$\pi\rho$ and $K\bar{K}$ Contribution to the Isoscalar Form Factor of the Nucleon*

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The isoscalar form factors of the nucleon are calculated assuming that the low-energy part of the spectral functions is determined by the three-pion ($\pi\rho$ in our approximation) and $K\bar{K}$ intermediate states. The effect of the ω and φ mesons is evaluated using the relativistic effective-range approximation. The $\langle\pi\rho|N\bar{N}\rangle$ and $\langle K\bar{K}|N\bar{N}\rangle$ amplitudes are considered in the approximation of a one-baryon exchange interaction. The results are compared with experiments and a qualitative discussion of the relative weight of the ω and φ contributions is given.

I. INTRODUCTION

THE computation of the isoscalar form factors of the nucleon is a long-standing problem of strong-interaction physics. The reason for this has been mainly the intrinsic difficulties associated with the three-pion state, which happens to be the lowest mass intermediate state and therefore the one supposed to dominate the form factor.[1] The extensive work on nucleon-pion scattering including unstable-particle production amplitudes by Ball *et al.*[2] enables one to calculate the 3π-meson contribution to the form factors in the approximation in which two of the 3π mesons are in a resonant state ρ. We consider this $\pi\rho$ state for 3π, which corresponds to neglecting F ($l=3$) and higher angular-momenta waves in the pion-pion system.

The existence of two vector mesons with the quantum numbers of the isoscalar electromagnetic current made possible a phenomenological analysis of the experimental data in terms of a Clementel-Villi–type formula with two poles corresponding to the mass of the ω and φ mesons.[3] The large experimental uncertainties, due mainly to the difficulty of measuring the neutron form factors, did not allow setting limits on the relative weight of the contribution of each of these vector mesons. This question becomes important now because it might lead to the justification of some assumptions related to the symmetry scheme based[4] on SU_3 or to the newly proposed selection rule for bosons (A symmetry[5]).

In Sec. II, we express the form factors in terms of N and D functions of the $K\bar{K}$ and $\pi\rho$ elastic-scattering amplitudes and the unphysical cuts in $\langle N\bar{N}|\pi\rho\rangle$ and $\langle N\bar{N}|K\bar{K}\rangle$. N, D and unphysical cuts are calculated in Sec. III, and a simplification of the form factors into the two-pole formulas is done. The poorly known coupling constants appearing in this calculation are discussed. Finally, in Sec. IV, the results are compared with experiments and a few conclusions are drawn.

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[1] G. F. Chew, R. Karplus, S. Gasiorowicz, and F. Zachariasen, Phys. Rev. **110**, 265 (1958); P. Federbush, M. L. Goldberger, and S. B. Treiman, Phys. Rev. **112**, 642 (1958); K. Kawarabayashi and A. Sato, Progr. Theoret. Phys. (Kyoto) **28**, 173 (1962).

[2] J. S. Ball, W. R. Frazer, and M. Nauenberg, Phys. Rev. **128**, 478 (1962); L. F. Cook, Jr., and B. W. Lee, Phys. Rev. **127**, 283 (1962).

[3] N. Gelfand, D. Miller, M. Nussbaum, J. Ratau, J. Schultz, J. Steinberger, and T. H. Tan, Phys. Rev. Letters **11**, 436 (1963); **11**, 438 (1963).

[4] Y. Ne'eman, Nucl. Phys. **26**, 222 (1961); M. Gell-Mann, Phys. Rev. **125**, 1067 (1962).

[5] J. B. Bronzan and F. E. Low, Phys. Rev. Letters **12**, 522 (1964).

II. APPROXIMATION AND FORMULATION OF THE φ AND ω CONTRIBUTION TO THE FORM FACTORS

In the usual manner we write the matrix element of the nucleon isoscalar electromagnetic vertex as

$$\langle\bar{p}p'|j_\mu(0)|0\rangle=(4p_0'\bar{p}_0)^{-1/2}\bar{u}(p')\times[\gamma_\mu F_1{}^S(q^2)+iF_2{}^S(q^2)\sigma_{\mu\nu}q^\nu]v(\bar{p}), \quad \text{(II.1)}$$

where p' and $\bar{p}$ are the four-momenta of the nucleon and antinucleon,

$$t=(\bar{p}+p')^2, \quad q=(\bar{p}+p'),$$

and

$$\bar{p}^2=p'^2=m^2.$$

The normalization of the form factors is then (we suppress the superscript "S" henceforth unless it becomes necessary to specify it)

$$F_1(0)=\tfrac{1}{2}, \quad F_2(0)=-0.06,$$

in the units of e and $e/2m$, respectively.

It is customary nowadays to analyze the experimental data in terms of electric and magnetic form factors[6] G_E and G_M which are related to F_1 and F_2 by

$$G_E=F_1+(t/4m^2)F_2, \quad G_M=F_1+F_2. \quad \text{(II.2)}$$

Latest measurements of the electron-proton scattering[7] have given strong support to the hypothesis that both electric and magnetic form factors tend to zero as the momentum transfer tends to infinity:

$$G_E{}^p \underset{t\to\infty}{=} O(t^{-1}), \quad G_M{}^p \underset{t\to\infty}{=} O(t^{-1}). \quad \text{(II.3)}$$

The result gives rise to the possiblity of writing un-

[6] E. J. Ernst, R. G. Sachs, and K. C. Wali, Phys. Rev. **119**, 1105 (1960).

