Magnetic Moments of the Baryons*

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The anomalous magnetic moments of the baryon octet are calculated in broken $SU(3)$ symmetry using low-energy pole dominance as a dynamical model and keeping only the lowest lying intermediate states, the psuedoscalar-meson–baryon states. By using dispersion theory, the anomalous moments are related to an energy integral over the $S_{1/2}$ and $P_{1/2}$ photomeson production amplitudes, which at low energy and for vanishing meson mass are exactly given by the pole terms. From this exact information we calculate the low-energy contribution to the anomalous moments, keeping all orders in baryon and meson mass splittings and using the $SU(3)$-symmetric strong-coupling constants. We are able to account for the dominant contribution to the proton and neutron magnetic moment and find in addition, for an F/D ratio$\sim$0.6, that $K(\Lambda)\sim 0.4K(n)$, in agreement with the observed value. The $SU(3)$ predictions for the other moments and the $\Sigma^0\rightarrow\Lambda+\gamma$ transition moment are found to be more badly violated.

I. INTRODUCTION

OUR purpose here is to present a dynamical calculation of the magnetic moments of the baryon octet in broken $SU(3)$ symmetry by applying the method of low-energy pole dominance.[1] It has been shown[2,3] that to all orders in the $SU(3)$-symmetric strong couplings and to first order in the electromagnetic coupling that all the static magnetic moments of the baryons, including the $\Sigma^0\rightarrow\Lambda^0+\gamma$ transition moment, can be expressed in terms of the proton and neutron magnetic moments. The question to which we now address ourselves is how are these predictions in the limit of exact $SU(3)$ symmetry altered by taking into account the observed splitting of the baryon and meson masses? To answer this we must appeal to a specific dynamical model which we shall now describe.

We assume that the charged baryons have an intrinsic Dirac moment $\mu_D=e/2M_B$ while for the uncharged baryons $\mu_D=0$. The anomalous part K_B of the total moment μ_T given by $K_B|e|/2M_B=\mu_T-\mu_D$ is to be accounted for in terms of strong-interaction corrections to the baryon electromagnetic current. We estimate these corrections by applying the method of low-energy pole dominance already successfully applied to the calculation of the electron anomalous moment g^e-2 and the anomalous moments of the nucleons, K_p and K_n.[1] The fundamental assumption of this method is that the static electromagnetic properties of a particle emerge predominantly as a consequence of the physics of the low-energy region. For the baryons this implies it is the lighter charged mesons in the cloud surrounding the baryon whose coupling with the electromagnetic field transforms like $Q=e(F_3+F_8/\sqrt{3})$ that are responsible for contributing the major part of the anomalous moment. In Ref. 1 the hypothesis of low-energy dominance was applied to nucleon moments with the result $K_p\simeq -K_n$ which followed from the pure isovector character of the electromagnetic coupling to the nucleon current in the limit of low-momentum transfer.

To estimate the magnitude of the low-energy contribution to the anomalous moments we use sidewise dispersion relations first used by Bincer in an examination of the electromagnetic properties of the nucleons.[4] Bincer was able to relate, as we shall spell out in more detail below, the anomalous static moment of a fermion to an energy integral over the $S_{1/2}$ and $P_{1/2}$ photomeson production amplitudes. The pole terms of the photomeson production amplitude for vanishing meson mass correspond at threshold to the *exact* amplitude. By including only the pole terms in the photomeson production amplitude and extending the energy integral only over the low-energy region this exact threshold behavior is incorporated into the calculation of the static moments. In agreement with the hypothesis of threshold dominance it is found in the case of the nucleons that the major contribution to K_p and K_n can be accounted for from the low-energy region $M\leq E\leq 1.7M$ of the photopion-production amplitude, with the result $K(p)\approx 1.5$, $K(n)\approx -1.6$, whereas $K^{\mathrm{expt}}(p)=1.79$, $K^{\mathrm{expt}}(n)=-1.91$. It is purely on the basis of this successful estimation of the nucleon moments, assuming threshold dominance, that we now attack the problem of computing the moments of the remaining members of the baryon octet.

