Minimal Electromagnetic Interaction and C, T Noninvariance*

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It is well known that the principle of minimal electromagnetic interaction determines a unique electromagnetic-interaction form for a spin-0 or spin-$\frac{1}{2}$ charged particle. In this paper, it is shown that the same principle, when applied to a spin-1 charged particle, leads to a minimal electromagnetic interaction that depends on two arbitrary real parameters: the charge and the magnetic moment. It is further shown that the minimal electromagnetic interaction of a system of N spin-1 particles of the same charge depends on the charge ϵ and an $(N\times N)$ Hermitian matrix, called the magnetic-moment matric M. Such a minimal electromagnetic interaction can be noninvariant under C and T. The general condition of C, T invariance, or noninvariance is analyzed. These considerations are extended to a system of N neutral spin-1 particles, assuming that the minimal electromagnetic interaction of such a system is not zero. Application to the observed ϕ^0, ρ^0, and ω^0 particles gives a C, T noninvariant minimal electromagnetic interaction, which, however, is invariant under P and CT. By making a further assumption concerning its transformation property under SU_3, this C, T noninvariant interaction assumes a simple and unique form. Some of its experimental consequences are discussed.

I. INTRODUCTION

It is well known that from the Lagrangian density $\mathfrak{L}_{\text{free}}$ for a free particle, one may obtain a gauge-invariant electromagnetic interaction $\mathfrak{L}_\gamma$ by replacing in $\mathfrak{L}_{\text{free}}$

$$\partial/\partial x_\mu \to \partial/\partial x_\mu - i\epsilon A_\mu\,, \tag{1}$$

which changes

$$\mathfrak{L}_{\text{free}} \to \mathfrak{L}_{\text{free}} + \mathfrak{L}_\gamma\,, \tag{2}$$

where ϵ is the charge of the particle and A_μ is the electromagnetic field. The $\mathfrak{L}_\gamma$, thus generated, is called the minimal electromagnetic interaction of the particle.[1]

Transformation (1), when applied to a spin-0 or spin-$\frac{1}{2}$ particle, leads to a unique form of minimal electromagnetic interaction.[2] The remarkable success of quantum electrodynamics for the charged leptons gives strong support to the principle of minimal electromagnetic interaction, which requires all electromagnetic interactions that exist in nature to be of the minimal form. The validity of this principle will be assumed in this paper.

We note that the principle of minimal electromagnetic interaction, when applied to a spin-1 particle, leads to an interaction form depending on two independent parameters[3]: the charge ϵ and the magnetic moment μ. Thus, e.g., in the absence of the strong interaction, the quadrupole moment of a spin-1 particle becomes completely determined by ϵ and μ, in analogy to the minimal electromagnetic interaction of a spin-$\frac{1}{2}$ particle which requires its magnetic moment to be determined by its charge.

To show this, let us first denote the usual free Lagrangian density of a spin-1 particle by

$$\mathfrak{L}_{\text{free}}{}^0(x) = -\tfrac{1}{2}G_{\lambda\nu}{}^*G_{\lambda\nu} - m^2\psi_\nu{}^*\psi_\nu\,, \tag{3}$$

where

$$G_{\lambda\nu} = \frac{\partial}{\partial x_\lambda}\psi_\nu - \frac{\partial}{\partial x_\nu}\psi_\lambda\,,$$

$$G_{\lambda\nu}{}^* = \frac{\partial}{\partial x_\lambda}\psi_\nu{}^* - \frac{\partial}{\partial x_\nu}\psi_\lambda{}^*\,,$$

ψ_ν and m are, respectively, the field operator and the mass of the particle, and $\psi_\nu{}^*$ is related to the Hermitian-conjugate field operator $\psi_\nu{}^\dagger$ by

$$\psi_\nu{}^* = +\psi_\nu{}^\dagger \quad \text{for} \quad \nu\neq 4\,,$$

and

$$\psi_4{}^* = -\psi_4{}^\dagger\,.$$

Throughout this paper, the fourth component of x_μ is pure imaginary (i.e., $x_4 = it$), and all repeated indices are to be summed over. The application of transformation (1) to $\mathfrak{L}_{\text{free}}{}^0$ leads to a particle of charge ϵ and magnetic moment $\mu = (\epsilon/2m)\times\text{spin}$.

The free Lagrangian density $\mathfrak{L}_{\text{free}}{}^0$ can also be ex-

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[1] Transformation (1) has been used since the beginning of quantum mechanics. The possibility of a nonminimal type of electromagnetic interaction has been discussed by W. Pauli, Rev. Mod. Phys. **13**, 203 (1941). See also M. Gell-Mann, Nuovo Cimento **4**, Suppl. 2, 848 (1956).

[2] It should be emphasized that in order to obtain the minimal electromagnetic interaction it is not allowed to add to the usual free Lagrangian density any additional, but fictitious, term which contains more derivatives than that in $\mathfrak{L}_{\text{free}}$. For example, if we add to the free Lagrangian of a spin-$\frac{1}{2}$ particle,

$$\mathfrak{L}_{\text{free}} = -\tfrac{1}{2}[\psi^\dagger\gamma_4\gamma_\lambda(\partial/\partial x_\lambda)\psi - (\partial\psi^\dagger/\partial x_\lambda)\gamma_4\gamma_\lambda\psi],$$

an additional term

$$-i\kappa(\partial\psi^\dagger/\partial x_\lambda)\gamma_4\sigma_{\lambda\nu}(\partial\psi/\partial x_\nu),$$

where $\sigma_{\lambda\nu} = (2i)^{-1}(\gamma_\lambda\gamma_\nu - \gamma_\nu\gamma_\lambda)$, then, while the free equation of motion remains unchanged, transformation (1) would lead to a particle with an anomalous magnetic moment $\epsilon\kappa$. Throughout this paper, we make the explicit restriction that the free Lagrangian of a spin-$\frac{1}{2}$ particle has only linear dependence on $\partial/\partial x_\mu$, and the free Lagrangian of a spin-1, or spin-0, particle has only quadratical dependence on $\partial/\partial x_\mu$.

[3] I have learned in a private conversation with Dr. G. Feinberg that this result was also known to him. See also H. C. Corben and J. Schwinger, Phys. Rev. **58**, 953 (1940).

pressed in other alternative forms, such as

$$\mathcal{L}_{\text{free}}{}^1 = -(\partial\psi_\nu^*/\partial x_\lambda)(\partial\psi_\nu/\partial x_\lambda) + (\partial\psi_\lambda^*/\partial x_\lambda)(\partial\psi_\nu/\partial x_\nu) - m^2\psi_\nu^*\psi_\nu. \quad (4)$$

It is easy to see that the action integral $\int \mathcal{L} d^4x$ is the same for both free Lagrangian densities; therefore, the free equation of motion is also the same. The application of transformation (1) to $\mathcal{L}_{\text{free}}{}^1$ leads to a particle of charge ϵ, but with *zero* magnetic moment.

By partial integrations,[4] it can be readily verified that the free Lagrangian density of a spin-1 particle can be expressed in the following general form:

$$\mathcal{L}_{\text{free}} = -(\partial\psi_\nu^*/\partial x_\lambda)(\partial\psi_\nu/\partial x_\lambda) + g(\partial\psi_\lambda^*/\partial x_\nu)(\partial\psi_\nu/\partial x_\lambda) + (1-g)(\partial\psi_\lambda^*/\partial x_\lambda)(\partial\psi_\nu/\partial x_\nu) - m^2\psi_\nu^*\psi_\nu, \quad (5)$$

where g is an arbitrary real parameter. We note that $\mathcal{L}_{\text{free}}{}^0$ and $\mathcal{L}_{\text{free}}{}^1$ correspond, respectively, to the special case $g=1$ and 0. The application of transformation (1) to this general form of free Lagrangian density leads to a minimal electromagnetic interaction $\mathcal{L}_\gamma$ which corresponds to a spin-1 particle of charge ϵ and gyromagnetic ratio g. The magnetic moment is given by

$$\mu = \text{spin} \times (\epsilon g/2m). \quad (6)$$

The quadrupole moment Q of this particle is uniquely determined by ϵ and g (in the absence of strong interactions and radiative corrections):

$$Q = \int (3z^2 - r^2)\rho d^3r = \epsilon(1-g)/m^2, \quad (7)$$

where ρ is the static charge density for the state $(\text{spin})_z = +1$.