[7] K. W. Chen, A. A. Cone, J. R. Dunning, Jr., S. G. Frank, N. F. Ramsey, J. K. Walker, and Richard Wilson, Phys. Rev. Letters **11**, 561 (1963).

subtracted dispersion relations for both isoscalar form factors in principle. However, it is hardly expected that the low-energy part of the spectral function exhausts the description of the charge and magnetic-moment form factors. We use subtracted dispersion relations. Using the units $m_\pi=1$, we have

$$G_E(t)=G_E(0)-\frac{t}{\pi}\int_9^\infty \frac{g_E(t')}{t'(t'-t)}dt',$$
$$G_M(t)=G_M(0)-\frac{t}{\pi}\int_9^\infty \frac{g_M(t')}{t'(t'-t)}dt', \tag{II.4}$$

where $g_E(t)=\mathrm{Im}G_E(t)$ and $g_M(t)=\mathrm{Im}G_M(t)$.

Let us define $g_i^1(t)$ and $g_i^2(t)$ $(i=E, M)$ to be the contributions to the imaginary part of the isoscalar form factors from $\pi\rho$ and $K\bar{K}$, respectively:

$$g_i=g_i^1+g_i^2.$$

One may expect[8] that $g_i^1(t)$ and $g_i^2(t)$ have the forms

$$g_i^1(t)=g_{\pi\rho\gamma}\rho_1(t)V_1^*(t)\Gamma_i^1(t), \quad t>9;$$
$$g_i^2(t)=\tfrac{1}{2}e\rho_2(t)V_2^*(t)\Gamma_i^2(t), \quad t>4m_K^2, \tag{II.5}$$

where $\Gamma_E^1(t)$ $[\Gamma_M^1(t)]$ and $\Gamma_E^2(t)$ $[\Gamma_M^2(t)]$ are proportional to the $N\bar{N}$-$\pi\rho$ and $N\bar{N}$-$K\bar{K}$, $J=1$, $I=0$ helicity amplitudes. Specifically, for the $N\bar{N}$-$K\bar{K}$ amplitude we have

$$\Gamma_E^2=-(\tfrac{1}{2}t^{1/2}/m)(T_+/q_K^2),$$
$$\Gamma_M^2=-(2\sqrt{2}m)^{-1}(T_-/q_K^2), \tag{II.6}$$
$$q_K^2=\tfrac{1}{4}t-m_K^2,$$

where T_+ and T_- are defined by formulas similar to Eqs. (3.11) and (3.12) of Ref. 8. $V_1(t)$ and $V_2(t)$ are the vertex functions which are defined by

$$\langle\pi\rho|j_\mu(0)|0\rangle=\frac{g_{\pi\rho\gamma}}{(4\omega_\pi\omega_\rho)^{1/2}}\epsilon_{\mu\nu\alpha\beta}q_1^\nu q_2^\alpha e^\beta(S)V_1(t),$$
$$\langle K\bar{K}|j_\mu(0)|0\rangle=\frac{e/2}{(4\omega_K\omega_{\bar{K}})^{1/2}}(q_1-q_2)_\mu V_2(t), \tag{II.7}$$

where $e^\beta(S)$ is the ρ-meson polarization vector and ω's are energies. Here, $\rho_1(t)$ and $\rho_2(t)$ are the phase-space factors for $\pi\rho$ and $K\bar{K}$; q_1 and q_2 are the momenta of ρ and π (K and $\bar{K}$ in the case of the $K\bar{K}$ channel). The explicit forms of ρ_1 and ρ_2 are

$$\rho_2(t)=2q_K^3/\sqrt{t}, \tag{II.8}$$

$$\rho_1(t)=\frac{\sqrt{t}}{\pi}\int_4^{(\sqrt{t}-1)^2} 2q_\pi^3(t,\sigma)\left(\frac{(\sigma-4)^3}{\sigma}\right)^{1/2} \times\frac{\gamma}{(m_\rho^2-\sigma)^2+\gamma^2(\sigma-4)^3/\sigma}$$
$$\equiv(2\sqrt{t})q_\rho^3, \tag{II.9}$$

where

$$q_\pi^2(t,\sigma)=[t-(\sqrt{\sigma}-1)^2][t-(\sqrt{\sigma}+1)^2]/4t,$$

which is the square of the spatial part of the momentum in the c.m. system of a pion and a compound particle which has total energy $\sqrt{\sigma}$ in its c.m. system.

The unitarity condition on the $N\bar{N}$-$K\bar{K}$ ($N\bar{N}$-$\pi\rho$) amplitude requires Γ_i^2 (Γ_i^1) to have the phase of $K\bar{K}$ ($\pi\rho$) in the region $4m_k^2<t<4m^2$ $(9<t<25)$, and we will assume that the effects of inelastic scattering are small so that this phase condition will continue to be approximately valid at higher energies. In addition to the right-hand singularities given by the unitary condition, the function $\Gamma_i^{1,2}$ will have left-hand cuts which are related to the physical singularities in the crossed channel.