II. CALCULATION OF THE ABSORPTIVE AMPLITUDE

Here we shall show how the static moment is related to the photomeson-production amplitude. Consider the transition amplitude for a virtual fermion of momentum

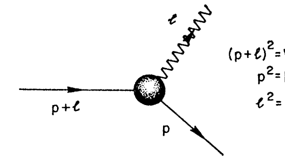

FIG. 1. Fermion-photon vertex.

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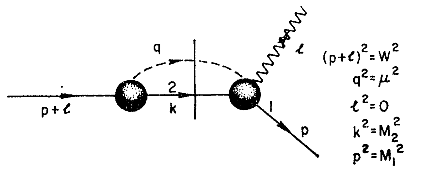

FIG. 2. Pseudoscalar-meson–baryon intermediate-state contribution to the absorptive part.

$p+l$ and invariant mass W, $W^2=(p+l)^2$, to produce a real fermion of momentum p, $p^2=M_1^2$ and a real photon of momentum l, $l^2=0$ (Fig. 1). Bincer[4] has shown that the most general form for this vertex consistent with Lorentz invariance, parity invariance, time reversal, and the generalized Ward identity is

$$\begin{aligned}e\bar{u}(p)\Gamma_\mu = e\bar{u}(p)[\gamma_\mu &+ (-K(W^2)i\sigma_{\mu\nu}l_\nu/2M_1+F_3{}^+(W^2)l_\mu) \\ &\times((M_1+p\!\!\!/+l\!\!\!/)/2M_1) \\ &+(-K^-(W^2)i\sigma_{\mu\nu}l_\nu/2M_1+F_3{}^-(W^2)l_\mu) \\ &\times((M_1-p\!\!\!/-l\!\!\!/)/2M_1)], \quad (1)\end{aligned}$$

where the invariant functions $K(W^2)$, $K^-(W^2)$, $F_3{}^\pm(W^2)$ are analytic functions in the cut W^2 plane with the branch cut extending from the threshold of the lightest intermediate state with the quantum numbers of the fermion to $+\infty$. We recognize from Eq. (1) in the limit $W\to M_1$, $K(M_1^2)$ as the anomalous moment of the fermion. From the analytic properties of $K(W^2)$ and the assumption $K(W^2)\to 0$ as $|W^2|\to\infty$ we may write an unsubtracted dispersion relation for $K(W^2)$

$$K(W^2)=\frac{1}{\pi}\int_{W_T{}^2}^{\infty}\frac{\mathrm{Im}K(W'^2)dW'^2}{W'^2-W^2}, \quad (2)$$

where $W_T{}^2=(M_2+\mu)^2$, the threshold for photomeson production, corresponds to the lightest intermediate state contributing to the absorptive amplitude $\mathrm{Im}K(W^2)$. Here, M_2 and μ are the masses of the intermediate baryon and meson. The low-energy contribution to the static moment may be gotten from (2) by extending the range of the energy integration only over the threshold region and evaluating Eq. (2) at $W^2=M_1^2$,

$$K(M_1^2)=\frac{1}{\pi}\int_{(M_2+\mu)^2}^{\Lambda(M_2+\mu)^2}dW^2\frac{\mathrm{Im}K(W^2)}{W^2-M_1^2}, \quad (3)$$

where $\Lambda>1$ is the cutoff.