The quantization of the theory $\mathcal{L}_{\text{free}}$, or $\mathcal{L}_{\text{free}} + \mathcal{L}_\gamma$, can be carried out either by using the Lagrangian form of quantum mechanics developed by Feynman,[5] or by following the canonical formalism. The simplest way to apply the canonical formalism is to use the ξ-limiting process,[6] in which one introduces an additional term $-\xi(\partial\psi_\lambda^*/\partial x_\lambda)(\partial\psi_\nu/\partial x_\nu)$ to the free Lagrangian, Eq. (5), and then takes the limit $\xi = 0+$. Both methods, of course, lead to the same physical results. [Details of the canonical formalism are given in Appendix A.]

Upon examining the structure of Feynman graphs, it emerges that the principle of minimal electromagnetic interaction can also be expressed as the mathematical requirement that the three-point vertex function for the electromagnetic interaction (to first order in ϵ, and in the absence of the strong interaction) should have *a minimal power dependence on its external momenta.* Thus, e.g., for a spin-$\frac{1}{2}$ particle, the minimal interaction gives a vertex function that does not contain any explicit momentum dependence. The Pauli-type extra-magnetic-moment term gives a linear momentum dependence. Such a term is nonminimal for the spin-$\frac{1}{2}$ particle, and is, therefore, eliminated by the principle of minimal electromagnetic interaction. For a spin-1 particle, the same principle requires the vertex function to be a linear function of its external momenta; consequently, both the magnetic moment μ and the charge ϵ are independent parameters, but the quadrupole moment becomes completely determined by ϵ and μ.

Recently, it has been suggested,[7,7a] in connection with the observed decay[8]

$$K_2{}^0 \to \pi^+ + \pi^-, \quad (8)$$

that, perhaps, the electromagnetic interaction $\mathcal{L}_\gamma$ of the strongly interacting particles is not invariant under either the particle-antiparticle conjugation C or the time reversal T, and reaction (8) is simply the radiative correction effects of $\mathcal{L}_\gamma$ on the usual CP-conserving weak interaction.

Part of the purpose of this paper is to analyze the possibility of a minimal, but C, T noninvariant, electromagnetic interaction. In Sec. 2, a system of N spin-1 particles of the same charge ϵ is considered. The minimal electromagnetic interaction $\mathcal{L}_\gamma$ of such a system depends on ϵ and a $N \times N$ Hermitian "magnetic-moment" matrix M. It is shown that, depending on the structure of M and the number N, the resulting minimal interaction $\mathcal{L}_\gamma$ may be noninvariant under C and T; all such minimal electromagnetic interactions are, however, invariant under P and CT. The general conditions for C, T invariance or noninvariance of $\mathcal{L}_\gamma$, in the absence of any strong interaction, are given in Sec. 3.

In Sec. 4, the same considerations are extended to a system of N neutral spin-1 particles. Assuming that such a system does have an electromagnetic interaction, its minimal electromagnetic interaction must depend only on a $N \times N$ antisymmetric Hermitian magnetic-moment matrix M.

Application to the observed ϕ^0, ρ^0, and ω^0 particles leads naturally to a C, T noninvariant minimal electromagnetic interaction. We make the *ad hoc* assumption that under the SU_3 group of transformations, this C, T noninvariant minimal electromagnetic current, called K_μ, transforms in the same way as the minimal electromagnetic current J_μ of the known baryon octet (which, because of its spin being $\frac{1}{2}$, must conserve C and T). The current K_μ, then, assumes a simple and unique form, given in Sec. 5. Several definite predictions can

[4] For a spin-0 or a spin-$\frac{1}{2}$ particle, the same method does not lead to any different form of $\mathcal{L}_{\text{free}}$; therefore, transformation (1) does give a unique form of electromagnetic interaction (cf. footnote 2).

[5] R. P. Feynman, Rev. Mod. Phys. **20**, 367 (1948).

[6] T. D. Lee and C. N. Yang, Phys. Rev. **128**, 885 (1962).

[7] J. Bernstein, G. Feinberg, and T. D. Lee, Phys. Rev. **139**, B1650 (1965).

[7a] Cf. also S. Barshay, Phys. Letters **17**, 78 (1965).

[8] J. H. Christenson, J. W. Cronin, V. L. Fitch, and R. Turlay, Phys. Rev. Letters **13**, 138 (1964). See also A. Abashian *et al.*, *ibid.* **13**, 243 (1964).

be derived in the limit of SU_3 symmetry. For example, both the C-noninvariant $|\Delta\mathbf{I}|=0$ decay

$$\phi^0 \to \omega^0+\gamma$$

and the C-noninvariant $|\Delta\mathbf{I}|=1$ decay

$$\phi^0 \to \rho^0+\gamma$$

should occur, and the ratio of these two decay rates is given by

$$\frac{m_\rho^2(m_\phi^2+m_\omega^2)(m_\phi^2-m_\omega^2)^3}{m_\omega^2(m_\phi^2+m_\rho^2)(m_\phi^2-m_\rho^2)^3}\cong 0.79\,, \tag{9}$$

where m_ρ, m_ϕ, and m_ω are the masses of ρ^0, φ^0, and ω^0.

This specific SU_3 transformation property of K_μ also requires the absence of any T-noninvariant term in the decay[9]

$$\Sigma^0 \to \Lambda^0+e^++e^-,$$

and that the decay

$$\eta^0 \to \pi^0+e^++e^-$$

is forbidden[10] in the single-photon-exchange approximation, provided SU_3 symmetry holds.

It should be emphasized that these consequences depend *only* on the *assumed* transformation property of the C, T noninvariant current K_μ under SU_3. From a phenomenological point of view, the SU_3 transformation property of K_μ could be arbitrary. Thus, the decays $\phi^0\to\omega^0+\gamma$, $\phi^0\to\rho^0+\gamma$, $\Sigma^0\to\Lambda^0+e^++e^-$, and $\eta^0\to\pi^0+e^++e^-$ can be used, experimentally, to determine the transformation property of K_μ under SU_3.

II. MINIMAL ELECTROMAGNETIC INTERACTION OF A SYSTEM OF N CHARGED SPIN-1 PARTICLES

Let us consider a system of N spin-1 particles of the same charge ϵ. The field operators $\psi_{1,\nu}, \psi_{2,\nu}, \cdots, \psi_{N,\nu}$ of these N particles can be represented by a $(N\times 1)$ column matrix

$$\psi_\nu=\begin{bmatrix}\psi_{1,\nu}\\ \psi_{2,\nu}\\ \vdots\\ \psi_{N,\nu}\end{bmatrix}. \tag{10}$$

Similarly to Eq. (5), the general form of the free Lagrangian density $\mathcal{L}_{\text{free}}$ for such a system is given by

$$\mathcal{L}_{\text{free}}=-(\partial\psi_\nu^*/\partial x_\lambda)(\partial\psi_\nu/\partial x_\lambda)+(\partial\psi_\lambda^*/\partial x_\nu)g(\partial\psi_\nu/\partial x_\lambda)+(\partial\psi_\lambda^*/\partial x_\lambda)(1-g)(\partial\psi_\nu/\partial x_\nu)-\psi_\nu^*m^2\psi_\nu \tag{11}$$

in which ψ_ν^* is related to the Hermitian conjugate $\psi_{a,\nu}^\dagger$ of the field operator $\psi_{a,\nu}$ $(a=1,2,\cdots,N)$ by

$$\psi_\nu^*=\pm(\psi_{1,\nu}^\dagger,\psi_{2,\nu}^\dagger,\cdots,\psi_{N,\nu}^\dagger)\,, \tag{12}$$

where the $+$ sign is for $\nu\neq 4$ and the $-$ sign is for $\nu=4$, g is an arbitrary $(N\times N)$ Hermitian matrix, and m^2 is a diagonal $(N\times N)$ real matrix,

$$m^2=\begin{bmatrix}m_1^2 & & & \\ & m_2^2 & & \\ & & \ddots & \\ & & & m_N^2\end{bmatrix}. \tag{13}$$

The diagonal elements of m^2 are the squares of the masses of these particles. The Hermiticity of g is required by the Hermiticity of $\mathcal{L}_{\text{free}}$.