Defining $B_i^j(t)$ $(j=1, 2, i=E, M)$ to be the result obtained by carrying out the integration over left cuts of $\Gamma_i^j(t)$,

$$B_i^j(t)=\frac{1}{\pi}\int_{L_j}^{-\infty}\frac{\mathrm{Im}\Gamma_i^j(t')}{t'-t}dt', \tag{II.10}$$

we can write the following solutions for the Γ's which will satisfy the phase condition

$$\Gamma_i^j(t)=\frac{1}{D_j(t)}\left[B_i^j(t)+\frac{1}{\pi}\int_{R_j}^{\Lambda_j}\frac{B_i^j(t')-(t/t')B_i^j(t)}{t'-t}\rho_j(t')N_j(t')dt'\right], \tag{II.11}$$

where $R_1=9$, $R_2=4m_K^2$ and N_j/D_j is defined by

$$N_1/D_1=\rho_1^{-1}(t)e^{\delta_{\pi\rho}}\sin\delta_{\pi\rho},$$
$$N_2/D_2=\rho_2^{-1}(t)e^{i\delta_K}\sin\delta_K. \tag{II.12}$$

The Λ_j's are cutoff parameters in each channel. These cutoffs are introduced because the contributions of various inelastic scatterings at higher energy are unknown. The choice of the Λ_j's are discussed in Sec. IV.

The function $1/D_1$ $(1/D_2)$ has the phase of $\pi\rho$ $(K\bar{K})$ scattering and is regular for $t<9$ $(t<4m_K^2)$. N_1 (N_2) is regular for $t>9$ $(t>4m_K^2)$.

A similar argument holds for vertex functions, and we have

$$V_1(t)=D_1(0)/D_1(t), \quad V_2(t)=D_2(0)/D_2(t). \tag{II.13}$$

Substituting (II.11) and (II.13) into (II.5) we get $g_i^j(t)$ in terms of N, D, and B as follows:

$$g_i^j(t)=C_j\rho_j(t)\frac{D_j(0)}{|D_j(t)|^2}\left[B_i^j(t)+\frac{1}{\pi}\int_{R_j}^{\Lambda_j}\frac{B_i^j(t')-(t/t')B_i^j(t)}{t'-t}\rho_j(t')N_j(t')dt'\right], \tag{II.14}$$

[8] W. R. Frazer and J. R. Fulco, Phys. Rev. **117**, 1603 (1960).

where

$$C_1 = g_{\pi\rho\gamma} \quad \text{and} \quad C_2 = \tfrac{1}{2}e.$$

A more careful derivation of Eq. (II.14) is given in the Appendix using the coupled N/D method.

III. *N*, *D*, AND *B* FUNCTIONS AND REPRODUCTION OF TWO-POLE FORMULAS

(A) *N*, *D*, and *B*

Denoting the $\pi\rho$ and $K\bar{K}$ scattering amplitudes by $T_1(t)$ and $T_2(t)$, we have

$$T_j(t) = N_j(t)/D_j(t). \qquad \text{(III.1)}$$

In the physical region, the effect of the left-hand singularities can be estimated by replacing the branch cut by a pole having appropriate position and residue. This assumption seems reasonable because for N-N scattering it leads to the well-known effective-range formulas. Making this approximation, we have

$$N_j = \lambda_j/(t+t_j), \qquad \text{(III.2)}$$

where $t_1 > -9$, $t_2 > -4m_K^2$. Then we get from (II.4) for

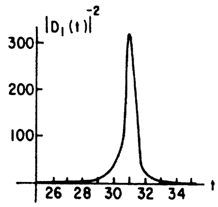

FIG. 1. $|D_1(t)|^{-2}$ around the resonance $t_1 = -5.0$, $\lambda_1^{-1} = 1.45$, cutoff $\Lambda_1 = 168(m_\pi^2)$.

a single channel,

$$D_j(t) = 1 - \frac{t}{\pi} P \int_{R_j}^{\infty} \frac{\rho_j(t')N_j(t')}{t'(t'-t)} dt' + iN_j(t)\rho_j(t)\theta(t-R_j)$$
$$\equiv \mathrm{Re}D_j(t) + i\,\mathrm{Im}D_j(t). \qquad \text{(III.3)}$$

We adjust the four parameters λ_1, λ_2, t_1, and t_2 such that each channel produces the ω and φ at the right positions $t_\omega = m_1^2 = 31.2$ and $t_\varphi = m_2^2 = 53.2$ with the right widths $\Gamma_\omega = 9$ MeV, and $\Gamma_\varphi = 4$ MeV. In other words,

$$\mathrm{Re}D_j(m_j^2) = 0,$$
$$\Gamma_\omega = \frac{N_1(t\omega)\rho_1(t\omega)}{m_1(\partial/\partial t)[\mathrm{Re}D_1(t\omega)]},$$
$$\Gamma_\varphi = \frac{N_2(t\varphi)\rho_2(t\varphi)}{m_2(\partial/\partial t)[\mathrm{Re}D_2(t\varphi)]}. \qquad \text{(III.4)}$$

The $|D_1(t)|^{-2}$ and $|D_2(t)|^{-2}$ which are obtained with a set of parameters $t_1 = -5.0$, $\lambda_1^{-1} = 1.45$, and $t_2 = 10^8$, $\lambda_2^{-1} = 0.67\times 10^{-6}$, are shown in Fig. 1 and Fig. 2.

As for the B functions, we take into account the contribution from the Feynman diagram shown in Fig. 3.

