Classification of All C-Noninvariant Electromagnetic Interactions and the Possible Existence of a Charged, but $C=1$, Particle*

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Assuming that the strong interaction H_{st} is invariant under the particle-antiparticle conjugation C, it is shown that all possible C-noninvariant electromagnetic interactions H_γ can be classified according to the anticommutator between C and the charge operator Q into two types: (1) $\{C,Q\}=0$ and (2) $\{C,Q\}\neq 0$. Discussions of the first type C-noninvariant minimal electromagnetic interaction have already been given in a previous paper. If $\{C,Q\}\neq 0$, then the operator C must be different from what is normally called the "charge-conjugation operator" C_γ which, by definition, changes any state of charge Q to that of $-Q$. Thus, $\{C_\gamma,Q\}=0$ and $C\neq C_\gamma$. As a consequence, there must exist, at least, a charged particle a^+ which is an eigenstate of C; its eigenvalue can always be chosen to be $+1$. Furthermore, in the framework of a Lorentz-invariant local-field theory, H_{st} and H_γ are invariant under $C_\gamma PT$, but not CPT. The $C_\gamma PT$ invariance requires the existence of another charged particle a^- which has the same mass as a^+ but the opposite charge. The a^- is also an eigenstate of C. The existence of such $a^\pm$ particles necessitates not only the C nonconservation of H_γ, but also the T noninvariance of H_{st}. The general algebraic relations between H_{st}, H_γ, and these symmetry operators are studied, and the properties of the particles $a^\pm$ are discussed. An explicit spin-$\frac{1}{2}$ model of $a^\pm$ based on the principle of minimal electromagnetic interaction is given. A possible unifying view connecting the present C, T noninvariance with the well-known C, P nonconservation of the weak interaction is discussed.

I. GENERAL DISCUSSIONS

In this paper, we assume that the following two propositions are valid:

(i) The strong interaction is invariant under the particle-antiparticle conjugation C.

(ii) The electromagnetic interaction is not invariant under the same particle-antiparticle conjugation operator C.

At present, there is good evidence that proposition (i) is correct. For instance, we may mention the recent study[1] of the equality between the energy distributions of π^+ and π^- in the annihilation of $\bar{p}$ and p,

$$\bar{p}+p\rightarrow\pi^++\pi^-+\cdots,\qquad(1)$$

which places an upper limit on the C-noninvariant amplitude to be not more than $\sim 1\%$ of the C-invariant amplitude. A similar upper limit of $\sim 2\%$ is obtained by studying the energy distributions of K^+ and K^- in the same $(\bar{p}+p)$ annihilation experiment. Further evidence of C invariance of the strong interaction comes from the smallness of the observed decay amplitude[2] of

$$K_2^0\rightarrow\pi^++\pi^-.\qquad(2)$$

Additional supporting evidence can also be obtained from the p-p double scattering experiments[3] and from the experiments on reciprocity relations in nuclear reactions.[4] (See theorems 2 and 6 in the subsequent sections.)

The transformation properties of the known strongly interacting particles under C can be determined from the various observed strong reactions: e.g.,

$$C|p\rangle=|\bar{p}\rangle,\quad C|n\rangle=|\bar{n}\rangle,$$
$$C|\pi^+\rangle=|\pi^-\rangle,\quad C|\pi^0\rangle=|\pi^0\rangle,\qquad(3)$$

Proposition (ii) is purely a theoretical possibility.[5,6] As has been pointed out in Ref. 5, this possibility is consistent with all existing experiments, and it gives a natural explanation for the smallness of the observed amplitude of reaction (2), which is about (α/π) times that of $K_1^0\rightarrow\pi^++\pi^-$.

Using the notations of Ref. 5, the electromagnetic currents of all strongly interacting particles can be written as

$$\mathcal{J}_\mu=J_\mu+K_\mu,\qquad(4)$$

where

$$CJ_\mu C^{-1}=-J_\mu\qquad(5)$$

and

$$CK_\mu C^{-1}=+K_\mu.\qquad(6)$$

Let us define

$$Q_J\equiv -i\int J_4d^3r\qquad(7)$$

and

$$Q_K=-i\int K_4d^3r.\qquad(8)$$

The total charge Q of the system is given by

$$Q=Q_J+Q_K.\qquad(9)$$

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[1] C. Baltay *et al.*, Phys. Rev. Letters **15**, 591 (1965).

[2] J. H. Christenson, J. W. Cronin, V. L. Fitch, and R. Turlay, Phys. Rev. Letters **13**, 138 (1964). See also, A. Abashian *et al.*, *ibid.* **13**, 243 (1964).

[3] A. Abashian and E. M. Hafner, Phys. Rev. Letters **1**, 255 (1958); C. F. Hwang, T. R. Ophel, E. H. Thorndike, and R. Wilson, Phys. Rev. **119**, 352 (1960).

[4] L. Rosen and J. E. Brolley, Jr., Phys. Rev. Letters **2**, 98 (1959); D. Bodansky *et al.*, *ibid.* **2**, 101 (1959).

[5] J. Bernstein, G. Feinberg, and T. D. Lee, Phys. Rev. **139**, B1650 (1965).

[6] Cf. also S. Barshay, Phys. Letters **17**, 78 (1965).

The operator C anticommutes with Q_J but commutes with Q_K:

$$CQ_J+Q_JC=0, \quad (10)$$

$$CQ_K-Q_KC=0. \quad (11)$$

All C-noninvariant electromagnetic interactions, independent of whether they are minimal or not, can be classified into two types:

(1) The operator Q_K is zero for all physically acceptable states. Thus, we have

$$CQ+QC=0.$$

(2) The operator Q_K has nonzero eigenvalues, and therefore

$$CQ+QC\neq 0. \quad (12)$$

The first type of minimal C-noninvariant electromagnetic interactions has been discussed in a previous paper[7] (hereafter called Paper I). In Paper I, the current K_μ is given by the derivative of the magnetic-moment matrix of a set of spin-1 particles; thus, the spatial integral Q_K of its fourth component is always zero.

