

## Nuclear Energy Levels of Fe<sup>56</sup> and Co<sup>59</sup>†

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Nuclear energy levels of Fe<sup>56</sup> and Co<sup>59</sup> have been determined by analyses of particles from the Co<sup>59</sup>(*p*, $\alpha$ ), Fe<sup>56</sup>(*p*,*p'*), and Co<sup>59</sup>(*p*,*p'*) reactions with a broad-range single-gap magnetic spectrograph. In the first 6.7 MeV of excitation energy, 100 levels of Fe<sup>56</sup>, and in the first 4.0 MeV, 62 levels of Co<sup>59</sup> were resolved and investigated. In the excitation-energy region of Fe<sup>56</sup> examined previously, new levels were found at 3.748 and 4.802 MeV and the previously reported level at 4.676 MeV was resolved into a doublet with energies of 4.675 and 4.688 MeV. Information on the spins of a few levels of Fe<sup>56</sup> is obtained from their cross sections in the aforementioned reactions. The energy dependence of the level density is found to agree qualitatively with theoretical expectations for a finite superconducting nucleus.

### I. INTRODUCTION

AS a part of a general investigation of the level density of Fe<sup>56</sup> by various methods, the energies of the excited levels of this nucleus were measured by high-resolution magnetic analyses of the particles emitted in the Fe<sup>56</sup>(*p*,*p'*) and Co<sup>59</sup>(*p*, $\alpha$ ) reactions. The levels of Fe<sup>56</sup> had been investigated previously up to an excitation energy of 5 MeV by magnetic analysis of the protons from the Fe<sup>56</sup>(*p*,*p'*) reaction.<sup>1,2</sup> Because of somewhat better energy resolution in this experiment, levels have been resolved up to an excitation energy of 6.7 MeV. Moreover, two different reactions were used for the determination of the energy levels of Fe<sup>56</sup> in order to reduce the probability of missing levels. In addition, energy levels of Co<sup>59</sup> were investigated by inelastic proton scattering in the excitation energy range 0–4 MeV.

In this paper we want to present mainly the experimental data. New information on the spins of some of the low-lying levels of Fe<sup>56</sup>, obtained from the intensities of the particle groups observed in the two reactions investigated, is discussed. The level density of Fe<sup>56</sup> is calculated from the data including a correction for missing levels. These results are discussed only briefly since a more detailed comparison with theory will be given in a later paper in conjunction with other relevant data.

### II. EXPERIMENTAL PROCEDURE

The measurements were made with the Argonne Tandem Van de Graaff and broad-range single-gap

magnetic spectrograph.<sup>3</sup> Measurements at two different angles were made for each reaction in order to check the proper variation of the energy of the particle groups with angle. Incident proton energies of 10.965 MeV for the Co<sup>59</sup>(*p*, $\alpha$ ) and Fe<sup>56</sup>(*p*,*p'*) reactions and 8.972 MeV for the Co<sup>59</sup>(*p*,*p'*) reaction were chosen in order to bring the highest resolvable particle groups just above the Coulomb barrier (the use of a still higher bombarding energy would decrease the intensity of resolvable particle groups because of the opening of additional exit channels).

Targets of iron enriched to 99.9% in Fe<sup>56</sup> (15–20  $\mu\text{g}/\text{cm}^2$  thick) and 99.99% pure cobalt (10–20  $\mu\text{g}/\text{cm}^2$  thick) were prepared by vacuum evaporation onto 35- $\mu\text{g}/\text{cm}^2$  carbon backing foils. Nuclear emulsion plates (Kodak NTA) of 50- $\mu$  thickness were used for recording the proton spectra, and Ilford KO plates of 50- $\mu$  thickness were used for the alpha-particle spectra. By proper development, proton and deuteron tracks were suppressed in the plates used for determination of alpha particles. On each plate, reference lines were marked at accurately known positions prior to exposure, and all measurements were made with respect to these reference lines. Measurements were made of the number of particles in consecutive strips which were 0.5 mm wide and 10 mm long. According to the calibration used, the projection on the base line of a point of  $\frac{1}{3}$  maximum height on the high-energy side of a peak was taken as the position of the corresponding particle group. The calibration of J. R. Erskine<sup>4</sup> based on a value of 5.3045 MeV for the  $\alpha$  particles of Po<sup>210</sup> was used. In addition, the accuracy and reproducibility of this calibration was checked by observing the positions of the second and third excited levels of C<sup>13</sup> in all (*p*,*p'*) runs. The energy

† Based on work performed under the auspices of the U. S. Atomic Energy Commission.

<sup>1</sup> A. Aspinall, G. Brown, and S. E. Warren, Nucl. Phys. **46**, 33 (1963).

<sup>2</sup> A. Sperduto and W. W. Buechner, Phys. Rev. **134**, B142 (1964).

<sup>3</sup> C. P. Browne and W. W. Buechner, Rev. Sci. Instr. **27**, 899 (1956).

<sup>4</sup> J. Erskine (private communication).

TABLE I. Comparison of the energies in MeV of the second and third excited states of  $C^{13}$  observed in the different measurements.

Reaction	Second excited state	Third excited state
Co( $p, p'$ ) at $60^\circ$	3.685	3.854
Co( $p, p'$ ) at $120^\circ$	3.683	
Fe( $p, p'$ ) at $60^\circ$	3.681	3.851
Fe( $p, p'$ ) at $140^\circ$	3.684	3.853
Carter and Motz <sup>a</sup>	3.684 <sub>3</sub>	

<sup>a</sup> See Ref. 5.

of the second excited level of  $C^{13}$  has been measured very accurately by Carter and Motz.<sup>5</sup> The experimental energies of the two levels in  $C^{13}$  for several runs along with the energy of the second excited level of Carter and Motz are shown in Table I and indicate a possible error of  $\pm 0.004$  MeV in the calibration.

Half-widths of 6–8 keV were observed for the proton groups and about 10–15 keV for the  $\alpha$ -particle groups (see Figs. 1–3). The poorer resolution for the  $\alpha$ -particle groups is partly due to their higher energy loss and straggling in the target. Most of the increase in half-

width for the  $\alpha$  particles, however, is probably due to the fact that these particles were observed at higher energies than the protons because of their larger Coulomb barrier. Since the relative energy resolution  $\Delta E/E$  of the spectrograph is approximately independent of the energy  $E$ , the linewidth  $\Delta E$  for the  $\alpha$  particles is larger.