Physics enters our calculation via the absorptive amplitudes $\mathrm{Im}K(W^2)$ in the threshold region. In the case of the baryons the only intermediate states contributing to $\mathrm{Im}K(W^2)$ in this region are pseudoscalar-meson–baryon (PS-B) states (Fig. 2). The thresholds for the vector-meson–baryon states lie higher in the mass spectrum and are a correction to the contributions from the lower lying states. Including only the contribution from the PS-B intermediate state we have as an *exact* expression for the absorptive part in the region near threshold,

$$\mathrm{Im}K(W^2)=\sum_{\substack{\text{spin}\\\text{states}}}\int_{-1}^{1}dx\,\rho(W^2)\bar{u}(p,s)J_\mu u(k,s') \times\bar{u}(k,s')\Gamma(W^2)\nu_\mu{}^2 \quad (4)$$

corresponding to the graph of Fig. 2. The factor $\rho(W^2)$ arises from purely kinematical considerations and is proportional to the available phase space for the intermediate state

$$\rho_2(W^2)=[(W^2+M_2{}^2-\mu^2)^2-4W^2M_2{}^2]^{1/2}/W^2. \quad (5)$$

The projection operator $\nu_\mu{}^2$ serves to project out the anomalous moment $K(W^2)$ from the vertex [Eq. (1)] and is explicitly given in Ref. 1. The factor $\bar{u}(k,s')\Gamma(W^2)$, corresponding to the vertex for a virtual baryon to create a real baryon and pseudoscalar meson, we approximate with its threshold value $g\bar{u}(k,s')i\gamma_5$ where g is the coupling constant. The photomeson-production amplitude, $\bar{u}(p,s)J_\mu u(k,s')$ for a baryon and meson of momentum k_α and $q_\alpha(k^2=M_2{}^2, q^2=\mu^2)$ to produce a baryon and photon of momentum p_α and $l_\alpha(p^2=M_1^2, l^2=0)$ is approximated by the pole terms (Fig. 3),

$$\begin{aligned}&\bar{u}(p,s)J_\mu u(k,s') \\ &=g\bar{u}(p,s)[\Gamma_\mu{}^1(i/(p\!\!\!/+l\!\!\!/-M_1))i\gamma_5+i\gamma_5(i/(k\!\!\!/-l\!\!\!/-M_2))\Gamma_\mu{}^2 \\ &\quad+(e_1-e_2)(i(2q_\mu-l_\mu)/(q-l)^2-\mu^2)i\gamma_5]u(k,s'), \quad (6) \\ &\quad\Gamma_\mu{}^j=e_j\gamma_\mu+(K_j/2M_j)\sigma_{\mu\nu}l_\nu, \quad j=1,2\end{aligned}$$

where $e_{1,2}$ is the sign of the charge on the final and intermediate baryon and $K_{1,2}$ are the anomalous moments of the final and intermediate baryons. In making this approximation we are assured that for vanishing meson mass Eq. (6) reproduces the exact amplitudes at threshold and thus provides a low-energy "anchor" for our calculation. The angular integration in Eq. (4) extends over the range of scattering angles $x=\mathbf{q}\cdot\mathbf{l}/|\mathbf{q}||\mathbf{l}|$ in the center-of-mass system for the scattering process for which $W=p_0+l_0=k_0+q_0$, $\mathbf{q}=-\mathbf{k}$, $\mathbf{p}=-\mathbf{l}$, and the sum is understood to include all contributing B-PS states.

Inserting Eq. (6) into Eq. (4) we obtain for the contribution of a single B-PS intermediate state to the absorptive part

$$\begin{aligned}\mathrm{Im}K_{12}(W^2)&=(g^2/4\pi)\rho_2(W^2)F_{12}(W^2), \\ F_{12}(W^2)&=e_1E_1(W^2)+e_2E_2(W^2) \\ &\quad+K_1K_1(W^2)+K_2K_2(W^2),\end{aligned} \quad (7)$$

where $E_{1,2}$ and $K_{1,2}$ arising from charge and anomalous

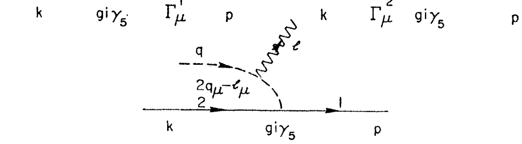

FIG. 3. Pole terms for photomeson production.

magnetic moment interaction in the photomeson-production amplitude are given in Appendix I. Equation (7) for the absorptive amplitude along with the $SU(3)$ predictions of the B-PS coupling constants in terms of the F/D ratio form the basis of our calculation in the next section.