The second type of C-noninvariant electromagnetic interactions will be studied in this paper. We note that if Eq. (12) holds, then the particle-antiparticle conjugation operator C must be *different* from what is normally called the "charge-conjugation operator" C_γ which, by definition, changes any state of charge Q to that of $-Q$: that is C_γ satisfies

$$C_\gamma Q+QC_\gamma=0, \quad (13)$$

but the operator C does not; consequently,

$$C\neq C_\gamma. \quad (14)$$

From Eq. (12), it follows that the operator

$$Q_K\neq 0, \quad (15)$$

which means Q_K must have at least one eigenstate, say $|a^+\rangle$, with a nonzero eigenvalue. We may write

$$Q_K|a^+\rangle=e|a^+\rangle, \quad (16)$$

where $e\neq 0$. The charge Q_K is related to the total charge Q by

$$Q_K=\tfrac{1}{2}[Q+CQC^{-1}]. \quad (17)$$

The strong-interaction Hamiltonian $H_{\rm st}$ satisfies the commutation relations

$$[H_{\rm st},Q]=0 \quad (18)$$

and

$$[H_{\rm st},C]=0. \quad (19)$$

Equation (18) follows from the total charge conservation and Eq. (19) is simply the proposition (i). By using Eq. (17), we find that $H_{\rm st}$ also satisfies

$$[H_{\rm st},Q_K]=0. \quad (20)$$

According to Eqs. (11), (19), and (20), the three operators $H_{\rm st}$, C, and Q_K mutually commute. Thus, the state $|a^+\rangle$ can be set to be also the eigenstate of $H_{\rm st}$ and C:

$$H_{\rm st}|a^+\rangle=E_a|a^+\rangle \quad (21)$$

and

$$C|a^+\rangle=\eta_c|a^+\rangle, \quad (22)$$

where, by a gauge transformation of the form $C\to C\exp(iQ_K\theta)$, the phase factor η_c can always be chosen to be unity,

$$\eta_c=1. \quad (23)$$

The state $|a^+\rangle$ is a charged state,[8] but it is also an eigenstate of C; its existence necessitates the C nonconservation. We note that in the absence of the electromagnetic interaction H_γ (i.e., $e=0$), Eq. (22) does not appear strange, and C is conserved. In the presence of H_γ, C conservation must be violated.

Another consequence is connected with the fact that the "CPT" operator derived in the usual "CPT" theorem[9] must be one which changes all particles of charge $+Q$ to that of $-Q$. Therefore, it *cannot* be the CPT operator[10] used in this paper; rather it should be identified as the $C_\gamma PT$ operator. The general algebraic relations between $H_{\rm st}$, H_γ and these symmetry operators will be investigated in this paper.

To make our subsequent analysis definite, we shall assume, in the following sections, two additional propositions:

(iii) All interactions can be described by a local-field theory which is invariant under the continuous inhomogeneous Lorentz transformations, and the usual relation between spin and statistics is valid.

(iv) The principle of minimal electromagnetic interaction holds; furthermore, the total electromagnetic current $\mathcal{J}_\mu$ can be expressed in terms of the "bare" field operators of various spin-$\frac{1}{2}$ and spin-0 particles only,[11] and the charges of these particles are all of the same unit e.

Propositions (ii) and (iv) require the electromagnetic interaction to be of the second type; i.e., $\{C,Q\}\neq 0$.

[7] T. D. Lee, Phys. Rev. **140**, B967 (1965).

[8] The total charge of the state $|a^+\rangle$ is $\langle a^+|Q|a^+\rangle$. By using Eqs. (10) and (22), we find $\langle a^+|Q_J|a^+\rangle=0$. Thus, $\langle a^+|Q|a^+\rangle=\langle a^+|Q_K|a^+\rangle=e\neq 0$.

[9] W. Pauli, *Niels Bohr and the Development of Physics* (Pergamon Press, London, 1955), and J. Schwinger, Phys. Rev. **91**, 720, 723 (1953); **94**, 1366 (1953). See also G. Lüders, Kgl. Danske Videnskab. Selskab, Mat Fys. Medd. **28**, No. 5 (1954).

[10] Throughout the paper, P and T refer, respectively, to the space-inversion and the time-reversal operators. Both operators do not change the charge of the particle. (See, however, Sec. V.) The operators C and P are both unitary operators, but T is not. For a definition of the T operator, see E. P. Wigner, Gött. Nachr. Math. Naturw. Kl. 546 (1932). In our discussions, we will often say that a certain Hamiltonian, say $H_\gamma(t)$, is invariant under T. Such a statement refers specifically to the Schrödinger representation in which H_γ is independent of t.

[11] Consequently, given the set of these spin-$\frac{1}{2}$ and spin-0 field operators, the structure of the current $\mathcal{J}_\mu$ is uniquely determined by the principle of minimal electromagnetic interaction. (See, especially, footnote 2 of Paper I.) The same problem can also be readily analyzed without this assumption by using the method developed in Paper I.

Thus, the state $|a^+\rangle$ must exist. From proposition (iii), it follows that there must exist another state $|a^-\rangle$ which has the same mass but opposite charge. The states $|a^+\rangle$ and $|a^-\rangle$ are not related by C.

It will be shown that, in the absence of the weak interaction, the state $|a^+\rangle$, or $|a^-\rangle$, cannot decay into any final states consisting of only known particles. Thus, among all such states the ones with the lowest mass m_a behave like metastable particles; they can only decay through the weak interaction. Because a^+ and a^- are strongly interacting particles, their mass m_a is not expected to be small.[12] The existence of such particles $a^\pm$ implies not only the C noninvariance of the electromagnetic interaction, but also the T noninvariance of the *strong* interaction. Nevertheless, it can be established that, in the limit $e=0$, reciprocity relations among all known particles remain valid, although the mathematical requirement of T invariance is violated. The role of the particles a^+ and a^- for the C, T noninvariance is somewhat similar to that of the neutrinos for the C, P noninvariance.

The details of the general consequences of these propositions and their applications to particles $a^\pm$ are given in Secs. II and III. The properties of the leptons and the weak interactions are discussed in Sec. IV.

A possible unifying view which connects the present C, T noninvariance and the well-known C, P nonconservation in weak interactions is discussed in Sec. V. In the Appendix, an explicit spin-$\frac{1}{2}$ model of such particles a^+ and a^- is given. Some further experimental consequences are discussed.

II. SYMMETRY AND ASYMMETRY PROPERTIES OF H_{st} AND H_γ

In this section we will analyze the consequences of propositions (i)–(iv). For clarity, all conclusions will be stated in the form of mathematical theorems.