The  $Q$  value of each of the various particle groups on the plates was calculated with the Argonne IBM 704 computer. Relativistic kinematics were used in the transformation program.

### III. RESULTS

Typical spectra of the particles from the various reactions are shown in Figs. 1 to 3. In the ( $p, p'$ ) reactions the parts of the spectra below an excitation energy of 1.5 MeV in  $Co^{59}$  and 3 MeV in  $Fe^{56}$  were not observed. In order to improve the resolution in the higher excitation-energy range, the energy scale in this range was expanded on the spectrograph with the consequence that the well-known low-excitation-energy groups fell outside the spectrograph range.

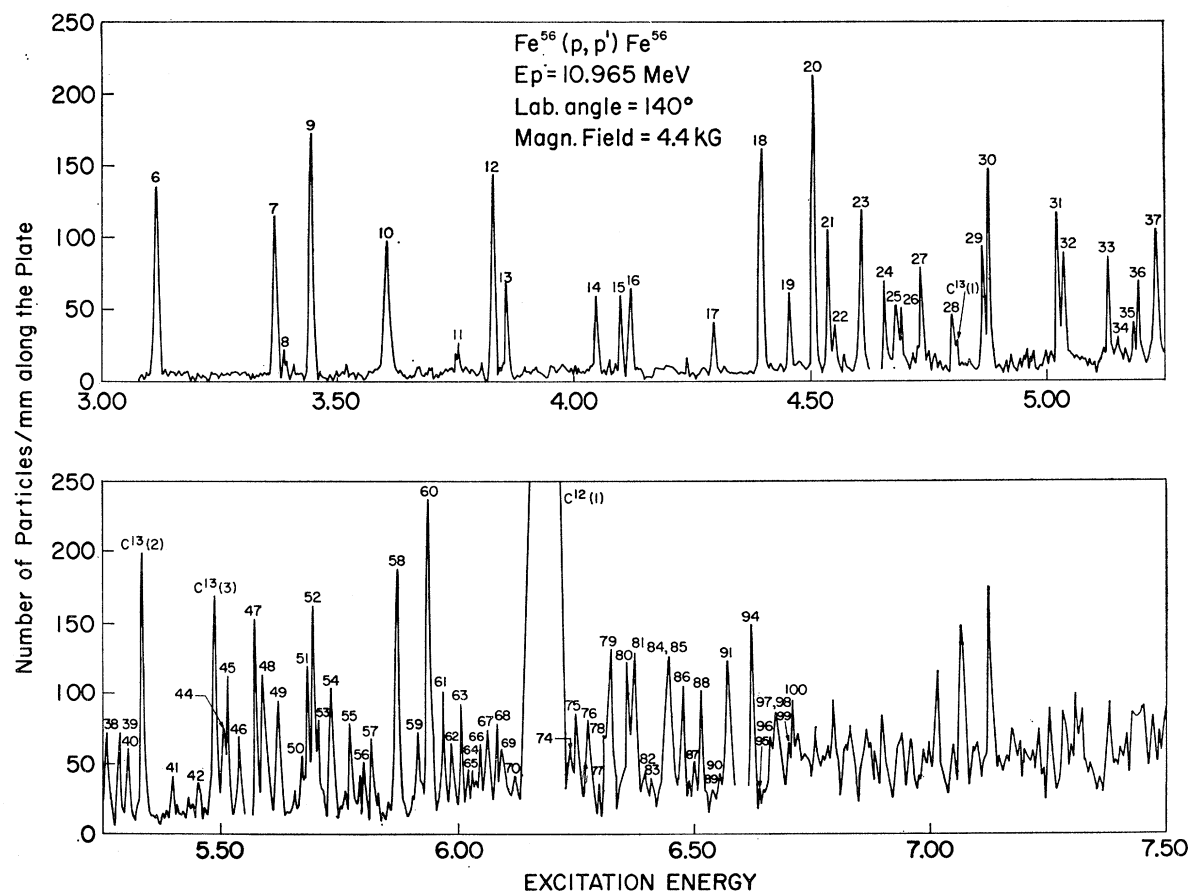


FIG. 1. Spectrum of protons inelastically scattered from  $Fe^{56}$ .

<sup>5</sup> R. E. Carter and H. T. Motz, Argonne National Laboratory Report No. ANL 6797, 1963, p. 179 (unpublished).

