in the Hilbert space

$$\mathcal{H}=\int_{\alpha=0}^{2\pi\,\oplus}\mathcal{H}_\alpha(d\alpha)^{1/2}.$$

Then $\psi(x)$ [as well as $\phi(x)$] possesses the symmetry property

$$G(\beta)\psi(x)G^\dagger(\beta)=e^{i\beta}\psi(x)$$

[$G(\beta)$, defined in terms of $G_\alpha(\beta)$ by

$$(\varphi,G(\beta)\varphi')=\int_0^{2\pi}(\varphi_{\alpha+\beta},G_\alpha(\beta)\varphi_\alpha')d\alpha$$

for φ, $\varphi'\epsilon\mathcal{H}$, is unitary]. However, the symmetry of $\psi(x)$ with respect to $H(\beta)$ is destroyed [unlike the case for $\phi(x)$]. The algebra formed by $\psi(x)$ is cyclically represented in $\mathcal{H}$ with $\Omega=\int^\oplus\Omega_\alpha(d\alpha)^{1/2}$ as a cyclic vector (which is, of course, not true for ϕ), as follows from an argument given in Sec. 2 of Ref. 6, since $(\Omega_0,\psi_0(x)\Omega_0)\neq0$ and there are vacuum states in $\mathcal{H}$ for which ψ has nonvanishing expectation values. Nevertheless, ψ does not contain particle states of mass zero since it transforms according to the same representation of the Lorentz group as ϕ.

We explain the failure of Goldstone's assertion in the example presented by the lack of a conserved current connected with G. The fact that the symmetry of the field ψ under the G transformation does not entail a conserved current is probably due to the reducibility of the field which in turn is necessary for the degeneracy of the vacuum state.

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$M(12)$ Predictions for pp Scattering

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The relations $A=-R'$, $A'=R$, $C_{KP}=0$, $C_{PP}=C_{KK}$ and $C_{NN}(\theta)=D(\theta)+D(\pi-\theta)-1$ between the standard parameters of pp scattering are derived from $M(12)$ invariance and compared with experiment. The applicability of $M(12)$ invariance to scattering processes is discussed.

RECENTLY the predictions of the $M(12)$-symmetry scheme[1] for four-particle reactions[2,3] have been discussed. Whereas some of these predictions are in disagreement with experiment and others contradict the basic principle of unitarity for the S-matrix, it has become evident that predictions for forward scattering or for low-energy reactions are in good agreement with experiment. We shall analyze the reasons for this and present some new results for nucleon-nucleon scattering.

Baryon-antibaryon scattering has been discussed[3] in the context of $M(12)$. From the results obtained there, one can easily cross over into the baryon-baryon channel. We now consider the important special case of pp scattering. In this case the amplitude can be written in the form

$$\begin{aligned}M=a(k,\theta)+ic(k,\theta)[\boldsymbol{\sigma}^{(1)}\mathbf{N}+\boldsymbol{\sigma}^{(2)}\mathbf{N}]+m(k,\theta)\boldsymbol{\sigma}^{(1)}\mathbf{N}\boldsymbol{\sigma}^{(2)}\mathbf{N}\\+g(k,\theta)[\boldsymbol{\sigma}^{(1)}\mathbf{P}\boldsymbol{\sigma}^{(2)}\mathbf{P}+\boldsymbol{\sigma}^{(1)}\mathbf{K}\boldsymbol{\sigma}^{(2)}\mathbf{K}]\\+h(k,\theta)[\boldsymbol{\sigma}^{(1)}\mathbf{P}\boldsymbol{\sigma}^{(2)}\mathbf{P}-\boldsymbol{\sigma}^{(1)}\mathbf{K}\boldsymbol{\sigma}^{(2)}\mathbf{K}],\quad(1)\end{aligned}$$

where we have used the notations of Moravcsik's book.[4] Expanding the $M(12)$ expression in terms of Pauli spinors and comparing with (1), we find the relation

$$h(k,\theta)=0.\quad(2)$$

In terms of measurable parameters, this means

$$A=-R',\quad(3)$$

$$A'=R,\quad(4)$$

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[1] K. Bardakci, J. M. Cornwall, P. G. O. Freund, and B. W. Lee, Phys. Rev. Letters 14, 48 (1965); B. Sakita and K. C. Wali, *ibid.* 14, 405 (1965); A. Salam, R. Delbourgo, and J. Strathdee, Proc. Roy. Soc. (London) A284, 146 (1965); see also M. Bég and A. Pais, Phys. Rev. Letters 14, 267 (1965).

[2] J. M. Cornwall, P. G. O. Freund, and K. T. Mahanthappa, Phys. Rev. Letters 14, 515 (1965); R. Blankenbecler, M. L. Goldberger, K. Johnson, and S. B. Treiman, *ibid.* 14, 518 (1965).

[3] S. Y. Lo, Phys. Rev. (to be published).

[4] M. J. Moravcsik, *The Two-Nucleon Interaction* (Clarendon Press, Oxford, England, 1963); D. Sprung, Phys. Rev. 121, 925 (1961).

$$C_{PP}=C_{KK}, \tag{5}$$

$$C_{KP}=0, \tag{6}$$

$$C_{NN}(\theta)=D(\theta)+D(\pi-\theta)-1, \tag{7}$$

whence

$$C_{NN}(90°)=2D(90°)-1. \tag{7'}$$

Relations (3)–(5) are valid at all energies, whereas (6) and (7) are derived using nonrelativistic kinematics.[4] Predictions (3) and (4) are compared with experimental data at 430 MeV[5] in Figs. 1 and 2. Concerning prediction (6), the experimental measurements are listed in Table I. Concerning (7′) experimentally[6]

TABLE I. Experimental results for the correlation coefficient C_{KP}.