The total Hamiltonian is assumed to be given by

$$H=H_{\text{free}}+H_\gamma+H_{\text{st}}+H_{\text{wk}}\,, \tag{24}$$

where H_{free} is the free-particle Hamiltonian but with the masses given by the observed physical masses, and H_γ, H_{st}, H_{wk} are, respectively, the electromagnetic, the strong-, and the weak-interaction Hamiltonians.

There are, by now, numerous experiments[13] which establish that both H_{st} and H_γ are invariant under the space-inversion operation P; i.e.,

$$[H_{\text{st}},P]=0 \tag{25}$$

and

$$[H_\gamma,P]=0\,. \tag{26}$$

Theorem 1. There exists an operator C_γ which satisfies

$$C_\gamma \mathcal{J}_\mu(x) C_\gamma^{-1}=-\mathcal{J}_\mu(x) \tag{27}$$

and

$$C_\gamma(H_{\text{free}}+H_\gamma)C_\gamma^{-1}=(H_{\text{free}}+H_\gamma)\,. \tag{28}$$

Proof. From proposition (iv), it follows that $(H_{\text{free}}+H_\gamma)$ is, by itself, separately invariant[10] under P, T, and C_γ, where C_γ satisfies Eq. (27). Theorem 1 is, then, proved.

Comparison between Eqs. (4), (6), and (27) shows that the operator C_γ is different from the particle-antiparticle conjugation operator C; i.e.,

$$C_\gamma \neq C\,.$$

The operator C_γ satisfies Eq. (13); therefore, C_γ is the charge-conjugation operator. It is this mismatch between these two conjugation operators C and C_γ, that gives rise to all the noninvariance properties of the combined Hamiltonian $(H_{\text{free}}+H_{\text{st}}+H_\gamma)$.

According to proposition (ii),

$$CH_\gamma C^{-1}\neq H_\gamma\,. \tag{29}$$

Since H_γ is invariant under T and P, we find

$$(CTP)H_\gamma(CTP)^{-1}\neq H_\gamma\,. \tag{30}$$

Instead, the usual "CTP" theorem[9] becomes

$$(C_\gamma TP)H_\gamma(C_\gamma TP)^{-1}=H_\gamma\,. \tag{31}$$

Theorem 2. The strong-interaction Hamiltonian satisfies

$$C_\gamma H_{\text{st}} C_\gamma^{-1}\neq H_{\text{st}}\,, \tag{32}$$

$$TH_{\text{st}}T^{-1}\neq H_{\text{st}}\,, \tag{33}$$

but

$$(C_\gamma T)H_{\text{st}}(C_\gamma T)^{-1}=H_{\text{st}}\,. \tag{34}$$

Proof. From proposition (i), we have

$$CH_{\text{st}}C^{-1}=H_{\text{st}}\,. \tag{35}$$

If $C_\gamma H_{\text{st}} C_\gamma^{-1}=H_{\text{st}}$, then we could have defined $C=C_\gamma$, and would violate proposition (ii); therefore, Eq. (32) is established. From proposition (iii), and Eqs. (30) and (31), it follows that

$$(C_\gamma TP)H_{\text{st}}(C_\gamma TP)^{-1}=H_{\text{st}}\,. \tag{36}$$

Equations (33) and (34) are direct consequences of Eqs. (25), (32), and (36).

Theorem 3. Both H_{st} and H_γ commute with Q_J and Q_K; i.e.,

$$[H_{\text{st}},Q_J]=[H_{\text{st}},Q_K]=0 \tag{37}$$

and

$$[H_\gamma,Q_J]=[H_\gamma,Q_K]=0\,. \tag{38}$$

[12] If $a^\pm$ does not decay through weak interactions, then a lower limit $m_a>5$ BeV can be set by using the recent experimental results of D. E. Dorfan, J. Eades, L. M. Lederman, W. Lee, and C. C. Ting, Phys. Rev. Letters **14**, 999 (1965). If $a^\pm$ does decay through the weak interaction, then the present lower limit of m_a becomes ~1 BeV, or ~1.5 BeV, depending on whether $a^\pm$ is a boson, or a fermion.

[13] We list but a few of the relatively recent such experiments: F. Boehm and E. Kankeleit, Calt-63-13 report (unpublished); Yu. G. Abov, P. A. Krupchitsky, and Yu. A. Oratovsky, Compt. Rend. Congr. Intern. Phy. Nucl., Paris, **1964**; L. Grodzins and F. Genovese, Phys. Rev. **121**, 228 (1961); R. E. Segel *et al.*, *ibid.* **123**, 1382 (1961); D. E. Alburger *et al.*, Phil. Mag. **6**, 171 (1961); R. Haas, L. B. Leipuner, and R. K. Adair, Phys. Rev. **116**, 1221 (1959); F. Boehm and U. Hauser, Nucl. Phys. **14**, 615 (1959); D. A. Bromley *et al.*, Phys. Rev. **114**, 758 (1959).

TABLE I. Symmetry and asymmetry properties of different interactions. [$H_{wk}{}^0$ includes only the $a^{\pm}$-independent part of the weak interaction.]

	H_{st}	H_{γ}	$H_{wk}{}^{0}$
P	✓	✓	×
C	✓	×	×
C_{γ}	×	✓	×
T	×	✓	✓
CT	×	×	×
$C_{\gamma}T$	✓	✓	×
CP	✓	×	✓
$C_{\gamma}P$	×	✓	✓
PT	×	✓	×
CPT	×	×	✓
$C_{\gamma}PT$	✓	✓	✓

Furthermore,

$$[Q_J,Q_K]=0. \tag{39}$$

Proof. Equation (37) is identical with Eqs. (18) and (20) proved in the previous section. To establish Eq. (38), we observe that from proposition (iv) and Eqs. (5) and (6)

$$[J_{\mu},Q_J]=[J_{\mu},Q_K]=0 \tag{40}$$

and

$$[K_{\mu},Q_J]=[K_{\mu},Q_K]=0. \tag{41}$$

Consequently, Q_J commutes with Q_K, and H_{γ} commutes with both Q_J and Q_K.

The symmetry and asymmetry properties of H_{st} and H_{γ} are given in Table I. The corresponding properties of H_{wk} will be discussed in Sec. IV.