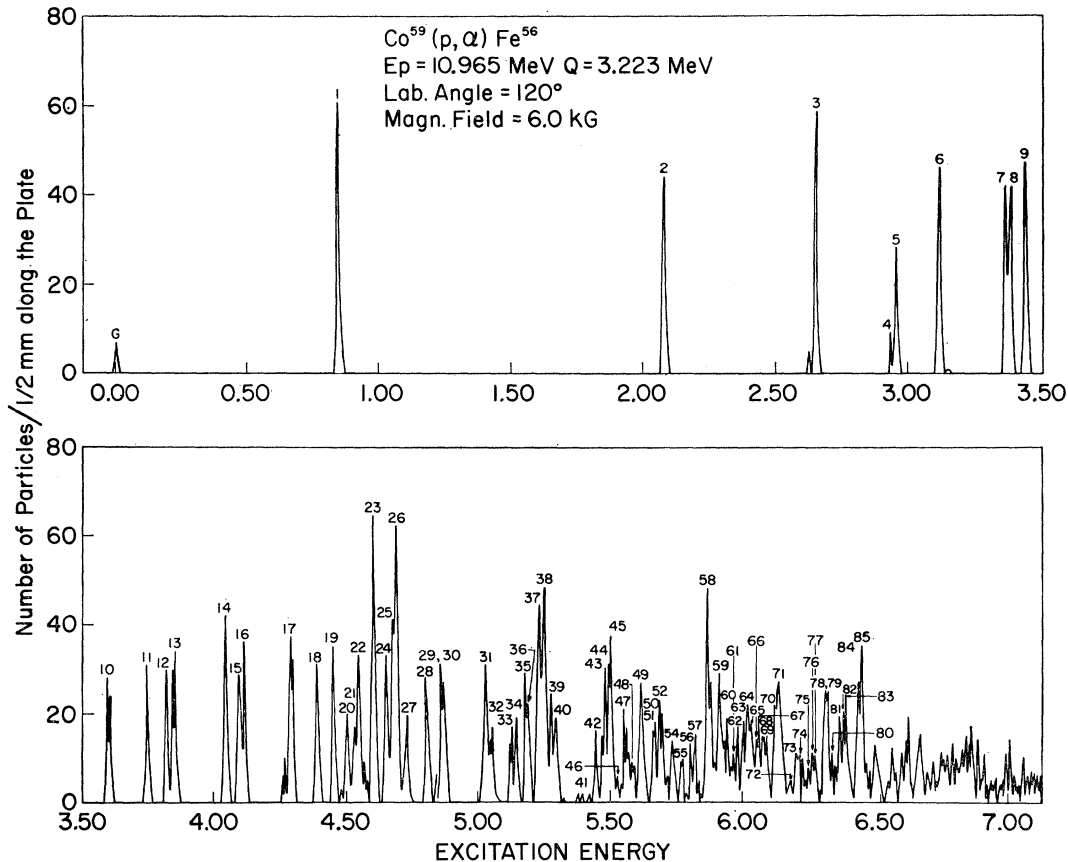


FIG. 2. Spectrum of  $\alpha$  particles from the reaction  $\text{Co}^{59}(p, \alpha)\text{Fe}^{56}$ .

The energies of the excited levels of  $\text{Fe}^{56}$  are listed in Table II. The first four columns of energy values are computed from the energies of proton groups observed at  $60^\circ$  and  $140^\circ$  in the  $\text{Fe}^{56}(p, p')$  reaction and  $\alpha$ -particle groups observed at  $90^\circ$  and  $120^\circ$  in the  $\text{Co}^{59}(p, \alpha)$  reaction. The intensities of each respective particle group are also listed in Table II. In addition, the weighted average of the four experimental determinations of the excitation energy of each level is given in the last column of Table II. Excitation energies for the  $\text{Co}^{59}(p, \alpha)\text{Fe}^{56}$  reaction were obtained by subtracting the ground-state  $Q$  value of 3.223 MeV from the measured  $Q$  values of the other observed alpha groups. The ground-state  $Q$  value was adjusted<sup>6</sup> to give the well-established value<sup>7</sup> of 845 keV for the first excited state of  $\text{Fe}^{56}$ . The energies of the excited levels of  $\text{Co}^{59}$  and the intensities of the corresponding particle groups are listed in Table III. In deriving the weighted energy

<sup>6</sup> We chose to use the first excited state rather than the ground state for the primary  $Q$ -value measurement because of its much larger cross section. The accuracies of the energies of the excited states do not depend to any extent on a possible small error in our determined ground-state  $Q$  value.

<sup>7</sup> *Nuclear Data Sheets*, compiled by K. Way (Printing and Publishing Office, National Academy of Sciences-National Research Council, Washington, D. C.), NRC 59-4-50, 59-4-51, and 59-4-58.

averages given in the last column of Table II, account was taken of the fact that the  $(p, p')$  results were obtained with higher energy resolution than the  $(p, \alpha)$  results. The variation in the heights of the peaks in the various spectra was considered also since the accuracy in determining the level position is related to the peak height.

Systematic and random errors arise in the determination of level energies. Systematic errors are due to the limited accuracy and reproducibility of the calibration, and in this experiment are probably less than 4 keV. Random errors are introduced due to the statistical uncertainty involved in the determination of the position of the particle groups. For well-separated groups of average intensity the random error is expected to be 2 keV, whereas, for partially overlapping groups, the random error might be as large as 8 keV. However, the random fluctuations between the excitation energies determined in the different experiments (see Tables II and III) are somewhat larger and indicate some additional, as yet unknown, source of random error.

#### IV. DISCUSSION

In Table IV the excitation energies of the levels of  $\text{Fe}^{56}$  found in this experiment are compared with

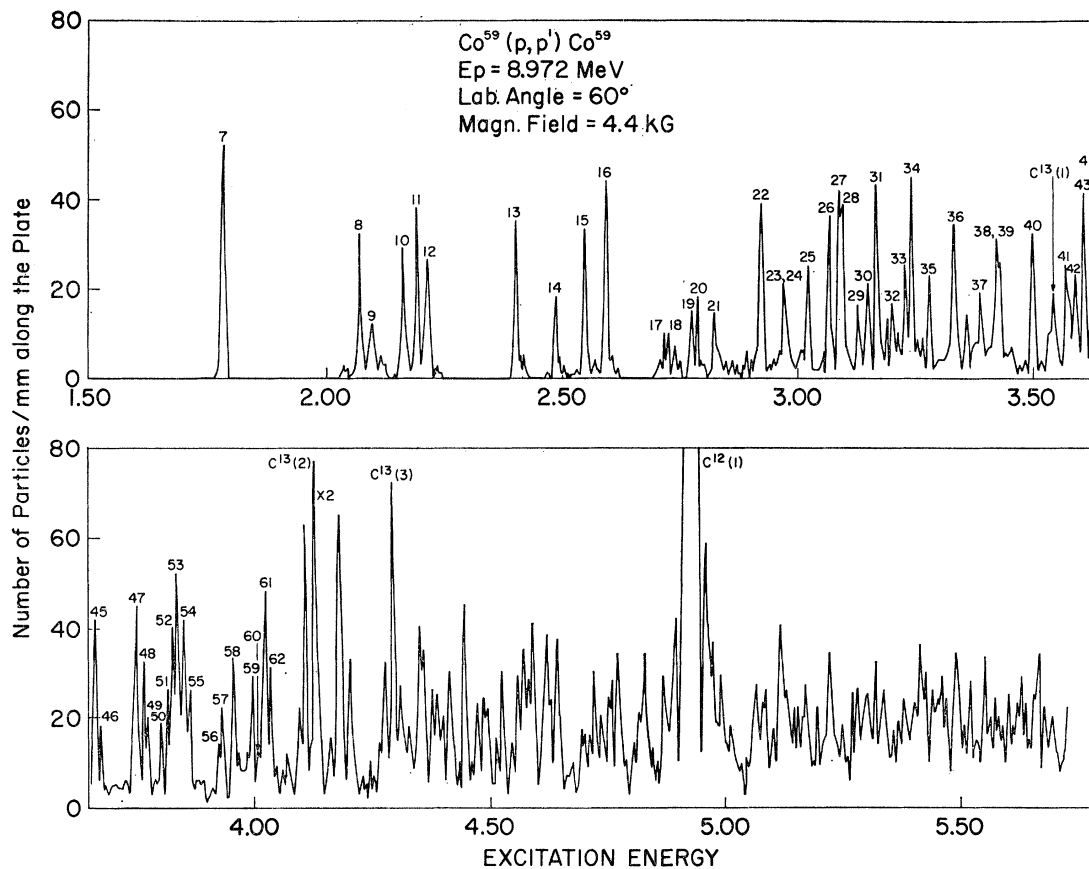


FIG. 3. Spectrum of protons inelastically scattered from  $\text{Co}^{59}$ .

