Collective Excitations in Pb²⁰⁷, Pb²⁰⁸, and Bi²⁰⁹[†]

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The nuclei Pb²⁰⁷, Pb²⁰⁸, and Bi²⁰⁹ are investigated by inelastic scattering of 42-MeV α particles. The previously noted similarity in the excitation of these three nuclei is studied in detail. The excitation of collective levels is strongly favored in this experiment. The measured angular distributions show equally strong octupole levels in the three nuclei, at 2.6-MeV excitation. Levels in Pb208 around 3.2 MeV are found with the same strength as the known 5⁻ level at 3.198 MeV in Pb²⁰⁸. The previously reported level at 4.3 MeV in Pb²⁰⁸ is shown to consist of two levels at 4.12 and 4.40 MeV and their angular distributions favor a 2⁺ assignment. These levels are also found in Pb207 with the same intensity as in Pb208. The conclusion is drawn that the excitations in these three nuclei are due to the collective excitations of the Pb²⁰⁸ core and that the coupling to the Pb²⁰⁸ core of a single proton or a single neutron hole is very weak. Collective parameters are extracted using the Austern-Blair model which is shown to give good agreement with the measured angular distributions. The Coulomb-excitation contribution to the inelastic scattering is demonstrated by extending the angular-distribution measurements to small angles.

INTRODUCTION

T has been known for several years that inelasticalpha-scattering experiments provide a good tool for the study of collective nuclear excitations.¹ It seemed interesting to use this tool to study the collective excitations in a spherical and double magic nucleus of which Pb²⁰⁸ is a good example. By comparing the inelastic scattering from Bi²⁰⁹ and Pb²⁰⁷ with that

TABLE I. Center-of-mass cross sections for 42-MeV α particles scattered from Pb²⁰⁸.

Excitation energy (MeV)	()	2.6	15	3.:	20	4.	12	4.	40	5	.6
$\theta_{\text{c.m.}}$ (deg) ± 0.1	σ/σ_R	Error (%)	$d\sigma/d\Omega$ (mb/sr)	Error (mb/sr)								
16.3	0.867	1.0	2.91	0.87	0.71ª		1.0ª		2.2ª			
17.3	0.916	1.0										
18.3	1.000	1.0	5.13	1.28	2.5ª		3.5ª		2.3ª			
19.4	1.039	1.0	8.70	0.69								
20.4	1.098	1.0			2.44	0.35	2.58	0.33	2.19	0.28		
21.4	1.114	1.0			0.41 *		0.6		1 2 -			
22.4	1.190	1.0	1 70	0.00	2.41*		0.6*		1.3*			
23.4	1.170	1.0	1.70	0.90	1.55"							
24.5	1.204	1.0	2.31	0.05	1.678		2 1 a		0.08		9 778	
23.5	1.203	1.0	2.40	1.10	2.15		2.1"		0.94		2.114	
20.5	1 113	1.0			2.15							
28.5	1 042	1.0	0.94	0.39	0.37*							
29.5	0.934	1.0	1.64	0.66	0.07		1.37	0.30	1.06	0.29	1.86ª	
30.5	0.891	1.0	1101	0.00			1.50	0.24	2100	0.2/	100	
32.6	0.796	1.0					0.38	0.15	0.27	0.01	1.64	0.15
34.6	0.666	1.0	3.15	0.22							1.17	0.16
36.6	0.533	1.0	2.15	0.17			0.85	0.14	0.71ª		1.04	0.13
38.7	0.442	1.0	2.52	0.17	0.60	0.11	0.49ª		0.33	0.12		
40.7	0.342	1.0	3.04	0.14	0.62	0.09						
42.7	0.278	1.0	2.25	0.10	0.32	0.05	0.39	0.07			0.66	0.09
44.8	0.247	1.2	2.01	0.10	0.34	0.05	0.69	0.07	0.60	0.07	0.42	0.06
46.8	0.179	1.2	1.514	0.064	0.33	0.04	0.54	0.05	0.34	0.04		
48.8	0.178	1.2	2.262	0.081	0.37	0.05	0.48	0.05	0.38	0.05		
50.8	0.139	1.4	1.930	0.077	0.27	0.04	0.35	0.06	0.32	0.04	0.57	0.05
52.9	0.107	1.4	1.425	0.063	0.19	0.03	0.37	0.04	0.39	0.03	0.39	0.03
54.9	0.084	1.4	1.089	0.049	0.23	0.03	0.28	0.03	0.34	0.03	0.42	0.03
50.9	0.070	1.4	0.984	0.040	0.15	0.03	0.33	0.03	0.30	0.03	0.30	0.03
50.9	0.003	2.0	0.900	0.047	0.13	0.02	0.22	0.03	0.10	0.03	0.23	0.03
63.0	0.030	$\frac{2.0}{2.0}$	0.555	0.032	0.03	0.02	0.10	0.02	0.10	0.02	0.17	0.03
67.0	0.033	$\frac{2.0}{2.0}$	0.423	0.024	0.06	0.01	0.14	0.02	0.13	0.02	0.12	0.02
01.0	0.000	2.0	0.140	0.041	0.00	0.01		0.04	0.10	0.04	0.12	0.02