III. APPLICATIONS

Since H_{st}, Q_J, and Q_K mutually commute, we may diagnolize these three matrices simultaneously. The eigenstates of H_{st} can, therefore, be classified according to their values of Q_J and Q_K.

Theorem 4. All presently known strongly interacting particles have $Q_K=0$.

Proof. From the observed C invariance of reaction (1), it follows that $C|p\rangle=|\bar{p}\rangle$, $C|\pi^+\rangle=|\pi^-\rangle$. Thus, both the nucleon and the pion must have $Q_K=0$. By using theorem 3, we conclude that all known strongly interacting nonstrange particle states, such as $\rho^{\pm}$, N^*, ϕ^0, etc., must have $Q_K=0$.

The Λ^0 has a total charge

$$Q(\Lambda)=Q_J(\Lambda)+Q_K(\Lambda)=0.$$

Under C, the state $|\Lambda^0\rangle$ changes into $C|\Lambda^0\rangle$ which remains an eigenstate of H_{st}. The state $C|\Lambda^0\rangle$ has the same mass m_{Λ}, but it has a charge $=[-Q_J(\Lambda)+Q_K(\Lambda)]$. The only known particles that have mass m_{Λ} are Λ^0 and $\bar{\Lambda}^0$. Thus

$$-Q_J(\Lambda)+Q_K(\Lambda)=0,$$

and consequently $Q_K=0$ for Λ^0. All the presently known strongly interacting particles can be connected to some Λ-nucleon-pion system through H_{st}. Theorem 4 is, then, proved by using theorem 3.

Let $|a^+\rangle$ be an eigenstate of H_{st} which has $Q_J=0$ and $Q_K=e$. The state $|a^+\rangle$ satisfies Eqs. (16), (21), and (22). Since H_{st} is invariant under $(C_{\gamma}T)$, the state $|a^-\rangle$, defined by

$$|a^-\rangle\equiv(C_{\gamma}T)|a^+\rangle, \tag{42}$$

must be an eigenstate of H_{st}, and it has the same eigenvalue as $|a^+\rangle$. From the definition of C_{γ}, we can conclude that the state $|a^-\rangle$ has $Q_J=0$ and $Q_K=-e$. The state $|a^-\rangle$ is also an eigenstate of C.

In the following, the particle a^+ (or a^-) refers to the state that has the lowest mass among all states with $Q_J=0$ and $Q_K=+e$ (or $-e$).

Theorem 5. In the absence of the weak interaction, the particles a^+ and a^- are stable.

Proof. Theorem 5 is a direct consequence of theorems 3 and 4.

The particles a^+ and a^- can be produced in pairs through H_{st}; e.g.,

$$p+p\rightarrow p+p+a^++a^-+\cdots. \tag{43}$$

It is important to note that since H_{st} is not invariant under either C_{γ} or T, neither $C_{\gamma}|a^{\pm}\rangle$ nor $T|a^{\pm}\rangle$ are eigenstates of H_{st}. Similarly, under either C_{γ} or T, a physical proton p does not transform into either $\bar{p}$ or p.

Theorem 6. In the absence of the electromagnetic interaction, the usual reciprocity relation holds for any strong reaction which consists of only the presently known particles.

Proof. From Eqs. (34) and (35), we find

$$C^{-1}(C_{\gamma}T)H_{st}(C_{\gamma}T)^{-1}C=H_{st}. \tag{44}$$

Under C, any known strongly interacting particle, say p, becomes its antiparticle $\bar{p}$. Under $(C_{\gamma}T)$, $\bar{p}$ is transformed back into p but with its spin and momentum directions inverted. Thus,

$$C^{-1}(C_{\gamma}T)|p(\mathbf{k},\mathbf{s})\rangle=\eta|p(-\mathbf{k},-\mathbf{s})\rangle, \tag{45}$$

where $\mathbf{k}$ and $\mathbf{s}$ denote, respectively, the momentum and the spin of the state and η is a phase factor. Similar relations hold for all strongly interacting particles with $Q_K=0$. Theorem 6 can be easily proved by using Eq. (44).

Similarly, one can prove that for any collision process consisting of only the known strongly interacting particles all other consequences of $C^{-1}(C_{\gamma}T)$ invariance are also identical to that of T invariance. Thus, the experimental results cited in Refs. 3 and 4 can still be used as supporting evidence of C invariance for H_{st}, even though H_{st} is not invariant under T.

The reciprocity relation does not hold for reactions that involve $a^{\pm}$. For example, the strong reaction

$$a^++p\rightarrow a^++n+\pi^+ \tag{46}$$

is related to

$$a^-+\bar{n}+\pi^-\rightarrow a^-+\bar{p} \tag{47}$$

through $C_\gamma T$, and is related to

$$a^- + n + \pi^+ \to a^- + p \tag{48}$$

through $C^{-1}(C_\gamma T)$. Neither one is the usual reciprocity relation.

An explicit spin-$\frac{1}{2}$ model of the particle $a^\pm$ is given in the Appendix.

IV. LEPTONS AND THE WEAK INTERACTION

The leptons have no strong interaction. Thus, the particle-antiparticle conjugation operator of the leptons is determined by the electromagnetic interaction; i.e.,

$$(C)_{\text{lepton}} = (C_\gamma)_{\text{lepton}}. \tag{49}$$

The C of the entire system is the product of $(C)_{\text{lepton}}$ times the operator C of the strongly interacting particles. All above theorems 1–6 remain valid, if J_μ (consequently, also H_γ) includes the known leptonic electromagnetic currents.

The weak-interaction Hamiltonian H_{wk} can be decomposed into

$$H_{\text{wk}} = H_{\text{wk}}^0 + H_{\text{wk}}^a, \tag{50}$$

where H_{wk}^0 consists only of the field operators of the known particles, and H_{wk}^a depends also on the particle $a^\pm$.