previous data.<sup>1,2</sup> The agreement is satisfactory since the deviations are well within the stated limits of error. Within the energy range investigated in the earlier experiments two new levels were found in the present investigation (Nos. 11 and 28). Both of these levels are only weakly excited in the  $(p, p')$  reaction and were missed in the earlier experiments. However, they are rather strongly excited in the  $\text{Co}^{59}(p, \alpha)$  reaction. Although our level No. 11 is well recognizable in Fig. 2 of Ref. 1, it was not positively assigned to  $\text{Fe}^{56}$  because of its low intensity. One of the levels previously reported proved to be a closely spaced doublet (Nos. 25 and 26) which we resolved because of our better resolution compared to Ref. 1.

In Table V the energy levels obtained in this experiment are compared with the results of Mazari *et al.*<sup>8</sup> The deviations are well within the stated limits of error indicating satisfactory agreement. Because of our better energy resolution five levels of Mazari *et al.*<sup>8</sup> were identified as unresolved doublets. In two cases, levels 27 and 28, and 38 and 39, the composite nature of the levels was already indicated in the work of Mazari *et al.*<sup>8</sup> Four levels which were only tentatively assigned

to  $\text{Co}^{59}$  previously<sup>8</sup> were confirmed in the present experiments.

The reactions discussed in this report proceed mainly through the formation of a compound nucleus<sup>9,10</sup> and, hence, it is possible to calculate their cross sections according to the statistical theory of nuclear reactions. On the basis of this theory the cross section for producing a certain final level depends on the following quantities:

- (1) Excitation energy and angular momentum of the final level.
- (2) Energy of the intermediate compound nuclei and their distribution in angular momentum. These properties are uniquely determined by specifying the projectile, target nucleus, and incident energy.
- (3) The properties of all the exit channels.

For a calculation of the formation cross section of a final state of excitation energy  $E^*$  and spin  $I$  one needs the following parameters: spins of the target nucleus and projectile, optical-model transmission coefficients for the incident particle and all possible outgoing particles over their energetically possible energy ranges,

<sup>8</sup> M. Mazari, A. Sperduto, and W. W. Buechner, Phys. Rev. **107**, 365 (1957).

<sup>9</sup> R. Sherr and F. P. Brady, Phys. Rev. **124**, 1928 (1961).

<sup>10</sup> B. L. Cohen and A. G. Rubin, Phys. Rev. **113**, 579 (1959).

TABLE II. Excitation energies of levels of Fe<sup>56</sup> and particle-group intensities determined from the Fe<sup>56</sup>(*p,p'*) and Co<sup>59</sup>(*p,α*) reactions with 10.966-MeV protons.

Level	Fe <sup>56</sup> ( <i>p,p'</i> )Fe <sup>56</sup>				Co <sup>59</sup> ( <i>p,α</i> )Fe <sup>56</sup>				Excitation energy (MeV) (weighted average)
	140°	60°	120°	90°	Excitation energy (MeV)	Relative intensity <sup>a</sup>	Excitation energy (MeV)	Relative intensity <sup>a</sup>	
G			0.000		0.2				
1			0.845		1.5				0.845±0.005
2			2.078		1.2				2.078±0.005
3			2.652		1.4		2.650	1.1	2.652±0.005
4			2.939		0.2		2.937	0.3	2.938±0.008
5			2.954		0.5		2.955	0.7	2.954±0.007
6	3.110	4.5	3.115		1.3		3.117	2.1	3.114±0.005 <sup>b</sup>
7	3.361	2.7	3.365		0.8		3.364	0.9	3.363±0.005
8	3.382	0.4	3.380		1.2		3.381	2.1	3.381±0.007
9	3.438	4.2	3.440		1.2		3.442	2.1	3.440±0.005 <sup>c</sup>
10	3.594	3.7	3.595		0.8		3.601	1.1	3.597±0.005 <sup>c</sup>
11	3.748	0.6	3.750	0.2	3.745	0.6	3.751	1.2	3.748±0.005
12	3.824	2.4	3.822	1.7	3.819	0.7	3.826	0.6	3.823±0.005
13	3.851	1.4	3.849	1.4	3.845	0.8	3.856	1.0	3.851±0.005
14	4.041	1.00	4.043	1.00	4.038	1.00	4.048	1.00	4.042±0.005
15	4.094	1.1	4.095	1.7	4.090	0.7	4.094	1.3	4.093±0.005
16	4.112	1.4	4.115	1.2	4.113	0.8	4.112	1.1	4.113±0.005
17	4.289	0.8	4.292	0.8	4.288	0.9	4.291	1.0	4.290±0.005
18	4.388	4.2	4.386	4.5	4.386	0.9	4.391	1.2	4.388±0.005
19	4.449	1.1	4.450	0.6	4.450	0.7	4.448	1.0	4.449±0.005
20	4.501	3.2	4.500	2.9	4.501	0.5	4.500	0.7	4.501±0.005
21	4.532	1.8	4.532	0.7	4.530	0.3	4.531	0.4	4.531±0.005
22	4.544	0.8	4.547	0.5	4.543	0.9	4.546	1.3	4.545±0.007
23	4.602	2.3	4.601	1.6	4.603	1.4	4.602	1.5	4.602±0.005
24	4.651	1.1	4.650	0.7	4.649	0.6	4.652	0.6	4.651±0.005
25	4.674	1.0	4.675	0.8	4.677	0.6	4.675	1.5	4.675±0.005
26	4.689	0.8	4.685	0.4	4.691	1.5	4.685	1.0	4.688±0.008
27	4.729	1.5	4.722	1.3	4.725	0.5	4.732	0.7	4.727±0.005
28	4.804	1.1	4.795	0.2	4.803	0.7	4.805	0.7	4.802±0.005
29	4.860	1.4	4.857	0.6	4.860	0.5	4.857	0.2	4.858±0.005
30	4.872	2.3	4.870	1.6	4.873	0.5	4.877	0.6	4.873±0.005
31	5.018	1.7			5.023	0.8	5.022	1.1	5.021±0.005
32	5.028	1.6			5.032	0.4	5.041	0.6	5.032±0.007
33	5.124	1.4	5.120	1.1	5.123	0.3	5.127	0.4	5.123±0.005
34	5.139	0.6	5.136	0.9	5.143	0.4	5.142	0.5	5.140±0.006
35	5.176	0.5	5.182		5.181	0.3	5.182	0.7	5.181±0.008
36	5.187	1.1			5.188	0.4	5.192	0.3	5.188±0.005
37	5.224	2.3	5.217	1.3	5.224	0.9	5.227	1.1	5.224±0.005
38	5.251	1.2	5.246	1.2	5.248	1.1	5.250	1.4	5.249±0.005
39	5.276	1.0	5.272	0.6	5.273	0.4			5.274±0.005
40	5.296	1.0	5.293	1.1	5.298	0.4			5.296±0.005
41	5.389	0.6	5.385	0.3	5.383	0.1			5.386±0.007
42	5.444	0.7	5.442	0.4	5.445	0.2			5.444±0.008
43	5.479	2.8	5.478	1.1	5.473	0.5			5.476±0.005
44	5.496	1.0	5.495	1.4	5.496	0.4			5.496±0.005
45	5.504	1.3	5.500	0.7	5.503	0.4			5.503±0.008
46	5.529	1.0	5.530	1.1	5.523	0.1			5.528±0.005
47	5.558	1.8	5.557	1.3	5.555	0.4			5.557±0.005
48	5.584	2.2	5.580	1.0	5.583	0.2			5.583±0.005
49	5.612	1.6	5.611	1.1	5.613	0.8			5.612±0.005
50	5.664	0.6	5.660	0.5	5.665	0.2			5.663±0.006
51	5.676	1.4	5.670	0.6	5.673	0.1			5.673±0.008
52	5.689	4.8	5.680	0.8	5.683	0.3			5.684±0.005
53	5.699	1.0	5.696	0.5	5.695	0.4			5.697±0.008
54	5.725	1.7	5.722	0.5	5.728	0.3			5.725±0.005
55	5.769	1.1	5.766	0.7	5.770	0.2			5.768±0.005
56	5.796	0.8	5.795	0.9	5.808	0.2			5.797±0.005
57	5.812	0.9	5.810	0.4	5.818	0.1			5.813±0.007
58	5.864	2.8	5.861	1.5	5.864	1.2			5.863±0.005
59	5.909	1.5	5.905	0.5	5.910	0.7			5.908±0.005
60	5.931	3.6	5.927	1.7	5.938	0.2			5.932±0.005
61	5.962	1.5	5.960	0.6	5.968	0.2			5.962±0.007
62	5.980	1.1	5.978	0.9	5.983	0.2			5.980±0.008
63	6.002	1.2	5.997	0.4	6.006	0.2			6.002±0.007
64	6.012	0.6	6.012	0.3	6.015	0.3			6.013±0.010
65	6.024	0.6	6.020	0.3	6.028	0.4			6.024±0.010
66	6.040	1.1	6.038	0.6	6.048	0.2			6.041±0.008
67	6.054	1.2	6.053	0.9	6.058	0.2			6.055±0.008

