Deformation of the Transition-State Nucleus in Energetic Fission*

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Fission-fragment anisotropy ratios, $W(170°)/W(90°)$, from fission induced by 42.8-MeV helium ions, have been measured for the targets Th^{230}, Th^{232}, Pa^{231}, U^{233}, U^{234}, U^{235}, U^{236}, U^{238}, Np^{237}, Pu^{239}, Pu^{240}, Pu^{242}, Am^{241}, Am^{243}, Cm^{244}, and Cf^{249}. By modifying values of the fission-to-neutron level-width ratio Γ_f/Γ_n from the literature, first-chance anisotropies were calculated to permit comparison of the targets at nearly uniform and rather high excitation energies. The resulting values of K_0^2 (the projection K of the total angular momentum I on the nuclear symmetry axis is assumed to have a Gaussian distribution, and K_0^2 is the squared standard deviation of the Gaussian) associated with first-chance fission in conjunction with the assumptions of rigid moments of inertia permitted evaluation of saddle deformations. The saddle deformations were found to be fairly insensitive to programmed variations in Γ_f/Γ_n values based on a semiempirical relation deduced for the (Z,A) dependence of the compound nucleus. The unrelieved disparity for the heaviest elements between the experimentally derived saddle deformations and those theoretically deduced from the conventional liquid-drop model has been interpreted to suggest a re-evaluation of the fissionability parameter, $(Z^2/A)_{\text{crit}}$. Qualitative extrapolation of the new data (assuming a nuclear level-density parameter $a=A/8$) yields $(Z^2/A)_{\text{crit}}\approx 44$–$45$ for saddle deformations based on first-chance anisotropies computed from experimental data (deleting higher chance fission contributions). This value compares favorably with results of a modification of the conventional liquid-drop model, which allows the surface tension to vary with the curvature of the nuclear surface.

I. INTRODUCTION

In heavy-element $(Z>90)$ fission, the compound-nucleus cross section is strongly dominated by fission and neutron emission for moderate excitation energies (<50 MeV). The fission-to-neutron level-width ratio, Γ_f/Γ_n, for the two competing processes is a rather sensitive function of the charge and mass of the target and for a particular target appears, however, to be rather independent of excitation energy.[1]

If a series of heavy-element targets is bombarded with helium ions of moderate but fixed energy, the variation of Γ_f/Γ_n with target species due to differences in excitation energy is held to a minimum. Since the net angular distribution of fission fragments is determined in part by Γ_f/Γ_n values for the several branches of a fission chain, the measurement of fragment anisotropies for various targets under these conditions provides an indirect means of deducing a possible dependence of Γ_f/Γ_n on Z and A, the charge and mass number, respectively, of the compound nucleus.

Implications for nuclear structure are least ambiguous when anisotropy measurements are made from a single fissioning species. In practice, however, the measurement of fragment anisotropies for specific fissioning species is complicated by the fact that for a given excited compound nucleus, fission may take place following the emission of one or more neutrons. This possibility of multichance fission leads to a composite angular distribution produced by several co-fissioning species. Moreover, for second- and later-chance fission the semi-Maxwellian spread in energy of the emitted neutrons leads to variation in the excitation energy of same-chance fissioning nuclei.

These complications make it very difficult to resolve, by experimental means, the gross angular distribution pattern into the component distributions from various multi-chance fission events. An indirect method of resolution consists of synthesizing a gross anisotropy from various-chance fission events by theoretical means, which is made to fit the experimentally measured anisotropy by variation of the appropriate parameters. The characteristics of first-chance fission events are then isolated to permit systematic comparison of different target species.

The research reported herein entails measurement of fission-fragment anisotropies from fission induced in heavy-element targets up to Cf^{249} by constant-energy helium ions. The increasing availability of trans-neptunium elements of high purity makes possible the use of a wider range of targets in this work than in similar experiments previously reported.[2,3] An empirical relation for the dependence of Γ_f/Γ_n on (Z,A) of the compound nucleus is deduced, and the parameters for a smooth fit are determined by means of a computer program. The first-chance anisotropies obtained in the computer calculation are then used to determine the saddle deformation at an essentially constant excitation energy for all of the target species studied. The saddle deformations are extrapolated to a new value for $(Z^2/A)_{\text{crit}}$, which is compared with a revised value based on a modification of the conventional liquid-