The $SU(3)$ symmetry is broken through the introduction of nondegenerate baryon and meson masses in Eq. (7). For simplicity let us examine the symmetric case and set all the meson masses equal to zero and assume the baryon masses degenerate $M_1=M_2=M$. Moreover, we shall evaluate the dynamical factor $F_{12}(W^2)$ at threshold $W^2=M^2(\mu=0)$ where $F_{12}(M^2)=e_1-e_2=$ charge on intermediate meson, simply a reflection of the Kroll-Ruderman theorem which implies that at threshold only the charged mesons contribute to photoproduction.[5] With these approximations Eq. (7) becomes

$$\mathrm{Im}K_{12}(W^2)=(e_1-e_2)(g^2/4\pi)((W^2-M^2)/4M^2), \quad (8)$$

and from the dispersion relation

$$K_{12}=\frac{1}{\pi}\int_{M^2}^{\Lambda M^2}\frac{\mathrm{Im}K_{12}(W^2)dW^2}{W^2-M^2}$$

one finds

$$K_{12}=(e_1-e_2)(g^2/4\pi)(\ln\Lambda/4\pi) \quad (9)$$

for the threshold contribution of one intermediate state to the baryon moment. Summing over all possible intermediate B-PS states and taking into account the correct isotopic-spin factors we obtain for the anomalous moments of the baryons and the $\Sigma^0\rightarrow\Lambda^0+\gamma$ transition moment:

$$\begin{aligned}
K(p)&=c(2g_{\pi N}^2+g_{\Lambda K}^2+g_{\Sigma K}^2),\\
K(n)&=c(-2g_{\pi N}^2+2g_{\Sigma K}^2),\\
K(\Lambda)&=c(-g_{\Lambda K}^2+h_{\Lambda K}^2),\\
K(\Sigma^-)&=c(-g_{\Sigma\pi}^2-2g_{\Sigma K}^2-g_{\Lambda\pi}^2),\\
K(\Sigma^0)&=c(-g_{\Sigma K}^2+h_{\Sigma K}^2),\\
K(\Sigma^+)&=c(g_{\Sigma\pi}^2+g_{\Lambda\pi}^2),\\
K(\Xi^-)&=c(-h_{\Sigma K}^2-h_{\Lambda\Sigma}^2-2g_{\Xi\pi}^2),\\
K(\Xi^0)&=c(-2h_{\Sigma K}^2+2g_{\Xi\pi}^2),\\
K(\Sigma^0,\Lambda)&=c(-g_{\Sigma K}g_{\Lambda K}+h_{\Sigma K}h_{\Lambda K}),
\end{aligned} \quad (10)$$

where $c=\ln\Lambda/4\pi$. From the expressions for the coupling constants in terms of the F/D ratio[6] follow the predictions of $SU(3)$ symmetry

$$\begin{aligned}
&K(\Sigma^+)=K(p), \qquad K(\Lambda)=\tfrac{1}{2}K(n),\\
&K(\Xi^0)=K(n), \qquad K(\Xi^-)=K(\Sigma^-)=-[K(p)+K(n)],\\
&K(\Sigma^0)=-\tfrac{1}{2}K(n),\ K(\Sigma^0,\Lambda)=-\tfrac{1}{2}\sqrt{3}K(n),
\end{aligned} \quad (11)$$

and as a consequence of our model

$$K(p)=(\ln\Lambda/12\pi)(10-8f+16f^2)(g^2/4\pi)$$
$$K(n)=(\ln\Lambda/\pi)2f(f-1)(g^2/4\pi), \quad g^2/4\pi\sim 15,$$

which for $f=F/D=\frac{2}{3}$ yields $K(p)/K(n)=-2.2$. This disagreement with the observed $K(p)/K(n)=-0.94$ is here attributed to the assumed mass degeneracy. If we assume a nondegenerate spectrum the πN state has a much lower threshold than ΛK or ΣK and in the threshold approximation it is the only contributing state, so $K(p)/K(n)=-1.0$, independent of F/D.