TABLE II (continued)

Level	$Fe^{56}(p,p')Fe^{56}$				$Co^{59}(p,\alpha)Fe^{56}$				Excitation energy (MeV) (weighted average)
	140°		60°		120°		90°		
	Excitation energy (MeV)	Relative intensity <sup>a</sup>	Excitation energy (MeV)	Relative intensity <sup>a</sup>	Excitation energy (MeV)	Relative intensity <sup>a</sup>	Excitation energy (MeV)	Relative intensity <sup>a</sup>	
68	6.074	1.0	6.076	1.3	6.073	0.2			6.074±0.008
69	6.086	1.1	6.093	0.2	6.088	0.2			6.089±0.010
70	6.109	0.8	6.120	0.6	6.120	0.3			6.118±0.010
71			6.138	0.9	6.138	0.4			6.138±0.007
72			6.174	0.9	6.173	0.2			6.174±0.007
73			6.200	0.2	6.203	0.3			6.201±0.010
74	6.224	1.1	6.225	0.5	6.229	0.1			6.226±0.008
75	6.244	1.4	6.239	0.4	6.251	0.1			6.243±0.010
76	6.266	1.8	6.264	0.8	6.263	0.2			6.265±0.008
77	6.294	0.4	6.285	0.2	6.288	0.05			6.289±0.010
78	6.304	0.9	6.305		6.308	0.3			6.306±0.010
79	6.315	1.9			6.318	0.3			6.316±0.008
80	6.352	1.8	6.349	0.5	6.353	0.1			6.351±0.008
81	6.364	2.1	6.359	1.3	6.368	0.2			6.364±0.007
82	6.385	0.8	6.380	0.6	6.383	0.2			6.382±0.008
83	6.404	0.6	6.400	0.1	6.393	0.2			6.397±0.008
84	6.429				6.433	0.5			6.432±0.008
85					6.448	0.5			6.448±0.008
86	6.464	1.6	6.462	0.6					6.463±0.008
87	6.489	0.8							6.489±0.010
88	6.509	1.2							6.509±0.008
89	6.529	0.5	6.526	0.4					6.527±0.010
90	6.549	0.5	6.540	0.3					6.543±0.010
91	6.558	2.5	6.552	0.4					6.555±0.010
92			6.563	0.9					6.563±0.010
93			6.593	0.2					6.593±0.012
94	6.615	2.0	6.612	0.7					6.613±0.010
95	6.635	0.4	6.625	0.2					6.630±0.012
96	6.655	0.9	6.650	1.4					6.652±0.010
97	6.665		6.660	0.6					6.662±0.010
98			6.670	0.3					6.670±0.012
99	6.695	0.8							6.695±0.012
100	6.700	0.8							6.700±0.012

<sup>a</sup> Intensity relative to 4.042-MeV level.

<sup>b</sup> Recent evidence indicates that this level is an unresolved doublet [P. F. Hinrichsen, M. H. Shapiro, and D. M. Van Patter, Bull. Am. Phys. Soc. 10, 427 (1965)].

<sup>c</sup> Note added in proof: Recent evidence indicates that levels 9 and 10 are doublets (M. H. Shapiro, P. F. Hinrichsen, R. Middleton, and R. K. Mohindra, Phys. Letters, to be published). Our  $Co^{59}(p,\alpha)Fe^{56}$  spectrum also shows some indication that level 10 (see Fig. 2) is a doublet.

and the level densities of all residual nuclei as a function of energy and angular momentum.