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[1] J. R. Huizenga and R. Vandenbosch, *Nuclear Reactions* (North-Holland Publishing Company, Amsterdam, 1962), Vol. 2, Chap. 2, p. 42.
[2] C. T. Coffin and I. Halpern, Phys. Rev. **112**, 536 (1958).
[3] R. Chaudhry, R. Vandenbosch, and J. R. Huizenga, Phys. Rev. **126**, 220 (1962).

drop model[4,5] to account for the variation of the nuclear surface tension as a function of the curvature of the nuclear surface.[6]

II. EXPERIMENTAL RESULTS

In the preparation of targets, target material was laid down on thin nickel foil of 100 $\mu g/cm^2$ or 150 $\mu g/cm^2$ thickness by volatilization or evaporation. The volatilization technique involved successive "flashings" of heavy-element nitrates or chlorides off a tantalum filament under high-vacuum ($\sim 10^{-6}$ mm Hg) conditions. The evaporation procedure utilized the general solubility of heavy-element chlorides in ethyl alcohol and was used for those isotopes of dangerously high specific activity. Prior to evaporation the target material was purified by ion-exchange elution if necessary and evaporated to dryness under dry-air jet. The salt was taken up in a minimal amount of water-free ethyl alcohol, which alcohol solution was evaporated as necessary to contain 3–5-μg target material per λ of solution. The target was prepared by depositing $\sim 3\lambda$ of ethyl alcohol solution on Ni foil from a micropipette and allowing to dry in air. Target thicknesses obtained by both methods were of the order of 10 $\mu g/cm^2$. Isotopic purity of the targets was better than 99% for targets through Pu^{239} and at least 90% for targets Pu^{240} and heavier.

Induced-fission experiments were carried out in an 18-in. scattering chamber with helium ions accelerated to the full energy, 42.8 MeV, of the 60-in. Argonne fixed-frequency cyclotron. In order to minimize error due to possible beam wobble, the plane of the target was oriented so as to bisect the angle between the two detectors. Total fission fragment anisotropy was measured by detectors set at 89° and 170° in the laboratory system of reference. Semiconductor surface-barrier detectors were used along with general experimental techniques described in previous reports,[3,7] with the following modification in the electronics system: Fission fragment pulses generated in the detectors were amplified in charge-sensitive preamplifiers and fed into separate amplifiers and multichannel pulse-height analyzers. Simultaneous control of the two analyzing systems was obtained by means of a common gating switch for the two analyzers and an elapsed-time clock. Dead-time measurements for the two analyzing systems were obtained by recording in each analyzer pulses from a common single generator capacitatively coupled into each detector input. Use of the two independent analyzers eliminated cross signals otherwise possible with a single analyzer operated in split-memory mode.

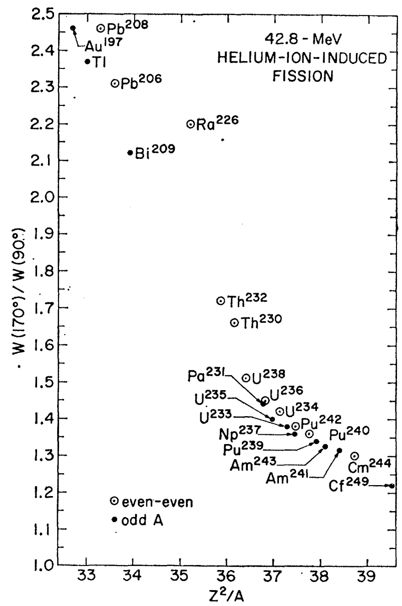

FIG. 1. Variation of fission-fragment anisotropies for helium-ion-induced fission in various target nuclides (as labeled) as a function of Z^2/A for the compound nucleus.

Following previously described methods,[7] differential cross section ratios were calculated for each target by summing counts in the fission peaks in the analyzer read-outs. These results obtained with 42.8-MeV helium ions for Th^{230}, Th^{232}, Pa^{231}, U^{233}, U^{234}, U^{235}, U^{236}, U^{238}, Np^{237}, Pu^{239}, Pu^{240}, Pu^{242}, Am^{241}, Am^{243}, Cm^{244}, and Cf^{249} are shown in Fig. 1, where the ratio of the count rates at 170° and 90°, $W(170°)/W(90°)$, in center of mass coordinates, is plotted as a function of Z^2/A for the compound nucleus. The quantity Z^2/A is proportional to the relative magnitudes of the electrostatic and surface energies of the compound nucleus and in the theory of the charged liquid drop model, relates to the fissionability of the compound nucleus. For comparison with lighter elements the $W(170°)/W(90°)$ anisotropies of Bi^{209} and Pb^{208} were also measured for helium-ion induced fission. These, together with Au^{197}, Tl, Pb^{206}, and Ra^{226} data from previous work,[3,8] are also included in Fig. 1.