The free equation of motion is *independent* of the matrix g, and is given by

$$\frac{\partial^2}{\partial x_\lambda\partial x_\lambda}\psi_\nu-\frac{\partial}{\partial x_\nu}\left(\frac{\partial}{\partial x_\lambda}\psi_\lambda\right)-m^2\psi_\nu=0. \tag{14}$$

Under the transformation (1), this free Lagrangian is changed to $(\mathcal{L}_{\text{free}}+\mathcal{L}_\gamma)$:

$$\mathcal{L}_{\text{free}}+\mathcal{L}_\gamma=-(D_\lambda^*\psi_\nu^*)(D_\lambda\psi_\nu)+(D_\nu^*\psi_\lambda^*)g(D_\lambda\psi_\nu)+(D_\lambda^*\psi_\lambda^*)(1-g)(D_\nu\psi_\nu)-\psi_\nu^*m^2\psi_\nu\,, \tag{15}$$

where

$$D_\lambda=\partial/\partial x_\lambda-i\epsilon A_\lambda\,,\quad D_\lambda^*=\partial/\partial x_\lambda+i\epsilon A_\lambda\,, \tag{16}$$

and $\mathcal{L}_\gamma$ describes the *minimal* electromagnetic interaction of this system.

By using the commutation rule

$$[D_\nu,D_\lambda]=-i\epsilon F_{\nu\lambda}\,, \tag{17}$$

where

$$F_{\nu\lambda}=\partial A_\lambda/\partial x_\nu-\partial A_\nu/\partial x_\lambda\,,$$

the equation of motion in the presence of electromagnetic field is found to be

$$D_\lambda D_\lambda\psi_\nu-iMF_{\nu\lambda}\psi_\lambda-D_\nu(D_\lambda\psi_\lambda)-m^2\psi_\nu=0\,, \tag{18}$$

where

$$M=\epsilon g=M^\dagger. \tag{19}$$

The matrices M and m^2 will be called the magnetic-moment matrix and the mass matrix, respectively.

We note that for this system there are two separate current-conservation laws. Let us define

$$\mathcal{J}_\mu^\epsilon\equiv i\epsilon[(D_\mu^*\psi_\nu^*)\psi_\nu-\psi_\nu^*(D_\mu\psi_\nu)-(D_\nu^*\psi_\nu^*)\psi_\mu+\psi_\mu^*(D_\nu\psi_\nu)] \tag{20}$$

and

$$\mathcal{J}_\mu^M\equiv i(\partial/\partial x_\nu)[\psi_\nu^*M\psi_\mu-\psi_\mu^*M\psi_\nu]\,, \tag{21}$$

where $\mathcal{J}_\mu^\epsilon$ is a current which depends only on the charge ϵ, and $\mathcal{J}_\mu^M$ is the current associated with the magnetic-moment matrix M. The current $\mathcal{J}_\mu^M$ clearly satisfies the

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[9] The same conclusions with respect to $\Sigma^0\to\Lambda^0+\gamma$ and $\eta^0\to\pi^0+\gamma$, where γ can be either real or virtual, have also been independently observed by N. Cabibbo. See N. Cabibbo, Phys. Rev. Letters **14**, 965 (1965). In Cabibbo's paper, the hypothesis that the C, T noninvariant electromagnetic current transforms like a member of an octet under SU_3 is implicitly assumed.

[10] I wish to thank Dr. G. Feinberg for pointing out this particular consequence to me.

conservation law

$$\partial \mathcal{J}_\mu{}^M / \partial x_\mu = 0. \quad (22)$$

By using the equation of motion [Eq. (18)], it can be verified that $\mathcal{J}_\mu{}^\epsilon$ also satisfies the conservation law

$$\partial \mathcal{J}_\mu{}^\epsilon / \partial x_\mu = 0. \quad (23)$$

The total current $\mathcal{J}_\mu$ is, by definition, the source of the electromagnetic field A_μ. From Eq. (15), we obtain

$$\mathcal{J}_\mu = \mathcal{J}_\mu{}^\epsilon + \mathcal{J}_\mu{}^M. \quad (24)$$

In terms of $\mathcal{J}_\mu$, the minimal electromagnetic interaction $\mathcal{L}_\gamma$ becomes

$$\mathcal{L}_\gamma = \mathcal{J}_\mu A_\mu + \epsilon^2 A_\mu A_\nu [\delta_{\mu\nu}(\psi_\lambda{}^* \psi_\lambda) - \psi_\mu{}^* \psi_\nu], \quad (25)$$

where $\delta_{\mu\nu} = 1$ if $\mu = \nu$, and 0 if $\mu \neq \nu$.

To quantize this theory it is simplest to use the ξ-limiting method[6] and to follow the canonical formalism. The details are given in Appendix A.

III. CONDITIONS FOR C, T NONINVARIANCE

In this section, we discuss the symmetry properties of the Lagrangian density

$$\mathcal{L}(x) = \mathcal{L}_{\text{free}}(x) + \mathcal{L}_\gamma(x) \quad (26)$$

under C, T, and P, in the absence of any strong interaction. The free Lagrangian $\mathcal{L}_{\text{free}}(x)$ and the minimal electromagnetic interaction $\mathcal{L}_\gamma(x)$ are given by Eqs. (11) and (25) respectively. For definiteness, we assume the transformation property of the electromagnetic field A_μ has already been determined by its interactions with other particles, such as the charged leptons:

$$CA_\mu(\mathbf{r},t)C^{-1} = -A_\mu(\mathbf{r}, t), \quad (27)$$

$$TA_\mu(\mathbf{r},t)T^{-1} = -A_\mu(\mathbf{r}, -t), \quad (28)$$

and

$$PA_\mu(\mathbf{r},t)P^{-1} = \mp A_\mu(-\mathbf{r}, t), \quad (29)$$

where the $-$ sign is for $\mu \neq 4$ and the $+$ sign is for $\mu = 4$. By using Eqs. (11) and (25), it can be readily verified that $\mathcal{L}(x)$ satisfies the requirements of CPT, P, and CT invariance; i.e.,

$$CT\mathcal{L}(\mathbf{r},t)T^{-1}C^{-1} = \mathcal{L}(\mathbf{r}, -t) \quad (30)$$

and

$$P\mathcal{L}(\mathbf{r},t)P^{-1} = \mathcal{L}(-\mathbf{r}, t), \quad (31)$$

provided[11]

$$CT\psi_\mu(\mathbf{r},t)T^{-1}C^{-1} = \eta_{CT}\tilde{\psi}_\mu{}^*(\mathbf{r}, -t) \quad (32)$$

and

$$P\psi_\mu(\mathbf{r},t)P^{-1} = \pm\eta_P\psi_\mu(-\mathbf{r}, t), \quad (33)$$

where the $+$ and $-$ signs are for $\mu \neq 4$ and $\mu = 4$, respectively, the phase factors η_{CT} and η_P can be any two unimodular complex numbers. Throughout the paper, we use $\sim$ to denote the transpose.

The problem of C, or T, invariance is connected with the question whether there exists a $(N \times N)$ unitary matrix U such that

$$U^\dagger M U = \tilde{M} \quad (34)$$

and

$$U^\dagger m^2 U = m^2, \quad (35)$$

where M and m^2 are given by Eqs. (19) and (13), respectively. If such a U exists, then

$$C\mathcal{L}(x)C^{-1} = \mathcal{L}(x), \quad (36)$$

provided

$$C\psi_\mu(x)C^{-1} = U\tilde{\psi}_\mu{}^*(x). \quad (37)$$

In this case, the Lagrangian density $(\mathcal{L}_{\text{free}} + \mathcal{L}_\gamma)$ is invariant under C. From (CT) invariance, it follows that the theory is also invariant under T. Conversely, if Eqs. (34) and (35) have no solution, then the corresponding minimal electromagnetic interaction is not invariant under C and T.

