geomagnetic latitudes. Here $r_{34}^{\Delta\epsilon}$ and $r_{34}^{\Delta R}$ represent the ratios of He^3 to He^4 in the energy/nucleon and rigidity intervals, respectively. The curves for the year 1963 have been derived from the He^4 spectrum at geomagnetic latitudes $\lambda=70°$ and $55°$, respectively,[28] when the average hourly rate of the Mt. Washington neutron monitor[47] was approximately 2300. The curves for 1961 were also derived from the He^4 spectrum, again at $\lambda=70°$ and $55°$, respectively,[25,48] when the average hourly Mt. Washington neutron monitor rate was approximately 2150. The curves were calculated for energy/nucleon intervals having a width of 100 MeV/nucleon. The estimated standard deviation of each point of the curves is approximately 20%. In the interval $0.15\leq\epsilon\leq1.0$ BeV/nucleon the variation in ψ_{34}' is greater than a factor of 5.

Figure 2(b) shows the corresponding calculation for the light and medium nuclei. On the basis of the experimental data[19,1,2] in the energy intervals considered, it was assumed in this calculation that the

[47] J. Lockwood, University of New Hampshire (private communication).

[48] M. M. Shapiro, B. Hildebrand, F. W. O'Dell, R. Silberberg, and B. Stiller, in *Proceedings of the Third International Space Science Symposium, Washington, D. C., 1962* (North-Holland Publishing Company, Amsterdam, 1963), p. 1097.

TABLE III. The reported He^3/He^4 ratios in the energy/nucleon interval $\Delta\epsilon$ and rigidity interval ΔR at geomagnetic latitudes λ, and average hourly Mt. Washington neutron monitor rates (n.m.r.). $\psi_{34}=r_{34}^{\Delta\epsilon}/r_{34}^{\Delta R}$.

Ratio	April 1961,[a] $\lambda=55°$, n.m.r.$=2152$[c]		August 1962,[b] $\lambda=70°$, n.m.r.$=2241$[c]	
$r_{34}^{\Delta\epsilon}$	$0.068_{-0.02}^{+0.03}$	260–360 MeV/nucleon	0.22 ± 0.06	160–370 MeV/nucleon
$r_{34}^{\Delta R}$	$0.17_{-0.07}^{+0.10}$	1.10–1.48 BV	0.32 ± 0.11	1.1–1.4 BV
ψ_{34}	0.4 ± 0.2		$0.7\ \pm0.2$	

[a] See Ref. 49. [b] See Ref. 8. [c] See Ref. 47.

spectrum of the medium nuclei does not differ significantly from that of He^4. In the energy/nucleon interval $0.15\leq\epsilon\leq1.0$ BeV/nucleon, the variation in ψ_{LM}' is approximately a factor of 2.

The qualitative predictions (b), (c), (e), and (f) are summarized in Table I. The magnitude of the variations can be evaluated quantitatively from Fig. 2 with the use of Eqs. (11) and (12).

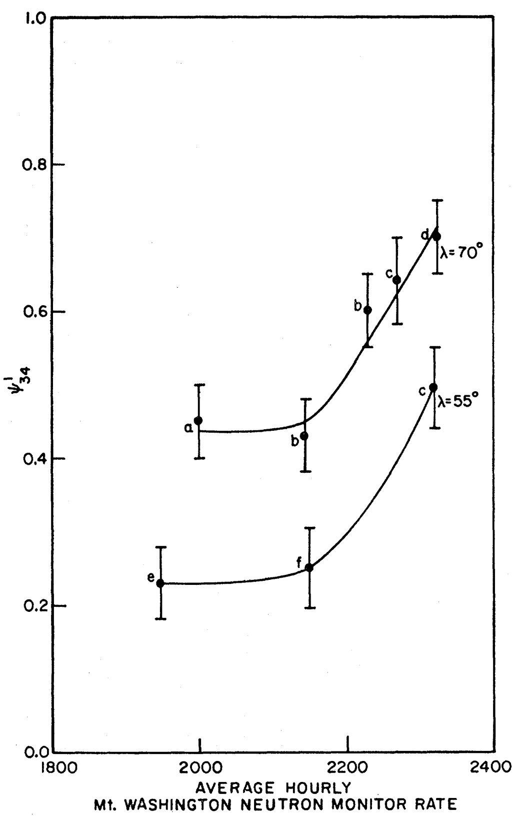

FIG. 3. The dependence of $\psi_{34}'=r_{34}^{\Delta\epsilon}/r_{34}^{\Delta R}$ of Eq. 14 on the total cosmic-ray intensity, as measured by the Mt. Washington neutron monitor rate. Here $r_{34}^{\Delta\epsilon}$ and $r_{34}^{\Delta R}$ are the ratios of He^3 to He^4 in the energy/nucleon intervals and rigidity intervals, respectively. The $\Delta\epsilon$ and ΔR intervals are taken to be identical, hence ψ_{34}' depends on the He^4 spectrum alone. The two curves are evaluated for geomagnetic latitudes $\lambda=70°$ and $55°$, respectively. ψ_{34}' has been evaluated for an energy/nucleon interval 250–350 MeV/nucleon. The point *a* is based on the spectrum obtained by Engler *et al.* (Ref. 52), *b* by Fichtel *et al.* (Ref. 25), *c* by Ormes and Webber (Ref. 28), *d* by Freier and Waddington (Ref. 29), *e* by McDonald (Ref. 53), and *f* by Shapiro *et al.* (Ref. 48).