For bismuth and lighter elements, the anisotropy is essentially that due to first-chance fission because the contribution of higher chance fission is negligible. For radium and heavier targets the anisotropy derives from fission events following any one of several generations of neutron emission, with first-chance fission again dominating for those species with largest values of Z^2/A. Further, the excitation energy available for fission in lead and bismuth is reduced because of the

[4] S. Cohen and W. J. Swiatecki, Ann. Phys. (N. Y.) **22**, 406 (1963).

[5] V. M. Strutinskii, N. Ya. Lyashchenko, and N. A. Popov, Nucl. Phys. **46**, 639 (1963); V. M. Strutinskii, Zh. Eksperim. i Teor. Fiz. **45**, 1900 (1963) [English transl.: Soviet Phys.—JETP **18**, 1305 (1964)].

[6] V. M. Strutinskii, Zh. Eksperim. i Teor. Fiz. **45**, 1891 (1963) [English transl.: Soviet Phys.—JETP **18**, 1298 (1965)]; private communication.

[7] R. Vandenbosch, H. Warhanek, and J. R. Huizenga, Phys. Rev. **124**, 846 (1961).

[8] J. E. Gindler, G. L. Bate, and J. R. Huizenga, Phys. Rev. **136**, B1333 (1964).

large amount of deformation energy expended in reaching the saddle point before fission. This improverishment of excitation energy accounts in part for the relatively large anisotropies exhibited in the figure by lead and bismuth by virtue of the copious amount of angular momentum brought in by 42.8-MeV helium ions.

On the other hand, from radium on up through the heavy elements, the fission thresholds are much smaller, with the result that the excitation energies are substantially larger than those of lead and bismuth and moreover tend to be approximately constant in value. The larger values of anisotropy for some of these elements are accounted for principally by contributions from later-chance fission events for which the excitation energy has been depleted by neutron boil-off. In this way the anisotropies for such targets as Ra^{226}, Th^{230}, and Th^{232} are built up primarily from the larger anisotropies associated with later-chance fission.

III. DISCUSSION

A more useful comparison of the fragment anisotropies would be provided if the anisotropies for first-chance fission alone were obtainable. Since resolution of various-chance anisotropies is not yet possible experimentally, an indirect method involving calculation of gross anisotropy as synthesized from multi-chance fission events is employed here. In order to synthesize the experimentally measured anisotropy from various-chance fission events, a computer program, MUCHFAN, was developed for calculating multi-chance fission anisotropy. This program incorporates the basic equations for calculation of excitation energy as developed in a previous report[8] for Ra^{226} anisotropies from helium-ion-induced fission. Anisotropy ratios in this program are calculated from the relation[9]

$$\frac{W(\theta)}{W(90°)}=\frac{\sum_{l=0}^{l_m}[l(2l+1)T_l]e^{-(\beta I^2\sin^2\theta)/2}J_0(\tfrac{1}{2}i\beta I^2\sin^2\theta)}{\sum_{l=0}^{l_m}[l(2l+1)T_l]e^{-\beta I^2/2}J_0(\tfrac{1}{2}i\beta I^2)}. \quad (1)$$

Transmission coefficients T_l were computed from the optical-model ABACUS II program[10] and supplied to the MUCHFAN program as input data.

In addition, an allowance is made in the MUCHFAN program for the distribution in energy of neutrons emitted before fission takes place. While the neutron energy distribution does not affect first-chance fission, a spread in the excitation energies for second- and higher chance fission is introduced which slightly modifies the corresponding anisotropies. In the program, excitation energies for second- and higher chance fission were incremented in fractions of MeV corresponding to stepwise integration under the neutron energy curve based on the semi-Maxwellian distribution of Blatt and Weisskopf,[11] $n(\epsilon)\sim\epsilon e^{-\epsilon/T}$, where T is the maximum nuclear temperature of the nuclide remaining after emission of a spectrum of neutrons of variable energy ϵ.