a These values represent upper limits to the cross section.

† Work performed under auspices of the U. S. Atomic Energy Commission.
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 ¹ J. S. Blair, Argonne National Laboratory Report ANL-6878 (unpublished) and references therein.

Excitation energy (MeV)	0		2.	60	3.4 + 3.	40 - 7	4.0 + 4.2	07 - 27
$\begin{array}{c} \theta_{\mathrm{c.m.}} \\ (\mathrm{deg}) \\ \pm 0.1 \end{array}$	σ/σ_R	Error (mb/sr)	$\frac{d\sigma/d\Omega}{({ m mb/sr})}$	Error (mb/sr)	$d\sigma/d\Omega$ (mb/sr)	Error (mb/sr)	$d\sigma/d\Omega$ (mb/sr)	Error (mb/sr)
14.3 15.3 16.3 17.3 18.3 19.4	$1.064 \\ 0.969 \\ 0.955 \\ 0.946 \\ 0.995 \\ 1.043$	1.2 1.2 1.2 1.2 1.2 1.2 1.2			4.5	• • ^{+1.5}	2.9	1.5
21.4 22.4 23.4	1.198 1.159 1.159	1.2 1.0 1.0	6.0 3.0	3.0	2.8 1.5	$-2.0^{+1.9}$ $-2.0^{+1.9}$ $1.0_{+0.8}$	3.7 5.2	2.0 1.4
25.3 28.5 29.5 30.6 32.6 34.6	$ 1.208 \\ 1.024 \\ 0.967 \\ 0.863 \\ 0.755 \\ 0.623 $	1.0 1.0 1.0 1.0 1.0 1.0 1.0	0.0 1.5 0.9 1.3 3.4 3.5	$\begin{array}{c} -0.4 \\ 0.6 \\ 0.3 \\ 1.0 \\ 0.3 \\ 0.3 \\ 0.3 \end{array}$	2.1 1.13 0.59	0.52 0.28	1.4 1.74 1.06 2.09	0.35 0.31 0.36
36.7 38.7 40.7	$\begin{array}{c} 0.023 \\ 0.482 \\ 0.420 \\ 0.353 \\ 0.321 \end{array}$	1.0 1.0 1.0 1.0	2.9 3.4 3.5 2.75	0.3 0.3 0.2 0.20	0.27 0.52	0.15 0.09	1.82	0.20
44.8 46.8 48.8 50 0	$\begin{array}{c} 0.321 \\ 0.250 \\ 0.205 \\ 0.174 \\ 0.147 \end{array}$	1.0 1.0 1.0 1.0	2.10 2.10 2.10 2.10	0.20 0.20 0.18 0.14	0.35 0.253	0.05 0.048	1.40 1.30 0.86	0.08 0.08
50.9 52.9 54.9 56.9 58.0	0.147 0.106 0.090 0.075 0.067	1.0 1.0 1.2 1.2	1.90 1.35 1.00 0.93	0.09 0.05 0.07 0.06 0.04	$\begin{array}{c} 0.217 \\ 0.194 \\ 0.153 \\ 0.160 \end{array}$	0.039 0.035 0.034 0.037	0.66 0.79 0.51	0.05 0.06 0.05
61.1 63.1 65.1 67.1	$\begin{array}{c} 0.007 \\ 0.049 \\ 0.041 \\ 0.033 \\ 0.029 \end{array}$	$ 1.2 \\ 1.2 \\ 1.4 \\ 1.4 \\ 1.4 $	0.89 0.76 0.59 0.43 0.37	0.04 0.03 0.02 0.04	0.099 0.093 0.076 0.078	0.018 0.018 0.014 0.026	0.396 0.345 0.289 0.266	0.032 0.028 0.023 0.037
69.1 71.1 73.1 75.1 77.1	$\begin{array}{c} 0.024 \\ 0.017 \\ 0.014 \\ 0.0134 \\ 0.0125 \end{array}$	1.4 1.4 2.0 2.0	0.37 0.27 0.22 0.17 0.15	0.02 0.02 0.02 0.03 0.02	0.089 0.035 0.044 0.030 0.020	0.011 0.010 0.009 0.008 0.005	0.208 0.144 0.136 0.119 0.099	0.015 0.016 0.011 0.012 0.011
79.1 81.2 85.2 89.2 91.2	$\begin{array}{c} 0.013 \\ 0.007 \\ 0.0048 \\ 0.0042 \\ 0.0032 \end{array}$	2.0 2.5 2.5 2.5 2.5	$\begin{array}{c} 0.130 \\ 0.096 \\ 0.082 \\ 0.065 \\ 0.055 \end{array}$	$\begin{array}{c} 0.009 \\ 0.009 \\ 0.015 \\ 0.006 \\ 0.007 \end{array}$	$\begin{array}{c} 0.029 \\ 0.025 \\ 0.0074 \\ 0.010 \\ 0.007 \end{array}$	$\begin{array}{c} 0.009 \\ 0.004 \\ 0.0027 \\ 0.002 \\ 0.002 \end{array}$	$\begin{array}{c} 0.117\\ 0.075\\ 0.019\\ 0.035\\ 0.028\end{array}$	0.013 0.014 0.002 0.004 0.003