If Eqs. (11) and (12) are applied to more than one set of daughter and parent species and the transmission factors and the shapes of the local interstellar spectra of one set are related to those of the other, respectively, then it is possible to evaluate the relative $f_{sp}(R_p)$ factors between the sets. $f_{sp}(R_p)$ yields information on injection of daughter species at the cosmic-ray sources or on acceleration in interstellar space.

In a recent paper[45] Kaplon and Skadron have considered a model of cosmic-ray sources whereby low-energy particles suffer many reflections in the source regions. In this model a considerable amount of fragmentation takes place at the source, particularly at low rigidities (of the order of 1.0 BV). Accordingly, the value of $r_{sp}^{\Delta R}$, the abundance ratio of the daughter-to-parent species in the rigidity interval ΔR, should increase with decreasing rigidity at low values of rigidity. A determination of $r_{sp}^{\Delta R}$ at different rigidities, at solar minimum and at a high geomagnetic latitude can provide a test between various models of both solar modulations and modulations in the cosmic-ray-source regions. The effects of various types of modulation on the abundance ratios $r_{sp}^{\Delta\epsilon}$ and $r_{sp}^{\Delta R}$, based on predictions (a) and (d) are summarized in Table II. The variations described in Table II for the L/M ratio will be smaller by a factor of approximately 3 than those corresponding to the case of He^3/He^4. The magnitude of the variations due to solar modulation can be determined quantitatively from Fig. 2 with the use of Eqs. (11) and (12).

III. APPLICATION TO RELATIVE ABUNDANCE EXPERIMENTS

A. He^3/He^4 Ratios

Several experiments on the abundance ratio of He^3/He^4 have been reported.[5–8,48–51] In some cases mass identification of the individual particles, which is necessary for an accurate assignment into energy/nucleon and rigidity intervals, was not achieved. A single method of mass identification has this shortcoming.

[49] B. Hildebrand, F. W. O'Dell, M. M. Shapiro, R. Silberberg, and B. Stiller, *Proceedings of the* 1963 *IUPAP Cosmic Ray Conference, Jaipur, India* (Tata Institute of Fundamental Research, Bombay, India, 1964), Vol. 3, p. 101.
[50] H. Aizu, *Proceedings of the* 1963 *IUPAP Cosmic Ray Conference, Jaipur, India* (Tata Institute of Fundamental Research, Bombay, India, 1964), Vol. 3, p. 90.
[51] W. R. Webber and J. Ormes, Phys. Rev. **138**, B416 (1965).

TABLE IV. The experimental values of $r_{LM}^{\Delta\epsilon}$, the ratio of light to medium nuclei in the energy/nucleon interval $\Delta\epsilon$, as a function of solar modulation, geomagnetic latitude, and energy.

Time	Aug. 1956[a]	June 1963[b]	Sept. 1956[c]	Aug. 1962[d]	Aug. 1958[e]	Sept. 1957[f]	Sept. 1959[g]
Mt. Washington neutron monitor rate[h]	2415	2325	2307	2240	2001	1977	1871
Geomagnetic latitude	55°	70°	55°	70°	61°	61°	55°
Energy interval (BeV/nucleon)							
0.2–0.4			0.5 ±0.15		0.6 ±0.15	0.4 ±0.1	0.7 ±0.15
0.2–0.7			0.31±0.1	0.54±0.11		0.37±0.05	0.51±0.07
0.4–0.8		0.28±0.08					
0.4–1.0	0.37±0.1				0.43±0.07		
>0.7			0.21±0.05	0.25±0.04		0.21±0.03	0.32±0.03
>1.5	0.2 ±0.08				0.28±0.05		

a See Ref. 56.
b See Ref. 26.
c See Ref. 55.
d See Ref. 4, evaluated by assuming the ratio $n_{HM}^{\Delta\epsilon}=0.32$.
e See Ref. 2.
f See Ref. 1.
g See Ref. 3.
h See Ref. 47.

Table III gives the results of Hildebrand *et al.*[49] and Dahanayake *et al.*[8] which are based upon the use of two independent methods of mass identification.

Figure 3 shows the variation of ψ_{34}', the ratio of $r_{sp}^{\Delta\epsilon}$ to $r_{sp}^{\Delta R}$ of Eq. (14), with the cosmic-ray intensity. The Mt. Washington neutron monitor rate[47] is taken as a measure of the total cosmic-ray intensity. The values of ψ_{34}' were calculated from Eq. (14) and the reported helium spectra[25,28,29,48,52,53] at geomagnetic latitudes $\lambda=70°$ and 55°, respectively. ψ_{34}' has been evaluated for an energy/nucleon interval 250–350 MeV/nucleon and a rigidity interval 1.10–1.38 BV. The value of ψ_{34}' is in agreement with predictions (h) and (i) of Sec. IIC. The values of ψ_{34} in Table III are consistent with the corresponding values of ψ_{34}' in Fig. 3. Some of the early experimental work on the He^3/He^4 ratios does not meet this internal consistency requirement.