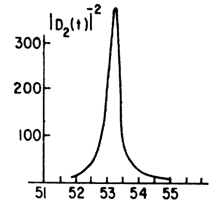

FIG. 2. $|D_2(t)|^2$ around the resonance $t_2 = 10^8$, $\lambda_2^{-1} = 0.67\times 10^{-6}$.

The various coupling constants are defined by the following interaction Lagrangian density:

$$\mathcal{L}_{\rm int} = g\bar{N}\boldsymbol{\tau}\gamma_5 N\boldsymbol{\pi} + g_{N\Lambda K}\bar{N}\Lambda K$$
$$+ g_{N\bar{N}\rho}\bar{N}\boldsymbol{\tau}\gamma_\mu N\boldsymbol{\rho}^\mu + g_{N\Sigma K}\bar{N}\gamma_5\boldsymbol{\tau}\boldsymbol{\Sigma}K + \text{H.c.}$$

(Here H.c. means Hermitian conjugate.) We will neglect the magnetic-type $(\sigma_{\mu\nu}q^\nu)$ interaction between nucleon and ρ meson in our calculation. This approximation will be justified by the results in Sec. IV.

It might be worthwhile to mention how one can avoid the complexity caused by projecting the partial wave for ρN system. This could be done as follows: Using the fact that the absorptive part of the isoscalar nucleon-photon vertex, denoted by $J_\mu{}^A$, is expressed by Eq. (III.5), we obtain two independent linear equations for g_E and g_M by taking traces[9] and separating g_E and g_M. Then equating them to the right-hand side of Eq. (II.5) in which $\Gamma_i{}^j$ corresponds to $B_i{}^j$, we get

$$J_\mu{}^A = -(\tfrac{1}{2}P_0')^{1/2}(2\pi)^{-3}\int\int d^3q_1\, d^3q_2\, \delta^4(q_1+q_2-q)$$
$$\times\langle p'|j_\mu(0)|q_1q_2\rangle^{I=0}\langle q_1q_2|j_\mu(0)|0\rangle^{I=0}v(\bar{p}), \qquad \text{(III.5)}$$

where the superscript "$I=0$" means the isotropic spin-zero component of the matrix element. We have abbreviated the sum over the ρ-meson polarization vector in the expression.

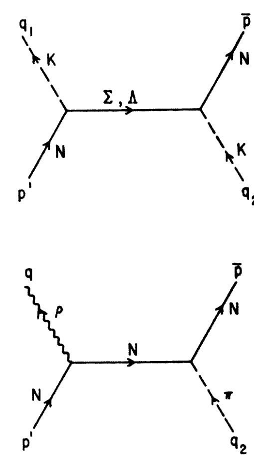

FIG. 3. Diagrams which contribute to $B_i{}^j(t)$. p', q_1, and q_2 are the 4-momenta of the indicated particles in the diagrams.

[9] S. D. Drell and F. Zachariasen, *Electromagnetic Structure of Nucleons* (Oxford University Press, London, 1961), Chap. III.

The explicit forms of the nucleon currents are

$$\langle p'|j_N(0)|\pi\rho\rangle^{I=0}=\sqrt{6}\frac{gg_{NN\rho}}{(8E_{p'}\omega_\pi\omega_\rho)^{1/2}}\bar{u}(p')\gamma_5\frac{1}{\mathbf{p}'-\mathbf{q}_1-m}\gamma_\mu e^\mu(s)\,,$$

$$\langle p'|j_N(0)|K\bar{K}\rangle^{I=0}=(g_{N\Lambda K}{}^2,3g_{N\Sigma K}{}^2)(4\omega_K\omega_{\bar{K}})^{-1/2}\bar{u}(p')\gamma_5(\mathbf{p}^{i'}-\mathbf{q}_1-m)^{-1}\gamma_\mu\,,$$

where ω_π, ω_ρ, ω_K, and $\omega_{\bar{K}}$ are the energies of π, ρ, K, and $\bar{K}$. The results obtained are

$$B_1{}^1(t)=(G/12mpq_\rho)[Q_0(z_\rho)-Q_2(z_\rho)]\,,$$

$$B_1{}^2(t)=(G/6pq_\rho t)[Q_2(z_\rho)+2Q_0(z_\rho)]\,,$$

$$B_2{}^1(t)=\frac{g_{NK\Lambda}{}^2}{4\pi}\frac{1}{8pq_K}\left[\frac{Q_0(z_\Lambda)}{3}+\tfrac{2}{3}Q_2(z_\Lambda)-\frac{m-m_\Lambda}{q_K}\frac{p}{m}Q_1(z_\Lambda)\right]+3\frac{g_{N\Sigma K}{}^2}{4\pi}\frac{1}{8pq_K}\left[\frac{Q_0(z_\Sigma)}{3}+\tfrac{2}{3}Q_2(z_\Sigma)-\frac{m-m_\Sigma}{q_K}\frac{p}{m}Q_1(z_{\Lambda\Sigma})\right],$$

$$B_2{}^2(t)=(g_{NK\Lambda}{}^2/4\pi)(24pq_K)^{-1}[Q_0(z_\Lambda)-Q_2(z_\Lambda)]+3(g_{N\Sigma K}{}^2/4\pi)(24pq_K)^{-1}[Q_0(z_\Sigma)-Q_2(z_\Sigma)]\,,$$

where

$$p=(\tfrac{1}{4}t-m^2)^{1/2}\,,\quad q_K=\tfrac{1}{4}(t-m_K{}^2)^{1/2}\,,$$

$m=$nucleon mass, $G=gg_{N\bar{N}\rho}/4\pi$, $m_\Lambda=\Lambda$ mass, and $m_\Sigma=\Sigma$ mass. The q_ρ are defined in (II.9) and the $Q_i(z)$ are the Legendre functions of the second kind.

