Algebra of Currents and Getting $SU(6)$ Results from $SU(4)$*

C. RYAN

Department of Physics and Astronomy, University of Rochester, Rochester, New York

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Within the framework of the algebra-of-currents approach of B. W. Lee, it is shown that apart from a slight complication the $SU(6)$ results for the ratios, G_A/G_V and μ_P/μ_n are still obtained when the assumption of $SU(3)$ invariance is relaxed and only isospin $[SU(2)]$ symmetry is required.

IT has now become clear that fundamental difficulties of principle are encountered by any attempt to understand the higher symmetry groups recently introduced[1] as groups of invariance either of a Lagrangian or of an S matrix. We refer here to the fact that these groups are compatible neither with the principle of unitarity on the one hand nor with charge-conjugation invariance,[2] as usually defined, on the other (both difficulties being simple direct consequences of the noncovariance of the free-particle equations). The alternative point of view which interprets these symmetries merely as sets of prescriptions dictating the form of the various matrix elements in leading approximation is, of course, unassailable, but even it appears unsatisfactory in view of the poor agreement between many of the predictions and the experimental facts. There remains, however, the perplexing problem of explaining the undoubted successes of this whole development while at the same time avoiding the troubles which it has encountered.

An encouraging advance in this direction has recently been achieved by B. W. Lee.[3] This work suggests that many of the results of the nonrelativistic $SU(6)$ theory[4] can be explained in terms of the algebra of the currents,[5] coupled with the assumption that the form factors of the currents are highly convergent and that $SU(3)$ is a good invariance of the dynamics of strong interactions. In particular, $SU(6)$ invariance of the strong interactions or any part of them is not assumed.

One of the more interesting questions prompted by Lee's work is whether the assumption of $SU(3)$ invariance of the strong interactions is essential to obtain his results. $SU(3)$ symmetry is after all quite badly broken in certain respects; it seems, therefore, worthwhile to discover whether any or all of the $SU(6)$ results survive the breaking of the unitary symmetry.

Authors[6] who assume the existence (in the senses explained above) of a group of invariance which merges the spin and the internal symmetry have already studied this question within that framework and have shown that the central results of the $SU(6)$ and $\tilde{U}(12)$ symmetries survive when these symmetries are scaled down to $SU(4)$ and $\tilde{U}(8)$, respectively, by reducing the internal symmetry from $SU(3)$ to $SU(2)$ (isospin invariance). We wish to report in this note that substantially the same is true when we adopt Lee's approach although there is a slight complication here, not present in Lee's treatment.

Our starting point is the algebra generated by the quantities

$$A_i^{(\alpha)}=\int \mathcal{A}_i^{(\alpha)}(x,t)d^3x\,,\quad A_i^{(0)}=\int \mathcal{A}_i^{(0)}(x,t)d^3x\,,$$
$$V_0^{(\alpha)}=\int \mathcal{V}_0^{(\alpha)}(x,t)d^3x\,, \tag{1}$$

where $\mathcal{A}_i^{(\alpha)}(x,t)$, $\mathcal{A}_i^{(0)}(x,t)$, and $\mathcal{V}_i^{(\alpha)}(x,t)$ are, respectively, the space components of the axial-vector isovector current, the space components of the axial-vector isoscalar current, and the time component of the vector isovector current (the indices i and α running over 1, 2, and 3). In a model based upon a fundamental isotopic doublet field $\psi(x,t)$, these currents are given by

$$\mathcal{A}_i^{(\alpha)}(x,t)=i\bar{\psi}(x,t)\gamma_i\gamma_5(\tau_\alpha/2)\psi(x,t)\,,$$
$$\mathcal{A}_i^{(0)}(x,t)=i\bar{\psi}(x,t)\gamma_i\gamma_5\psi(x,t)\,,$$
$$\mathcal{V}_0^{(\alpha)}(x,t)=\bar{\psi}(x,t)\gamma_4(\tau_\alpha/2)\psi(x,t)\,; \tag{2}$$

the quantities $A_i^{(\alpha)}$, $A_i^{(0)}$, and $V_0^{(\alpha)}$ then obey the commutation relation

$$[A_i^{(\alpha)},A_j^{(\beta)}]=i\delta_{ij}\epsilon_{\alpha\beta\gamma}V_0^{(\gamma)}+\tfrac{1}{2}i\delta_{\alpha\beta}\epsilon_{ijk}A_k^{(0)}\,. \tag{3}$$

We now assume that this commutation relation is still satisfied by the physical-current integrals (1) even though the model from which it is derived may have no physical validity.

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[2] Cf., Riazuddin, L. K. Pandit, and S. Okubo (to be published).

[3] B. W. Lee, Phys. Rev. Letters 14, 676 (1965); 14, 850 (E) (1965).