The following theorem gives a few sufficient conditions for C, T invariance:

Theorem 1. The Lagrangian density $\mathcal{L}_{\text{free}} + \mathcal{L}_\gamma$ is invariant under C, if (i) the magnetic-moment matrix M is real, or if (ii) the matrices M and m^2 commute, or if (iii) the number of spin-1 particles N is ≤ 2.

Proof. We may choose U to be the unit matrix in case (i), and $U = V\tilde{V}$ in case (ii) where V is the $(N \times N)$ unitary matrix that satisfies $V^\dagger m^2 V = m^2$ and $V^\dagger M V$ = diagonal matrix.

In case (iii), $U = 1$ if $N = 1$. For $N = 2$, we may choose U to be the diagonal matrix

$$U = \begin{pmatrix} \exp(i\theta_{12}) & 0 \\ 0 & \exp(-i\theta_{12}) \end{pmatrix}, \quad (38)$$

where θ_{12} is the phase of the matrix element M_{12},

$$M_{12} = |M_{12}| \exp(i\theta_{12}),$$

if $M_{12} \neq 0$. If $M_{12} = 0$, this case reduces to the previous case (ii). Theorem 1 is, then, proved.

In the general case $N \geq 3$, it is convenient to consider a lattice of N points, labeled $1, 2, \cdots, N$. For each nonvanishing off-diagonal matrix element M_{ab} we draw a line between the points a and b on the lattice. A set of n points $(L_1, L_2, \cdots, L_n)$ is defined to form a *cycle* if the corresponding n matrix elements $M_{L_1L_2}$, $M_{L_2L_3}$, $\cdots$, $M_{L_{n-1}L_n}$ and $M_{L_nL_1}$ are all different from zero, so that between these points there are n lines which form a closed polygon. For each cycle $(L_1, L_2, \cdots, L_n)$, we

[11] Equations (32) and (33) are only special solutions. To obtain the general solution, we note that the Lagrangian density $\mathcal{L} = (\mathcal{L}_{\text{free}} + \mathcal{L}_\gamma)$ is also invariant under some other unitary transformations $\psi_\mu(x) \to S\psi_\mu(x)$ which are unrelated to space-time transformations, nor to the particle-antiparticle conjugation [e.g., $S = \exp(i\theta)$, where θ is a constant]. Let G be the group of all such transformations S. The replacement of P by $P_s = (SP)$ makes Eq. (31) unchanged, provided P satisfies Eq. (33) and S is any member of G. The set of all such operators P_s is the general solution of the space inversion operator [at least, so far as $(\mathcal{L}_{\text{free}} + \mathcal{L}_\gamma)$ is concerned]. Identical considerations can be applied to the operator CT. If $\mathcal{L}$ satisfies the invariance requirements of particle-antiparticle conjugation and time reversal, then the same consideration also leads to the general solutions for the operators C and T.

define an angle

$$\Theta(L_1,L_2,\cdots,L_n)\equiv\theta_{L_1L_2}+\theta_{L_2L_3}+\cdots+\theta_{L_{n-1}L_n}+\theta_{L_nL_1}, \quad (39)$$

where θ_{ab} is the corresponding phase angle of the nonvanishing matrix element M_{ab}:

$$M_{ab}=|M_{ab}|\exp(i\theta_{ab}). \quad (40)$$

Theorem 2. If the mass matrix m^2 is nondegenerate, then the necessary and sufficient condition for $\mathcal{L}_{\text{free}}+\mathcal{L}_\gamma$ to be invariant under C is

$$\Theta(L_1,\cdots,L_n)=\pi\times\text{integer}, \quad (41)$$

for all possible cycles $(L_1,\cdots,L_n)$ that can be constructed from the magnetic-moment matrix M.

Because of CT invariance, the same condition also applies to T invariance.

Proof. Equation (35) and the nondegenerate condition of m^2 require U to be a diagonal matrix. From Eq. (34) we have

$$M_{ab}U_{bb}=U_{aa}M_{ba} \quad (42)$$

for all a and b. Equation (43) has a solution only if Eq. (41) is satisfied for all cycles. This proves theorem 2.

By using theorem 2, one can easily construct a minimal electromagnetic interaction that is noninvariant under C and T. As an example, we may take $N=3$ and consider a magnetic-moment matrix M for which the set (1,2,3) forms a cycle; i.e., M_{12}, M_{23}, and M_{31} are all different from zero. The minimal electromagnetic interaction becomes C, T noninvariant if

$$\theta_{12}+\theta_{23}+\theta_{31}\neq\pi\times\text{integer}, \quad (43)$$

provided the masses m_1, m_2, and m_3 are all different.

Remarks

It has been stated in Ref. 7 that if a Lagrangian density, in the absence of the electromagnetic interaction, satisfies the invariance requirement of time reversal, then the corresponding minimal electromagnetic interaction must also satisfy the requirement of T invariance, since $(\partial/\partial x_\mu - i\epsilon A_\mu)$ transforms in the same way as $\partial/\partial x_\mu$ under T. Thus, a C, T noninvariant minimal electromagnetic interaction $\mathcal{L}_\gamma(x)$ implies that the free Lagrangian density $\mathcal{L}_{\text{free}}(x)$ also violates the following conditions:

$$C\mathcal{L}_{\text{free}}(x)C^{-1}=\mathcal{L}_{\text{free}}(x), \quad (44)$$

and

$$T\mathcal{L}_{\text{free}}(\mathbf{r},t)T^{-1}=\mathcal{L}_{\text{free}}(\mathbf{r},-t). \quad (45)$$

We note that independent of whether $\mathcal{L}_{\text{free}}(x)$ satisfies Eqs. (44) and (45) or not, the free equation of motion [Eq. (14)] is always *invariant* under C and T. On the other hand, if $\mathcal{L}_{\text{free}}(x)$ does not satisfy Eqs. (44) and (45), then the minimal electromagnetic interaction $\mathcal{L}_\gamma(x)$ cannot satisfy the requirement of C, T invariance; in addition, the equation of motion in the presence of the electromagnetic field [Eq. (18)] must also be noninvariant under C and T. Thus, there is no inconsistency if the minimal electromagnetic interaction of a system of spin-1 particles violates C, T invariance.

The minimal electromagnetic interaction of a system of spin-$\frac{1}{2}$, or spin-0, particles is (by itself) always invariant under C, P, and T.

Throughout this section, the operator C denotes the charge conjugation operator C_γ. It should be emphasized that the charge conjugation operator C_γ may, or may not, be the same operator as the particle antiparticle conjugation operator C_{st}, which is determined by the strong interaction. If $C_\gamma\neq C_{\text{st}}$, then even though the electromagnetic interaction $\mathcal{L}_\gamma$ may be invariant under C_γ, it can still violate the C_{st} symmetry.

IV. MINIMAL ELECTROMAGNETIC INTERACTION OF A SYSTEM OF NEUTRAL SPIN-1 PARTICLES

In this section, our discussions will be extended to a system of N neutral spin-1 particles by assuming that the minimal electromagnetic interaction $\mathcal{L}_\gamma$ of such a system is not zero. The most direct way to obtain $\mathcal{L}_\gamma$ is to regard the principle of minimal electromagnetic interaction as the mathematical requirement of a minimal power dependence of the vertex function on its external momenta. Thus, $\mathcal{L}_\gamma$ contains only the magnetic-moment matrix M.

A more formal approach is to consider the system as the limiting case $\epsilon=0$ of a corresponding system of N spin-1 charged particles. Let us start with the field operator ψ_μ given by Eq. (10). In the limit $\epsilon\to 0$, but keeping $M\neq 0$ and finite [consequently, $g=(M/\epsilon)\to\infty$], Eqs. (25) and (18) become, respectively,

$$\mathcal{L}_\gamma=\mathcal{J}_\mu{}^M A_\mu, \quad (46)$$

and

$$\frac{\partial}{\partial x_\lambda}\left(\frac{\partial\psi_\nu}{\partial x_\lambda}-\frac{\partial\psi_\lambda}{\partial x_\nu}\right)-iMF_{\nu\lambda}\psi_\lambda-m^2\psi_\nu=0. \quad (47)$$

To reduce the system to that of only N independent neutral particles, we impose the subsidiary condition

$$\psi_\mu=\tilde{\psi}_\mu{}^*. \quad (48)$$

By using Eq. (21) and the subsidiary condition, we find

$$M=-\tilde{M} \quad (49)$$

and

$$\mathcal{J}_\mu{}^M=2i\frac{\partial}{\partial x_\nu}(\tilde{\psi}_\nu M\psi_\mu). \quad (50)$$

The magnetic-moment matrix is, therefore, a purely imaginary antisymmetric matrix.