$$z_\rho=\frac{2[(q_\rho{}^2+1)t/4]^{1/2}+1}{2pq_\rho}\,,$$

$$z_\Lambda=\frac{\frac{1}{2}t-m_K{}^2+(m_\Lambda{}^2-m^2)}{2pq_K}\,,$$

$$z_\Sigma=\frac{\frac{1}{2}t-m_K{}^2+(m_\Sigma{}^2-m^2)}{2pq_K}\,.$$

(B) Two-Pole Formulas and Coupling Constants

It is necessary to let the widths of Γ_ω and Γ_φ go to zero in order to reduce $G_i(t)$ to two-pole formulas. Since $\mathrm{Re}D(t)$ becomes zero at the resonance $t=t_r$, $\mathrm{Re}D(t)\simeq(t-t_r)(d/dt)[\mathrm{Re}D(t_r)]$ is good approximation near the resonance. Then from Eq. (III.3)

$$\frac{1}{|D(t)|^2}\simeq\frac{1/[\mathrm{Re}D'(t_r)]^2}{(t-t_r)^2+N^2\rho^2/[\mathrm{Re}D'(t_r)]^2}\,.\tag{III.6}$$

Therefore, in the limit $N(t_r)\rho(t_r)/\mathrm{Re}D'(t_r)\rightarrow0$, we have

$$\frac{1}{|D(t)|^2}\rightarrow\frac{\pi\delta(t-t_r)}{N(t_r)\rho(t_r)\,\mathrm{Re}D(t_r)}\,.\tag{III.7}$$

Substituting Eq. (III.7) into Eq. (II.14) for $t_r=t_\omega$ and $t_r=t_\varphi$ and performing the integration in Eq. (II.4), we get

$$G_E(t)=0.5\left[1-d_E{}^1-d_E{}^2+\frac{d_E{}^1}{1-t/t_\omega}+\frac{d_E{}^2}{1-t/t_\varphi}\right],$$

$$G_M(t)=0.44\left[1-d_M{}^1-d_M{}^2+\frac{d_M{}^1}{1-t/t_\omega}+\frac{d_M{}^2}{1-t/t_\varphi}\right].\tag{III.8}$$

Redefining

$$\bar{\Gamma}_i{}^j(t)=D_j(t)\Gamma_i{}^j(t)\,,$$

we can express the $d_i{}^j$'s as follows:

$$d_E{}^1=\frac{2\bar{\Gamma}_E{}^1(t_\omega)}{t_\omega N_1(t_\omega)\,\mathrm{Re}D'_1(t_\omega)}\frac{g_{\pi\rho\gamma}}{e}\,,$$

$$d_E{}^2=\frac{2\bar{\Gamma}_E{}^2(t_\varphi)}{t_\varphi N_1(t_\varphi)\,\mathrm{Re}D'_2(t_\varphi)}\,,$$

$$d_M{}^1=\frac{\bar{\Gamma}_M{}^1(t_\omega)}{0.44t_\omega N_1(t_\omega)\,\mathrm{Re}D'_1(t_\omega)}\frac{g_{\pi\rho\gamma}}{e}\,,$$

$$d_M{}^2=\frac{\bar{\Gamma}_M{}^2(t_\varphi)}{0.44t_\varphi N_1(t_\varphi)\,\mathrm{Re}D'_2(t_\varphi)}\,.$$

How much information we have for coupling constants is the next problem to be discussed. There are four coupling constants whose values are not well established, viz., $g_{\pi\rho\gamma}$, $g_{N\bar{N}\rho}$, $g_{N\Lambda K}$, and $g_{N\Sigma K}$.

As for $g_{N\bar{N}\rho}$, we use the value obtained from the ρ-meson contribution to isovector form factors[10]:

$$g_{N\bar{N}\rho}{}^2/4\pi=4.4\,.$$

The same constant determined from the existence of spin-orbit forces is between 4 and 7.[11]

The octet model[4] provides the fermion and boson coupling in terms of a constant d and g which is pion-nucleon coupling constant, $g^2/4\pi=14$:

$$g_{N\Lambda K}/(4\pi)^{1/2}=-(1/\sqrt{3})(3-2d)g\,,$$

$$g_{N\Sigma K}/(4\pi)^{1/2}=(2d-1)g\,.$$

According to Ref. 12 the lower and upper limits for $g_{\pi\rho\gamma}$ are as follows:

$$0.2<|g_{\pi\rho\gamma}/e|<2.1\,.$$

Since the lower limit is obtained under the assumption that ω dominates the nucleon form factors, we take

$$|g_{\pi\rho\gamma}/e|<2.1\,.\tag{III.9}$$

[10] J. S. Ball and D. Y. Wong, Phys. Rev. **133**, B179 (1964).
[11] J. J. Sakurai, Phys. Rev. **119**, 1784 (1960).
[12] S. M. Berman and S. D. Drell, Phys. Rev. **133**, B791 (1964).