[4] F. Gürsey and L. Radicati, Phys. Rev. Letters 13, 173 (1964); B. Sakita, Phys. Rev. 136, B1756 (1964); A. Pais, Phys. Rev. Letters 13, 175 (1964).

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The next step is to take the matrix element of this relation (3) between the zero-momentum states of various physical particles. The states which we consider are those of the nucleon N and the (3,3) resonance N^*, and we denote the former by $|\sigma,s\rangle$, where σ is the isospin and s is the spin index. In order to derive the consequences of taking these matrix elements we insert a complete set of states between the two operators in the commutator on the left-hand side, but we immediately make the assumption that this complete set may be well approximated by taking into account only the two lowest lying groups of states, namely those of the nucleon and N^* themselves. With this approximation and the definition

$$\langle\sigma',s'|A_i^{(\alpha)}|\sigma,s\rangle=Gu^\dagger(\sigma',s')\sigma_i(\tau_\alpha/2)u(\sigma,s)\,, \qquad (4)$$

where $u(\sigma,s)$ is a constant four-component spinor representing the nucleon, we deduce that

$$G=\pm 5/3\,, \qquad (5)$$

which apart from the uncertainty of sign is the $SU(6)$ result.[7]

Turning now to the question of magnetic moments, we define the isoscalar and isovector magnetic-moment operators, respectively, by the expressions

$$\mathfrak{M}_i^{(0)}=\tfrac{1}{2}\epsilon_{ijk}\int x_j\mathcal{V}_k^{(0)}(x,t)d^3x+\mu\int \mathcal{T}_i^{(0)}(x,t)d^3x\,, \qquad (6)$$

and

$$\mathfrak{M}_i^{(\alpha)}=\tfrac{1}{2}\epsilon_{ijk}\int x_j\mathcal{V}_k^{(\alpha)}(x,t)d^3x+\mu\int \mathcal{T}_i^{(\alpha)}(x,t)d^3x\,. \qquad (7)$$

Here $\mathcal{V}_k^{(0)}$ and $\mathcal{V}_k^{(\alpha)}$ are the spatial components of the isoscalar vector current and of the isovector vector current, respectively, while $\mathcal{T}_k^{(0)}$ and $\mathcal{T}_k^{(\alpha)}$ are likewise the spatial components of the isoscalar tensor covariant and of the isovector tensor covariant, respectively. The inclusion of the second terms in (6) and (7) is quite essential, for they introduce terms corresponding to[8] $L=0$ without which the magnetic moments would be exactly zero.[9] Such terms will be present if the quarks have intrinsic anomalous magnetic moments.

From the assumed commutation relations of the currents, we deduce that the following commutation relation is satisfied:

$$[A_i^{(\alpha)},\mathfrak{M}_j^{(\beta)}]=i\delta_{ij}\epsilon_{\alpha\beta\gamma}\mu\int S^{(\gamma)}(x,t)d^3x +\tfrac{1}{2}i\delta_{\alpha\beta}\epsilon_{ijk}\mathfrak{M}_k^{(0)}+\cdots\,, \qquad (8)$$

where $S^{(\alpha)}(x,t)$ is the isovector scalar density and the omitted terms play no role in what follows. Then, on defining

$$\langle\sigma',s'|\mathfrak{M}_i^{(0)}|\sigma,s\rangle=\mu_0u^\dagger(\sigma',s')\sigma_iu(\sigma,s)\,, \qquad (9)$$

and

$$\langle\sigma',s'|\mathfrak{M}_i^{(\alpha)}|\sigma,s\rangle=\mu_1u^\dagger(\sigma',s')\sigma_i(\tau_\alpha/2)u(\sigma,s)\,, \qquad (10)$$

and taking the matrix elements of Eq. (8) between the zero momentum states of the nucleon and N^* as before, we find that

$$\mu_1/\mu_0=G\,, \qquad (11)$$

if μ_0 is assumed not to vanish.

In order to deduce the ratio of the magnetic moments of the proton and neutron we require a definition of the charge operator Q. We adopt the definition

$$Q=\tfrac{1}{6}V_0^{(0)}+V_0^{(3)}\,. \qquad (12)$$

This choice is motivated by the fact that the nucleon and N^* states can be regarded as constituting the basis of a 20-dimensional representation of $U(4)$, and if they are so regarded the above will be the form of the charge operator. It follows then that the physical magnetic moment operator is

$$\mathfrak{M}_i=\tfrac{1}{6}\mathfrak{M}_i^{(0)}+\mathfrak{M}_i^{(3)}\,, \qquad (13)$$

and consequently we have

$$\frac{\mu_p}{\mu_n}=-\tfrac{3}{2}\quad\text{if}\quad G=5/3\,;$$

$$\frac{\mu_p}{\mu_n}=-\tfrac{2}{3}\quad\text{if}\quad G=-5/3\,. \qquad (14)$$

The first of these is the well-known $SU(6)$ result.[10] Thus, apart from this indeterminacy of sign, Lee's procedure leads to essentially the same results, when the requirement of $SU(3)$ symmetry of the strong interactions is relaxed and only isospin [$SU(2)$] invariance is assumed.

