A Parity for Very Weak Interactions

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Relevance of the charge-parity quantum number A to very weak interactions is studied. It appears that nonleptonic decays conserve AP, when P is real parity; a similar rule is postulated for the hadron currents in leptonic decays. Observed relations between $\Lambda \rightarrow p\pi$ and $\Xi^- \rightarrow \Lambda\pi$ verify the AP rule with a deviation of order 15% from A symmetry in the baryon octet. Simple relations are predicted between $\Xi \rightarrow (\Lambda,\Sigma)$ and $(\Lambda,\Sigma) \rightarrow N$ leptonic decays. The structure of nonleptonic baryon decays is almost completely specified by introduction of one further symmetry postulate, which does not fit readily into the SU_3 framework. The AP rule sheds no light on $K_2{}^0 \rightarrow 2\pi$ decay.

THE quantum number A of charge parity provides a selection rule for bosons on both strong and electromagnetic interactions.[1] One can in fact take A to be strictly conserved by electromagnetic interactions and to be violated only by such medium-strong interactions as produce the baryon mass asymmetry. It is natural to inquire whether A has any relevance to very weak interactions (VWI). We again assume the VWI to strictly obey selection rules on A, with failures attributable to the medium-strong interactions.

For the present question the obvious test cases $K_1{}^0 \rightarrow 2\pi$ and $K_2{}^0 \rightarrow 3\pi$ are completely opposite in their behavior: $A_f = -A_i$ for the first, $A_f = A_i$ for the second. On the other hand, $(AC)_f = \epsilon_T(AC)_i$ for both transitions where $\epsilon_T = \pm 1$ is the unknown signature under time reversal. Using $CT = P$, we have

$$AP = +1 \quad (1)$$

as the selection rule for very weak decays.[2] Although Eq. (1) provides a suggestive formal relation between real and charge parities, it is rather empty physically; for example, it says nothing against $K_2{}^0 \rightarrow 2\pi$, which appears to be strongly inhibited but not completely absent.[3]

Another question of interest is whether relative charge parity is a useful quantum number for baryons, as is suggested by the formal approach.[4] Nonleptonic decay of baryons provides an opportunity to study this question in connection with Eq. (1), which relates the asymmetry parameters of $\Xi \rightarrow \Lambda + \pi$ and $\Lambda \rightarrow N + \pi$ decays. Agreement with observation indicates that A is well conserved by VWI among baryons of the lowest octet if anticipated minor deviations of the octet from A symmetry are used to interpret certain experimental ratios.

Equation (1) halves the number of independent terms allowed by the $\Delta I = \frac{1}{2}$ rule for baryon decay. A single additional assumption (called W invariance below) is sufficient to remove most of the remaining arbitrariness and yield an interaction form in good qualitative accord with observation. The operation of W invariance does not occur naturally in SU_3, thus suggesting a more complicated charge symmetry for very weak interactions.

Extension of Eq. (1) to leptonic decay of hadrons can ultimately be tested by the simple relations it predicts among baryon leptonic decays. Under this extension the divergence of the W meson must also satisfy Eq. (1) if it mediates VWI.

1. BARYON SYMMETRY UNDER *A*

The operator A can be defined in a general fashion that applies both to bosons and fermions[4]; its effect on an ideal baryon octet is then

$$A\Sigma^i = -(\Sigma^{-i})^c, \quad A\Lambda = \Lambda^c, \quad A\Xi^- = p^c, \quad A\Xi^0 = -n^c, \quad (2)$$

where $A^2 = 1$ and the superscript c means antiparticle. The relative signatures in Eq. (2) are fixed by convention; the absolute signature of the array then distinguishes two classes of octets in just the same sense as the relative real parities of nucleon states. By definition, take the signs in Eq. (2) to represent positive charge parity.

The application of Eq. (1) to baryon decay requires the physical baryon octet to be a reasonably pure state of charge parity, say $\Psi(+)$. The purity cannot be perfect, since the mass difference $2\Delta = (M_\Xi - M_N)$ is distinctly nonvanishing. On the other hand, matrix-element deviations associated with this failure are of order $\Delta/M \approx \frac{1}{6}$, which gives a good account of A failure in boson transitions.[5] In the next sections we wish to assume that admixtures to $\Psi(+)$ in the baryon octet do not exceed this order.