If H_γ violates C invariance, then reaction (2) can be attributed to the radiative correction effect, and there is no particular reason to assume, in addition, that H_{wk}^0 has a (CP)-noninvariant amplitude which happens to be $0(\alpha)$ times that of the (CP)-conserving amplitude. We will, therefore, assume

$$[H_{\text{wk}}^0, CP] = 0. \tag{51}$$

In the framework of a local-field theory, H_{wk}^0 is a function depending only on the "bare"-field operators of the known particles. Therefore, we expect that

$$[H_{\text{wk}}^0, C_\gamma P] = 0. \tag{52}$$

From $(C_\gamma PT)$ invariance

$$(C_\gamma PT) H_{\text{wk}}^0(t) (C_\gamma PT)^{-1} = H_{\text{wk}}^0(-t), \tag{53}$$

it follows that

$$T H_{\text{wk}}^0(t) T^{-1} = H_{\text{wk}}^0(-t), \tag{54}$$

$$C^{-1}(C_\gamma T)[H_{\text{wk}}^0(t)](C_\gamma T)^{-1} C = H_{\text{wk}}^0(-t), \tag{55}$$

and

$$CPT H_{\text{wk}}^0(t) T^{-1} P^{-1} C^{-1} = H_{\text{wk}}^0(-t). \tag{56}$$

The H_{wk}^0 is, of course, not invariant under either C, C_γ, or P.

These properties of H_{wk}^0 are summarized in Table I. The remaining $a^\pm$-dependent part H_{wk}^a is totally unknown. The main question is whether $a^\pm$ can decay through H_{wk}^a. It seems reasonable that there should be weak decay processes such as

$$a^\pm \to l^\pm + \text{neutrino} + \cdots,$$

where $l = e$ or μ, and other possible decay modes. An analysis of the various possibilities will be given elsewhere.

V. A UNIFYING VIEW OF DISCRETE SYMMETRY VIOLATIONS

From the above discussions, it is clear that the operators associated with the various discrete symmetry operations can be different for different interactions. For example, we may define

$$C_{\text{st}} = C, \tag{57}$$

$$T_{\text{st}} = C^{-1}(C_\gamma T), \tag{58}$$

$$P_{\text{st}} = P_\gamma = P, \tag{59}$$

and

$$T_\gamma = T. \tag{60}$$

Thus,

$$P_{\text{st}} C_{\text{st}} T_{\text{st}} = P_\gamma C_\gamma T_\gamma. \tag{61}$$

The H_{st} is invariant under P_{st}, C_{st}, and T_{st}, while $(H_{\text{free}} + H_\gamma)$ is invariant under P_γ, C_γ, and T_γ. The fact that $C_{\text{st}} \neq C_\gamma$ and $T_{\text{st}} \neq T_\gamma$ gives rise to the "C" "T" noninvariance of the combined Hamiltonian $(H_{\text{free}} + H_\gamma + H_{\text{st}})$.

In the absence of H_γ, we cannot differentiate a charged particle from a neutral particle. Thus, there is no difficulty in accepting the a^+ and a^- as eigenstates of C_{st}. The difference between $a^\pm$ and the known particles can be attributed to, say, the baryon number N. We may assign

$$N = 0 \tag{62}$$

for $a^\pm$, and require that

$$C_{\text{st}} N + N C_{\text{st}} = 0. \tag{63}$$

Under C_{st}, a baryon with $N=1$ must transform into an antibaryon with $N=-1$. The known $N=0$ mesons can be regarded as composites of baryons and antibaryons; their transformation properties under C_{st} are determined by those of the baryons.

The C_γ satisfies

$$C_\gamma Q + Q C_\gamma = 0. \tag{13}$$

Consequently, neither a^+ nor a^- can be an eigenstate of C_γ, which results in the mismatch between C_{st} and C_γ.

The same view can also be extended to the weak interaction. We note that in the absence of H_γ and the leptonic mass terms in H_{free}, it is *not* possible to differentiate μ^-, e^-, ν_μ, and ν_e through either their mass differences or their charge differences.

We will now require that H_{wk} be invariant under C_{wk}, P_{wk}, and T_{wk}, where

$$T_{\text{wk}} = T_{\text{st}} \tag{64}$$

and

$$P_{\text{wk}} C_{\text{wk}} T_{\text{wk}} = P_{\text{st}} C_{\text{st}} T_{\text{st}}. \tag{65}$$

[From Eqs. (54) and (55), it follows that if we consider only the $a^\pm$-independent part H_{wk}^0, then an equally good choice can be given by $T_{\rm wk}=T_\gamma$ and $P_{\rm wk}C_{\rm wk}T_{\rm wk}=P_\gamma C_\gamma T_\gamma$].

The $C_{\rm wk}$ must be *independent* of the space-time transformation, and, in analogy to Eqs. (63) and (13),

$$C_{\rm wk}L+LC_{\rm wk}=0\,, \tag{66}$$

where L is one of the lepton numbers.[14]

For clarity, we will consider only the leptonic part of the weak interaction. (All following considerations can, of course, be applied to the nonleptonic part of $H_{\rm wk}$ as well.) Let

$$H_{\rm wk}=(1/\sqrt{2})g_\mu(I_\lambda^* j_\lambda+I_\lambda j_\lambda^*+j_\lambda^* j_\lambda)\,, \tag{67}$$

where g_μ is the μ-decay coupling constant,

$$j_\lambda=i\sum_{l=e,\mu}\psi_l^\dagger\gamma_4\gamma_\lambda(1+\gamma_5)\psi_{\nu_l}\,, \tag{68}$$

$$j_\lambda^*=i\sum_{l=e,\mu}\psi_{\nu_l}^\dagger\gamma_4\gamma_\lambda(1+\gamma_5)\psi_l\,, \tag{69}$$

and I_λ, I_λ^* depend only on the strongly interacting particles. In Eqs. (68) and (69), we find it convenient to represent the neutrino fields by 4-component operators.