The origin of the indeterminacy of sign in the present case as against Lee's calculation is not difficult to isolate. The essential point is that the operators defined in Eq. (1) generate the Lie algebra of the group $SU(4)$ while the operators $\mathfrak{M}_i^{(0)}$ and $\mathfrak{M}_i^{(\alpha)}$ are tensor operators under this group. It follows from this that the procedure of taking matrix elements which we have employed will yield a consistent result, provided only that the states which we choose constitute the basis of a representation of $SU(4)$.[11] The states which we have chosen, namely, the nucleon and the N^*, constitute a basis of a 20 representation of $SU(4)$ but, because at the $SU(2)$ (isospin) level there is no distinction between a representation and its conjugate, they can also be placed in a 20* representation. There ought then to be two solutions to our problem and this is indeed what we have found. At the $SU(6)$ level, there is no corresponding

[7] F. Gürsey, A. Pais, and L. Radicati, Phys. Rev. Letters **13**, 299 (1964).

[8] See, for example, B. W. Lee, Phys. Rev. Letters **14**, 850 (1965) and M. Gell-Mann, *ibid.* **14**, 77 (1965).

[9] A complete discussion of this point will be presented in a forthcoming paper.

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[11] For a proof of this statement see S. Okubo, Phys. Letters **17**, 172 (1965).

indeterminancy, for the states of the octet of baryons and the decuplet of baryon resonances belong uniquely in a 56 representation of $SU(6)$, and cannot be put in a 56* representation. It is rather remarkable, though, that all $SU(3)$ symmetry adds to $SU(2)$ is the fixing of the signs of G_A/G_V and μ_1/μ_0 while leaving their absolute values unaltered.

In a subsequent paper we shall discuss in detail the implications which are involved in the present approach. In particular the approximation scheme employed will be examined and additional results will be presented.

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Unstable-Particle Scattering and the Strip Approximation

IAN T. DRUMMOND
Department of Physics, University of California, Berkeley, California
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We examine, in this paper, the problem of formulating a bootstrap calculation when one of the scattering particles is unstable. Having defined the unstable-particle scattering amplitude as an S-matrix pole residue, we go on to discuss its analytic structure and point out that it may be determined from the usual Landau rules. We conclude that although the instability of the external particle complicates the structure it does not do so too severely. Therefore, we are free to postulate that, in analogy with the stable case, the unstable-particle amplitude exhibits Regge asymptotic behavior. This assumption leads us to construct a strip approximation to the amplitude which is a crossing-symmetric superposition of Regge pole terms. We point out that this approximation exhibits, in some respects, satisfactory analytic structure. In particular it takes quite well into account certain anomalous threshold effects. It satisfies a quasi-Mandelstam representation which we use to explore the analytic structure of the corresponding partial-wave amplitudes and their continuation to arbitrary angular momentum. We use certain simple discontinuity formulas to obtain dynamical equations for the partial-wave amplitudes and are consequently able to construct, formally, a complete bootstrap scheme. Finally, we mention some difficulties and unsolved problems.

I. INTRODUCTION

At the present time qualitative success has been achieved in some simple calculations involving strongly interacting particles.[1–7] More elaborate calculations have been proposed and attempted; for example, the various forms of the strip approximation to the π-π scattering amplitude.[8–11] Most of these calculations make use of elastic unitarity. It has always been intended, however, to improve on this situation by introducing some inelastic effects explicitly. In some calculations this has already been done.[12–14]

Inelastic effects due to the presence of two-body channels can be discussed by means of a finite-matrix formalism which is a simple generalization of that used for elastic calculations.[15] Most inelastic effects, however, are associated with the presence of many-particle channels. The formalism necessary for discussing these channels exactly must involve infinite matrices of a complicated kind.[16–21] It would be convenient, therefore, to have an approximate method for dealing with many-particle systems which is as analogous as possible to that for two-particle systems. The purpose of this paper is to outline such a method.

The idea, which is not new, on which the method is based, is that the dynamics of many-body systems is dominated by resonance-resonance or particle-resonance configurations. For example, the four-pion system is, for suitable ranges of the center of mass energy, dominated by the π-ω and ρ-ρ configurations of the pions. Similarly the $\pi\pi N$ system is dominated by the π-N^* and ρ-N configurations of the particles. The experimental support for this idea may be summed up by pointing to the impressive qualitative success of even very simple isobar models.[22]

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