IV. RESULTS AND CONCLUSIONS

Let us first look at the experiments. Four groups, Hofstadter *et al.*,[13] Kirson,[14] Balachandran *et al.*,[15] and Dunning *et al.*,[16] have tried to fit experimental data to two-pole formulas with (III.8) with and without subtraction, assuming the contributions from φ and ω to be dominant.

The common feature of these four papers is that the signs of d_E^1 and d_M^1 are positive and those of d_E^2 and d_M^2 are negative except in the case of Ref. 15 where the quoted error is compatible with opposite signs for d_E^2 and d_M^2.

Recalling the fact that we have two cutoffs Λ_1 and Λ_2 and flexible coupling constants d and $g_{\pi\rho\gamma}$, three questions may be raised: First, are there any results from this calculation which are independent of this arbitrariness? Second, is it possible to reproduce any of these d_i^j ($i=E, M$, $j=1, 2$) obtained by these four groups? Third, are there any other possible sets of d_i^j which fit to experiment in the low-momentum-transfer region?

Let us start with the first. There is a result independent of the ambiguities; namely, d_E^2 and d_M^2 have lower limits:

$$d_E^2>0.87\,,\quad d_M^2>1.5\,. \tag{IV.1}$$

The answer to the second question is clear now. We have only positive values for d_E^2 and d_M^2. Therefore, it is impossible to reproduce any of the d_i^2 obtained by the four groups.

So the third question becomes important. There are three independent groups to be considered:

(1) ω dominant ($|d_i^1|\gg d_i^2$, $i=E, M$),
(2) φ dominant ($|d_i^1|\ll d_i^2$),
(3) φ and ω having equal contributions ($|d_i^1|\sim d_i^2$).

It is easily seen that it is impossible to get a good ω dominant fit because of the condition (IV.1).

We make the following observations: (1) The ratios $R_1=d_E^1/d_M^1$ and $R_2=d_E^2/d_M^2$ are independent of the coupling constants and depend only on the cutoffs Λ_1 and Λ_2, respectively. (Strictly speaking, this is true for R_2 in the approximation $M_\Sigma=M_\Lambda$. However, R_2 is insensitive to the mass difference.) (2) The R's are monotonically increasing functions of the Λ's and have the ranges $0.38<R_1<0.6$ for $100<\Lambda_1<220$ and $0.57<R_2<1.0$ for $100<\Lambda_2<10^6$. Λ_1 is required to be smaller than 220 in order to satisfy both $t_1<-9$ and $\Gamma_\omega=9.0$ at $t_\omega=31.2$. The above observations together with restrictions (IV.9) and (IV.1) lead us to conclude that only the φ-dominant solution can account for the proton data in the region $t\leq 60$.

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TABLE I. Two sets of parameters (d, $g_{\pi\rho\gamma}/e$, Λ_1, and Λ_2) which give good fits to the data are listed. The residues of the poles obtained from the sets are also listed.

Set \ Parameters	d_E^1	d_E^2	d_M^1	d_M^2	d	$g_{\pi\rho\gamma}/e$	Λ_1	Λ_2
A	0.182	1.0	0.333	1.69	0.7	0.077	216	377
B	−0.660	2.33	−1.20	3.49	0.97	−0.28	216	722

Using the isovector form factor obtained by Hofstadter,

$$G_E{}^V(t)=0.5\left[1-1.26+1.26\frac{1}{1-t/18}\right],$$
$$G_M{}^V(t)=2.353\left[1-1.09+1.09\frac{1}{1-t/18}\right], \tag{IV.2}$$

the proton form factors, $G^P=G^S+G^V$, are shown in Fig. 4 for sets A and B whose parameters are given in Table I. The experimental data in Fig. 4 are from Chen *et al.*[7] Both $G_E{}^P$ and $G_M{}^P$ become too small in the high-momentum-transfer region.

Though sets A and B together with Eq. (IV.2) give very good agreement with experiments for $G_E{}^{P\prime}(0)$, $G_M{}^{P\prime}(0)$ and $G_M{}^{n\prime}(0)$, they give a higher value for $G_E{}^{n\prime}(0)$ [$G_E{}^{n\prime}(0)=0.045$ F^2 for set A which should be compared with the experimental value 0.021 ± 0.001 F^2]. Considering the fact that this calculation has employed the simplest approximation (neglecting rescattering terms, etc.), this qualitative agreement with experiment can be appreciated.

The calculation with finite width for φ and ω showed that a Clemental-Villi type formula is very good approximation in this momentum transfer region. For both sets A and B, the threshold condition $G_M(4m^2)=G_E(4m^2)$ is not satisfied; however, this is not surprising because both the low and high ($t>4m^2$) energy parts of the spectral function are equally important at $t=4m^2$. Nevertheless, it is noticed that the imaginary part of the form factors satisfies the threshold condition in our formulation automatically.