TABLE II. Center-of-mass cross sections for 42-MeV α particles scattered from Pb²⁰⁷.

of Pb²⁰⁸ one can study, respectively, the effect of a single proton and a single neutron hole on the collective excitation properties. In particular, it should enable one to test the weak-coupling core-excitation model.²

A comparison will be made with the inelastic-electronscattering results of Kendall et al.,3 and inelastic proton scattering of Cohen et al.4 at 23 MeV, and of Stovall et al.⁵ at 40 MeV; the inelastic neutron scattering of Cranberg et al.,⁶ Stelson et al.,⁷ and Towle et al.⁸; the inelastic α scattering on Pb²⁰⁸ of Satchler *et al.*⁹ and

- ² A. de-Shalit, Phys. Rev. 122, 1530 (1961).
 ³ H. Kendall and J. Oeser, Phys. Rev. 130, 245 (1963).
 ⁴ B. L. Cohen and S. W. Mosko, Phys. Rev. 106, 995 (1957).
 ⁵ T. Stovall and N. M. Hintz, Phys. Rev. 135, B330 (1964);
 M. P. Fricke and G. R. Satchler, *ibid*. 139, B567 (1965).
 ⁶ L. Cranberg and C. D. Zafiratos (private communication).
 ⁷ P. H. Stelson, R. L. Robinson, H. J. Kim, J. Rapaport, and
 G. R. Satchler, Nucl. Phys. 68, 97 (1965).
 ⁸ J. L. Towle and W. B. Gibson, Nucl. Phys. 44, 256 (1963), and
 W. B. Gibson (1974).
- W. B. Gibson (private communication). ⁹ G. R. Satchler, H. W. Broek, and J. L. Yntema, Phys. Letters 16, 52 (1965).

Bruge et al.¹⁰ A comprehensive review of the nuclei in the lead region has recently been given by Hyde et al.¹¹

One can use the distorted-wave Born approximation (DWBA) formalism¹² or the adiabatic, parametrized phase-shift model of Austern and Blair¹³ to extract collective parameters from the experimental data. It was of particular interest to see whether the Austern-Blair model would correctly calculate the inelastic scattering from very heavy nuclei. The DWBA formalism allows one to calculate the effect of the Coulomb-excitation contribution to the inelastic scattering.9

¹⁰ G. Bruge, J. C. Faivre, H. Faraggi, J. Saudinos, and G. Vallois, Phys. Letters 14, 221 (1965). ¹¹ E. K. Hyde, I. Perlman, and G. T. Seaborg, *The Nuclear*

Properties of the Heavy Elements (Prentice-Hall, Inc., Englewood

Cliffs, New Jersey, 1964). ¹² R. H. Bassel, G. R. Satchler, R. M. Drisko, and E. Rost, Phys. Rev. **128**, 2693 (1962); N. Austern, in *Selected Topics in Nuclear Theory* (International Atomic Energy Agency, Vienna, 1963). ¹³ N. Austern and J. S. Blair, Ann. Phys. 33, 15 (1965).