In the absence of any strong interaction, the question of C, T invariance is, according to the discussions given in the previous section, whether there exists a matrix

U which satisfies Eqs. (34) and (35). In order to satisfy the subsidiary condition, Eq. (48), we must also require in the present case

$$U = \text{real}. \tag{51}$$

Theorem 3. The Lagrangian density $(\mathcal{L}_{\text{free}}+\mathcal{L}_\gamma)$ is, by itself, invariant under C and T (i) if $N=2$, or (ii) for arbitrary N, provided the matrices M and m^2 commute.

From CT invariance, the same conclusion also applies to T invariance.

Proof. In order to have $\mathcal{L}_\gamma \neq 0$, the system must consist of two or more neutral spin-1 particles. We consider first the simplest case $N=2$. From Eq. (49), it follows that

$$M = \mu\sigma_y\,, \tag{52}$$

where

$$\sigma_y = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix} \tag{53}$$

and μ is a real constant. In this case, the minimal electromagnetic interaction has a unique form. Independent of the mass matrix, the equation of motion, Eq. (47), becomes invariant under C and T, provided

$$C\psi_\mu C^{-1} = \eta_c \sigma_z \psi_\mu\,, \tag{54}$$

where

$$\sigma_z = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \tag{55}$$

and η_c can be either $+1$ or -1. Case (i) is then proved.

To establish case (ii), we note that any antisymmetric Hermitian matrix M can be transformed into a block-diagonal form by a *real orthogonal* matrix V:

$$\tilde{V}MV = \begin{bmatrix} \mu_1\sigma_y & & & \\ & \mu_2\sigma_y & & \\ & & \ddots & \\ & & & \mu_n\sigma_y \end{bmatrix} \tag{56}$$

if $N=2n=$ even, or

$$\tilde{V}MV = \begin{bmatrix} \mu_1\sigma_y & & & & \\ & \mu_2\sigma_y & & & \\ & & \ddots & & \\ & & & \mu_n\sigma_y & \\ & & & & 0 \end{bmatrix} \tag{57}$$

if $N=2n+1$, where $\mu_1\cdots\mu_n$ are real numbers. The condition that M commutes with m^2 implies

$$\tilde{V}m^2V = m^2. \tag{58}$$

Let us define

$$U = VD\tilde{V},$$

where D is a real diagonal matrix,

$$D = \begin{bmatrix} \sigma_z & & & \\ & \sigma_z & & \\ & & \ddots & \\ & & & \sigma_z \end{bmatrix} \tag{59}$$

if $N=$ even, and

$$D = \begin{bmatrix} \sigma_z & & & & \\ & \sigma_z & & & \\ & & \ddots & & \\ & & & \sigma_z & \\ & & & & 1 \end{bmatrix} \tag{60}$$

if $N=$ odd. It can be readily verified that U satisfies Eqs. (34), (35), and (51). Thus, theorem 3 is proved.

Theorem 4. The minimal electromagnetic interaction of a *nondegenerate* system of N neutral spin-1 particles violates C invariance and T invariance, if and only if a set of numbers $(L_1,L_2,\cdots,L_n)$, where n is an odd integer, can be selected from $(1,2,\cdots,N)$ such that the n matrix elements $M_{L_1L_2}$, $M_{L_2L_3}$, $\cdots$, $M_{L_nL_1}$ are all different from zero.

Here, the violation of C, T invariance, again, means that the equation of motion, Eq. (47), is not invariant under C and T in the *absence* of any other interaction.

Proof. The antisymmetry property of M implies that the phase θ_{ij} of its off-diagonal matrix element M_{ij} must be $\pm(\pi/2)$. Theorem 4 can, then, be proved by using the same arguments as those used in proving theorem 2.

Theorem 4 can also be proved by noticing that each nonvanishing matrix element M_{ij} gives rise to the transition $\psi_i^0 \rightleftarrows \psi_j^0+\gamma$. Assuming C invariance, the nondegeneracy of these N particles requires each particle to have a definite C value. The C values of ψ_i^0 and ψ_j^0 must, therefore, differ by a minus sign, if $M_{ij}\neq 0$. This is not possible if $(L_1,L_2,\cdots,L_n)$ forms a cycle and n is odd.

It should be emphasized that so far we have only studied the C, T invariance, or noninvariance, of the minimal electromagnetic interaction $\mathcal{L}_\gamma$ by itself. For clarity, let us denote these operators by C_γ and T_γ. In the presence of the strong interaction $\mathcal{L}_{\text{st}}$, even though $\mathcal{L}_{\text{st}}$ may be invariant under the particle-antiparticle conjugation operator C_{st} and $\mathcal{L}_\gamma$ may be invariant under the charge-conjugation operator C_γ, it is possible that the sum $(\mathcal{L}_{\text{st}}+\mathcal{L}_\gamma)$ can still violate both the C_{st} and the C_γ invariances. As an example, we may take the simple case $N=2$. Theorem 3 states that $\mathcal{L}_\gamma$ is invariant under the charge conjugation C_γ. Let us assume that the strong interaction $\mathcal{L}_{\text{st}}$ of these particles is invariant under a different particle-antiparticle conjugation operator C_{st} which is different from C_γ:

$$C_{\text{st}}\psi_\mu C_{\text{st}}^{-1} = \eta_c' \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \psi_\mu\,, \tag{61}$$

where η_c' can be either $+1$ or -1. By using Eq. (54), we find that the sum $(\mathcal{L}_{\text{st}}+\mathcal{L}_\gamma)$ must violate C invariance. Since the strong interaction is stronger than $\mathcal{L}_\gamma$, it is useful to identify the particle-antiparticle operator C with that determined by the strong interaction; i.e.,

$$C = C_{\text{st}}. \tag{62}$$

The C_{st} (and T_{st}) noninvariant amplitude is, in this sense, regarded as generated by the minimal electromagnetic interaction. An explicit example of such a C_{st}, T_{st} noninvariance is given in Appendix B.

V. APPLICATION TO ϕ^0, ω^0, AND ρ^0

The mesons ϕ^0, ω^0, and ρ^0 form a convenient system of neutral vector mesons for our consideration. Let us represent their respective field operators by ϕ_ν, ω_ν, and ρ_ν, and define

$$\psi_\nu = \begin{pmatrix} \phi_\nu \\ \omega_\nu \\ \rho_\nu \end{pmatrix} . \tag{63}$$

The minimal electromagnetic interaction of this system depends only on the magnetic-moment matrix

$$M = i\begin{pmatrix} 0 & a & b \\ -a & 0 & c \\ -b & -c & 0 \end{pmatrix} , \tag{64}$$

where a, b, and c are all real parameters.

In the following, we will extend our discussions to include also strong interactions. For definiteness, we assume that the strong interaction is invariant under C and T, where $C = C_{st}$ and $T = T_{st}$. Furthermore, we assume that the minimal electromagnetic interaction of ϕ^0, ρ^0, and ω^0 is not zero. The minimal electromagnetic interaction of all other particles is assumed to be invariant under C_{st} and T_{st}.

From the known strong interaction $\mathcal{L}_{st}$, the C_{st} values of ϕ^0, ω^0, and ρ^0 have all been determined[7] to be -1. Thus, if $M \neq 0$, the sum $(\mathcal{L}_{st} + \mathcal{L}_\gamma)$ violates C, T invariance. Since $\mathcal{L}_{st}$ is stronger than $\mathcal{L}_\gamma$, it is useful to attribute such violations as due to $\mathcal{L}_\gamma$, independent of whether the minimal electromagnetic interaction is, or is not, by itself invariant under C and T, where $C = C_\gamma$ and $T = T_\gamma$. [See Appendix B.]