Weighting factors for the angular distributions from various-chance fission events were initially based on Γ_f/Γ_n ratios obtained from previously published estimates.[12] As applied to a given chain of neutron evaporation and fission branching, the Γ_f/Γ_n ratios were assumed to be independent of excitation energy and spin of the compound nucleus. While these assumptions are not strictly valid, the errors introduced by them in the present results are not serious. The anisotropies of most of the reported nuclei are strongly dominated by first-chance fission so that possible errors deriving from higher chance fission are substantially reduced. With a given set of Γ_f/Γ_n values, the anisotropies were calculated and compared with the experimentally observed values. A systematic variation of K_0^2 was programmed into the computer which changed the value of K_0^2 used in the anisotropy calculation of Eq. (1) until the calculated net anisotropy agreed with the experimental value.

In order to test the sensitivity of the calculated angular distributions to possible uncertainties in the values of Γ_f/Γ_n employed, a semiempirical expresssion

$$\Gamma_n/\Gamma_f=\alpha e^{(\beta Z^2/A)+\gamma A]} \quad (2)$$

was deduced, which assumes an exponential dependence on (Z,A) of the compound nucleus. By varying the parameters α, β, and γ, different values of Γ_n/Γ_f were programmed into the anisotropy calculation.

Equation (2) represents the essential form of the (Z,A) dependence of Γ_n/Γ_f shown in Fig. 2 of Ref. 12. A theoretical justification of Eq. (2) is partially suggested by the use of a constant-temperature level-density model of the nucleus in a small and fixed energy interval where the nuclei are compared. Hence with certain approximations[1] one obtains

$$\Gamma_n/\Gamma_f=(2TA^{2/3}/k)e^{[(E_f'-B_n')/T]}, \quad (3)$$

where T is the nuclear temperature, k is a constant, and E_f' and B_n' are the respective fission threshold and neutron binding energies corrected for the energy gap to the excited level threshold. E_f' and B_n' may be written as (Z,A) dependent in the form

$$E_f'=C_1-C_2(Z^2/A), \quad (4)$$
$$B_n'=C_3-C_4A,$$

[9] G. L. Bate, R. Chaudhry, and J. R. Huizenga, Phys. Rev. **131**, 722 (1963).

[10] The authors are indebted to E. H. Auerbach and C. E. Porter of Brookhaven National Laboratory for the revised version ABACUS II optical-model program.

[11] J. M. Blatt and V. F. Weisskopf, *Theoretical Nuclear Physics* (John Wiley and Sons, Inc., New York, 1952), p. 368.

[12] R. Vandenbosch and J. R. Huizenga, *Proceedings of the Second United Nations International Conference on the Peaceful Uses of Atomic Energy* (United Nations, Geneva, 1958), Vol. 15, p. 284, paper P688.

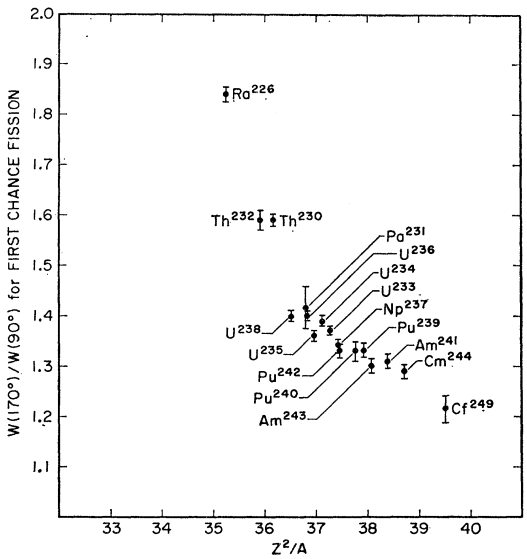

FIG. 2. First-chance anisotropies calculated for synthesis of experimental values of total anisotropy by variation of parameters to minimize fluctuations, as a function of Z^2/A for the compound nucleus.

where the C's are constant but Z-dependent in the second of the two equations. Substitution of Eqs. (4) into (3) then gives an exponential function of the form (2), but with the A dependence entirely compressed into the exponent. The close relationship of Eq. (2) via Eqs. (4) to Eq. (3) supports its application in this experiment to a series of targets at essentially constant excitation energy for first-chance fission but does not fully account for its validity with different Z values.

Initial values of the parameters α, β, γ in Eq. (2) were determined by approximation to the curves (Fig. 2) of Vandenbosch and Huizenga.[12] Although the angular distributions were found to be relatively insensitive to subsequent variations in the parameters, the distribution of first-chance anisotropies, $W(170°)/W(90°)$, with Z^2/A tended to be smoother for certain values of the parameters.