Excitation energy (MeV)	0		2	.50 + .67	3.	1
$\begin{array}{c} \theta_{\mathrm{c.m.}} \\ (\mathrm{deg}) \\ \pm 0.1 \end{array}$	σ/σ_R	Error (%)	$\frac{d\sigma/d\Omega}{({ m mb/sr})}$	Error (mb/sr)	$d\sigma/d\Omega$ (mb/sr)	Error (mb/sr)
$14.3 \\ 16.3 \\ 18.3 \\ 20.4 \\ 22.4 \\ 24.4 \\ 26.5 \\ 28.5$	$\begin{array}{c} 0.945\\ 0.900\\ 0.858\\ 0.981\\ 1.064\\ 1.094\\ 1.143\\ 1.016\\ \end{array}$	$1.0 \\ 1.0 $	5.06 5.48 4.21 7.07 3.56 0.34 0.73	$\begin{array}{c} -2.0^{+0.80} \\ -2.0^{+0.80} \\ -1.5^{+0.55} \\ 0.78 \\ -1.5^{+0.64} \\ -1.0^{+0.04} \\ 0.27 \end{array}$	2.25 1.95 0.94* 1.65 1.08* 0.45*	$-1.5^{+0.67} -1.0^{+0.47}$ 0.31
30.5 32.6 33.6 34.6 36.6 38.7 40.7 42.7 44.8 46 8	$\begin{array}{c} 0.928\\ 0.826\\ 0.790\\ 0.726\\ 0.577\\ 0.480\\ 0.408\\ 0.333\\ 0.267\\ 0.219\end{array}$	$ \begin{array}{c} 1.0\\ 1.0\\ 1.0\\ 1.0\\ 1.0\\ 1.0\\ 1.0\\ 1.0\\$	4.66 2.66 2.63 2.65 3.03 2.95 2.11 1.98	0.14 0.24 0.16 0.21 0.30 0.18 0.10 0.08	$\begin{array}{c} 0.43 \\ 0.60 \\ 0.58 \\ 0.408 \\ 0.390 \\ 0.342 \end{array}$	0.19 0.14 0.12 0.045 0.058 0.041
48.8 50.8 52.9 54.9 56.9 58.9 61.0 63.0 65.0 67.0	$\begin{array}{c} 0.189\\ 0.155\\ 0.120\\ 0.094\\ 0.082\\ 0.073\\ 0.0562\\ 0.0440\\ 0.0435\\ 0.0324\\ \end{array}$	$\begin{array}{c} 1.2 \\ 1.2 \\ 1.2 \\ 1.4 \\ 1.4 \\ 1.4 \\ 1.4 \\ 1.4 \\ 1.4 \\ 1.4 \\ 1.4 \\ 1.4 \\ 1.4 \\ 1.4 \end{array}$	$1.94 \\ 1.95 \\ 1.46 \\ 0.94 \\ 1.03 \\ 1.01 \\ 0.85 \\ 0.68 \\ 0.56 \\ 0.46$	0.08 0.08 0.06 0.05 0.05 0.05 0.05 0.04 0.03 0.03 0.02	$\begin{array}{c} 0.343\\ 0.247\\ 0.143\\ 0.135\\ 0.190\\ 0.136\\ 0.118\\ 0.072\\ 0.075\\ 0.066\end{array}$	$\begin{array}{c} 0.034\\ 0.027\\ 0.027\\ 0.022\\ 0.021\\ 0.019\\ 0.016\\ 0.010\\ 0.011\\ 0.010\\ \end{array}$

TABLE III. Center-of-mass cross sections for 42-MeV α particles scattered from Bi²⁰⁹.

a These values represent upper values to the cross section.