The experimental data in Table III support the qualitative predictions of (b), (c), (e), and (f) of Sec. IIC:

(b) For a fixed energy/nucleon interval $\Delta\epsilon$, the ratio of He^3 to He^4 in this energy/nucleon interval $r_{34}^{\Delta\epsilon}$, increases with geomagnetic latitude at a given time in the solar cycle.

(c) For a fixed $\Delta\epsilon$ and geomagnetic latitude λ, $r_{34}^{\Delta\epsilon}$ increases with a decreasing solar modulation that is rigidity-dependent, (and will be constant for a velocity-dependent modulation).

(e) The ratio of He^3 to He^4 in the rigidity interval $r_{34}^{\Delta R}$ is independent of geomagnetic latitude.

(f) $r_{34}^{\Delta R}$ is independent of solar modulation if the latter depends only on rigidity, (if the latter has some velocity dependence, $r_{34}^{\Delta R}$ should increase with increasing solar modulation).

The results (f) and (c) support the existence of a solar modulation which is largely rigidity-dependent in the interval 1.2–1.8 BV.

The recent data of Balasubrahmanyan *et al.*[54] at 70 MeV/nucleon indicate a very low He^3/He^4 ratio. This result is consistent with prediction (a) of Sec. IIC, if the solar modulation is largely rigidity-dependent. At such low energies, however, the effect of ionization loss becomes important, which may also lead to a low ratio.

B. L/M Ratios

The mass-to-charge ratio is different for the light and medium nuclei, the ratios of atomic mass to atomic number being $(A/Z)_L=2.2$ and $(A/Z)_M=2.0$, respectively. Hence we may expect the transmission through the solar system and geomagnetic fields to affect $r_{LM}^{\Delta\epsilon}$, the observed ratio of light to medium nuclei in the energy/nucleon interval $\Delta\epsilon$. The ratio of daughter-to-parent rigidities is taken to be $K_{LM}=1.1$. In this case, as contrasted with the case of helium, the average rigidity of the daughter nuclei, upon fragmentation, is larger than that of the parent nuclei and thus the observed $r_{LM}^{\Delta\epsilon}$ may be expected to be larger than the ratio of intensities in local interstellar space.

The experimental L/M values at low energies reported in the literature[1–4,26,55,56] have only somewhat better statistics than the work on He^3/He^4. The L/M ratio rather than the $L/(M+H)$ ratio is used here, since the intensity of heavy elements has not been determined in the lowest energy interval of some of the experiments considered below. Table IV gives the experimental results at geomagnetic latitudes 55°, 61°, and 70°, which include measurements at various values of energy/nucleon in the years 1956–1963. The corresponding average hourly Mt. Washington neutron monitor counts are also given. The data of Table IV

[52] A. Engler, F. Foster, T. L. Green, and J. Mulvey, Nuovo Cimento **20**, 1157 (1961).
[53] F. B. McDonald, Phys. Rev. **116**, 462 (1959).
[54] V. K. Balasubrahmanyan, K. A. Brunstein, G. H. Ludwig, F. B. McDonald, and R. A. Palmeira, Annual Meeting of the American Geophysical Union, April, 1965, Washington, D. C. (unpublished) and private communication.
[55] D. E. Evans, Nuovo Cimento **27**, 394 (1963).
[56] F. B. McDonald and W. R. Webber, J. Geophys. Res. **67**, 2119 (1962).

TABLE V. The experimental values of $r_{LM}^{\Delta R}$, the ratio of light to medium elements in the rigidity interval ΔR, and ψ_{LM}, the ratio of $r_{LM}^{\Delta\epsilon}/r_{LM}^{\Delta R}$, as a function of rigidity, geomagnetic latitude, and solar modulation.

Time	September 1957[a]		March 1958[b]	September 1958[c]	
Mt. Washington neutron monitor rate[d]	1977		1886	1871	
Geomagnetic latitude	61°		41°	55°	
Ratio	$r_{LM}^{\Delta R}$	ψ_{LM}	$r_{LM}^{\Delta R}$	$r_{LM}^{\Delta R}$	ψ_{LM}
Rigidity interval (BV)					
1.3–2.7	0.32±0.05	1.15±0.2		0.38±0.05	1.34±0.2
>2.7	0.25±0.04	0.84±0.2		0.37±0.03	0.86±0.2
>4.5			0.24±0.05		

[d] See Ref. 1. r See Ref. 44. [c] See Ref. 57. [d] See Ref. 47.

are in agreement with predictions (a), (b), and (c) of Sec. IIC.

(a) $r_{LM}^{\Delta\epsilon}$, the ratio of light to medium elements in the energy/nucleon interval $\Delta\epsilon$, increases with decreasing energy at a fixed geomagnetic latitude and solar modulation.

(b) $r_{LM}^{\Delta\epsilon}$ increases with decreasing geomagnetic latitude at a fixed rigidity and solar modulation.

(c) $r_{LM}^{\Delta\epsilon}$ increases with increasing solar modulation at a fixed energy and geomagnetic latitude for a modulation with a large rigidity dependence.