The results of such calculations according to the formalism of Douglas and MacDonald<sup>11</sup> are shown in Figs. 4 and 5 for the two reactions  $Fe^{56}(p,p')$  and  $Co^{59}(p,\alpha)$  studied in this experiment. Transmission coefficients for  $\alpha$ -particles, protons, and neutrons were calculated from the respective optical potentials of Huizenga and Igo,<sup>12</sup> Perey,<sup>13</sup> and Bjorklund and Fernbach.<sup>14</sup> A constant temperature ( $T=1.4$  MeV) and spin-cutoff factor-type level density ( $2\sigma^2 = T\mathcal{G}_{\text{rigid}}/\hbar^2$  where  $\mathcal{G}_{\text{rigid}}$ =rigid-body moment of inertia with  $r_0=1.20$  F) were used for simplicity since the results are very insensitive to the choice of the level-density parameters. The details of the calculation are described elsewhere.<sup>15</sup>

<sup>11</sup> A. C. Douglas and N. MacDonald, Nucl. Phys. 13, 382 (1959).

<sup>12</sup> J. R. Huizenga and G. Igo, Nucl. Phys. 29, 473 (1962).

<sup>13</sup> F. G. Perey, Phys. Rev. 131, 745 (1963).

<sup>14</sup> F. Bjorklund and S. Fernbach, Phys. Rev. 109, 1295 (1958).

<sup>15</sup> H. K. Vonach, Habilitationsschrift, München, 1964 (unpublished).

The relative cross sections for single levels of particular spin are plotted in Figs. 4 and 5 as a function of level excitation energy for the reactions  $Fe^{56}(p,p')$  and  $Co^{59}(p,\alpha)$ , respectively. There is a characteristic difference in the spin dependence of the cross sections between the two reactions. Levels with large angular momenta are excited in the  $Co^{59}(p,\alpha)$  reaction with much larger cross sections than they are in the  $Fe^{56}(p,p')$  reaction. This is due to the large target spin of  $Co^{59}(\frac{7}{2})$  and the fact that  $\alpha$  particles can carry away considerably more angular momentum than protons. From a comparison of the intensities of levels observed in the above two reactions, it is possible to make a rough classification of the levels, according to their spins, into three groups. These groups are:

(1) Levels which are very weakly excited in both reactions probably have spin 0.

(2) Levels which are of about average intensity in both reactions probably have a spin in the range 1–5.

(3) Levels which are very weakly excited in the  $Fe^{56}(p,p')$  reaction and have at least an average in-

tensity in the Co<sup>59</sup>(*p*, $\alpha$ ) reaction probably have a spin larger than 5.

Before discussing the experimental results according to these rules it must be mentioned that the results of Figs. 4 and 5 apply to cross sections averaged over an energy interval which is large compared to the level

TABLE III. Excitation energies of levels of Co<sup>59</sup> and particle-group intensities determined from the Co<sup>59</sup>(*p*,*p'*) reaction with 8.972-MeV protons.

Level	60°		120°		Average (MeV)
	Excitation energy (MeV)	Relative intensity <sup>a</sup>	Excitation energy (MeV)	Relative intensity <sup>a</sup>	
7	1.745	3.9	1.746	3.0	1.745±0.005
8	2.065	1.9	2.065	2.0	2.065±0.005
9	2.085	1.3	2.091	1.2	2.088±0.005
10	2.157	2.0	2.158	1.4	2.157±0.005
11	2.187	2.3	2.187	2.0	2.187±0.005
12	2.207	1.8			2.207±0.005
13	2.396	1.9	2.396	2.0	2.396±0.005
14	2.480	1.00	2.480	1.00	2.480±0.005
15	2.543	1.6	2.542	1.7	2.542±0.005
16	2.586	2.6	2.585	2.4	2.585±0.005
17	2.715	0.4	2.713	0.2	2.714±0.010
18	2.720	0.4	2.720	0.3	2.720±0.010
19	2.768	0.7	2.770	0.5	2.769±0.007
20	2.784	0.6	2.780	1.2	2.782±0.007
21	2.818	1.0	2.825	2.4	2.821±0.006
22	2.915	2.3	2.913	2.9	2.914±0.005
23			2.958	1.4	2.958±0.005
24	2.976		2.973	0.7	2.973±0.012
25	3.016	1.5	3.016	1.9	3.016±0.005
26	3.062	1.6	3.061	1.6	3.061±0.005
27	3.082	2.1	3.083		3.082±0.005
28	3.090	1.6			3.090±0.012
29	3.125	0.7	3.121	1.1	3.123±0.007
30	3.140	1.1	3.140	1.3	3.140±0.007
31	3.161	2.1	3.161	2.2	3.161±0.005
32	3.195	0.8	3.196	1.3	3.195±0.005
33	3.223	1.2	3.225	1.1	3.224±0.005
34	3.236	2.0	3.235	2.0	3.235±0.005
35	3.275	1.0	3.275	1.4	3.275±0.005
36	3.325	2.4	3.327	2.2	3.326±0.005
37	3.384	1.1	3.383	1.8	3.383±0.006
38	3.414	1.0	3.418	1.2	3.416±0.005
39	3.420	1.5	3.430	2.0	3.425±0.010
40	3.492	1.8	3.493	2.6	3.492±0.005
41	3.565	1.7	3.566	2.2	3.565±0.005
42	3.582	1.0	3.583	0.7	3.582±0.008
43	3.600	1.9	3.600	2.1	3.600±0.005
44	3.618	1.3	3.622	1.3	3.620±0.010
45	3.655	2.1	3.655	2.8	3.655±0.005
46	3.667	0.6	3.665	1.1	3.666±0.007
47	3.740	2.3	3.742	3.4	3.741±0.005
48	3.760	0.9			3.760±0.005
49	3.765	0.8			3.765±0.010
50	3.797	0.7			3.797±0.005
51	3.812	0.8			3.812±0.005
52	3.819	1.4			3.819±0.008
53	3.830	2.0			3.830±0.008
54	3.845	1.8			3.845±0.008
55	3.857	0.9			3.857±0.008
56	3.917	0.6			3.917±0.010
57	3.925	0.7			3.925±0.010
58	3.950	1.4			3.950±0.005
59	3.986	1.4			3.986±0.008
60	4.000	0.5			4.000±0.012
61	4.015	2.0			4.015±0.008
62	4.030	1.1			4.030±0.008

<sup>a</sup> Intensity relative to level No. 14 at 2.480 MeV.