Using the notations of Ref. 7, the total (minimal) electromagnetic current $\mathcal{J}_\mu$ of all particles can be written as

$$\mathcal{J}_\mu = J_\mu + K_\mu , \tag{65}$$

where K_μ is the current generated by ϕ^0, ρ^0, and ω^0,

$$K_\mu = 2i \frac{\partial}{\partial x_\nu} (\tilde{\psi}_\nu M \psi_\mu) , \tag{66}$$

and J_μ is that generated by all other existing fields. Under the particle-antiparticle conjugation (defined by the strong interaction),

$$C_{st} J_\mu(x) C_{st}^{-1} = -J_\mu(x) \tag{67}$$

and

$$C_{st} K_\mu(x) C_{st}^{-1} = +K_\mu(x) . \tag{68}$$

Both J_μ and K_μ satisfy the conservation law:

$$\partial J_\mu / \partial x_\mu = 0 \tag{69}$$

and

$$\partial K_\mu / \partial x_\mu = 0 . \tag{70}$$

Under the isospin rotation, J_μ and K_μ transform as sums of $I=0$ and 1 components:

$$J_\mu = J_\mu^s + J_\mu^v \tag{71}$$

and

$$K_\mu = K_\mu^s + K_\mu^v , \tag{72}$$

where the superscripts s and v indicate, respectively, the isoscalar and isovector properties of these currents. From Eq. (64), we note that $K_\mu^v = 0$ if $b = c = 0$, and $K_\mu^s = 0$ if $a = 0$.

Under the SU_3 group of transformations, the current J_μ, in the limit of perfect SU_3 symmetry, transforms like the

$$2O_1^1 - O_2^2 - O_3^3 \tag{73}$$

member of an octet O_i^j (apart from a possible additional unitary singlet term).

In the same limit of SU_3 symmetry and under the approximation of a perfect degeneracy between ϕ^0 and ω^0, one can construct[12] from the states $|\phi^0\rangle$ and $|\omega^0\rangle$ the following two states:

$$|v_8^0\rangle \equiv \sqrt{\tfrac{2}{3}}\, |\phi^0\rangle - \sqrt{\tfrac{1}{3}}\, |\omega^0\rangle \tag{74}$$

and

$$|v_1^0\rangle = \sqrt{\tfrac{1}{3}}\, |\phi^0\rangle + \sqrt{\tfrac{2}{3}}\, |\omega^0\rangle , \tag{75}$$

where $|v_1^0\rangle$ and $|v_8^0\rangle$ transform, respectively, like a unitary singlet and the isoscalar $(I=0)$ member of a unitary octet under SU_3.

We now make the further assumption that, under the SU_3 transformations, the $C_{st} = +1$ current K_μ also transforms like (73). It can be readily verified that, by using Eqs. (74) and (75), this assumption requires the parameters a, b, c in Eq. (64) to be related by

$$a = b = (c/\sqrt{2}) .$$

The magnetic-moment matrix M becomes

$$M = i\mu \begin{pmatrix} 0 & 1 & 1 \\ -1 & 0 & \sqrt{2} \\ -1 & -\sqrt{2} & 0 \end{pmatrix} , \tag{76}$$

where μ is a real parameter. The minimal C, T noninvariant electromagnetic interaction $K_\mu A_\mu$ now acquires a unique form.

In Ref. 7, the various possible experimental tests of an arbitrary C, T noninvariant electromagnetic interaction have been extensively analyzed from a phenomenological point of view. The present simple form of a minimal C, T noninvariant interaction $K_\mu A_\mu$ has

[12] The decomposition of ϕ^0 and ω^0 to a SU_3 singlet and a SU_3 octet has been discussed by many authors: M. Gell-Mann, Phys. Rev. **125**, 1067 (1962); S. Okubo, Phys. Letters **5**, 165 (1963); J. J. Sakurai, Phys. Rev. **132**, 434 (1963). The particular forms of v_1^0 and v_8^0, given by Eqs. (74) and (75), have been used by F. Gürsey, T. D. Lee, and M. Nauenberg, Phys. Rev. **135**, B467 (1964).

several definite consequences with regard to these experimental tests.

1. The current K_μ contains both an isoscalar $K_\mu{}^s$ and an isovector $K_\mu{}^v$ part. Thus, the C-noninvariant $|\Delta\mathbf{I}|=0$ reactions, such as

$$\phi^0 \to \omega^0+\gamma \tag{77}$$

and

$$\eta^0 \to 2\pi^0+\gamma\,, \tag{78}$$

as well as the C-noninvariant $|\Delta\mathbf{I}|=1$ reactions, such as

$$\phi^0 \to \rho^0+\gamma\,, \tag{79}$$

etc., can occur.

The existence of both $K_\mu{}^s$ and $K_\mu{}^v$ implies that there is a π^+, π^- asymmetry in

$$\eta^0 \to \pi^++\pi^-+\gamma\,, \tag{80}$$

$$\rho^0 \to \pi^++\pi^-+\gamma \tag{81}$$

[both of which depend on $K_\mu{}^s$], and also in

$$\varphi^0 \to \pi^++\pi^-+\gamma \tag{82}$$

and

$$\omega^0 \to \pi^++\pi^-+\gamma \tag{83}$$

[both of which depend on $K_\mu{}^v$].

Another consequence is the existence of both the $I=0$ and the $I=2$ final $C=-1$ three-pion states in

$$\eta^0 \to \pi^++\pi^-+\pi^0. \tag{84}$$

The phenomenological analysis of the $\pi^\pm$ asymmetry in this reaction has been discussed elsewhere.[13]

2. In the limit of perfect SU_3 symmetry, the ratio of the matrix elements for the C-violating decays,

$$\phi^0 \to \omega^0+\gamma\,,$$
$$\phi^0 \to \rho^0+\gamma\,,$$

are given by that of the corresponding matrix elements of M [Eq. (76)]. Thus, we have

$$\begin{aligned}\text{Rate}(\phi^0\to\omega^0+\gamma):&\text{Rate}(\phi^0\to\rho^0+\gamma)\\ &=[m_\rho^{-2}(m_\phi^2+m_\rho^2)(m_\phi^2-m_\rho^2)^3]^{-1}\\ &\quad\times[m_\omega^{-2}(m_\phi^2+m_\omega^2)(m_\phi^2-m_\omega^2)^3]\cong 0.79.\end{aligned} \tag{85}$$

3. A much more difficult experiment is to study the decay

$$\omega^0 \to \rho^0+\gamma\,. \tag{86}$$

Assuming SU_3 symmetry, we find

$$\begin{aligned}\text{Rate}(\omega^0\to\rho^0+\gamma):&\text{Rate}(\phi^0\to\rho^0+\gamma)\\ &=2[m_\phi^{-5}(m_\phi^2+m_\rho^2)(m_\phi^2-m_\rho^2)^3]^{-1}\\ &\quad\times[m_\omega^{-5}(m_\omega^2+m_\rho^2)(m_\omega^2-m_\rho^2)^3]\cong 1.7\times10^{-3},\end{aligned} \tag{87}$$

which is, at present, too small to be of any practical use.

4. Another consequence is the *absence* of any T-noninvariant effect in the decay

$$\Sigma^0 \to \Lambda^0+e^++e^- \tag{88}$$

in the single-photon-exchange approximation, if SU_3 symmetry holds. To prove this, we recall that both J_μ and K_μ transform like (73) under SU_3. Thus, its off-diagonal matrix element between Σ^0 and Λ^0 is related to its diagonal element at arbitrary 4-momentum transfer q_λ; e.g.,

$$\langle\Lambda^0|J_\mu+K_\mu|\Sigma^0\rangle=(\sqrt{3}/2)\langle n|J_\mu+K_\mu|n\rangle. \tag{89}$$

Since the charge and the magnetic-moment form factors of n are both real due to Hermiticity, the same must also hold for the transition-matrix element

$$\langle\Lambda^0|J_\mu+K_\mu|\Sigma^0\rangle.$$

5. Identical reasoning leads to the conclusion that

$$\eta^0 \not\to \pi^0+e^++e^- \tag{90}$$

in the single-photon-exchange approximation, if SU_3 symmetry holds.