The calculated first-chance anisotropies showing the least scatter on manual variation of the parameters are shown in Fig. 2 as a function of Z^2/A. The experimental errors are reproduced in Fig. 2 since the largest contributions derive from first-chance fission, especially for the heavier elements. The anisotropies shown for the targets Th^{230} through Cf^{249} were calculated with parameter values $\alpha=13\,000$, $\beta=-1.2$, and $\gamma=0.14$. The value for Ra^{226} was calculated with parameters based on data from a previous report.[8]

It is of particular interest to use the values of K_0^2 concomitant with first-chance anisotropies, for calculation of saddle deformations. A theoretical development of the Fermi-gas model leads to the relation $K_0^2=t\mathscr{I}_{\text{eff}}/\hbar^2$ where t is the thermodynamic temperature and $\mathscr{I}_{\text{eff}}$ is the effective moment of inertia of the fissioning nucleus at the saddle point:

$$\mathscr{I}_{\text{eff}}=\mathscr{I}_{\parallel}\mathscr{I}_{\perp}/(\mathscr{I}_{\perp}-\mathscr{I}_{\parallel}), \tag{5}$$

where $\mathscr{I}_{\perp}$ and $\mathscr{I}_{\parallel}$ are the moments of inertia about an axis perpendicular and parallel to the fission axis, respectively. If the projection K of the total angular momentum I on the nuclear symmetry axis is assumed to have a Gaussian distribution, then K_0^2 is the squared standard deviation of the Gaussian. Assuming an energy-temperature relation,[13]

$$E=at^2-t, \tag{6}$$

where $a=A/8$ and A is the conventional mass number, the effective moment of inertia may be calculated from the values of K_0^2 computed for the first-chance anisotropies shown in Fig. 2. Application of the Fermi-gas model with a rigid moment of inertia to the present results is warranted on the basis of the moderately large excitation energies at which $\mathscr{I}_{\text{eff}}$ is measured. Under these conditions, pairing correlations should be relatively unimportant and the moment of inertia nearly equal to the rigid-body value.

The saddle deformation is calculated as the ratio $\mathscr{I}_{\text{sph}}/\mathscr{I}_{\text{eff}}$, where $\mathscr{I}_{\text{sph}}$ is the rigid moment of inertia of a sphere of equal volume. The radius parameter used for calculation of the rigid moment is 1.216×10^{-13} cm. The saddle deformations calculated in this manner for the various target nuclides are shown in Fig. 3 as a function of Z^2/A. The errors indicated correspond to the experimental uncertainties in the measurement of the $W(170°)/W(90°)$ anisotropies due to counting statistics.

The solid curve shown in Fig. 3 represents the theoretical dependence of $\mathscr{I}_{\text{sph}}/\mathscr{I}_{\text{eff}}$ on Z^2/A, based on charged liquid-drop model calculations by Cohen and Swiatecki[4] and by Strutinskii,[5] using a value of $(Z^2/A)_{\text{crit}}=50.1$. On comparing the saddle deformations from first-chance anisotropies with the theoretical curve, it is seen that for the heavy elements the former points fall consistently below the theoretically predicted locus by margins considerably in excess of experimental error. In particular, the disparity is not alleviated for the heaviest elements, where points from the new data of Pu^{240}, Pu^{242}, Am^{241}, Am^{243}, Cm^{244}, and Cf^{249} are shown. It may be emphasized that this conclusion stands independent of possible small errors in the computed first-chance anisotropies because of the large values of Γ_f/Γ_n for these heavy nuclides.

The unrelieved disparity between the experimentally deduced saddle deformations and those theoretically predicted, especially for the heaviest elements, may be interpreted to suggest the need for a re-examination of the theoretical model. In particular, saddle deformations obtained with the conventional liquid-drop model are, in making comparison with the experimental data,

[13] K. J. LeCouteur and D. W. Lang, Nucl. Phys. 13, 32 (1959).

