Nuclear Matrix Elements in the First-Forbidden 2.2-MeV β Transition of La^{140}†

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The angular dependence of the circular polarization of the 1.6-MeV γ radiation following the 2.2-MeV β transition of La^{140} has been measured. The analysis of these data, together with β-γ directional-correlation and spectrum-shape data, was made using exact electron wave functions and taking into consideration finite-nuclear-size effects. The nuclear beta matrix elements determined in this manner show that the deviations of the 2.2-MeV β transition from the Coulomb approximation is caused by a cancellation effect of the vector-type matrix elements and not by a selection-rule effect. The experimentally determined ratio of the $\int i\boldsymbol{\alpha}$ and the $\int \mathbf{r}$ matrix elements does not agree well with the value predicted by Fujita on the basis of the conserved-vector-current theory. The discrepancy may be due to the presence of higher order effects in the 2.2-MeV β transition of La^{140}.

I. INTRODUCTION

In recent years, there has been a growing interest in the studies of first-forbidden β transitions with the aim of determining the nuclear matrix elements in beta decay. The majority of first-forbidden β transitions, in particular $\Delta I=0$ transitions, are well described by the so-called Coulomb or ξ approximation.[1] The determination of the individual matrix elements, however, is facilitated if the β transition exhibits a deviation from the Coulomb approximation. These β transitions are characterized by (i) spectral shapes that deviate from the allowed shape, (ii) larger than usual ft values, and (iii) large anisotropies of the β-γ directional correlation when the β transition is followed by a gamma ray.

First-forbidden beta transitions from medium-heavy nuclei may show deviations from the predictions of the Coulomb approximation for two reasons[1,2]: (i) a selection-rule effect exists which causes the reduction of the matrix elements of tensor rank $\lambda=0$ and $\lambda=1$, relative to the $\int B_{ij}$ matrix element of tensor rank $\lambda=2$, (ii) a cancellation of certain otherwise large nuclear matrix elements. The decays of Sb^{124}, Eu^{152}, and Eu^{154}, which are of the type $3^-(\beta)2^+(\gamma)0^+$, have been studied so far. In the case of Sb^{124}[3] and Eu^{152},[4] the observed deviations from the Coulomb approximation seem to arise because the nuclear matrix elements of tensor rank $\lambda=1$ ($\int \mathbf{r}$, $\int i\boldsymbol{\alpha}$, and $\int i\boldsymbol{\sigma}\times\mathbf{r}$) are suppressed, relative to the $\int B_{ij}$ matrix element of rank $\lambda=2$.

The individual contribution of these four matrix elements can be determined[2] by analyzing the results from the measurements of (i) the $\log ft$ value, (ii) the shape factor of the β spectrum $C(W)$, (iii) the energy dependence of the β-γ directional-correlation coefficient $A_2(W)$, and (iv) the angular dependence of the β-γ circular-polarization correlation $P_c(\theta)$. In this paper, measurements of the angular dependence of the circular polarization of the 1.6-MeV gamma ray, following the 2.2-MeV first-forbidden β transition of La^{140} ($T_{1/2}=40$ h) are reported.

The β transitions of La^{140} to the excited states of Ce^{140} are of particular interest, since Ce^{140} has a closed neutron shell and the initial and final states of the nucleon that transforms in the β decay belong to different major shells. Thus, the usual Δj selection-rule arguments for the suppression of the vector-type matrix elements,[3] $\int i\boldsymbol{\alpha}$, $\int \mathbf{r}$, and $\int i\boldsymbol{\sigma}\times\mathbf{r}$, relative to the tensor-type matrix element $\int B_{ij}$, cannot be applied in this case.