TABLE IV. Energy levels of Fe<sup>56</sup>.

Level	Excitation energy (MeV)		
	This expt.	Aspinall <i>et al.</i> <sup>a</sup>	Sperduto and Buechner <sup>b</sup>
1	0.845	0.842	0.845
2	2.078	2.079	2.085
3	2.652	2.652	2.658
4	2.938	2.933	2.940
5	2.954	2.955	2.958
6	3.114	3.117	3.119
7	3.363	3.365	3.369
8	3.381	3.384	3.388
9	3.440	3.441	3.445
10	3.597	3.597	3.601
11	3.748		
12	3.823	3.819	3.830
13	3.851	3.844	3.856
14	4.042	4.035	4.042
15	4.093	4.087	(4.094)
16	4.113	4.108	(4.115)
17	4.290	4.289	(4.295)
18	4.388	4.387	(4.392)
19	4.449	4.450	(4.453)
20	4.501	4.502	(4.507)
21	4.531	4.533	(4.535)
22	4.545		(4.587)
23	4.602	4.601	(4.606)
24	4.651	4.651	(4.657)
25	4.675	4.676	(4.680)
26	4.688		
27	4.727	4.728	(4.743)
28	4.802		
29	4.858	4.866	
30	4.873		
31	5.021	5.017	
32	5.032	5.044	
33	5.123	5.130	
34	5.140		
35	5.181		
36	5.188	5.191	

<sup>a</sup> Reference 1.  
<sup>b</sup> Reference 2.

width of the compound nucleus. This criterion has not been fulfilled in the present measurements. Hence, the effect of cross-section fluctuations cannot be neglected. However, for the angles used in this experiment the actual cross sections fluctuate from the average cross

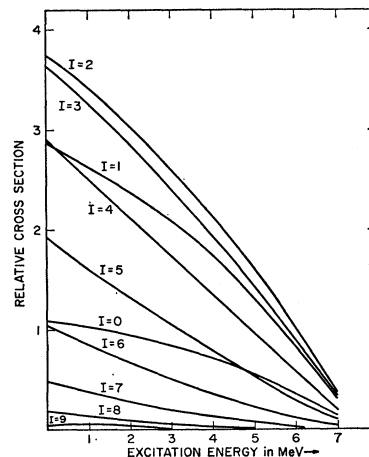


FIG. 4. Statistical-theory calculation of the relative cross section for exciting a level of excitation energy  $E^*$  and angular momentum  $I$  in the Fe<sup>56</sup>(*p*,*p'*) reaction induced with 11-MeV protons.

TABLE V. Energy levels of Co<sup>59</sup>.

Level	Excitation energy (MeV)	
	This expt.	Mazari <i>et al.</i> <sup>a</sup>
7	1.745	1.743
8	2.065	2.061
9	2.088	2.086
10	2.157	2.152
11	2.187	2.183
12	2.207	2.205
13	2.396	2.397
14	2.480	2.477
15	2.542	2.540
16	2.585	2.585
17	2.714	2.711
18	2.720	
19	2.769	2.770
20	2.782	2.781
21	2.821	2.822
22	2.914	2.911
23	2.958	2.955
24	2.973	2.964
25	3.016	3.015
26	3.061	3.058
27	3.082	3.081
28	3.090	
29	3.123	3.120
30	3.140	3.148
31	3.161	3.159
32	3.195	3.192
33	3.224	3.222
34	3.235	3.233
35	3.275	3.273
36	3.326	3.323
37	3.383	3.379
38	3.416	3.412
39	3.425	
40	3.492	3.490
41	3.565	3.560
42	3.582	
43	3.600	3.602
44	3.620	
45	3.655	3.654
46	3.666	

<sup>a</sup> Reference 8.

sections by at most a factor of 2 for spin 0 levels and considerably less (about 20–30%) for levels of higher angular momentum.<sup>16</sup> Therefore, the comparison of experimental cross sections with the values of the cross sections of Figs. 4 and 5 is a qualitatively valid procedure. From these comparisons we are able to make some comments on the spin of a few of the excited levels of Fe<sup>56</sup>. Level No. 4 at 2.938 MeV is very weakly excited in both the<sup>1,2</sup> Fe<sup>56</sup>(*p, p'*) and the Co<sup>59</sup>(*p, α*) reactions and probably has spin 0. This spin value is consistent with the fact that this level is not excited in either the decay of Mn<sup>56</sup> or Co<sup>56</sup>. A 0<sup>+</sup> level has been observed in Cr<sup>52</sup> at a similar excitation energy. Levels No. 7 and 8 are at 3.363 and 3.381 MeV, respectively. One of these levels is excited in the decay of Mn<sup>56</sup> and has a spin of 2<sup>+</sup> according to the investigation of its decay scheme. In *Nuclear Data Sheets*,<sup>7</sup> this level has been identified with level No. 8. Our data strongly suggest that this 2<sup>+</sup> level is level No. 7. Level No. 7 is

<sup>16</sup> H. K. Vonach, A. Katsanos, and J. R. Huizenga, *Phys. Rev. Letters* **13**, 88 (1964).

excited with about average intensity in both the Fe<sup>56</sup>(*p, p'*) and Co<sup>59</sup>(*p, α*) reactions, which supports a spin assignment of 2. Level No. 8 is excited with about the same intensity as level No. 7 in the Co<sup>59</sup>(*p, α*) reaction but is only weakly excited in the Fe<sup>56</sup>(*p, p'*) reaction. Therefore, the spin of level No. 8 is 6 or higher. Level No. 11 is populated with average intensity in the Co<sup>59</sup>(*p, α*) reaction and very weakly excited in the Fe<sup>56</sup>(*p, p'*) reaction indicating that it has a spin of 6 or higher in agreement with the fact that this level is not populated in Co<sup>56</sup> decay.