EXPERIMENT AND RESULTS

The experiment was performed with 42-MeV α particles accelerated in the University of Washington 60-in. cyclotron. The beam was brought into a 60-in. scattering chamber. The scattered particles were detected with Li-drifted silicon semiconductor counters. The targets, approximately 1–1.4 mg/cm² thick were

prepared by vacuum evaporation.¹⁴ The Pb²⁰⁸ was enriched to 97.98% with a contamination of 0.82%Pb²⁰⁷ and 1.15% Pb²⁰⁶. The Pb²⁰⁷ was enriched to 92.93% with a contamination of 4.63% Pb²⁰⁸ and 2.44%Pb²⁰⁶. The main contamination in all targets consisted of O and C which resulted in gaps in the measured angular distributions. The over-all energy resolution ranged from 90 to 110 keV. The angular spread ranged

TABLE IV. A list of levels in Pb²⁰⁷, Pb²⁰⁸, and Bi²⁰⁹ excited by 42-MeV α particles. The first column gives the excitation energy; the second column gives the orbital angular momentum transfer; the third column is the value $\beta_I R$ derived from the Austern-Blair model, and column 4 gives the transition strength in single-particle Weisskopf units as defined in the text.

			Pb^{208}				Bi ²⁰⁹				
$\stackrel{E_{ ext{exc}}}{(ext{MeV})}$	Ι	β _I R (F)	G	E_{exc} (MeV)	Ι	$egin{array}{c} eta_I R \ (F) \end{array}$	G	E_{exc} (MeV)	Ι	$\beta_I R$ (F)	G
 2.6	3	0.74	30.0	2.615	3	0.57	30.4	$1.6 \\ 2.50 \\ 2.67 \}$	3	0.76	${23.0 \\ 7.8}$
$3.4 \\ 3.7 $	5	0.34	7.5	3.198	5	0.32	7.0	3.1	5	0.32	7.0
4.08 4.27	2 (4)	0.55	9.0 8.5	4.11 4.40 4.75	$\begin{pmatrix} 2\\(4)\\2\\(4) \end{pmatrix}$	0.58	9.0 9.0	4.1			
5.2 5.5 5.8				5.6	(3)	0.4	9	5.4			

¹⁴ Obtained from Oak Ridge National Laboratory, Isotopes Development Center.



CHANNELS

F1G. 1. Energy spectrum of 42-MeV α particles scattered at 50 deg, for Pb²⁰⁷, Pb²⁰⁸, and Bi²⁰⁹.



FIG. 2. Level scheme showing the known levels in Pb³⁰⁷, Pb³⁰⁸, and Bi³⁰⁹ below 4 MeV (Refs. 6 and 8) together with all the levels excited in this experiment (preceded by an asterisk). More levels were found by inelastic neutron scattering.



FIG. 3. Angular distribution for the elastic scattering of 42-MeV α particles divided by Rutherford scattering for Pb²⁰⁷, Pb²⁰⁸, and Bi²⁰⁹.



FIG. 4. Angular distribution for the inelastic scattering of 42-MeV α particles from the 2.615-MeV level in Pb²⁰⁸. The solid line is the calculated angular distribution calculated with Eq. (1), using the parameter $l_A=19.7$; $\Delta l_A=0.88$, $\delta=0.25$, $l_{\delta}=21.3$, and $\Delta l_{\delta}=1.05$. [The dashed line is a DWBA calculation including Coulomb excitation taken from Ref. 9 with $\beta_{\delta}R=0.88F$.]

0.0

ñ



FIG. 5. Angular distribution for the inelastic scattering of 42-MeV α particles from the doublet at 3.1 MeV in Bi²⁰⁹. The solid line is the angular distribution calculated with Eq. (1) for the parameters given in Fig. 4.

40

θ_{c. m.}

50

60

20

from 0.5 to 1.0 deg. Figure 1 shows some typical energy spectra.

The measurement of inelastic cross sections at angles smaller than approximately 25 deg, where the elastic scattering peak is very large, was considerably complicated by a large background in the energy spectrum arising from a low-energy tail due to slit scattering and reactions in the detector. At a scattering angle of 20 deg the spectra were measured with a magnetic spectrometer¹⁵ in order to reduce some of this background. The cross section measured in this way agreed with the value obtained with the solid-state detectors, but resulted in a much smaller error for those few points.

Figure 2 shows the levels excited in this experiment. No complete angular distributions could be obtained for the levels at 4.75 MeV in Pb²⁰⁸, the 4.8, 5.2, 5.5, and 5.8 MeV levels in Pb²⁰⁷, for the weak level at 1.6 MeV in Bi²⁰⁹ and the groups of levels around 4.1 and 5.4 MeV in Bi²⁰⁹.