In conclusion, we emphasize that our calculation indicates the φ contribution to the isoscalar form factors to be larger than the ω contribution. For instance, in the case of the parameter set A, the percentage contributions of φ, ω, and hard core (higher mass contribution) to charge and total magnetic moment are as follows:

Set A:

State	Charge	Magnetic moment
φ	73.3%	55.5%
ω	13.35	11.1
Higher mass	13.35	33.4

This gives support to the empirical selection rule which has been proposed by Bronzan and Low.[5] The

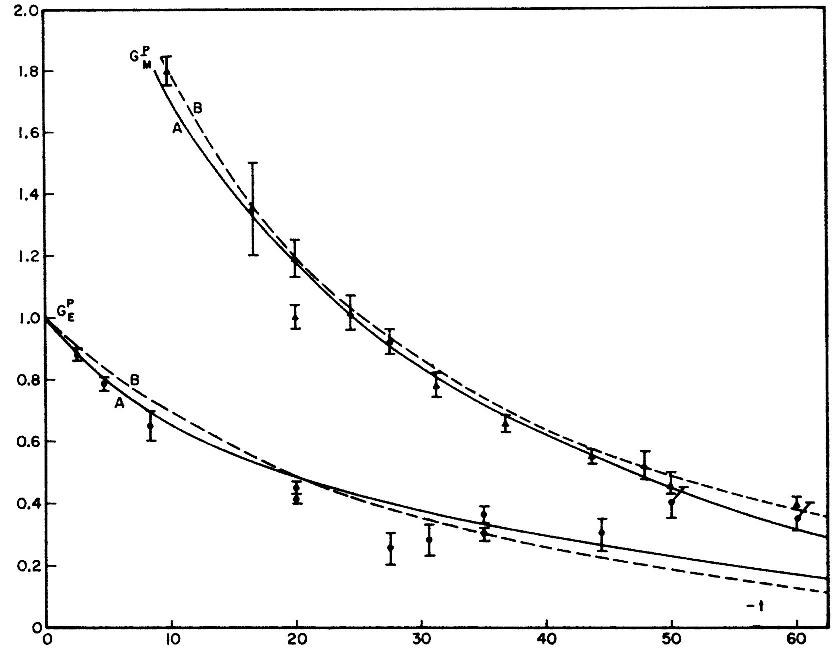

FIG. 4. The experimental magnetic-moment and charge form factors for the proton, $G_M{}^P$ and $G_E{}^P$, are shown by ▲ and ●, respectively, with error bars representing the statistical error. The $G_M{}^P$ and $G_E{}^P$ obtained from the sets A and B are shown. The units are $m_\pi{}^2=1$.

rough estimates of the ratio

$$R=\Gamma(\omega\rightarrow e^{+}+e^{-})/\Gamma(\varphi\rightarrow e^{+}+e^{-})\simeq(m_\omega/m_\varphi)(g_{\pi\rho\gamma}/e)^2$$

using sets A and B are $\sim 1/300$ and $\sim 1/25$, respectively. This can be directly checked from the decays involved or eventually with colliding beams of electrons and positrons.

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APPENDIX

In this appendix, we shall go through the coupled-channel N/D method in order to see the approximation taken in writing Eq. (II.14) more clearly. This kind of formalism has been given by other authors.[17,18] We would like to approach it a little differently.[19] (We assume the instability of ρ can be completely contained in the phase-space integral and the anomalous threshold which may occur will be neglected.)

We denote the relativistically invariant $J=1$ partial-wave amplitude for scattering $j\rightarrow i$ as T_{ij}:

$$T_{ij}=\begin{array}{c} \\ \gamma \\ N\bar{N} \\ \pi\rho \\ K\bar{K}\end{array}\!\!\begin{array}{c}\begin{array}{cccc}\gamma & N\bar{N} & \pi\rho & K\bar{K}\end{array}\\ \begin{bmatrix} T_{11} & T_{12} & T_{13} & T_{14}\\ T_{21} & T_{22} & T_{23} & T_{24}\\ T_{31} & T_{32} & T_{33} & T_{34}\\ T_{41} & T_{42} & T_{43} & T_{44}\end{bmatrix}\end{array}\equiv N_{iK}(D^{-1})_{Kj}. \quad \text{(A1)}$$

T_{24}, for instance, is the $N\bar{N}$-$K\bar{K}$ scattering amplitude. In compact matrix notation this is

$$T=ND^{-1}. \quad \text{(A1')}$$

It is well known that T_{ij} has cuts in the complex plane of the square of total energy t, in the center-of-mass system. Let us denote the right-hand cut by R_{ij} ($R_{ij}<t<\infty$), which arises from unitarity in the t channel, and the left-hand cut by L_{ij} ($\text{Re}t<R_{ij}$) which contains all other singularities of the scattering amplitude, T_{ij}.

In Eq. (A1′), we define N and D such that N contains only a left cut and D contains only a right cut. Using unitarity, $\text{Im}T_{ij}{}^{-1}=-\rho_i\delta_{ij}$, and (A1′), we obtain

$$N(t)=\frac{1}{\pi}\int_L \frac{\text{Im}T(t')}{t'-t}D(t')dt, \quad \text{(A2)}$$

$$D(t)=I-\frac{t}{\pi}\int_R \frac{\rho(t')N(t')}{t'(t'-t)}dt'. \quad \text{(A3)}$$

Substituting $D(t)$ in (A3) into $N(t)$ in (A2) and defining the left-hand-cut contribution to $T_{ij}(t)$ by

$$B_{ij}(t)=\frac{1}{\pi}\int_{L_{ij}} \frac{\text{Im}T_{ij}(t')}{t'-t}dt',$$

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[19] This approach has been made through private communications with Dr. J. R. Fulco.

we finally get

$$N_{ij}(t)=B_{ij}(t)+\frac{1}{\pi}\int_{L_{ij}}\frac{B_{iK}(t')-(t/t')B_{iK}(t)}{t'-t}\rho_K(t')N_{Kj}(t')dt'. \quad \text{(A4)}$$

Since the vertices have no left cut, we get

$$B_{i1}(t)=B_{1i}(t)=0. \quad \text{(A5)}$$

Substituting the requirement (A5) into (A4), we get

$$N_{1i}=C_i \quad \text{(constant)}, \quad \text{(A6)}$$

which can be identified as the coupling constant[20] in each channel.