Beam energy (MeV)	c.m. angle	C_{KP}
382[a]	90°	0.63±0.10
400[b,c]	90°	0.32±0.09
400[b]	60°	0.60±0.46
450[b]	90°	0.37±0.14
52[d]	90°	0.13±0.11

[a] A. Ashmore, A. Diddens, and G. Huxtable, Proc. Phys. Soc. (London) 73, 957 (1959).
[b] E. Engels, Jr., T. Bowen, J. W. Cronin, R. McIlwain, and L. Pondrom, Phys. Rev. 129, 1858 (1963).
[c] The 382-MeV $\theta=90°$ point of the Liverpool group (Ref. a) and the 400-MeV $\theta=90°$ point of the Princeton group (Ref. b) present a strong discrepancy. Our relation (5) obviously favors the result of the Princeton group.
[d] Keigo Nisimura, Junpei Sanda, Phillippe Catillon, Kiyoji Fukunaga, Takeo Hasegawa, Hiromi Hasai, Norio Ruy, D. C. Worth, and Hitoshi Imada, Progr. Theoret. Phys. (Kyoto) 30, 719 (1963).

at $E=310$ MeV, $C_{NN}(90°)=0.75\pm0.11$ and $2D(90°)-1=-0.15\pm0.25$, whereas at $E=650$ MeV, $C_{NN}(90)°=0.93\pm0.20$ and $2D(90°)-1=0.86\pm0.34$.

From $M(12)$ invariance for neutron-proton scattering, we also find $h(k,\theta)=0$, and consequently, relations

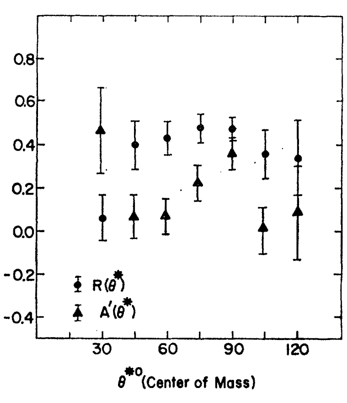

FIG. 1. R and A' plotted versus the c.m. scattering angle at 430 MeV (see Ref. 5).

[5] R. F. Roth, Princeton University thesis 1964 (unpublished).
[6] R. Wilson, *The Nucleon-Nucleon Interaction, Experimental and Phenomenological Aspects* (Interscience Publishers, Inc., New York, 1963), Tables 5-5 and A-4.

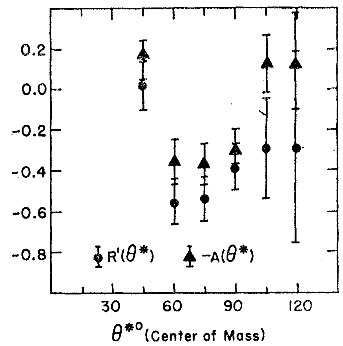

FIG. 2. R' and $-A$ plotted versus the c.m. scattering angle at 430 MeV. R' is not separately measured but calculated from R,A and A' (see Ref. 5).

(3)–(7′). Furthermore the four amplitudes for np scattering are fully determined in terms of the four amplitudes of pp scattering. Thus np scattering experiments can provide extra tests for $M(12)$ invariance in nucleon-nucleon scattering.[7]

Now we come to the question of why the agreement of $M(12)$ predictions for forward low-energy scattering are better than the predictions for arbitrary angles and energies. It has been emphasized that the main $M(12)$-symmetry–breaking agency is the kinetic energy of particles. At low energies[8] we expect these effects to be small. In forward scattering the four-momentum of the exchanged particles (mesons) is $=0$ and therefore the $k_\mu k_\nu$ type terms in their propagators [which break $M(12)$-symmetry] disappear. Away from the forward direction "kineton emission"[9] becomes important and has to be included.

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[7] It should be emphasized that the relations for pp and np scattering derived in this paper minimize $SU(3)$-breaking effects since no mass differences occur between the different connected amplitudes. They, therefore, can be regarded as "genuine $M(12)$" tests.

[8] At this point it may be worthwhile to emphasize that in evaluating kinematical functions dictated by $M(12)$ at thresholds great care is to be exercised. Thus, e.g., in meson-baryon scattering at the threshold $s=(m_M+m_B)^2$, a straightforward extrapolation of the kinematic functions defined in Ref. 2 would lead to the vanishing of the contributions from the 143′ and 5940 amplitudes and to the incorrect relation in scattering lengths $a_{\pi^+p}=a_{K^+p}$. Assuming, however, the $T_{143'}$ amplitude to have a kinematical pole that compensates the kinematical function's zero at $s=(m_M+M_B)^2$ avoids this prediction. If one furthermore assumes 143 exchange to dominate near threshold, one recovers the predictions expected from a vector-meson exchange model $a_{K^+p}=2a_{\pi^+p}=-2a_{\pi^-p}$ which are in good agreement with experiment.

[9] P. G. O. Freund, Phys. Rev. Letters 14, 803 (1965); R. Oehme, *ibid.* 14, 664 (1965).