From the data in Table II the level density of Fe<sup>56</sup> can be determined in the most direct way by simply counting the number of levels per energy interval. However, two kinds of systematic errors have to be considered. Firstly, some levels may escape detection due to their low cross section. Secondly, some closely spaced levels may remain unresolved because of the finite energy resolution of the experiment. In accordance with the discussion of group intensities in the preceding sections, errors of the first type can be neglected. Levels of spin 1–8 should have cross sections of comparable intensity in the Co<sup>59</sup>(*p, α*) reaction. The probability for occurrence of levels with spin zero and higher than 8, which might partially be missed, amounts only to a few percent for reasonable values of the spin-cutoff parameter  $\sigma$ . The second type of error is more serious, especially in the higher excitation energy region. In order to estimate the number of unresolved pairs of levels one has to make certain assumptions about the distribution of level spacings  $W(d)$ .

As adjacent levels very probably do not have the same spin and parity, the effect of Wigner repulsion can be neglected, and it appears reasonable to assume an exponential distribution of level spacings according to a completely random distribution of levels,

$$W(d/\bar{d}) = \exp\{- (d/\bar{d})\}, \quad (1)$$

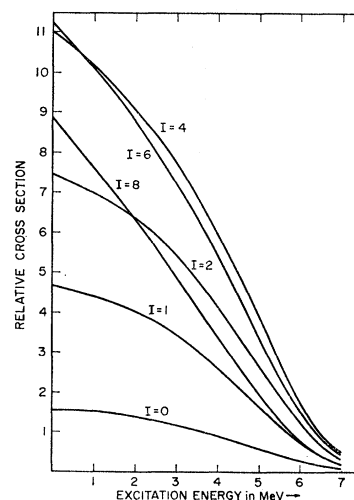


FIG. 5. Statistical-theory calculation of the cross section for exciting a level of excitation energy  $E^*$  and angular momentum  $I$  in the Co<sup>59</sup>-(*p, α*)Fe<sup>56</sup> reaction induced with 11-MeV protons.



where  $\bar{d}$  is the average level spacing. This distribution is also consistent with experimental data. The statistical distribution of 55 level spacings of Fe<sup>54,55,56,57,58</sup> and Fe<sup>59</sup> are plotted in Fig. 6. An interval of 1-MeV length and average spacing  $\bar{d}$  of 100 keV has been taken from the level scheme of each nucleus and combined in the distribution of Fig. 6. The experimental data agree well with the theoretical results of Eq. (1).

If one estimates from the experimental energy resolution that levels with spacings larger than  $\Delta E_1$  are resolved with certainty and levels with spacings smaller than  $\Delta E_2$  are unresolved, one obtains for the theoretical level spacing of Eq. (1) the following corrected level-density expression,

$$\rho_{\text{obs}} \exp(\Delta E_2/\bar{d}) < \rho_{\text{corr}} < \rho_{\text{obs}} \exp(\Delta E_1/\bar{d}). \quad (2)$$

In Fig. 7(a) the uncorrected experimental level density is compared with the level density corrected according to Eq. (2) when  $\Delta E_1$  and  $\Delta E_2$  are 8 and 4 keV, respectively.

This level density will be compared in detail with the predictions of theoretical models in a later paper. At

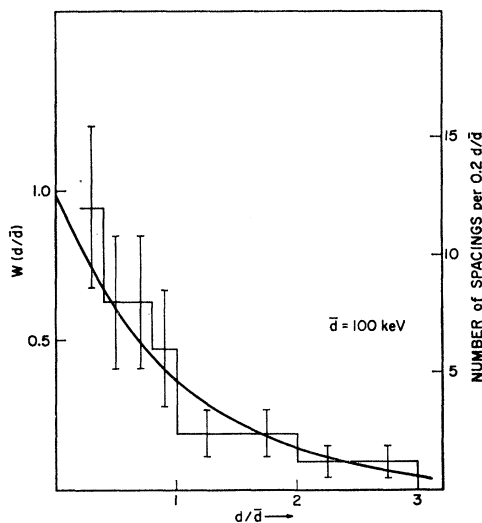
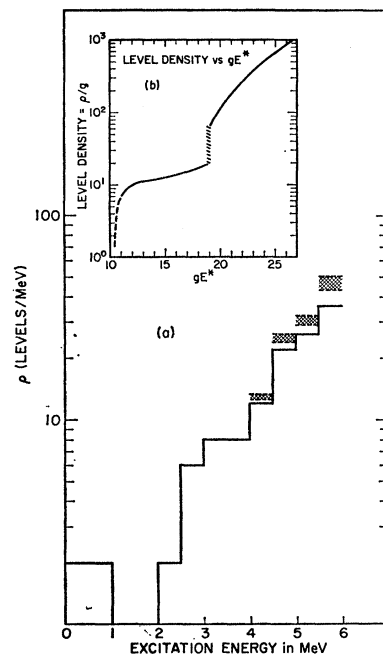


FIG. 6. Statistical distribution of level distances. Histogram: statistical distribution of 55 level distances from the level schemes of Fe<sup>54,55,56,57,58</sup> and Fe<sup>59</sup>. The histogram is drawn in terms of the number of spacings per 0.2  $\bar{d}/\bar{d}$ . Curve: theoretical level distance distribution if the levels are distributed at random.

FIG. 7. (a) Level density of Fe<sup>56</sup> as determined by direct level counting from the results of this experiment (Table II). The horizontal lines of the histogram represent uncorrected level density; the cross-hatching represents level density corrected for unresolved levels. The upper and lower limits to the cross-hatched region represent upper and lower limits to the corrected level density. (b) Level density of a finite superconducting nucleus according to Rich and Griffin (Fig. 2 of Ref. 17).



this time we want only to mention that the structure in the experimental level density of Fe<sup>56</sup> is qualitatively in agreement with the predictions of J. J. Griffin<sup>17</sup> for a finite superconducting nucleus. The predictions of Griffin are shown in Fig. 7(b), where for Fe<sup>56</sup> with  $a=A/8$  the onset of 2-quasiparticle states comes at about 2.5 MeV and 4-quasiparticle states at about 4.3 MeV. These energies agree well with the steps in the experimental level density as a function of energy and are also consistent with a pairing energy of about<sup>18</sup> 3 MeV as determined from mass differences of odd-odd and even-even nuclei if the effect of blocking is taken into account.<sup>19</sup>

#### ACKNOWLEDGMENT

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<sup>17</sup> M. Rich and J. J. Griffin, Phys. Rev. Letters **11**, 19 (1963).

<sup>18</sup> P. E. Nemirovski and Y. Adamchuk, Nucl. Phys. **39**, 551 (1962).

<sup>19</sup> H. K. Vonach, R. Vandenbosch, and J. R. Huizenga, Nucl. Phys. **60**, 70 (1964).