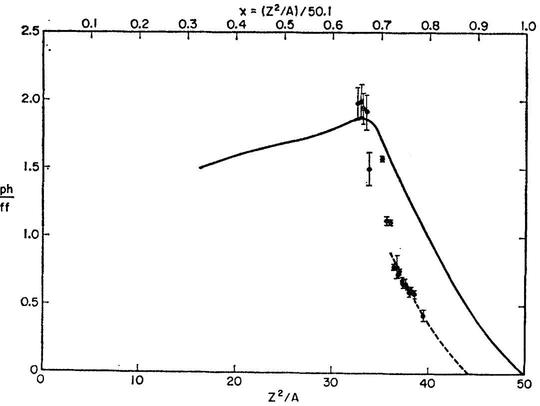

FIG. 3. Saddle deformations computed from first-chance anisotropies as a function of Z^2/A for the compound nucleus. Solid curve represents theoretical predictions based on charged liquid-drop-model calculations using $(Z^2/A)_{crit}=50.1$. Dashed curve shows extrapolation from Pa^{231} and heavier elements to zero saddle deformation for revised estimate of $(Z^2/A)_{crit}$.

particularly sensitive in the heavy-element region to the parameter $(Z^2/A)_{crit}$, which relates to the surface-tension energy of the nucleus.

In the region of the heaviest targets, it is possible to remove the above-mentioned disparity between ex-experimentally and theoretically deduced saddle deformations by reducing the level density parameter a of the transition state nucleus. Hence, calculations of $\mathscr{I}_{eff}$ have been performed also with $a=A/20$. The resulting values $\mathscr{I}_{sph}/\mathscr{I}_{eff}$ for the nuclei with the largest values of Z^2/A are approximately midway between the plotted points and solid line of Fig. 3. For example, $\mathscr{I}_{sph}/\mathscr{I}_{eff}$ for Fm^{253} changes from $0.42(a=A/8)$ to $0.72(a=A/20)$. These values are to be compared with the theoretical value of 1.05. Although it is possible by this arbitrary procedure to bring some of the experimental data into agreement with existing theory, it is not possible to obtain agreement with all data. Already for $a=A/20$ the values of $\mathscr{I}_{sph}/\mathscr{I}_{eff}$ for the lightest elements are much larger than those of theory. Over-all agreement between experiment and theory can be obtained only if a is a strong function of A. With the relationship $a=A/c$, c must increase from about 8 for the lightest elements to something greater than 40 for the heaviest elements. This is not a very likely possibility since it is known that for deformed nuclei (and the transition state nuclei are deformed nuclei) the relationship $a=A/8$ appears to hold over most of the periodic table.[13]

In order to investigate the optimum value of $(Z^2/A)_{crit}$ for a fit to the data of Fig. 3 (assuming $a=A/8$), a graphical comparison of the data with the solid curve was made, in which the squares of the lateral deviations were minimized. Weighting factors were assigned in proportion to the fission to neutron branching ratio for each target, and in inverse proportion to the statistical uncertainty of the experimental data. The targets Ra^{226}, Th^{230}, and Th^{232} were not used in the comparison because the derived first-chance fission anisotropies of these nuclei have larger uncertainties due to the importance of multichance fission in this region.

The results of such a least-squares analysis are shown as the dashed curve of Fig. 3. The curve fitted to the data is extrapolated to $(Z^2/A)_{crit}=44.3$ required by the conditions of the fit, which corresponds to zero saddle deformation ($\mathscr{I}_{eff}\rightarrow\infty$). The qualitative nature of this result should be emphasized, subject as it is not only to reading errors in the graphical comparison, but also to error arising from an assumed constant dilation factor over the region analyzed. Moreover, this type of extrapolation is not independent of theoretical model, since the deviations in the analysis were measured from the solid-lined curve shown in the figure for the conventional liquid-drop model. Nevertheless, the fitted curve and extrapolation probably do not represent a serious perturbation from the true function, and the accuracy remains such as to raise serious question concerning the validity of $(Z^2/A)_{crit}=50.1$ used hitherto in conventional liquid-drop-model calculations.

An approximate upper limit for $(Z^2/A)_{crit}$ based on the experimental data (assuming $a=A/8$) may be obtained by basing the extrapolation on a curve fitted to saddle deformations calculated directly from the experimental anisotropies. Such a calculation assumes 100 percent first chance fission and thus leads to inflated deformations because the contributions from higher chance fission are actually associated with smaller values of K_0^2. The extrapolation for least-squares fit to saddle deformations based on uncorrected experimental anisotropies gives $(Z^2/A)_{crit}=44.9$. This may be regarded as an approximate experimental upper limit for $(Z^2/A)_{crit}$ not only because of the neglect of higher chance fission contributions to the anisotropies, but also because the process of curve fitting does not accommodate the small increase in curve slope for decreasing $(Z^2/A)_{crit}$. However, it should be emphasized that it is not possible at this time to justify completely the use of the equation $a=A/8$ for the level density parameter of the transition-state nucleus. Use of $a=A/20$ gives $(Z^2/A)_{crit}$ of about 47–48. From a measurement of the fission barrier of Tl^{201}, Burnett *et al.*[14] deduced a value of 48.4 ± 0.5 MeV for $(Z^2/A)_{crit}$.