The $N\bar{N}$ and γ intermediate state would be negligible because of its higher mass and electromagnetic nature. Neglecting this state is equivalent to setting $\rho_1(t)=\rho_2(t)=0$ in the equations (A3) and (A4).

Then from Eq. (A3) we have

$$D_{1i}=\delta_{1i},\quad D_{2i}=\delta_{2i},\quad d\equiv|D|=D_{33}D_{44}-D_{34}D_{43}.$$

We assume time-reversal invariance $T_{ij}=T_{ji}$ which would be satisfied automatically if $B_{ij}=B_{ji}$. Then

$$T_{42}=T_{24}=\frac{1}{d}(-N_{23}D_{34}+N_{24}D_{33}),$$
$$T_{14}=T_{41}=\frac{1}{d}(N_{14}D_{33}-N_{13}D_{34}),\ \text{etc.} \quad \text{(A7)}$$

At this point we can see that the approximation given in Sec. II is reproduced easily just by setting $B_{ij}=0$, except B_{44}, B_{33}, B_{24}, and B_{23}; namely, we neglect the interactions raised by exchanging the particles in the channel (ij) except as listed above. Then N_{21}, N_{31}, N_{41}, N_{34} and N_{43} have to satisfy the following equations:

$$N_{l1}=\frac{1}{\pi}\int\frac{B_{l3}(t')-(t/t')B_{l3}(t)}{t'-t}\rho_3(t')N_{31}(t')dt'\times(\delta_{l2}+\delta_{l3})+\frac{1}{\pi}\int\frac{B_{l4}(t')-(t/t')B_{l4}(t)}{t'-t}\rho_4(t')N_{41}(t')dt'\times(\delta_{l2}+\delta_{l4}),\quad (l=2,3,4);$$

$$N_{ij}=\frac{1}{\pi}\int\frac{B_{ii}(t')-(t/t')B_{ii}(t)}{t'-t}\rho_i(t')N_{ij}(t')dt',$$

(i and j are either 3 or 4).

[20] L. Castillejo, R. H. Dalitz, and F. J. Dyson, Phys. Rev. **101**, 453 (1956); S. C. Frautschi, *Regge Poles and S Matrix Theory* (W. A. Benjamin, Inc., New York, 1963), Chap. III.

It is easily seen that N_{31}, N_{41}, N_{34}, and N_{43} have a trivial solution, namely

$$N_{31}=N_{41}=N_{34}=N_{43}=0. \quad \text{(A8)}$$

Accordingly, we have

$$N_{21}=0.$$

Substituting (A8) into (A3), we get

$$D_{21}=D_{31}=D_{41}=D_{34}=D_{43}=0.$$

Now let us write the photon-nucleon vertex explicitly for this case:

$$\mathrm{Im}T_{12}=T_{13}{}^*\rho_3T_{32}+T_{14}{}^*\rho_4T_{42}.$$

Substituting (A7) into the above equation, we get

$$\mathrm{Im}T_{12}=C_3\rho_3\frac{N_{23}}{|D_{33}|^2}+C_4\rho_4\frac{N_{24}}{|D_{44}|^2}, \quad \text{(A9)}$$

where the explicit forms of N_{23} and N_{24} are given from (A4):

$$N_{23}=B_{23}+\frac{1}{\pi}\int\frac{B_{23}(t')-(t/t')B_{23}(t)}{t'-t}\rho_3(t')N_{33}(t')dt',$$
$$N_{24}=B_{24}+\frac{1}{\pi}\int\frac{B_{24}(t')-(t/t')B_{24}(t)}{t'-t}\rho_4(t')N_{44}(t')dt'.$$

It is clearly seen that D_{33} (D_{44}) and N_{33} (N_{44}) are the N and D functions of the $\pi\rho$ ($K\bar{K}$) scattering amplitude by writing T_{33} and T_{44} explicitly using (A1). Recalling that D_1 (D_2) and N_1 (N_2) in Eq. (II.14) are the N and D functions for the $\pi\rho$ ($K\bar{K}$) scattering amplitude and that $B_i{}^1$ ($B_i{}^2$) corresponds to B_{33} (B_{44}), we see that (A9) is equal to (II.14).

Now what change will be caused if we take into account the B_{34} and B_{43} terms which allow ω and φ mixing? (Since the experiment shows that ω and φ mixing is small, we will not take it into account in our calculation. However, it is done here for the benefit of further application of this formalism to some other physical processes.)

$$D_{34}\neq0\neq D_{43}\quad N_{34}\neq0\neq N_{43}.$$

Therefore, the photon-nucleon vertex becomes

$$\mathrm{Im}T_{12}=|d|^{-2}[(-C_3D_{34}+C_4D_{33})^*\rho_4(N_{24}D_{33}-N_{23}D_{34})+(C_3D_{44}-C_4D_{43})^*\rho_3(N_{23}D_{44}-N_{24}D_{43})].$$