In the calculations made in Sec. IIID a quantitative estimate is made of the relative fraction of light to medium elements transmitted in the same energy/nucleon interval [i.e., the transmission fraction $T_L(R_L)/T_M(R_M)$], at $\lambda=55°$ at a time between solar maximum and solar minimum (1961) in the energy interval 200–400 MeV/nucleon. The result is $T_L(R_L)/T_M(R_M)=1.2\pm0.2$. Another estimate has been made starting from the experimental M spectra and Eq. (15) for ψ_{LM}'. This yields $T_M(R_L)/T_M(R_M)=1.5\pm0.2$. These results indicate that variations in the transmission fraction with geomagnetic latitude, solar cycle, and energy may account for much, if not most, of the experimental variations in $r_{LM}^{\Delta\epsilon}$, the ratio of light to medium elements in the energy/nucleon interval $\Delta\epsilon$. It is not necessary to assume a longer path length for low-energy cosmic rays in interstellar material to account for the increase of $r_{LM}^{\Delta\epsilon}$ at lower energies.

The $r_{LM}^{\Delta R}$ ratio of light to medium elements in rigidity intervals has been reported in three experiments, given in Table V. Interpreting these data in the light of the predictions (f), (d), (g), (h), and (i) of Sec. IIC, we can conclude:

(f) The ratio $r_{LM}^{\Delta R}$ in a fixed rigidity interval does not increase with decreasing solar modulation. This favors a rigidity rather than energy/nucleon dependent solar modulation.

(d) The ratio $r_{LM}^{\Delta R}$, at a fixed time in the solar cycle and within a single experiment, appears rather independent of energy, particularly when compared with the energy dependence of the $r_{LM}^{\Delta\epsilon}$ ratio which was explored above; there seem to be systematic differences between the experimental results listed in Table V, however. But because of the combination of several factors such as $S_p(R_p)/S_p(R_s)$, the ratio of local interstellar intensities at rigidities R_p and R_s, and $T_s^I(R_s)/T_p^I(R_s)$, the relative fraction of daughter-to-parent elements transmitted through the interplanetary fields at a rigidity R_s, as well as the rigidity or energy dependence of the interstellar path length $\bar{x}$ entering into prediction (d) of Sec. IIC, no conclusions can be drawn concerning the separate factors at the present time.

(g), (h), and (i). The ratio of light to medium elements in the energy/nucleon interval to that in the rigidity interval, ψ_{LM}, should increase with decreasing rigidity, with increasing solar modulation, and with decreasing geomagnetic latitude. The trends of Table V support these predictions and are in quantitative agreement with the values of ψ_{LM}' displayed in Fig. 2(b).

Because the average charge to mass ratios of H and M nuclei are almost identical, Eq. (11) predicts that the transmission through the interplanetary and geomagnetic fields will not affect $n_{HM}^{\Delta\epsilon}$, the ratio of heavy to medium nuclei in energy/nucleon interval $\Delta\epsilon$, in the energy region where the interaction cross sections remain essentially constant. On the other hand, should the variation of the ratio of light to medium elements in the energy/nucleon interval $\Delta\epsilon$, $r_{LM}^{\Delta\epsilon}$, be explicable in terms of an increasing interstellar path length with decreasing energy, then $n_{HM}^{\Delta\epsilon}$ will be expected to increase. The experimental $n_{HM}^{\Delta\epsilon}$ ratios,[1–3,55–57] however, do not indicate systematic trends and appear essentially constant with geomagnetic latitude, time, and energy above 200 MeV/nucleon.

C. Rigidity Dependence of Solar Modulation

The detailed nature and functional dependence of the solar modulation are not known. In principle they can be determined from the relative modulation of protons and that of helium, when compared in the same energy/nucleon and rigidity intervals at different times of the solar cycle. The main difficulty in this method of

[57] H. Aizu, E. Tamai, M. Koshiba, and E. Lohrmann, J. Phys. Soc. Japan 17, Suppl. A-III, 34 (1962).

determining the form of the modulation is due to uncertainties in the large correction for secondary protons and albedo protons in the low-energy region of the proton spectrum. The relative modulation of protons and helium as determined by Webber and McDonald[27] at solar minimum and maximum, respectively, can be fitted by a single curve as a function of rigidity. At the same values of energy/nucleon, however, the relative modulation at solar maximum and solar minimum is different for protons and helium. These data hence strongly favor a rigidity-dependent solar modulation. On the other hand, the data of Meyer and Vogt[58] and Fichtel *et al.*[25] on the proton spectrum, as analyzed by Webber,[46] disagree with a rigidity-dependent modulation, favoring a velocity dependence instead.

The rigidity or velocity dependence of the solar modulation can be determined also from the ratio of particles belonging to the daughter and parent species, as shown in Sec. IIC. Figure 4 shows the dependence of the abundance ratios $r^{\Delta\epsilon}{}_{LM}(\Delta\epsilon\approx 200\text{–}700$ MeV/nucleon) and $r_{34}{}^{\Delta\epsilon}(\Delta\epsilon\approx 200\text{–}350$ MeV/nucleon) as a function of the Mt. Washington hourly neutron monitor rate. The available information favors a solar modulation mechanism that is predominantly rigidity dependent:

(1) According to Table IV and Fig. 4(a), the ratio of light to medium elements in the energy/nucleon interval $\Delta\epsilon$, $r_{LM}{}^{\Delta\epsilon}$, decreases with decreasing solar modulation. Such a variation supports a rigidity-dependent solar modulation [(c) of Sec. IIC].