The ground state of La^{140} has been assigned a spin 3^-, according to the measurements by Petersen and Shugart.[5] The relevant features of the decay scheme of La^{140} are illustrated in Fig. 1. Accurate measurements of the β spectra of La^{140} have been reported by Langer and Smith,[6] who have found a nonstatistical shape for the 2.2-MeV β group with a large ft value, $\log ft=9.5$. The β-γ directional correlation for the 2.2-MeV β—1.6-MeV γ cascade has been measured by Alberghini

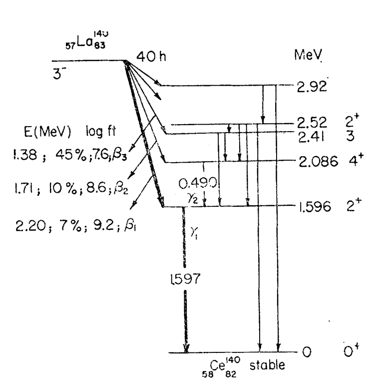

Fig. 1. Decay of La^{140}.

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1 T. Kotani and M. Ross, Progr. Theoret. Phys. (Kyoto) **20**, 643 (1958); Phys. Rev. **113**, 622 (1959).

2 T. Kotani, Phys. Rev. **114**, 795 (1959).

3 P. Alexander and R. M. Steffen, Phys. Rev. **124**, 150 (1961).

4 P. Alexander and R. M. Steffen, Phys. Rev. **128**, 1783 (1962).

5 F. R. Petersen and N. A. Shugart, Bull. Am. Phys. Soc. **5**, 343 (1960).

6 L. M. Langer and D. R. Smith, Phys. Rev. **119**, 1308 (1960).

and Steffen,[7] and also by Bhattacherjee and Mitra,[8] and by Newsome and Fischbeck.[9] A measurement of the β-γ circular-polarization correlation of the 2.2-MeV β–1.6-MeV γ cascade has been reported by Estulin and Petushkov.[10] Their results, however, do not provide sufficient information, since the correlation was measured at one angle θ only. The analysis of the experimental data (including the present measurement) on the β-γ cascade shows that there is a cancellation of nuclear matrix elements in the β decay of La^{140}.

II. EXPERIMENTAL METHODS AND RESULTS

The sources of La^{140} were prepared by evaporation from Ba^{140} Cl_2 in HCl solution obtained from Oak Ridge National Laboratory. We thus used a Ba^{140}-La^{140} source in which La^{140} is in equilibrium with its parent Ba^{140}, which has a half-life of 13 days. New sources of La^{140} were prepared from fresh samples every week, and during a week a drop of source material was added to the source every three days, so that the strength of the source remained fairly constant during several weeks of measurement. The sources were deposited on 1-mil Mylar foils.

The circular polarization of the 1.6-MeV gamma ray was analyzed by the method of Compton scattering on polarized electrons.[11] The scattering magnet and the associated electronic circuits have been described before.[3,4] The circular-polarization correlation (averaged over energy), $\bar{P}_c(\theta)$, was measured simultaneously at four different angles, $\theta=100°$, 127°, 143°, and 154°, using the magnet aperture A of Fig. 4, of Ref. 3. The thickness of the Pilot B disks used in the beta-detector was 0.51 in. The other details of the beta and gamma detectors were as previously described. The four beta single-channel analyzers were adjusted to accept beta rays above 1.71 MeV corresponding to an average beta energy $\bar{W}=4.59$ in units mc^2. The average resolving time of the four fast-coincidence circuits was about $2\tau_0=12$ nsec. The data were recorded automatically on IBM cards. The IBM card punch system used has been described by Alexander.[12] The magnetic field was reversed every 15 min, and the digital information, stored in the four coincidence and five singles counting scalers, was punched on an IBM card. The raw data were corrected for the presence of genuine γ-γ coincidences and for chance coincidences including higher order corrections.

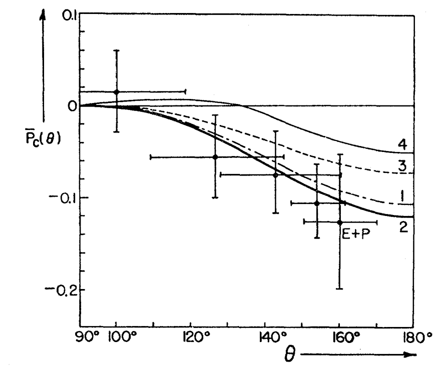

FIG. 2. Degree of circular polarization of 1.6-MeV gamma radiation $\bar{P}_c(\theta)$ as a function of the angle θ. The point labeled $E+P$ refers to the measurement of Ref. 10. The four curves are theoretical values calculated with the parameter sets defined in Table I.