We define

$$C_{\rm wk}\psi_e(x)C_{\rm wk}^{-1}=\psi_{\nu_\mu}(x)\,, \tag{70}$$

$$C_{\rm wk}\psi_\mu(x)C_{\rm wk}^{-1}=\psi_{\nu_e}(x)\,, \tag{71}$$

and

$$C_{\rm wk}^2=1\,. \tag{72}$$

The leptonic current $j_\lambda(x)$ transforms into $j_\lambda^*(x)$ under $C_{\rm wk}$; i.e.,

$$C_{\rm wk}j_\lambda(x)C_{\rm wk}^{-1}=j_\lambda^*(x)\,. \tag{73}$$

The operator $C_{\rm wk}$ anticommutes with L, provided we assign

$$L=1 \quad \text{for} \quad e^-,\ \nu_e,\ \mu^+,\ \bar\nu_\mu$$

and

$$L=-1 \quad \text{for} \quad e^+,\ \bar\nu_e,\ \mu^-,\ \nu_\mu\,. \tag{74}$$

In order that $H_{\rm wk}$ be invariant under $C_{\rm wk}$, we must have

$$C_{\rm wk}I_\lambda(x)C_{\rm wk}^{-1}=I_\lambda^*(x)\,. \tag{75}$$

The simplest way to study the action of $C_{\rm wk}$ operating on nonleptons is to use the quark model[15] or any one of the triplet models.[16] Let (ψ_1,ψ_2,ψ_3) be the field operators of the triplet $(\alpha_1^{q+1},\alpha_2^{q},\alpha_3^{q})$, where the superscripts denote their charges. The corresponding currents I_λ and I_λ^* are given by

$$I_\lambda^*=i\psi_1^\dagger\gamma_4\gamma_\lambda(G_V+G_A\gamma_5)(\cos\theta\psi_2+\sin\theta\psi_3) \tag{76}$$

and

$$I_\lambda=i(\cos\theta\psi_2+\sin\theta\psi_3)^\dagger\gamma_4\gamma_\lambda(G_V+G_A\gamma_5)\psi_1\,, \tag{77}$$

where G_V and G_A are real parameters and θ is the Cabibbo angle.[17] Equation (75) can be satisfied if

$$C_{\rm wk}\psi_1C_{\rm wk}^{-1}=\cos\theta\psi_2+\sin\theta\psi_3\,. \tag{78}$$

The remaining component $(\sin\theta\psi_2-\cos\theta\psi_3)$ is an eigenvector of $C_{\rm wk}$. Thus, we find

$$C_{\rm wk}H_{\rm wk}C_{\rm wk}^{-1}=H_{\rm wk}\,, \tag{79}$$

$$P_{\rm wk}H_{\rm wk}P_{\rm wk}^{-1}=H_{\rm wk}\,, \tag{80}$$

and

$$T_{\rm wk}H_{\rm wk}(t)T_{\rm wk}^{-1}=H_{\rm wk}(-t)\,, \tag{81}$$

where $P_{\rm wk}=P_{\rm st}C_{\rm st}C_{\rm wk}^{-1}$.

Each interaction H_i (i=strong, or γ, or weak) is invariant[18] under its own P_i, C_i, and T_i. The observed violation of these discrete symmetries is due to

$$P_\gamma=P_{\rm st}\neq P_{\rm wk}\,, \tag{82}$$

$$T_\gamma\neq T_{\rm st}=T_{\rm wk}\,, \tag{83}$$

$$P_{\rm st}C_{\rm st}T_{\rm st}=P_\gamma C_\gamma T_\gamma=P_{\rm wk}C_{\rm wk}T_{\rm wk}\,, \tag{84}$$

and consequently,

$$C_{\rm st}\neq C_\gamma\neq C_{\rm wk}\neq C_{\rm st}\,. \tag{85}$$

If the identity of a particle could be taken for granted, then it would be possible to define T, the pure time reversal, and P, the pure space inversion, unambiguously. However, the distinguishability between different particles depends on their interactions, and degeneracies occur if certain interactions are absent. It is, therefore, not possible to give a unique definition of P and T without any reference to some specific interactions.

At large distances, because of its long-range character, the electromagnetic force predominates. Thus, in all collision processes, the asymptotic conditions are determined by the physical masses and the electromagnetic properties of the incoming and outgoing particles. It is, therefore, convenient to identify $T=T_\gamma$ and $P=P_\gamma$, since $(H_{\rm free}+H_\gamma)$ is invariant under both T_γ and P_γ. On the other hand, the internal structures of these particles are determined mainly by $H_{\rm st}$, which makes it convenient to associate the particle-antiparticle conjugation operator C with $C_{\rm st}$. It is because of these particular identifications, that $H_{\rm st}$ becomes noninvariant under T, H_γ becomes noninvariant under C, and $H_{\rm wk}$ becomes noninvariant under C and P.

[14] Without Eq. (66), the invariance requirement of $H_{\rm wk}$ can be trivially satisfied by choosing $C_{\rm wk}=1$, $P_{\rm wk}=P_{\rm st}C_{\rm st}$, and $T_{\rm wk}=T_{\rm st}$.

[15] M. Gell-Mann, Phys. Letters **8**, 214 (1964); G. Zweig, CERN (unpublished report).

[16] See, e.g., T. D. Lee, Nuovo Cimento **35**, 933 (1965) for a summary of the different triplet models.

[17] N. Cabibbo, Phys. Rev. Letters **10**, 531 (1963).

[18] We note that if the physical masses of particles are not degenerate, then P_γ, C_γ, and T_γ can be uniquely determined by $(H_{\rm free}+H_\gamma)$, apart from a gauge phase factor. Both $H_{\rm st}$ and $H_{\rm wk}$ are invariant under some other groups of unitary transformations, denoted by $G_{\rm st}$ and $G_{\rm wk}$, respectively, which are not connected with the space-time transformations. For the $H_{\rm st}$, we have the isospin transformations; for the $H_{\rm wk}$, we have a $U_2\times U_2$ group of transformations. [See T. D. Lee, Nuovo Cimento **35**, 945 (1965).] From any special solution of C_i, P_i, T_i (i=strong or weak) we can easily obtain the general solution by transforming $C_i\to SC_i$, $P_i\to S'P_i$, and $T_i\to S''T_i$ where S, S', and S'' are members of the group G_i, provided Eqs. (63), (66), and (82)–(84) are satisfied.

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APPENDIX

In this Appendix, we give an explicit model of the particles a^+ and a^- by assuming that they are spin-$\frac{1}{2}$ fermions and that their strong interactions are invariant under the usual SU_3 group of transformations. Consequently, the particles a^+ and a^- must belong to certain irreducible representations of the SU_3 group. The simplest case is that the a^+ and a^- are both unitary singlets. It is also possible that there might be neutral a's, so that (a_1^+,a_2^0,a_3^0) and $(a_1^-,\bar{a}_2^0,\bar{a}_3^0)$ could form two unitary triplets, etc.