(2) According to Table III and Fig. 4(b), the ratio of He^3 to He^4 in the energy/nucleon interval $\Delta\epsilon$, $r_{34}{}^{\Delta\epsilon}$, increases with decreasing solar modulation, supporting a rigidity-dependent solar mechanism [(c) of Sec. IIC]. Point (e) on Fig. 4(b), however, must be regarded with some caution, since it was determined indirectly, by a comparison of proton and helium spectra at different latitudes.

(3) According to Tables III and V, $r_{34}{}^{\Delta R}$ does not decrease and $r_{LM}{}^{\Delta R}$ does not increase with decreasing solar modulation, supporting a rigidity-dependent solar mechanism [(f) of Sec. IIC].

(4) According to Table IV, $r_{LM}{}^{\Delta\epsilon}$, the ratio of light to medium elements in the energy/nucleon interval $\Delta\epsilon$, increases with decreasing energy by a factor of approximately 2. If such an increase is attributed entirely to an increase in $\bar{x}$ with decreasing energy then $\bar{x}$ also must increase by the same factor of 2 in the energy interval 0.3 to 1.5 BeV/nucleon. This increase in $\bar{x}$ should also be reflected in a decrease in $n_{HM}{}^{\Delta\epsilon}$, the ratio of heavy to medium elements, in the same energy interval by 30%. The experimental values of $n_{HM}{}^{\Delta\epsilon}$ tend to be constant; a change of 30% is off by one standard deviation from the experimental values of $n_{HM}{}^{\Delta\epsilon}$. Hence the variation of $r_{LM}{}^{\Delta\epsilon}$ favors a rigidity-dependent solar modulation [(a) of Sec. IIC].

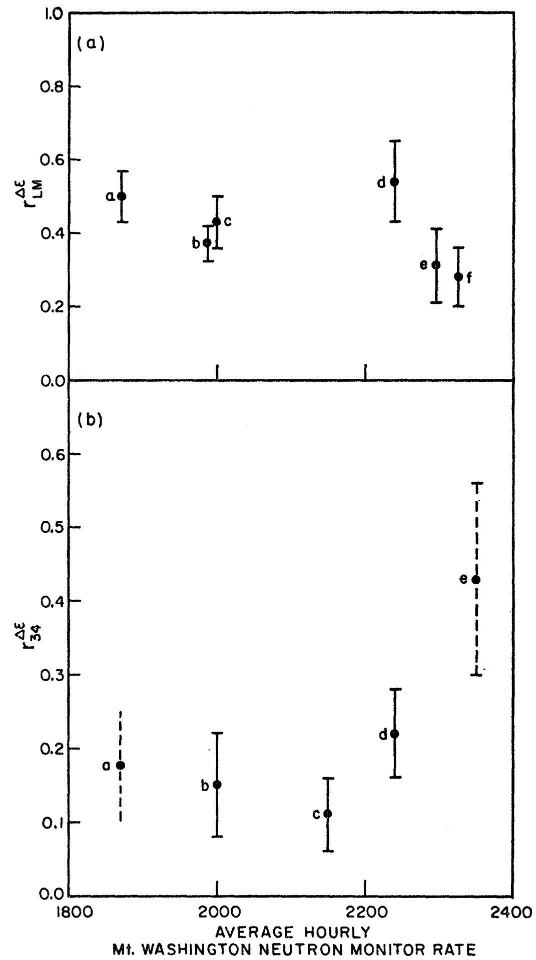

FIG. 4. The dependence of (a) $r_{LM}{}^{\Delta\epsilon}$ and (b) $r_{34}{}^{\Delta\epsilon}$ on the total cosmic-ray intensity as indicated by the Mt. Washington average hourly neutron monitor rate. $r_{LM}{}^{\Delta\epsilon}$ represents the ratio of light $(3\leq Z\leq 5)$ to medium $(6\leq Z\leq 9)$ nuclei in the energy/nucleon intervals $\Delta\epsilon$ of approximately 200–700 MeV/nucleon and $r_{34}{}^{\Delta\epsilon}$ the ratio of He^3 to He^4 in the energy/nucleon intervals of approximately 200–350 MeV/nucleon. The data here have been corrected for geomagnetic modulation. In Fig. 4(a), the point *a* is the ratio $r_{LM}{}^{\Delta\epsilon}$ obtained by Koshiba *et al.* (Ref. 3), *b* by Aizu *et al.* (Ref. 1), *c* by Foster and Debenedetti (Ref. 2), *d* by Badhwar *et al.* (Ref. 4), (assuming that the ratio $n_{HM}{}^{\Delta\epsilon}=0.32$), *e* by Evans (Ref. 55), and *f* by Balasubrahmanyan and McDonald (Ref. 26). In Fig. 4(b), the point *a* is the ratio $r_{34}{}^{\Delta\epsilon}$ obtained by Aizu (the standard deviation here has not been stated) (Ref. 50), *b* by Foster and Mulvey (Ref. 7), *c* by Hildebrand *et al.* (Ref. 49), *d* by Dahanayake *et al.* (Ref. 8), and *e* by Webber and Ormes by an indirect subtraction method, based on the comparison of proton and helium spectra at different latitudes (Ref. 51).