Strutinskii[6] has modified the conventional (constant-surface-tension) liquid-drop model to account for the blurring of the edge of the nucleus by means of a parameter Γ which introduces a perturbation in the nuclear surface tension, from the conventionally assumed constant value, as a function of the mean curvature of the effective nuclear surface. A value of $\Gamma=-0.1$ was derived from symmetric fission data in the element

[14] D. S. Burnett, R. C. Gatti, F. Plasil, P. B. Price, W. J. Swiatecki, and S. G. Thompson, Phys. Rev. 134, B952 (1964).

region Pb-Po. For the nuclear radius parameter $r_0=1.2\times10^{-13}$ cm and $\Gamma=-0.1$, Strutinskii calculates $(Z^2/A)_{\rm crit}\approx 43\text{-}47$. This value compares favorably with the values deduced above as extrapolations of the best fits for experimentally measured saddle deformations in the heavy-element region. This agreement lends further support to the validity of the modified liquid-drop model of the nucleus, which allows for the surface tension to vary with the curvature of the nuclear surface.

ACKNOWLEDGMENTS

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Isobaric Analogue States in Heavy Nuclei. I. Molybdenum Isotopes*

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Isobaric analogue states observed as compound-nucleus resonances in proton elastic scattering and (p,n) reactions have been studied using the target isotopes Mo^{92}, Mo^{94}, Mo^{95}, Mo^{96}, Mo^{97}, Mo^{98}, and Mo^{100}. The observed resonances which occur at high excitations in the target-plus-proton system have been analyzed using a Coulomb-plus-single-level formula. The spectroscopic information for these resonances is compared with the analogue states in the target-plus-neutron system as observed in (d,p) reactions on the same targets. Level separations, l-value determinations and corresponding spectroscopic factors show remarkable agreement with the (d,p) data. A survey of the experimentally determined Coulomb displacement energies (ΔE_c) has been performed via the relation $\Delta E_c=C\,Z/A^{1/3}$. It is found that C is fairly constant for all the molybdenum isotopes.

1. INTRODUCTION

RECENTLY, the existence of isobaric analogue states at very high excitations in heavy nuclei was verified by observing them as compound-nucleus resonances.[1,2] A detailed description[3] of isobaric analogue resonances has shown that their existence can be understood within the framework that has been used to explain the occurrence of isobaric analogue states formed by the (p,n) charge-exchange mechanism.[4] Consequently, the existence of analogue states as compound states is to be expected in all nuclei.

Because of the simple relation between states belonging to the same isobaric spin multiplet, it follows that there is also a simple relation between the resonances observed in proton-induced reactions and the corresponding states seen in neutron capture via (d,p) stripping on the same target nucleus. Information gained by observing analogue resonances in proton-induced reactions is essentially equivalent to the information gained from (d,p) stripping, so that proton reactions via analogue states constitute an alternative method for nuclear-spectroscopic studies.

The present paper is the first of a series of papers in which the possibility of using analogue-state resonances as a tool in nuclear spectroscopy is exploited. The experimental results obtained here for the seven molybdenum target isotopes Mo^{92}, Mo^{94}, Mo^{95}, Mo^{96}, Mo^{97}, Mo^{98}, and Mo^{100} concern the resonances in the corresponding technetium isotopes produced as compound systems in proton-induced reactions. This study yields information on the spins and parities of the low-lying states of the Mo isotopes belonging to the same isobaric multiplets as the resonances in the compound system. A comparison between the present results and the available (d,p) data illustrates how powerful a technique the analogue-state method of studying nuclear spectra can be in practice.

The observations of analogue-state resonances provide a measure of Coulomb energy which is more accurate than previous methods of measuring this quantity. The systematics of Coulomb energies for the Mo isotopes are discussed in Sec. 4.

2. THEORY

Elastic scattering of protons through nuclear states can be a useful means for measuring the properties of

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