We consider first the case in which the $a^\pm$ are unitary singlets. As an illustration of some possible forms of strong interactions between the $a^\pm$ and the known spin-$\frac{1}{2}$ baryon octet $B_k{}^j$ (where j, k=1, 2, 3) we may give the *phenomenological* Lagrangian density:

$$L_{\rm int}(x)=ig_1m_N{}^{-3}[B^\dagger{}_k{}^j\gamma_4\gamma_\mu\gamma_5B_j{}^k]\times\left[\psi^\dagger{}_a\gamma_4\gamma_5\frac{\partial\psi_a}{\partial x_\mu}-\frac{\partial\psi^\dagger{}_a}{\partial x_\mu}\gamma_4\gamma_5\psi_a\right]+g_2m_N{}^{-3}\times\left[B^\dagger{}_k{}^j\gamma_4\gamma_\mu\frac{\partial B_j{}^k}{\partial x_\nu}-\frac{\partial B^\dagger{}_k{}^j}{\partial x_\nu}\gamma_4\gamma_\mu B_j{}^k\right][\psi^\dagger{}_a\gamma_4\sigma_{\mu\nu}\psi_a],\quad \text{(A1)}$$

where the dimensionless coupling constants g_1 and g_2 are real and large, m_N is the mass of the nucleon, all repeated indices are to be summed over, μ, ν vary from 1 to 4, and j, k vary from 1 to 3. The ψ_a and $B_k{}^j$ are, respectively, the field operators of a^+ and the spin-$\frac{1}{2}$ baryon octets, and $\psi^\dagger{}_a$, $B^\dagger{}_j{}^k$ their respective Hermitian conjugate operators. The $\gamma_1, \gamma_2, \cdots, \gamma_5$ are five mutually anticommuting (4×4) Hermitian matrices, and $\sigma_{\mu\nu}=(2i)^{-1}(\gamma_\mu\gamma_\nu-\gamma_\nu\gamma_\mu)$. For definiteness, let us choose γ_1, γ_2, γ_3 to be real and γ_4, γ_5 to be pure imaginary. In addition to $L_{\rm int}(x)$ there are also other strong interacactions between the known particles and that between the a's.

The electric currents are, in the absence of the strong interaction, given by[19]

$$K_\mu=ie\psi^\dagger{}_a\gamma_4\gamma_\mu\psi_a \qquad \text{(A2)}$$

and

$$J_\mu=ie(\psi^\dagger{}_p\gamma_4\gamma_\mu\psi_p-\psi^\dagger{}_{\Xi^-}\gamma_4\gamma_\mu\psi_{\Xi^-}+\cdots)\,, \qquad \text{(A3)}$$

where $\psi_p=B_3{}^1$, $\psi_{\Xi^-}=B_1{}^3$, etc., and $\cdots$ represents the currents due to other known spin-$\frac{1}{2}$ and spin-0 particles.

Similar expressions can be easily written down if the a's are unitary triplets[20] (or octets, etc.) instead of being unitary singlets. We note that the $C=+1$ current K_μ is a unitary singlet if a^+ and a^- are unitary singlets; in footnote 20, it is shown that K_μ remains a unitary singlet, even if the a^+ and a^- are not themselves unitary singlets.

Under C, C_γ, and T, the field operators transform like

$$CB_k{}^j(\mathbf{r},t)C^{-1}=B^\dagger{}_j{}^k(\mathbf{r},t)\,, \qquad \text{(A4)}$$

$$C_\gamma B_k{}^j(\mathbf{r},t)C_\gamma{}^{-1}=B^\dagger{}_j{}^k(\mathbf{r},t)\,, \qquad \text{(A5)}$$

$$TB_k{}^j(\mathbf{r},t)T^{-1}=\gamma_1\gamma_2\gamma_3B_k{}^j(\mathbf{r},-t)\,, \qquad \text{(A6)}$$

$$C\psi_a(\mathbf{r},t)C^{-1}=\psi_a(\mathbf{r},t)\,, \qquad \text{(A7)}$$

$$C_\gamma\psi_a(\mathbf{r},t)C_\gamma{}^{-1}=\psi^\dagger{}_a(\mathbf{r},t)\,, \qquad \text{(A8)}$$

and

$$T\psi_a(\mathbf{r},t)T^{-1}=\gamma_1\gamma_2\gamma_3\psi_a(\mathbf{r},-t)\,, \qquad \text{(A9)}$$

where, for simplicity, all phase factors are chosen to be unity.

It can be readily verified that $L_{\rm int}$ is a Hermitian operator and it satisfies

$$CL_{\rm int}(\mathbf{r},t)C^{-1}=+L_{\rm int}(\mathbf{r},t)\,, \qquad \text{(A10)}$$

$$C_\gamma L_{\rm int}(\mathbf{r},t)C_\gamma{}^{-1}=-L_{\rm int}(\mathbf{r},t)\,, \qquad \text{(A11)}$$

and

$$TL_{\rm int}(\mathbf{r},t)T^{-1}=-L_{\rm int}(\mathbf{r},-t)\,. \qquad \text{(A12)}$$

Thus, the resulting $H_{\rm st}$ is invariant under $(C_\gamma T)$, C, and $(C_\gamma PT)$, but it is not invariant under either C_γ, T, or CPT.

[19] It should be noted that because of the derivative couplings in the strong interaction, Eq. (A1), there are the induced electromagnetic currents whose amplitudes are proportional to eg_1 and eg_2. In this and other similar cases, the total interaction Hamiltonian H is not a linear sum $(H_{\rm st}+H_\gamma+H_{\rm wk})$. Our general supposition, expressed in Sec. V, remains valid provided

$$H_{\rm st}\equiv\lim H,\quad \text{at}\quad e=g_{\rm wk}=0,$$
$$H_\gamma\equiv\lim H,\quad \text{at}\quad g_{\rm st}=g_{\rm wk}=0,$$

and

$$H_{\rm wk}\equiv\lim H,\quad \text{at}\quad e=g_{\rm st}=0,$$

where $g_{\rm st}$ and $g_{\rm wk}$ represent, respectively, all strong- and weak-interaction constants. Each interaction H_i (i=strong, γ, and weak), thus defined, remains invariant under its own C_i, T_i, and P_i.

I wish to thank Professor L. Van Hove for raising the question of derivative couplings.

