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Alpha Clusters in a Spherical Nucleus

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The probability of occurrence of an alpha cluster in a Po^{212} nucleus is calculated on the basis of the assumption that the alpha cluster is always formed of a pair of equivalent protons and a pair of equivalent neutrons, each pair coupling to zero angular momentum. It is shown that the calculated probability can be incorporated with Winslow's surface-well model to explain the decay rate of Po^{212}.

I. INTRODUCTION

THE existence of cluster structure in nuclei was first conceived by Wheeler.[1] The same idea is also employed in a cluster model, formulated by Wildermuth and Kanellopoulos,[2] which explains satisfactorily several properties of nuclear states in light- and medium-weight nuclei, such as spin, parity, reduced width, etc. The fact that alpha decay often leads to the ground state or a low excited state of a daughter nucleus suggests that a parent nucleus consists, part of the time, of an alpha cluster and a "daughter cluster" which overlap not too extensively. In this paper we shall calculate the probability of such a configuration for a spherical even-even nucleus.

II. PROBABILITY OF OCCURRENCE OF AN ALPHA CLUSTER

Let the shell-model wave functions of a parent nucleus, its daughter nucleus and an alpha cluster (in the parent nucleus)be $\Psi_P(JM)$, $\Psi_D(J'M')$, and $\Psi_\alpha(LM_L)$, respectively, where J and M are the total angular momentum and its z component of the parent nucleus, etc. In a zeroth-order approximation the probability amplitude for the occurrence of an alpha cluster of angular momentum L and M_L is given by

$$A_0=\sum_{\text{spin coordinates}}\int \Psi_P^*(JM)N\mathcal{A}\times\Big[\sum_{M_L}(J'M-M_LLM_L|J'LJM)\times\Psi_D(J'M-M_L)\Psi_\alpha(LM_L)\Big]d\mathbf{r}_1\cdots d\mathbf{r}_A\,, \quad (1)$$

where $\mathcal{A}$ is an antisymmetrizer; N is a normalization constant; $(J'M-M_LLM_L|J'LJM)$ is a Clebsch-Gordan coefficient; and $\mathbf{r}_i$ $(i=1,\cdots,A)$ are the coordinates of the A nucleons. The normalized and antisymmetric wave function

$$N\mathcal{A}\Big[\sum_{M_L}(J'M-M_LLM_L|J'LJM)\times\Psi_D(J'M-M_L)\Psi_\alpha(LM_L)\Big] \quad (2)$$

describes a system consisting of an alpha cluster and a daughter cluster. It is noted that the wave function of the daughter cluster is approximated by that of the daughter nucleus.

The summation and integration in (1) can be carried out over the coordinates of the nucleons in the closed subshells common to both the parent and daughter nuclei. We thus obtain

$$A_0=\sum_{\text{spin coordinates}}\int \psi_P^*(JM)\times N'\mathcal{A}\Big[\sum_{M_L}(J'M-M_LLM|J'LJM)\times\psi_D(J'M-M_L)\Psi_\alpha'(LM_L)\Big]d\mathbf{r}_1\cdots d\mathbf{r}_n\,, \quad (3)$$

where $\psi_P(JM)$ describes the $n=n_1+n_2$ remaining nucleons in the parent, n_1 (n_2) being the number of remaining protons (neutrons); $\psi_D(J'M-M_L)$ describes the $n-4$ remaining nucleons in the daughter nucleus; N' is a normalization constant; $\Psi_\alpha'(LM_L)$ is obtained from $\Psi_\alpha(LM_L)$, when the latter is expanded in terms of single-particle wave functions of the (parent or daughter) nucleus, by deleting the components that share single-particle states with the closed subshells common to both the parent and daughter nuclei; and the summation and integration are over the coordinates of the n nucleons. It should be noted that $\Psi_\alpha'(LM_L)$ is a linear combination of wave functions corresponding to different internal states of the alpha cluster.

For an even-even nucleus in its ground state, the probability amplitude for the occurrence of an alpha cluster of zero angular momentum and a daughter cluster in its ground state is

$$A_0=\sum_{\text{spin coordinates}}\int \psi_P^*(00)\times N'\mathcal{A}[\psi_D(00)\Psi_\alpha'(00)]d\mathbf{r}_1\cdots d\mathbf{r}_n\,. \quad (4)$$

For the case where $\psi_P(00)$ contains no more than one proton and one neutron subshell, it can be shown that, when $\psi_P(00)$ and $\psi_D(00)$ are approximated by shell-model wave functions of seniority zero, (4) gives

$$A_0=N'\left(\frac{n_1(n_1-1)}{2}\right)\left(\frac{n_2(n_2-1)}{2}\right)\times\left(\frac{2j_1+3-n_1}{(n_1-1)(2j_1+1)}\right)^{1/2}\left(\frac{2j_2+3-n_2}{(n_2-1)(2j_2+1)}\right)^{1/2}\times a(l_1j_1l_1j_1,0;l_2j_2l_2j_2,0)\,, \quad (5)$$

[1] J. A. Wheeler, Phys. Rev. 52, 1107 (1937).
[2] K. Wildermuth and Th. Kanellopoulos, Nucl. Phys. 7, 150 (1958).

where l_1 and j_1 (l_2 and j_2) are the single-particle orbital and total angular momentum of the proton (neutron) subshell; the fourth and fifth factors are coefficients of fractional parentage (in the $n_1 \to n_1-2$ and $n_2 \to n_2-2$ fractional parentage expansions, respectively); and $a(l_1j_1l_1j_1,0;l_2j_2l_2j_2,0)$ is a coefficient in the expansion of $\Psi_\alpha(00)$ in terms of single-particle wave functions:

$$\Psi_\alpha(00)=\sum_{\lambda,\,j_aj_bj_cj_dl_al_bl_cl_d} a(l_aj_al_bj_b,\lambda;l_cj_cl_dj_d,\lambda)\times\{\sum_\mu(\lambda\mu\lambda-\mu|\lambda\lambda00)\psi_p(l_aj_al_bj_b\lambda\mu)\times\psi_n(l_cj_cl_dj_d\lambda-\mu)\},\quad(6)$$

where $\psi_p(l_aj_al_bj_b\lambda\mu)$ is the wave function of the protons with angular momenta j_a and j_b coupling to a total angular momentum λ and corresponding z component μ, etc.