Other well-known levels below 4 MeV such as the Pb²⁰⁷ levels at 0.570 MeV (5/2⁻), 0.894 MeV (3/2⁻), 1.634 MeV (13/2⁺), and 2.338 MeV (7/2⁻) and the level in Bi²⁰⁹ at 0.900 MeV (7/2⁻) were not at all seen in this work. This is consistent with their assumed single-particle character.

The differential cross sections for those levels that were well separated are tabulated in Tables I, II, and III. The sum of the differential cross sections are given for the 2.50- and 2.67-MeV groups in Bi²⁰⁹ and the 3.4- and 3.6-MeV levels in Pb²⁰⁷, because the error in the cross sections for the individual levels would be much larger even though they were clearly separated. The strength of the individual levels is indicated in Table IV. The angular distributions for the ground state and the 2.615 level in Pb²⁰⁸ measured by Satchler *et al.*⁹ and Bruge *et al.*¹⁰ are in good agreement with our results. Figures 3–9 show the angular distributions.

Angular distributions were calculated¹⁶ with the theory of Austern and Blair¹³ which is a good approximation to the DWBA formalism for strongly absorbed particles and for excitation energies low enough for the adiabatic approximation to be valid. The similarity in the calculated results has been pointed out before¹ and a comparison with a DWBA calculation⁹ for the 3^- level in Pb²⁰⁸ is shown in Fig. 4.

The Austern-Blair model expresses the inelastic scattering amplitudes in terms of the elastic-scattering



FIG. 6. Angular distribution of the inelastic scattering of 42-MeV α particles from the 3.198-MeV level in Pb²⁰⁸. The solid line is the angular distribution calculated with Eq. (1) using the parameters given in Fig. 4.

¹⁵ D. K. McDaniels, W. Brandenberg, G. W. Farwell, and D. L. Hendrie, Nucl. Instr. Meth. 14, 263 (1961).

 $^{^{16}\,\}mathrm{A}$ copy of the IBM 7090 computer program can be obtained from the author.



FIG. 7. Angular distribution for the inelastic scattering of 42 MeV α -particles for the sum of the 4.07- and 4.27-MeV levels in Pb²⁰⁷ and the sum of the 4.12- and 4.40-MeV levels in Pb²⁰⁸.

phase shifts η_l :

$$(d\sigma/d\Omega) (0 \rightarrow I) = \sum_{M_I} |f_{I,M_I;0,0}|^2 (\beta_I R)^2,$$

$$f_{I,M_I;0,0} = -\frac{1}{2}i \sum_{l,l'} i^{l-l'} (2l'+1)^{1/2} e^{i(\sigma_l+\sigma_{l'})}$$

$$\times \langle l'I, -M_I, M_I | l0 \rangle \langle l' l00 | l0 \rangle$$

$$\times (\partial \eta_l / \partial l) Y_l^{-M_I}(\theta, 0),$$
(1)

where $\bar{l} = \frac{1}{2}(l+l')$. The elastic-scattering cross section is calculated through a parametrized phase-shift analysis¹⁷:

$$f_{\rm el}(\theta) = (i/2k) \sum_{l=0}^{\infty} (2l+1)(1-\eta_l e^{2i\sigma_l}) P_l(\cos\theta),$$

where $\sigma_l = \arg\Gamma(1+l+in)$ and $\eta_l = A_{le}^{2i\delta_l}$ with $A_l = \{1 + \exp[(l_A - l)/\Delta l_A]\}^{-1}$ and $\delta_l = \delta\{1 + \exp[(l - l_{\delta})/\Delta l_{\delta}]\}^{-1}$. By varying the phase-shift parameters l_A , Δl_A , δ , l_{δ} , and Δl_{δ} until a good fit to the elastic-scattering cross section is obtained a set of phase shifts η_l is defined (see Fig. 10). These same phase shifts are used in Eq. (1) to calculate the inelastic scattering cross section. Figure 11 shows the calculated shapes of the angular distributions for different values of orbital angular-momentum transfer *I* together with the calculated elastic-scattering angular distribution. It is surprising to notice that although the elastic-scattering angular distribution shows very little structure the inelastic angular distributions exhibit pronounced maxima and minima. This structure allows us to assign

spins and parities to the measured levels. The parities are determined from the phasing of the angular distribution (the parity-changing transition gives an angular distribution out of phase with the nonparitychanging transition). The difference between an I=3and I=5 transition shows up at small angles where the I=5 transition has a plateau and it is this region which has been used in the determination of the Ivalue in Figs. 4, 5, and 6.