[20] To show this, let us consider first the functional relation between Q_K and I_z, where I_z is the z component of the isospin operator $\mathbf{I}$. Since the eigenvalues of both Q_K and I_z are additive quantum numbers, there can only be linear relations between these two operators; i.e., $Q_K=\lambda I_z+A$, where λ is a constant and A must be another additive quantum operator. By applying this relation to all known particles, for which $Q_K=0$, we find $\lambda=0$. Thus, Q_K is independent of I_z. By extending this argument to include other components of $\mathbf{I}$, it can be established that $[Q_K,\mathbf{I}]=0$. Similarly, it can be shown that Q_K is an SU_3 unitary singlet. By proposition (iv), it follows, then, that K_μ is also a singlet under either the SU_3 or the isospin transformations.

It may be instructive to give an explicit example of nonsinglet a particles. Let us assume the existence of a unitary triplet of spin 0, or $\frac{1}{2}$, particles (a_1^+,a_2^0,a_3^0) where all the a's are of $Q_K=+\frac{1}{2}$, the a_1 and a_2 are of $I=\frac{1}{2}$, $N=S=0$, and a_3 is of $I=0$, $N=0$ but $S=-1$. The total charge Q is given by

$$Q=I_z+\tfrac{1}{2}(N+S)+Q_K.$$

This triplet has, therefore, total charges (+, 0, 0). It becomes one with total charges (0, +, +) under $C_{\rm st}$ and a different set with total charges (−, 0, 0) under $T_{\rm st}$. The $C_{\rm st}$ and $T_{\rm st}$ invariances, then, generate from the triplet (a_1^+,a_2^0,a_3^0) a total of four different set of triplets. Nevertheless, the current K_μ, given by the minimal electromagnetic interaction of all these triplets, remains a unitary singlet.

The currents J_μ and K_μ transform in the same way under either C_γ or T, but differently under C:

$$C_\gamma J_\mu(\mathbf{r},t)C_\gamma^{-1}=-J_\mu(\mathbf{r},t)\,, \tag{A13}$$

$$C_\gamma K_\mu(\mathbf{r},t)C_\gamma^{-1}=-K_\mu(\mathbf{r},t)\,, \tag{A14}$$

$$TJ_\mu(\mathbf{r},t)T^{-1}=-J_\mu(\mathbf{r},-t)\,, \tag{A15}$$

$$TK_\mu(\mathbf{r},t)T^{-1}=-K_\mu(\mathbf{r},-t)\,, \tag{A16}$$

$$CJ_\mu(\mathbf{r},t)C^{-1}=-J_\mu(\mathbf{r},t)\,, \tag{A17}$$

but

$$CK_\mu(\mathbf{r},t)C^{-1}=+K_\mu(\mathbf{r},t)\,. \tag{A18}$$

Thus, the H_γ is invariant[19] under C_γ, T, and $C_\gamma PT$, but it is not invariant under either C or CPT.

Remarks

1. From Eqs. (A4) and (A5), we see that C and C_γ both change a "bare" p to a "bare" $\bar{p}$. By using L_{int} and in the absence of H_γ, it can be verified explicitly that, because of the virtual creations and absorptions of the pair (a^+,a^-), a "physical" p is changed into a "physical" $\bar{p}$ only under C but not under C_γ. Similarly, a "physical" p remains a "physical" p under $C^{-1}(C_\gamma T)$ but not under T.

2. As remarked earlier, the K_μ associated with the $a^\pm$ particles must be a unitary singlet. However, even if $a^\pm$ exists, there may still be other "first type" C_{st}-symmetry violating interactions H_γ, and for which K_μ may not be a unitary singlet. From a phenomenological point of view, one should decompose the current K_μ into members of different representations of the SU_3 group:

$$K_\mu=(K_\mu)_1+(K_\mu)_8+\cdots\,, \tag{A19}$$

where $(K_\mu)_1$ is a unitary singlet and $(K_\mu)_8$ is a member of a unitary octet, etc. The experimental consequences of $K_\mu=(K_\mu)_8$ have already been examined in Paper I.

If the current $K_\mu=(K_\mu)_1$, then under the isospin transformations, K_μ must transform like an isoscalar, and therefore it satisfies the $|\Delta\mathbf{I}|=0$ rule.

We first list the consequences of K_μ satisfies the $|\Delta\mathbf{I}|=0$ rule[21]:

$$\phi^0 \nleftrightarrow \rho^0+\gamma\,, \tag{A20}$$

$$\omega^0 \nleftrightarrow \rho^0+\gamma\,, \tag{A21}$$

and there is *no* π^+, π^- asymmetry in either

$$\omega^0(\text{or }\phi^0)\rightarrow\pi^++\pi^-+\pi^0\,, \tag{A22}$$

or

$$\omega^0(\text{or }\phi^0)\rightarrow\pi^++\pi^-+\gamma\,. \tag{A23}$$

There remain, however, π^+, π^- asymmetries in

$$\rho^0\rightarrow\pi^++\pi^-+\gamma\,, \tag{A24}$$

$$\eta^0\rightarrow\pi^++\pi^-+\gamma\,, \tag{A25}$$

and

$$\eta^0\rightarrow\pi^++\pi^-+\pi^0\,. \tag{A26}$$

The 3π final state in reaction (A26) contains only the $I=1$, $C=+1$ and the $I=0$, $C=-1$ states.

Futhermore, in the single-photon-exchange approximation, we find

$$\eta^0 \nleftrightarrow \pi^0+e^++e^- \tag{A27}$$

and there are no observable "T" noninvariant effects [i.e., $C^{-1}(C_\gamma T)$-noninvariant effects] in

$$\Sigma^0\rightarrow\Lambda^0+e^++e^-\,. \tag{A28}$$

Another consequence of $K_\mu=(K_\mu)_1$ is that, in the limit of perfect SU_3 symmetry and by using $(C_\gamma PT)$ invariance,

$$\phi^0 \nleftrightarrow \omega^0+\gamma\,. \tag{A29}$$

It should be noted that identical results concerning reactions (A27) and (A28) can be derived in the limit of perfect SU_3 symmetry if $K_\mu=(K_\mu)_8$. However, by studying reactions (A20)–(A26) and (A29) it is possible to differentiate whether $K_\mu=(K_\mu)_1$ or $K_\mu=(K_\mu)_1+(K_\mu)_8$.

[21] For a detailed phenomenological analysis of these reactions, see Ref. 5.