(5) The abundance ratio of light to medium elements $r_{LM}{}^{\Delta R}$ has been reported[1,44,57] for three rigidity intervals ΔR. The measurements indicate little or no variation in $r_{LM}{}^{\Delta R}$ with rigidity. If the path length $\bar{x}$ does not vary with rigidity, then these results favor a rigidity-dependent solar modulation mechanism [(d) of Sec. IIC].

(6) Evaluating the path length $\bar{x}$ from Eq. (11), using the data[1–3,7,49,50] at the time of high solar activity

[58] P. Meyer and R. Vogt, Phys. Rev. 129, 2275 (1963).

with the assumption of a velocity-dependent modulation, and correcting for the effects of geomagnetic modulation, yields a higher $\bar{x}$ from $r_{LM}^{\Delta\epsilon}$ than from $r_{34}^{\Delta\epsilon}$ by a factor of 1.5±0.6. If the path length $\bar{x}$ is evaluated from the abundance ratios $r_{LM}^{\Delta\epsilon}$ and $r_{34}^{\Delta\epsilon}$ under the assumption of rigidity-dependent solar modulation, approximately equal values of $\bar{x}$ are obtained (Sec. IIID).

(7) The ratio of He^3 to He^4, in the energy/nucleon interval $\Delta\epsilon$, $r_{34}^{\Delta\epsilon}$ may decrease with decreasing energy[54] (see Sec. IIIA). This would favor a rigidity-dependent solar modulation [(a) of Sec. IIC].

These considerations individually are sufficiently uncertain so that conclusions based upon each alone are rather weak. The uncertainties point up the need of further work. However, taken collectively these seven arguments each tend toward the same conclusion and thus the existing data support the predominance of a rigidity-dependent solar modulation mechanism.

If these data are considered sufficient evidence for a predominantly rigidity-dependent solar modulation, then the available proton and helium spectra at solar minimum[25–29] may indicate that the local interstellar helium and proton spectra differ in shape.

D. Interstellar Path Length, Local Interstellar Spectra and Transmission

Equations (11) and (12) may be solved to yield information on the mean path length of cosmic rays in interstellar matter, the relative transmission fractions in the geomagnetic field and interplanetary space, as well as the shape of the local interstellar cosmic-ray spectra at low values of energy.

As an applied example of these methods, the equations have been solved for two sets of available input data corresponding to different times in the solar cycle. The first set[49] corresponds to April 1961 at a geomagnetic latitude of $\lambda=55°$ and the second set[8] corresponds to August 1962 at $\lambda=70°$. The abundance ratios of helium isotopes are given in Table III. Values of light to medium element abundances of 0.55±0.1 and 0.4±0.1 correspond to the first and second sets, respectively, in the energy interval $\Delta\epsilon=200$–400 MeV/nucleon (see Table IV).

In this example we make the further simplifying assumptions and approximations:

(i) For small values of the traversed interstellar material $\bar{x}$, (below 8 g/cm^2) the diffusion relations reduce to

$$\rho_{34}(\bar{x})=c_{34}\bar{x}, \tag{16}$$

$$\rho_{LM}(\bar{x})=c_{LM}\bar{x}. \tag{17}$$

The constants c_{34} and c_{LM} are 0.035 and 0.091 cm^2/g, respectively, (see Appendix).

(ii) $f_{34}(R_4)=f_{LM}(R_M)=1$. The implications of equating these factors to unity are stated in the assumptions (a), (b), (c), and (d) at the beginning of Sec. IIB. If a considerable amount of fragmentation occurs in the source region, particularly at low rigidities, as has been suggested by Kaplon and Skadron,[45] these factors are not unity. A test of the assumptions is proposed at the end of Sec. IIC.

(iii) The geomagnetic transmission fractions $T^\lambda(R)$ are evaluated by comparing the helium spectrum of Shapiro *et al.*[48] at a geomagnetic latitude $\lambda=55°$ with that obtained by Fichtel *et al.*[25] at $\lambda=70°$ at a corresponding time in the solar cycle (1961), assuming that the geomagnetic effects are negligible in the latter case above 1.1 BV. It was estimated that in the interval of interest, $\langle T^\lambda(R_3)/T^\lambda(R_4)\rangle_{\text{av}}=0.6\pm0.2$. Assuming that the spectrum of medium nuclei is similar to that of helium, we obtained $\langle T^\lambda(R_L)/T^\lambda(R_M)\rangle_{\text{av}}=1.06\pm0.04$.

(iv) The solar modulation (or interplanetary) transmission fraction was assumed to be of the form suggested by Parker[17]:

$$T^I(R)=a\exp(-c_p/R^2\beta)\,, \tag{18}$$

in which a is a proportionality constant, c_p is a constant at a fixed time, representing various parameters of the magnetic fields of the solar system, and β is the velocity of the particle corresponding to rigidity R. The insensitivity of this assumption was tested by repeating the calculations with a purely rigidity-dependent transmission fraction. Both calculations yield similar results for the path length of cosmic rays in interstellar matter, and for the shape of the local interstellar spectra.[59]

Equation (11) is applied to the abundance ratios $r_{34}^{\Delta\epsilon}$ and $r_{LM}^{\Delta\epsilon}$ using the above assumptions and approximations.