Strong emphasis has been put on the experimental point at 20 deg since the error for this point is small, as has been explained in the previous section.

Since there are no free parameters in Eq. (1), a unique value for $(\beta R)^2$ is found by normalizing the calculated cross section to the experimental value. The quality of the fit allows an accuracy of 10% for $(\beta R)^2$. Table IV collects these values of $\beta_I R$ together with the transition probabilities in Weisskopf units using the relation:

$$\frac{B(E\lambda)}{B_{\rm sp}(\lambda)} = \frac{9Z^2}{4\pi(2\lambda+1)} \left(\frac{3+\lambda}{3}\right)^2 \frac{(\beta_\lambda R)^2}{(1.2A^{1/3})^2} \,.$$

DISCUSSION

By comparing the levels excited in this experiment with those excited by inelastic neutron scattering^{6,8} one can readily see that the inelastic α scattering is very selective. The most prominent peak in the three



FIG. 8. Angular distribution for the inelastic scattering of 42-MeV α particles from the 4.11 level in Pb²⁰⁸. The solid lines are the angular distributions calculated with Eq. (1) for I=2 and I=4.

¹⁷ J. A. McIntyre, K. H. Wang, and L. C. Becker, Phys. Rev. **117**, 1337 (1960); J. Alster and H. E. Conzett, *ibid*. **136**, B1023 (1964); and *ibid*. **139**, B50 (1965).

nuclei is found at an excitation energy of approximately 2.6 MeV. Cohen and Mosko⁴ first noted the same effect for inelastic scattering of 23 MeV protons. The 2.615-MeV level has long been known to be a collective octupole vibrational level.^{3,4,18} Within the energy resolution of 100 keV the 2.60-MeV peak in Pb207 appears as a single level whereas in Bi²⁰⁹ two closely spaced groups occur at 2.50 and 2.67 MeV. The 2.50-MeV group contributes approximately one-fourth to the sum of the cross sections for these levels. The experimental cross sections for these three "2.6-MeV levels" are plotted in Fig. 12. It is seen that the angular distributions are identical in shape as well as in magnitude.¹⁹ Table IV also shows that the calculated $\beta_I R$ values are identical for these transitions. The $\beta_I R$ values are consistent with those found by Satchler et al.,9 Kendall et al.,3 Stelson et al.,7 and Cranberg et al.6 and those reported by Fricke et al.5

The next level excited in Pb²⁰⁸ is identified with the previously known 5⁻ level at 3.198 MeV. In Pb²⁰⁷ the next excited levels are at 3.4 and 3.6 MeV and in Bi²⁰⁹ at approximately 3.1 MeV (possibly a doublet). Table IV shows that the transitions in this set also have the same (βR) values.



FIG. 9. Angular distribution for the inelastic scattering of 42-MeV α particles from the 4.40-MeV level in Pb²⁰⁸. The solid lines are the angular distributions calculated with Eq. (1) for I=2 and I=4.

From the striking identity in cross sections one can conclude that the strong I=3 octupole transition in Pb²⁰⁸ is also present in Pb²⁰⁷ and in Bi²⁰⁹ and that the addition of a single neutron hole or a proton does not affect the collective excitation of the Pb²⁰⁸ core.²⁰ The same effect seems to hold for the I=5 transitions in these three nuclei.

Within the specific predictions of the weak-coupling model² one would expect the octupole strength in Pb²⁰⁷ to be split among two levels with spin $J_1 = \frac{5}{2}^+$ and $J_2 = \frac{7}{2}^+$ with an intensity ratio of $(2J_1+1)/(2J_2+1)$ arising from the coupling of the p_2^1 neutron hole with the 3⁻ core excitation. Analogously one should expect seven levels with spins $\frac{3}{2}^+$, $\frac{5}{2}^+$, \cdots , $\frac{15}{2}^+$ for the octupole strength in Bi²⁰⁹. This experiment shows that this splitting is less than 80 keV for Pb²⁰⁷. In Bi²⁰⁹ the levels are divided in two groups separated by 150 keV. If the 2J+1 rule is valid one would expect the lower spin levels to be grouped above the higher spin levels since the 2.67-MeV group is much less excited than the 2.50-MeV group. In this region more levels were found by neutron scattering.⁶

Several authors have reported a level in Pb^{208} at 4.3 MeV. Kendall and Oeser³ measured the inelastic electron scattering from Pb^{208} and assigned a 4⁺ to a level at 4.3 MeV. Satchler *et al.*⁹ tentatively assigned a



FIG. 10. Angular distribution for the elastic scattering of 42-MeV α particles divided by Rutherford scattering for Pb²⁰⁵. The solid line is the parametrized phase-shift model fitted with the parameters given in Fig. 4.