A quantity $\delta(\theta)$ was computed from the relation

$$\delta(\theta)=[N^+(\theta)-N^-(\theta)]/[N^+(\theta)+N^-(\theta)],$$

where $N^+(\theta)$ and $N^-(\theta)$ represent the corrected β-γ coincidence rates taken with the magnetic induction in the (+) and (−) direction, respectively. The quantities $N^\pm(\theta)$ were also normalized by dividing the β-γ coincidence rates by the product of the β- and γ-singles rates to remove the first-order effects. The data were accumulated in a total period of about five months.

The degree of circular polarization of the 1597-keV gamma rays was then calculated from the relation

$$\bar{P}_c(\theta)=\delta(\theta)/\bar{E}(h\nu).$$

The average polarization efficiency of the analyzer $\bar{E}$(1.6 MeV) was calculated by graphical methods.

The experimental values of $\bar{P}_c(\theta)$ are shown in Fig. 2. The points are not corrected for the finite angular resolution of the instrument. The horizontal bars shown with the experimental points indicate the finite angular resolution of the apparatus, while the vertical error bars show the rms statistical error. The value of $P_c(\theta)$, reported by Estulin and Petushkov,[10] was measured at $\theta=160°$ and $\bar{W}=4.2$. Our results are in satisfactory agreement with their measurements.

III. ANALYSIS OF DATA

The angular dependence of the beta-gamma circular polarization was combined with the energy dependence of the beta-gamma directional correlation measured by Alberghini and Steffen[7] and the shape correction factor measured by Langer and Smith[6] to determine the matrix elements contributing to the 2.2-MeV β transition of La^{140}. The calculations were performed on an IBM-7094 computer, using the procedure which has been described by Simms.[13]

The analysis was made on the basis of the following nuclear matrix-element parameters (notation of

[7] J. Alberghini and R. M. Steffen, Phys. Letters 7, 85 (1963).
[8] S. K. Bhattacherjee and S. K. Mitra, Phys. Rev. 131, 2611 (1963).
[9] R. W. Newsome and H. J. Fischbeck, Phys. Rev. 133, B273 (1964).
[10] I. V. Estulin and A. A. Petushkov, Nucl. Phys. 36, 334 (1962).
[11] H. Schopper, Nucl. Instr. 3, 158 (1958).
[12] P. Alexander, Nucl. Instr. 14, 288 (1961).
[13] P. C. Simms, Phys. Rev. 138, B784 (1965).

Kotani[2]):

$$\xi' y = -C_V \int i\boldsymbol{\alpha} \Big/ C_A \int B_{ij},$$

$$x = -C_V \int \mathbf{r} \Big/ C_A \int B_{ij},$$

$$u = C_A \int i\boldsymbol{\sigma}\times\mathbf{r} \Big/ C_A \int B_{ij},$$

$$Y = \xi' y - \xi(x+u),$$

$$\xi = \alpha Z/2R = 13.2 \quad (\text{for La}^{140}),$$

$$R = \text{nuclear radius} \quad (R = 0.0158 \text{ for La}^{140}).$$

The electron radial wave functions of Bhalla and Rose[14] were used and finite-nuclear-size effects were included. The criterion for acceptable sets of matrix elements was based on a χ^2 test of the experimental data. Sets of matrix elements, yielding values of $A_{22}(W)$, $P_c(\theta)$, and $C(W)$ that agreed with the experimental results with a probability of 30% or less, were rejected.

One important point in the analysis was that the value of $\bar{P}_c(\theta)$ was calculated by numerical integration over the energy range of the experiment. In several papers, $\bar{P}_c(\theta)$ has been calculated at the average energy of the experiment. Such a procedure is unsatisfactory and would have led to very different results in the present evaluation of the La^{140} data.