III. CALCULATION OF THE MAGNETIC MOMENTS

Next we take in account the nondegeneracy of the mass spectrum of the baryons and mesons neglecting the electromagnetic splittings. From the dispersion integral we compute the moments

$$K_i=\frac{1}{\pi}\int_{s_i}^{\Lambda s_i}\frac{\mathrm{Im}K_i(W^2)dW^2}{W^2-M_i^2}, \quad (12)$$

where i ranges from 1 to 9 corresponding to the 8 baryon moments and the $\Sigma^0\rightarrow\Lambda$ transition moment and s_i is the lowest threshold of the photomeson-production amplitude contributing to the ith moment. For a given intermediate state

$$\mathrm{Im}K_{12}(W^2)=(g^2/4\pi)\rho_2(W^2)F_{12}(W^2=W_T^2),$$

and we evaluate the contribution from the pole terms $F_{12}(W^2)$ at threshold $W^2=W_T^2$ so as not to emphasize the high-energy region. This term depends on the masses M_1, M_2, and μ, the charges e_1 and e_2 and the anomalous moments K_1 and K_2 of the contributing states. If we include the energy dependence given by the pole terms, we obtain results for the nucleon moments which are in disagreement with experiments. This is to be expected since away from threshold the pole terms need not approximate the exact amplitude. The major contribution to symmetry breaking arises as a consequence of the nondegeneracy of the thresholds of the competing processes. Since F_{12} is a constant we may perform the integral over the phase-space factor $\rho_{12}(W^2)$ and obtain from Eq. (12) the equation for the moments in broken $SU(3)$

$$K_i=F_i+\sum_{j=1}^{9}A_{ij}K_j, \quad (13)$$

where F_i arises from the electric interactions and A_{ij} from the anomalous moment interactions in the photo-meson-production amplitude. For the B-PS coupling constants we assume the $SU(3)$ symmetric values which give all the coupling constants in terms of $(g_{\pi N}/4\pi)\sim 15.0$ and f. There are two adjustable input parameters: Λ, the cutoff, chosen so as to approximately reproduce the observed nucleon moments, and f. In the degenerate case with all baryon masses equal and all meson masses equal the solutions K_i of Eq. (13) recover the complete symmetry [Eq. (10)]. With the baryon and meson masses set to their experimental

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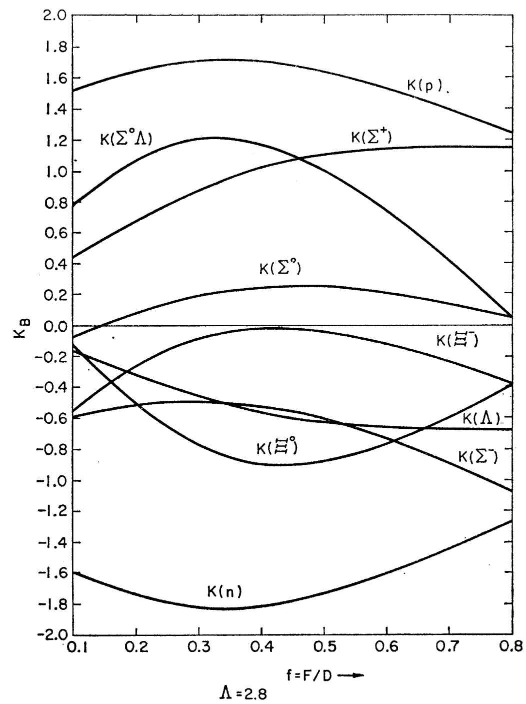

FIG. 4. Anomalous magnetic moments as a function of F/D.

values[7] we obtain the solutions shown in Fig. 4 given as a function of f.