¹⁸ J. C. Carter, W. T. Pinkston, and W. W. True, Phys. Rev. **120**, 504 (1960); A. M. Lane, E. D. Pendlebury, Nucl. Phys. **15**, 39 (1960).

 ¹² (1967), ¹³ (1967).
 ¹⁹ J. Alster, in *Proceedings of the Congrès International de Physique Nucléaire*, edited by P. Gugenberger (CNRS, Paris, 1964).

²⁰ M. Baranger, in *Proceedings of the Congrès International de Physique Nucléaire*, edited by P. Gugenberger (CNRS, Paris, 1964).



FIG. 11. A graph showing the calculated value of $(d\sigma/d\Omega) \times (0 \to I)/(\beta_I R)^2$, for several values of *I*, using Eq. (1), together with the shape of the calculated elastic scattering divided by the Rutherford scattering; all in arbitrary units.

spin and parity of 3⁻. In an inelastic proton scattering experiment²¹ the spin and parity is given as a mixture of 2^+ and 5^- . In the present experiment two levels are clearly separated at 4.12 and 4.40 MeV. In the above mentioned experiments the energy resolution was not good enough to separate the two levels. In Figs. 8 and 9 the experimental cross sections are compared with a calculation of I=2 and I=4. The 2⁺ assignments seem most likely although 4⁺ cannot be excluded. In Pb²⁰⁷ two levels show up at 4.07 and 4.27 MeV and from Table IV one can see that the $\beta_I R$ value for the sum of these two levels is approximately equal to the sum of the two levels in Pb²⁰⁸. In Bi²⁰⁹ there are too many closely spaced levels from 4.0 to 5.0 MeV to enable one to correlate them with specific levels in Pb²⁰⁸. The 5.6-MeV angular distribution in Pb²⁰⁸ agrees best with an I=3 transition. We tentatively assign a 3⁻ to this level.

Gillet et al.22 have calculated excitation energies and transition probabilities in Pb²⁰⁸ using a particle-hole calculation. The calculated transition probability of 17.9 for the level at 2.6 MeV is the only one that is reasonable in connection with this experiment, which is consistent with the value found by other groups.^{3,6,7,9}



FIG. 12. Differential cross sections for 42-MeV α particles scattered from the 2.6-MeV levels in Pb²⁰⁷, Pb²⁰⁸, and Bi²⁰⁹.

The contribution of the Coulomb excitation to the inelastic scattering amplitude was calculated by Satchler et al.⁹ for the 3⁻ level in Pb²⁰⁸. It shows up as a destructive interference with the largest effect at the minimum in the angular distribution at 28 deg. This deep minimum which was first unambiguously found in this work cannot be reproduced in detail by any combination of parameters in the Austern-Blair model or DWBA calculation which does not include Coulomb excitation. This effect should enable one to distinguish between an I=2 and I=4 transition since the Coulomb excitation contribution for higher I values is strongly reduced.

In conclusion one can say that α particles excite those levels in Pb²⁰⁷, Pb²⁰⁸, and Bi²⁰⁹ that are collective with respect to the ground state and that these excitations are very little affected by the addition of a single proton or a neutron hole. The inelastic scattering of α particles from Pb²⁰⁴ and Pb²⁰⁶ are now under investigation.

The Austern-Blair model gives a good general description of inelastic α scattering from low-lying levels in heavy nuclei but although it properly accounts for the Coulomb distortion in the ingoing and outgoing channels it does not include the Coulomb excitation contribution to the scattering amplitude.

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 ²¹ A. Scott, J. Harries, and M. P. Fricke, Bull. Am. Phys. Soc.
 10, 527 (1965); and A. Scott (private communications).
 ²² V. Gillet, A. M. Green, and E. A. Sanderson, Phys. Letters

^{11, 44 (1965).}