As already mentioned in Sec. I, all these experimental consequences, Eqs. (77)–(90), depend *only* on the *assumed* SU_3 transformation property of the C, T noninvariant current K_ν, and are, otherwise, independent of the detailed structure of K_ν. The particular form of K_ν given in this section can be regarded as a simple, but explicit, model of such a minimal C, T noninvariant electromagnetic current. In this model, the current K_ν depends only on one parameter μ. A speculative possibility which relates this parameter μ to the electromagnetic properties of other vector mesons $K^{*\pm}$, K^{*0}, and $\rho^\pm$ is given in Appendix B.

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APPENDIX A

In this section, we discuss the canonical formalism for the system of N spin-1 charged fields. We shall use the ξ-limiting method[6] and start with the free Lagrangian given by Eq. (11), plus an additive term $-\xi(\partial\psi_\mu{}^*/\partial x_\mu)(\partial\psi_\nu/\partial x_\nu)$:

$$\begin{aligned}\mathfrak{L}_{\text{free}}=&-\left(\frac{\partial\psi_\nu{}^*}{\partial x_\mu}\right)\left(\frac{\partial\psi_\nu}{\partial x_\mu}\right)+\left(\frac{\partial\psi_\mu{}^*}{\partial x_\nu}\right)g\left(\frac{\partial\psi_\nu}{\partial x_\mu}\right)\\ &-\left(\frac{\partial\psi_\mu{}^*}{\partial x_\mu}\right)(g+\xi-1)\left(\frac{\partial\psi_\nu}{\partial x_\nu}\right)-\psi_\mu{}^*m^2\psi_\mu\,,\end{aligned} \tag{A1}$$

where ξ is a positive number. Let η be the metric of the Hilbert space [see Eq. (18) of Ref. 6]. Equation (12)

[13] The suggestion to use the $\pi^\pm$ asymmetry in $\eta^0\to3\pi$ as a possible test of C noninvariance for interactions stronger than the weak interaction was made by R. Friedberg, T. D. Lee, and M. Schwartz (unpublished). Some discussions of such an asymmetry were given by T. D. Lee and L. Wolfenstein, Phys. Rev. **138**, B1490 (1965). A more detailed analysis has been discussed by T. D. Lee, Phys. Rev. **139**, B1415 (1965).

is replaced by

$$\psi_\nu^* = \pm\eta^{-1}(\psi_{1,\nu}^\dagger,\psi_{2,\nu}^\dagger,\cdots,\psi_{N,\nu}^\dagger)\eta\,, \qquad \text{(A2)}$$

where $\pm$ signs are for $\nu\neq 4$ and $\nu=4$, respectively, and † denotes the Hermitian conjugation.

Let us denote the spatial and the time components of ψ_ν and ψ_ν^* by $(\boldsymbol{\psi}, i\psi_0)$ and $(\boldsymbol{\psi}^*, i\psi_0^*)$, respectively. The conjugate momenta of $\boldsymbol{\psi}$, $\boldsymbol{\psi}^*$, ψ_0, and ψ_0^* are, respectively, $\boldsymbol{\pi}$, $\boldsymbol{\pi}^*$, π_0, and π_0^*. Furthermore, $\boldsymbol{\pi}$ and π_0 are $(N\times 1)$ column matrices (like $\boldsymbol{\psi}$ and ψ_0), while $\boldsymbol{\pi}^*$ and π_0^* are $(1\times N)$ row matrices (like $\boldsymbol{\psi}^*$ and ψ_0^*). The free Hamiltonian density H_{free} can be readily obtained from Eq. (A1). We find

$$\begin{aligned}H_{\text{free}} = {}& \boldsymbol{\pi}^*\cdot\boldsymbol{\pi} - \xi^{-1}\pi_0^*\pi_0 + \boldsymbol{\psi}^*\cdot m^2\boldsymbol{\psi} - \psi_0^* m^2 \psi_0 \\ & -[\tilde{\boldsymbol{\pi}}\cdot g(\nabla\psi_0) + (\nabla\psi_0^*)\cdot g\tilde{\boldsymbol{\pi}}^*] + (\nabla\times\boldsymbol{\psi}^*)\cdot(\nabla\times\boldsymbol{\psi}) \\ & -[\tilde{\pi}_0\xi^{-1}(g+\xi-1)(\nabla\cdot\boldsymbol{\psi}) + (\nabla\cdot\boldsymbol{\psi}^*)\xi^{-1}(g+\xi-1)\tilde{\pi}_0^*] \\ & -(\partial\psi_i^*/\partial x_j)(g-1)(\partial\psi_j/\partial x_i) - \xi^{-1}(\nabla\cdot\boldsymbol{\psi}^*)(g-1) \\ & \times(g+\xi-1)(\nabla\cdot\boldsymbol{\psi}) + (\nabla\psi_0^*)(g^2-1)\cdot(\nabla\psi_0)\,, \qquad \text{(A3)}\end{aligned}$$

where i, or j, varies from 1 to 3.

It is convenient to make the Fourier analysis:

$$\boldsymbol{\psi} = \sum_{\mathbf{k},t} \Omega^{-1/2}[Q_t(\mathbf{k})\mathbf{e}_k^{\,t}]\exp(i\mathbf{k}\cdot\mathbf{r}) + \sum_{\mathbf{k}} \Omega^{-1/2}[Q_l(\mathbf{k})\hat{k}]\exp(i\mathbf{k}\cdot\mathbf{r})\,, \qquad \text{(A4)}$$

$$\psi_0 = \sum_{\mathbf{k}} \Omega^{-1/2} Q_0(\mathbf{k})\exp(i\mathbf{k}\cdot\mathbf{r})\,, \qquad \text{(A5)}$$

$$\boldsymbol{\pi} = \sum_{\mathbf{k},t} \Omega^{-1/2}[P_t(\mathbf{k})\mathbf{e}_k^{\,t}]\exp(-i\mathbf{k}\cdot\mathbf{r}) + \sum_{\mathbf{k}} \Omega^{-1/2}[P_l(\mathbf{k})\hat{k}]\exp(-i\mathbf{k}\cdot\mathbf{r})\,, \qquad \text{(A6)}$$

and

$$\pi_0 = \sum_{\mathbf{k}} \Omega^{-1/2} P_0(\mathbf{k})\exp(-i\mathbf{k}\cdot\mathbf{r})\,, \qquad \text{(A7)}$$

where $\mathbf{e}_k^{\,1}$, $\mathbf{e}_k^{\,2}$, and $\hat{k} = |\mathbf{k}|^{-1}\mathbf{k}$ form a right-handed orthonormal set of unit vectors. In terms of these Fourier components, the free Hamiltonian becomes

$$\int H_{\text{free}} d^3r = \sum_{\mathbf{k}} [H_T(\mathbf{k}) + H_L(\mathbf{k})]\,, \qquad \text{(A8)}$$

where

$$H_T(\mathbf{k}) = \sum_t [P_t^*(\mathbf{k})P_t(\mathbf{k}) + Q_t^*(\mathbf{k})\omega^2 Q_t(\mathbf{k})]\,, \qquad \text{(A9)}$$

ω^2 is a diagonal matrix, whose diagonal matrix elements are $(k^2+m_a^2)^{1/2}$, and $a=1,2,\cdots,N$. The longitudinal part $H_L(\mathbf{k})$ is given by

$$\begin{aligned}H_L(\mathbf{k}) = {}& P_l^*P_l - \xi^{-1}P_0^*P_0 + Q_l^* m^2 Q_l - Q_0^* m^2 Q_0 \\ & -k^2\xi^{-1}Q_l^*(g-1)(g+2\xi-1)Q_l + k^2 Q_0^*(g^2-1)Q_0 \\ & -ik[\tilde{P}_l g Q_0 - Q_0^* g \tilde{P}_l^*] \\ & -i\xi^{-1}k[\tilde{P}_0(g+\xi-1)Q_l - Q_l^*(g+\xi-1)\tilde{P}_0^*]\,, \qquad \text{(A10)}\end{aligned}$$

where all operators P_l, P_0, Q_l, and Q_0 are referred to the same $\mathbf{k}$, and $k=|\mathbf{k}|$. The commutation relations at equal time are given by

$$[(P_\alpha)_a,(Q_\beta)_b] = -i\delta_{\alpha\beta}\delta_{ab}\,, \qquad \text{(A11)}$$

$$[(P_\alpha^*)_a,(Q_\beta^*)_b] = -i\delta_{\alpha\beta}\delta_{ab} \qquad \text{(A12)}$$

and all other commutators are zero, where the index α, or β, denotes t, l, and 0, and the index a, or b, denotes $1,2,\cdots,N$.