Four typical sets of matrix-element parameters that represent the experimental data within our criterion are listed in Table I to illustrate the results of the analysis. The observables computed from these sets of parameters are shown in Figs. 2–4, together with the experimental data. Sets 1–3 are similar insofar as the vector-type ($\lambda=1$) matrix elements are larger than the tensor-type matrix element $\int B_{ij}$. Furthermore, these sets give rise to a significant cancellation in the expression for the matrix-element parameter Y. Set 1 gives good agreement with the experimental data while sets 2 and 3 result in progressively less satisfactory agreement. However, none of these three sets can be rejected on the basis of the present experimental data.

It is interesting to compare these results with the prediction of Fujita[15] based on the conserved-vector-current (CVC) theory. Fujita gives the following theoretical estimate for the ratio Λ_{CVC} of the vector-type matrix elements:

$$\Lambda_{\text{CVC}} = \int i\boldsymbol{\alpha} \Big/ \int \mathbf{r} = (7/6)\alpha Z/R + (W_0 - 2.5) = 34.4.$$

This quantity is tabulated for the sets under consideration in Table I. One notices that the better the fit to the experimental data, the poorer the agreement with the prediction of Fujita. This trend was quite evident throughout the analysis. It would seem that the prediction of Fujita is more likely to be too small than too large; therefore, the disagreement is significant. This point will be considered further in the subsequent discussion.

Set 4 is quite different, and it has been included to illustrate the importance of the energy dependence of the circular polarization. Notice that in this set, the $\lambda=1$ matrix elements are an order of magnitude smaller than the $\lambda=2$ matrix element. Many sets of matrix elements of this type were found which agree perfectly with the prediction of Fujita. In fact, this type of solution was found by relaxing the requirements for agreement with the experimental data and imposing the requirement for agreement with the prediction of Fujita. Note that the agreement with the shape and directional correlation is not too bad. However, it is in very poor agreement with the circular-polarization correlation data. The predicted energy dependence of $P_c(W)$ for $\theta=180°$ is shown in Fig. 5. The energy dependence for set 4 is very different from that for sets 1, 2, and 3. In this case, as in many others, the energy dependence of P_c is more sensitive than the angular dependence. Since it is very difficult to determine the absolute magnitude of the circular polarization, it is evident that experimental data on the energy dependence of P_c would be much more useful in distinguishing between these two types of solutions. An experiment of this type is being planned in this laboratory.

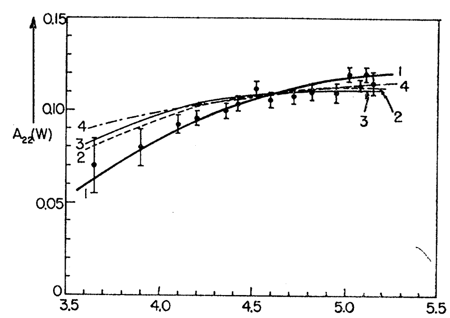

FIG. 3. Beta-gamma anisotropy factors $A_{22}(W)$ measured by Alberghini and Steffen (Ref. 7). The four curves are theoretical values calculated with the parameter sets defined in Table I.

TABLE I. Sets of nuclear matrix-element parameters.

Set	Y	$\xi' y$	x	u	Λ_{CVC}
1	+9.0	127	5.9	3.0	21.5
2	+3.3	106	3.9	3.9	27.1
3	+1.2	55	1.75	2.3	31.4
4	−3.13	−2.34	−0.068	0.128	34.4

[14] C. P. Bhalla and M. E. Rose, ORNL Oak Ridge National Laboratory Report No. 3207 (unpublished).

[15] J. I. Fujita, Phys. Rev. 126, 202 (1962).

Set 4 also illustrates the importance of calculating the average value $\bar{P}_c(\theta)$ rather than the value of $P_c(\theta)$ at the average energy. Notice in Fig. 5 that at the average energy of the experiment ($W=4.59$), set 4 gives a result which is very similar to sets 1, 2, and 3. If the common approximation of calculating P_c at the average energy had been made, set 4 would have been in agreement with the data and a very different type of solution would have been accepted.