III. APPLICATION TO Po^{212}

A simple case for application is Po^{212}. In Po^{212} there are two protons and two neutrons outside closed shells. Therefore we expect this nucleus to be one in which an alpha cluster and a daughter cluster, if formed, overlap least.

We shall use harmonic-oscillator wave functions with the approximation that the oscillator constants for alpha clusters and nucleons are equal.[3] Furthermore we shall assume that the alpha cluster is always formed of a pair of equivalent protons and a pair of equivalent neutrons, each pair coupling to zero angular momentum. The plausibility of the assumption is suggested by the fact that there is a large depression of the state of zero angular momentum in the spectrum of two equivalent, identical nucleons interacting with the ∂ force.[4] Thus the normalization constant N', in the case of Po^{212}, is given by

$$N'=\{\sum_{j_al_aj_cl_c}[a(l_aj_al_aj_a,0;l_cj_cl_cj_c,0)]^2\}^{-1/2},\quad(7)$$

where the summation is over the proton states $2p_{1/2}$, $2p_{3/2}$, $1f_{5/2}$, $1f_{7/2}$, and $0h_{9/2}$, and the neutron states $3s_{1/2}$, $2d_{3/2}$, $2d_{5/2}$, $1g_{7/2}$, $1g_{9/2}$, and $0i_{11/2}$. In an oscillator potential only these single-particle states are available to the constituent nucleons of an alpha cluster of lowest (total) energy in a Po^{212} nucleus.

Taking a pure configuration (two $0h_{9/2}$ protons and two $1g_{9/2}$ neutrons outside closed shells), we find, from (5) and (7), $A_0^2=5.8\times10^{-4}$. To demonstrate the effect of configuration mixing on A_0,[5] we take for nucleons outside closed shells the following mixture of configurations:

protons: $0.950(0h_{9/2})^2+0.050(1f_{7/2})^2$,

neutrons: $0.874(1g_{9/2})^2+0.020(2d_{5/2})^2+0.106(0i_{11/2})^2$.

We then find $A_0^2=4.1\times10^{-3}$. This value represents an increase by a factor of 7.1 over the case of no configuration mixing. In our calculations we used the same oscillator constant for both alpha clusters and nucleons, so the admixing of the proton configuration $(0i_{13/2})^2$ was ruled out by conservation of energy.

Winslow[6] has shown that the alpha decay constant calculated from the surface-well model is much larger than the one calculated from the traditional model. For Po^{212}, if the width of the well ΔR ranges from 0.5×10^{-13} to 2.5×10^{-13} cm, and the outer radius R of the well is, somewhat arbitrarily, assigned a value 9.25×10^{-13} cm, the ratio p_0 of the experimental decay constant to the one calculated from the surface-well model ranges from 2.2×10^{-3} to 1.9×10^{-2} [see Fig. 2(b) in Ref. 6]. Harada[5] has determined, from Igo's optical potentials of lead for alpha particles, an interaction radius as 10^{-12} cm. Hence a larger value for R should perhaps be used, thus lowering p_0.

A rough estimate of the probability of occurrence of an alpha cluster in a nuclear surface region is obtained by multiplying A_0^2 by the ratio of the volume of the surface region to the volume of the nucleus; the ratio ranges from 0.15 to 0.61, if the nuclear radius is equal to the value assumed for R and the surface thickness lies in the range assumed for ΔR. Thus, for instance, the probability of occurrence of an alpha cluster in a surface region of 0.5×10^{-13}-cm thickness is 6.2×10^{-4}. It therefore seems that in more exact calculations the experimental decay constant of Po^{212} can be explained, with a reasonable choice of R and ΔR, as a product of two factors[7]: (a) the probability of occurrence of an alpha cluster in a surface well, and (b) the probability of decay when an alpha cluster is already formed in the surface well.

[3] F. C. Chang, Phys. Rev. **137**, B1420 (1965).

[4] B. R. Mottelson, in *The Many-Body Problem*, edited by C. DeWitt (Dunod Cie., Paris, 1959), p. 259.

[5] The effect of configuration mixing on alpha decay rate has been investigated by several authors: K. Harada, Progr. Theoret. Phys. (Kyoto) **26**, 667 (1961). H. D. Zeh and H. J. Mang, Nucl. Phys. **29**, 529 (1962). J. O. Rasmussen, Nucl. Phys. **44**, 93 (1963). H. J. Mang, Ann. Rev. Nucl. Sci. **14**, 1 (1964).

[6] G. H. Winslow, Phys. Rev. **96**, 1032 (1954).

[7] Both Harada and Mang (Ref. 5) have shown that the alpha-decay constant depends on two factors: (a) the probability that the parent nucleus consists of an alpha particle and a residual nucleus at a fixed distance from each other, and (b) the probability of penetration through the Coulomb barrier.