Equation (18) is inserted into Eq. (11), setting $R_4=R_M=1.7$ BV. R_3 and R_L are evaluated from $R_s=K_{sp}R_p$. For each rigidity R_i the corresponding velocity β_i is calculated. This leaves c_p [the constant of Eq. (18)] and the interstellar path length $\bar{x}$ as the only unknowns to be determined.

(a) The mean path length of cosmic rays in interstellar matter is (at energies of about 300 MeV/nucleon) $\bar{x}=5.1\pm2$ g/cm^2. The data of Dahanayake *et al.*[8] when treated similarly, yield $\bar{x}=4.7\pm2$ g/cm^2. This value of mean interstellar path length is consistent with the value 2.7±0.5 g/cm^2 obtained at energies >700 MeV/nucleon.[42–44,1,3]

(b) The fraction of He^3 transmitted at a rigidity of about 1.3 BV relative to that of He^4 at a rigidity of 1.7 BV is $\langle T_3(R_3)/T_4(R_4)\rangle_{\text{av}}=0.38\pm0.2$ at the top of the atmosphere and $\langle T_3^I(R_3)/T_4^I(R_4)\rangle_{\text{av}}=0.63\pm0.3$ outside the earth's magnetosphere at a geomagnetic latitude $\lambda=55°$ in 1961. From Eq. (11) we obtain the result that $r_{34}^{\Delta\epsilon}$, the ratio of He^3 to He^4 in the energy/nucleon interval $\Delta\epsilon$, is less than $\rho_{34}(\bar{x})$, i.e., is less than

[59] B. Hildebrand and R. Silberberg, Regional IQSY Symposium, organized by the Consejo Latinoamericano de Radiacion Cosmica y Fisica del Espacio, 1964, Buenos Aires, Argentina (unpublished).

the ratio of He^3 to He^4 produced by fragmentation in a path length $\bar{x}$. The data of Dahanayake *et al.*[8] at $\lambda=70°$, in August 1962, indicate $\langle T_3{}^I(R_3)/T_4{}^I(R_4)\rangle_{av}\approx 1$, i.e., the effect of solar modulation on $r_{34}{}^{\Delta\epsilon}$ is negligible in the 160- to 370-MeV/nucleon energy interval at solar minimum.

(c) The fraction of light to medium elements transmitted at rigidities of $R_L=1.1\ R_M$ and $R_M=1.7$ BV, respectively, is $\langle T_L(R_L)/T_M(R_M)\rangle_{av}=1.2\pm0.2$. As anticipated from Eq. (11), we obtain $r_{LM}{}^{\Delta\epsilon}>\rho_{LM}(\bar{x})$, i.e., the abundance ratio of light to medium elements in the energy/nucleon interval $\Delta\epsilon$ is greater than would be produced by fragmentation in a path length $\bar{x}$.

The above transmission fractions and Eq. (18) lead to a value of $\langle T_3{}^I(R_3)/R_4{}^I(R_3)\rangle_{av}=1.18\pm0.15$ as the relative fraction of He^3 to He^4 transmitted through the interplanetary fields at a rigidity $R_3\approx1.3$ BV. Inserting this into Eq. (12) yields the following conclusions on the local interstellar rigidity spectrum of He^4:

(d) The relative intensities of He^4 in local interstellar space at $R_4=1.7$ BV to that at $R_3=\frac{3}{4}R_4$ is $\langle S_4(R_4)/S_4(R_3)\rangle_{av}=0.6\pm0.4$. Assuming a power-law spectrum of the form

$$S(R)=bR^{-q}, \tag{19}$$

where b is a constant in the interval of interest (1.2–1.7 BV), the above value corresponds to $q=1.8\pm2$. The data of Dahanayake *et al.*[8] lead to $\langle S_4(R_4)/S_4(R_3)\rangle_{av}=1.5\pm0.8$. The latter value appears too high, since the observed He^4 spectrum of Ormes and Webber[28] near solar minimum yields $\langle 0_4(R_4)/0_4(R_3)\rangle_{av}=1.0$ and any residual solar modulation would imply that $\langle S_4(R_4)/S_4(R_3)\rangle_{av}$ has a value less than 1.0. The preliminary analysis of the unpublished data of the NRL group obtained in summer 1963 at a geomagnetic latitude of 70° is in agreement with the value of $\langle S_4(R_4)/S_4(R_3)\rangle_{av}$ as deduced from the 1961 data of the NRL group. Thus the local interstellar spectrum tends to increase as the rigidity decreases. This result is not inconsistent with the rigidity dependence ($q=2.5$) observed at higher energies.[60] The slope of the local interstellar spectrum in the low-energy region ($q=1.8\pm2$) remains uncertain because of inadequate statistics and is also consistent with a flat spectrum ($q\approx0$). A determination of the He^3/He^4 ratio at high geomagnetic latitude, at solar minimum, will help to determine the shape of the local interstellar He^4 spectrum $S_4(R)$.

IV. SUMMARY

(1) The analysis of the abundance ratios of cosmic rays which are generically related can be used to separate the effects of source and acceleration mechanisms, fragmentation in interstellar space, transmission through the interplanetary fields, and transmission through the geomagnetic field.