Assuming for the moment that higher order matrix elements can be neglected and that the absolute magnitude of the circular polarization is correct, solutions of type 4 can be excluded and significant limits can be set on the matrix-element parameters:

$$47 \leq \xi' y \leq 163\,,$$
$$2.5 \leq x \leq 8.6\,,$$
$$1.2 \leq u \leq 4.4\,.$$

In terms of the nuclear matrix elements that contribute

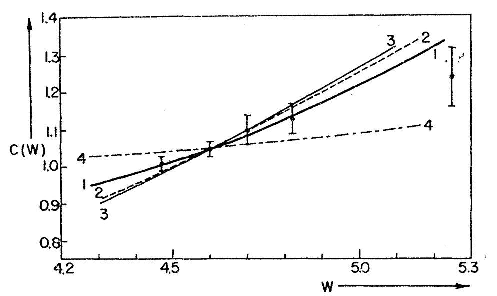

FIG. 4. The shape correction factor $C(W)$ as measured by Langer and Smith (Ref. 6). The four curves are theoretical values calculated with the parameter sets defined in Table I.

to the 2.2-MeV β transition of La^{140} one obtains

$$0.01 \leq \int B_{ij}/R \leq 0.03\,,$$

$$-2.6 C_A \int B_{ij}/R \leq C_V \int i\boldsymbol{\alpha} \leq -0.75 C_A \int B_{ij}/R\,,$$

$$-8.6 C_A \int B_{ij}/R \leq C_V \int \mathbf{r}/R \leq -2.7 C_A \int B_{ij}/R\,,$$

$$1.2 \int B_{ij}/R \leq \int i\boldsymbol{\sigma} \times \mathbf{r}/R \leq 4.4 \int B_{ij}/R\,.$$

It should be noted that the uncertainties associated with the individual matrix elements are interrelated.

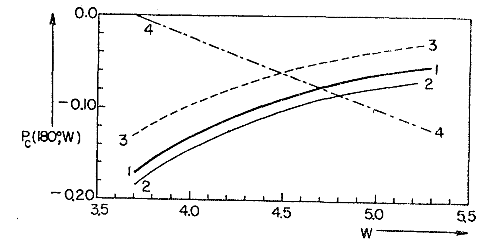

FIG. 5. The energy dependence of the circular polarization $P_c(180°, W)$ at 180°, computed with the parameter sets of Table I.

Thus, for a given value, e.g., for $\xi' y$, the errors in x and u are restricted to a much smaller range. For example,

$$\xi' y = 81.6\,, \quad 4.1 \leq x \leq 5.0\,, \quad 1.5 \leq u \leq 2.5\,.$$

This strong restriction of the x and u values occurs because there must be the proper cancellation in Y to fit the experimental data.

The present analysis of the La^{140} β-decay data gives a considerably different result from that reported by Newsome and Fischbeck,[9] although the experimental data used are similar in both cases.

The analysis shows that a matrix-element cancellation effect and not a selection-rule effect is responsible for the strong deviation of the 2.2-MeV β transition of La^{140} from the Coulomb approximation. The absence of a selection-rule effect in La^{140} is easily understood. These selection rules are known to be either the j selection rule or the K selection rule. In the La^{140} nucleus ($Z=57$, $N=83$), the shell model predicts a configuration with one hole in the proton shell, $g_{7/2}$, and one neutron in the state, $f_{7/2}$, above the filled shell $N=82$. Thus the 57th proton and the 83rd neutron in La^{140} are in different major shells and a j selection rule cannot reduce the vector-type matrix elements, relative to the $\int B_{ij}$ matrix element. The possibility of a K selection rule is ruled out for a nucleus like La^{140}, which is close to a filled shell. It thus appears that the cancellation effect is responsible for the deviation of the 2.2-MeV β transition from the Coulomb approximation.

Since the vector-type matrix elements are not appreciably reduced in this β transition, yet mutual cancellations make the transition probability small, the question arises whether the influence of higher order (third-forbidden) matrix elements should be considered.[13] It is quite possible that the inclusion of third-forbidden matrix elements may remove the disagreement with the predictions of Fujita. An analysis of the data, including higher order matrix elements, is in progress and will be reported at a later date.