We introduce the canonical transformation

$$\begin{aligned}(p_l)_a &= (P_l)_a + ik[Q_0^*(g-1)]_a\,, \\ (p_0)_a &= (P_0)_a - ik[Q_l^*(g-1)]_a\,, \\ (p_l^*)_a &= (P_l^*)_a - ik[(g-1)Q_0]_a\,, \\ (p_0^*)_a &= (P_0^*)_a + ik[(g-1)Q_l]_a\,, \\ q_l &= Q_l\,, \qquad q_l^* = Q_l^*\,, \\ q_0 &= Q_0\,, \quad \text{and} \quad q_0^* = Q_0^*\,.\end{aligned}$$

It can be verified that p_α, q_α, p_α^*, and q_α^* satisfy the same commutation relations as P_α, Q_α, P_α^*, and Q_α^*. In terms of these new canonical variables, the Hamiltonian $H_L(k)$ becomes *independent* of the matrix g:

$$\begin{aligned}H_L(\mathbf{k}) = {}& p_l^*p_l - \xi^{-1}p_0^*p_0 + q_l^*m^2q_l - q_0^*m^2q_0 \\ & -ik[\tilde{p}_l q_0 + \tilde{p}_0 q_l - q_0^*\tilde{p}_l^* - q_l^*\tilde{p}_0^*]\,. \qquad \text{(A13)}\end{aligned}$$

The subsequent developments for H_{free} and $H_{\text{free}}+H_\gamma$ are exactly the same as those given in Ref. 6. In the limit $\xi=0+$, the vertex functions in the Feynman graphs are the same ones as those that can be directly written down by using the $\mathcal{L}_\gamma$ given by Eq. (25).

APPENDIX B

In this Appendix, we discuss the minimal electromagnetic interaction $\mathcal{L}_\gamma$ of the ϕ^0, ρ^0, ω^0 system in the limit of perfect degeneracy; i.e.

$$m_\rho = m_\omega = m_\phi\,. \qquad \text{(B1)}$$

Theorem 3 states that for such a degenerate system $(\mathcal{L}_{\text{free}}+\mathcal{L}_\gamma)$ is, by itself, invariant under C and T, where $C=C_\gamma$ and $T=T_\gamma$. Thus, we may regard the C, T noninvariance as brought in by the strong interaction $\mathcal{L}_{\text{st}}$, which, by itself, is also invariant under C and T, but $C=C_{\text{st}}$ and $T=T_{\text{st}}$. [See, e.g., Eq. (62).] The combination $(\mathcal{L}_{\text{st}}+\mathcal{L}_{\text{free}}+\mathcal{L}_\gamma)$, of course, violates both the C_{st}, T_{st} invariances and the C_γ, T_γ invariances.

To see more explicitly the C_γ, T_γ invariance property of $(\mathcal{L}_{\text{free}}+\mathcal{L}_\gamma)$ in this limit of complete degeneracy, we may start with the general magnetic-moment matrix M given by Eq. (64). By using Eq. (57), we find the transformation

$$V\psi_\nu = \chi_\nu = \begin{pmatrix}\chi_{1,\nu}\\ \chi_{2,\nu}\\ \chi_{3,\nu}\end{pmatrix} \qquad \text{(B2)}$$

changes the matrix M to

$$\tilde{V}MV=\tfrac{1}{2}i\begin{bmatrix} 0 & \mu & 0 \\ -\mu & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}, \qquad \text{(B3)}$$

where $\frac{1}{2}\mu=(a^2+b^2+c^2)^{1/2}$. It is useful to define a complex field X_ν:

$$X_\nu \equiv (1/\sqrt{2})(\chi_{1,\nu}+i\chi_{2,\nu}). \qquad \text{(B4)}$$

In terms of X_ν, the current K_λ becomes

$$K_\lambda = i\mu\frac{\partial}{\partial X_\nu}(X_\nu^* X_\lambda - X_\lambda^* X_\nu). \qquad \text{(B5)}$$

Thus, the current K_λ can be considered to be one generated by the magnetic moment of a single complex field X_ν. The C_γ,T_γ invariance property of $K_\lambda A_\lambda$, by itself, is, then, obvious.

So far, the parameter μ in Eq. (B5) is independent of the electromagnetic properties of all other particles such as $\rho^\pm$, K^{*0}, $K^{*\pm}$, etc. The electromagnetic currents of these other particles are all contained in the $C_{\rm st}=-1$ current J_μ given by Eq. (67). The above representation of K_μ suggests a possible unified formulation of K_μ and J_μ. For simplicity, let us only consider the system of vector mesons. We may introduce a (4×1) column matrix field operator V_μ:

$$V_\mu=\begin{bmatrix} K_\mu^{*+} \\ K_\mu^{*0} \\ \rho_\mu^{+} \\ X_\mu \end{bmatrix}, \qquad \text{(B6)}$$

where K_μ^{*+}, K_μ^{*0}, and ρ_μ^+ are, respectively, the field operators of K^{*+}, K^{*0}, and ρ^+. The electromagnetic current of this system can then be written as

$$\mathcal{J}_\mu=\mathcal{J}_\mu^{e}+\mathcal{J}_\mu^{M}, \qquad \text{(B7)}$$

where [cf., Eqs. (20) and (21)]

$$\mathcal{J}_\mu^{M}=i\frac{\partial}{\partial x_\nu}[V_\nu^*\mathfrak{M}V_\mu - V_\mu^*\mathfrak{M}V_\nu] \qquad \text{(B8)}$$

and, to first order in e,

$$\mathcal{J}_\mu^{e}=ie[(\partial V_\nu^*/\partial x_\mu)QV_\nu-(\partial V_\nu^*/\partial x_\nu)QV_\mu] + \text{conjugate terms}, \qquad \text{(B9)}$$

where

$$\mathfrak{M}=\begin{bmatrix} \mu(K^{*+}) & 0 & 0 & 0 \\ 0 & \mu(K^{*0}) & 0 & 0 \\ 0 & 0 & \mu(\rho^+) & 0 \\ 0 & 0 & 0 & \mu \end{bmatrix}, \qquad \text{(B10)}$$

$$Q=\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}, \qquad \text{(B11)}$$

and $\mu(K^{*+})$, $\mu(K^{*0})$, and $\mu(\rho^+)$ are, respectively, the (total) magnetic moments of K^{*+}, K^{*0}, and ρ^+.

In the presence of the strong interaction, the current $\mathcal{J}_\mu^{M}$ is a mixture of the $C_{\rm st}=-1$ current J_μ and the $C_{\rm st}=+1$ current K_μ. To illustrate a possible connection between μ and the other magnetic moments $\mu(K^{*0})$, $\mu(\rho^+)$, etc., we may consider the symmetry group U_4, which transforms the four components of V_μ into linear combinations of each other. The free Lagrangian $\mathfrak{L}_{\rm free}$ is invariant under this U_4 group of transformations, provided these vector mesons are all degenerate. (The usual SU_3 group is *not* a subgroup of this U_4 group.) The matrix $\mathfrak{M}$ can assume a definite form, if the current $\mathcal{J}_\mu^{M}$, like $\mathcal{J}_\mu^{e}$, has simple properties under this U_4 group of transformations. For example, the particular requirement

$$\mathfrak{M}=\mu_1 Q+\mu_2, \qquad \text{(B12)}$$

where μ_1, μ_2 are constants, leads to the relations

$$\mu=\mu(K^{*0}) \qquad \text{(B13)}$$

and

$$\mu(K^{*+})=\mu(\rho^+). \qquad \text{(B14)}$$