(2) Variations in the experimentally determined He^3/He^4 and L/M cosmic-ray abundance ratios are explained in terms of variations in geomagnetic transmission and in solar modulation. The He^3/He^4 ratio in the energy/nucleon interval 0.2 to 1.0 BeV/nucleon decreases with increasing geomagnetic modulation, an increasing rigidity-dependent solar modulation, and decreasing energy. The L/M ratio changes in the opposite way.

(3) At a geomagnetic latitude at 55° and with a solar modulation as in April 1961, the relative transmission of He^3 to He^4 through the combination of interplanetary and geomagnetic fields (in the energy/nucleon interval 250–350 MeV/nucleon) was found to be

$$\langle T_3(R_3)/T_3(R_4)\rangle_{av}=0.38\pm0.2.$$

The transmission fraction through the geomagnetic field was estimated to be $\langle T^\lambda(R_3)/T^\lambda(R_4)\rangle_{av}=0.6\pm0.2$, and through the solar system fields $\langle T_3{}^I(R_3)/T_4{}^I(R_4)\rangle_{av}=0.63\pm0.3$.

(4) The effects of transmission must be considered in the evaluation of mean interstellar path length from low-energy cosmic-ray data. The application of the analysis based upon the limited available data yields a mean interstellar path of 5 ± 2 g/cm² in the region 200–400 MeV/nucleon. This result is consistent with the results of measurements at higher energies. It is not necessary to assume a longer path length in interstellar matter for the low-energy particles to explain the increasing ratio of light to medium elements in the same energy/nucleon interval with decreasing energy, although the possibility of an increase in path length by a factor of 2 with decreasing energy cannot be excluded.

(5) The intensity of cosmic-ray helium per unit rigidity interval at 1.7 BV in local interstellar space (i.e., before solar modulation) is 0.6 ± 0.4 of that at 1.3 BV. If the local interstellar helium spectrum in a narrow rigidity interval is approximated by $1/R^q$, then $q=1.8\pm2.0$ in the region 1.2–1.7 BV. This result is not inconsistent with the value of $q=2.5$ for higher-energy cosmic rays, though the result is also consistent with a relatively flat spectrum.

(6) The analysis permits the determination of the degree of residual solar modulation at solar minimum, the velocity or rigidity dependence of the solar modulation, and the modulations occurring in the cosmic-ray-source regions. The available data favor a rigidity-dependent solar modulation over a velocity-dependent one above a rigidity of 1.2 BV.

(7) Statistically meaningful experiments on the ratios of He^3/He^4 and L/M are required at high geomagnetic latitudes, especially at solar minimum for a good determination of local interstellar spectra.

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[60] F. B. McDonald and W. R. Webber, Phys. Rev. **115**, 197 (1959).

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APPENDIX: EVALUATION OF $\rho_{sp}(\bar{x})$

$\rho_{sp}(\bar{x})=N_s(\bar{x})/N_p(\bar{x})$ is the ratio of the differential intensities of fragmented daughter isotopes or elements s, to that of the predominant parent species p, after a passage through $\bar{x}$ g/cm^2 of interstellar material. Both $N_s(\bar{x})$ and $N_p(\bar{x})$ are evaluated by use of the one-dimensional diffusion equations.[31,32,39]

Table VI presents the assumed relative source composition used,[42,43] for the very heavy elements with $Z\geq 20$, heavy elements (H) with $10\leq Z\leq 19$, medium elements (M) with $6\leq Z\leq 9$, light elements (L) with $3\leq Z\leq 5$, and for He^4 and He^3. The numerical values of the inelastic-collision mean free paths, also given in Table VI, are obtained from the work of Badhwar *et al.*[42,43] except for the λ_4 and λ_3. The value of λ_4 is taken from the experimental work of Riddiford and Williams[61] while λ_3 has been assumed to be equal to λ_4.

TABLE VI. Assumed relative source composition and inelastic-collision mean free paths in hydrogen.

Species	Very heavy	Heavy	Medium	Light	He^4	He^3
Relative abundance	27	51	190	0	2510	0
λ (g/cm^2)	2.62	4.10	6.55	8.7	17.9	17.9

Table VII gives the fragmentation parameters for inelastic collision with hydrogen P_{ij}. Again, the numerical values are obtained from the work of Badhwar *et al.*[42,43] except for P_{LL}, P_{L4}, and P_{L3}. The latter have been assigned on the basis of the experimental data of Friedlander *et al.*[41]

TABLE VII. Fragmentation parameters for inelastic collision with hydrogen.

Parent elements or isotopes	Daughter elements or isotopes: Very heavy	Heavy	Medium	Light	He^4	He^3
Very heavy	0.44	0.30	0.09	0.12	0.598	0.113
Heavy		0.28	0.32	0.27	0.795	0.122
Medium			0.32	0.48	0.611	0.224
Light				0.25	1.1	0.4
He^4						0.599

The relationships between ρ_{sp} and $\bar{x}$ can be approximated by $\rho_{LM}=c_{LM}\bar{x}$ with $c_{LM}=0.091$ and $\rho_{34}=c_{34}\bar{x}$ with $c_{34}=0.035$ for $\bar{x}<8$ g/cm^2. The presence of components other than hydrogen in interstellar gas has been neglected.

[61] L. Riddiford and A. W. Williams, Proc. Roy. Soc. (London) **257**, 316 (1960).