We see that the nucleon moments $K(p)\simeq -K(n)\sim 1.6$ are reproduced within 15% of the observed values largely independent of f since it is the πN state which dominates. If we use $f=0.6$ and $\Lambda=2.8$ then $K(\Lambda)\simeq 0.4K(n)$ in agreement with the experimental value, $K(\Lambda)=-0.5\pm 0.3$.[8] Our calculation of the other moments indicates a larger violation of the $SU(3)$ symmetric predictions: $K(p)=1.5$, $K(n)=-1.6$, $K(\Lambda)=-0.66$, $K(\Sigma^-)=-0.7$, $K(\Sigma^0)=0.2$, $K(\Sigma^+)=1.2$, $K(\Xi^-)=-0.1$, $K(\Xi^0)=-0.8$, $K(\Sigma\Lambda)=0.75$.

The magnitude of the moments depends approximately logarithmically on Λ, the cutoff. The sensitivity of these results on f is indicated in Fig. 4. One can see that the $SU(3)$ predictions [Eq. (11)] are not well obeyed with the exception of those for $K(\Lambda)$ and $K(\Sigma^+)$ with $f\sim 0.6$. The relation $K(\Sigma^0)=\frac{1}{2}[K(\Sigma^+)+K(\Sigma^-)]$ which follows from $SU(2)$ symmetry,[9] is, of course, preserved while the Okubo relation[3] $\mu(\Sigma^0\Lambda)=\frac{1}{6}\sqrt{3}\times[\mu(\Sigma^0)+3\mu(\Lambda)-2\mu(\Xi^0)-2\mu(n)]$ obtained by including octet transformation properties to the current operator $S_1^1+S_{13}^{13}$ is not preserved in our calculation since we have included all orders in the baryon and meson mass splittings.

In conclusion, we remark that this calculation represents a first approximation to a more realistic calculation that includes the effects of symmetry breakings on the B-PS coupling constants presumably determined through a bootstrap mechanism.[10] An improved calculation would include the energy dependence of the full photomeson production amplitudes and higher-mass baryon-meson states. The primary success of the present calculation rests on the correct estimation of the nucleon moments on a dynamical basis and agreement with the measured Λ moment.

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APPENDIX

The contributions from the pole terms are given by

$$E_1(W^2)=(2M_1/A_1^{-2})[\mu^2M_1-\Delta B-(M_1/2W^2)(\mu^2A_1^+-2W^2\Delta^2-A_1^-A_2^-)],$$

$$E_2(W^2)=(-M_1/2W^2A_1^{-2})[2A_1^-A_2^-M_1+(2A_1^-/(A_2^+-\mu^2))(M_1\mu^2(3W^2-M_2^2)-4W^2\Delta+M_1A_2^{-2}+4W^2M_2Q_1(z))],$$

$$K_1(W^2)=-(1/2W^2A_1^-)[A_1^-A_2^-+2W^2\Delta^2-\mu^2A_1^+],$$

$$K_2(W^2)=-(M_1/2W^2M_2(A_2^+-\mu^2))\times[\mu^2(4W^2M_1M_2-M_1^2A_2^+)+M_1^2A_2^{-2}-4W^2M_2\Delta B+4W^4M_2^2Q_1(z)],$$

where

$$\Delta=M_1-M_2,$$

$$A_{1,2}^{\pm}=W^2\pm M_{1,2}^2,$$

$$B=W^2-M_1M_2,$$

$$z=(M_2^2-\mu^2+W^2)((M_2^2-\mu^2+W^2)^2-4W^2M_2^2)^{-1/2},$$

$$Q_1(z)=\tfrac{1}{2}z\ln[(z+1)/(z-1)]-1.$$

For the case $\mu^2=0$, $M_1=M_2=M$, $W=M$ the above expressions imply

$$E_1=-E_2=1,\quad K_1=K_2=0.$$

[7] We set the mass of any member of an isomultiplet equal to the average mass and in any diagram involving the $\Sigma^0\to\Lambda$ transition moment we set $M_\Sigma=M_\Lambda$.

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