There are three main objections to consider: (i) The $\frac{3}{2}^+$ decuplet is not in fact matched by a reverse decuplet, so that it must represent an almost complete lack of A symmetry—a roughly equal mixture of $\psi(+)$ and $\psi(-)$. (ii) The electromagnetic mass differences do not display anything like A symmetry, which for example predicts that $M(\Xi^-) - M(\Xi^0) \equiv \delta M_\Xi = \delta M_N \equiv M(p) - M(n)$. (iii) To be an eigenstate of AC, the Λ must have a vanishing

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[1] D. C. Peaslee and M. T. Vaughn, Phys. Rev. 119, 460 (1960); J. B. Bronzan and F. E. Low, Phys. Rev. Letters 12, 522 (1964).
[2] A similar but different suggestion is that $RP = +1$, where R is the SU_3 reflection generator; B. W. Lee, Phys. Rev. Letters 12, 82 (1964).
[3] J. H. Christenson *et al.*, Phys. Rev. Letters 13, 138 (1964).
[4] D. C. Peaslee, Phys. Rev. 117, 873 (1960).
[5] S. K. Kundu (to be published).

magnetic moment, instead of the measured value $\sim\frac{1}{2}\mu_n$. We answer these along the following lines: (i) The baryon octet is the simplest representation of the charge group and therefore may not display all the details revealed in a more complicated representation like the decuplet. That is, the possibility of A symmetry may occur for the octet by a sort of special degeneracy which is lifted in the decuplet. (ii) The nuclear electromagnetic mass differences are strongly influenced by the $\frac{3}{2}^+$ decuplet as possible intermediate states. Hence the δM violate A symmetry strongly without implying the same for the nucleons themselves. (iii) Failure of A symmetry to order Δ/M yields a Λ magnetic moment of order $2(\Delta/M)\mu_N\approx\frac{1}{3}\mu_N$, as observed.

Existence of A as a meaningful quantum number is tantamount to the assumption that hypercharge is proportional to the third component of an I-independent second isotopic vector, $I_z'=\frac{1}{2}Y$. In this case one immediately obtains the charge octet as the basic fermion; the same is true in SU_3, where Y is a scalar. Only higher representations differ in dimension for scalar and vector Y. An octet can always have accidental A symmetry even if Y is really a charge scalar.

The electromagnetic mass formula should contain terms as high as quadratic in the charge coordinates:

$$M=a_0+(a_1Y+b_1I_z)+a_2Y^2+b_2YI_z+c_2I_z^2. \quad (3)$$

The a terms are unobservable in the face of strong mass deviations. For perfect A symmetry $b_1=0$ and $\delta M_\Xi=\delta M_N$ in contradiction with experiment. Strong interference with the $\frac{3}{2}^+$ decuplet as an intermediate state will obviously distort this symmetry, so that the effective formula becomes

$$\delta M=(b_1+b_2y)I_z+c_2I_z^2 \quad (4)$$

with b_1 and b_2 the same order of magnitude.[6] Experiment indicates that $b_1=-3.8$ MeV, $b_2=+2.5$ MeV, and $c_2=1.0$ MeV. It is interesting to note that the mass relation

$$\delta M_\Xi-\delta M_N=M(\Sigma^-)-M(\Sigma^+) \quad (5)$$

requires only linearity in Y of the I_z coefficient. This is a weaker assumption than full SU_3 symmetry, on which Eq. (5) was first obtained.[7]

Under strict SU_3 symmetry, failure of A invariance is indicated not only by nonvanishing μ_Λ but by nonvanishing μ_n as well.[7] It should be noted, however, that this result depends critically on that feature of the SU_3 scheme which is in poorest agreement with observation, namely, the complete equivalence (i.e., cancellation for $\mu=0$) in strength and secondary electromagnetic effects of π-baryon and K-baryon interactions.

[6] An elegant formulation has been given by L. A. Radicati *et al.*, Phys. Rev. Letters **14**, 160 (1965). Their coefficients s_1 and e_1 represent noninvariance of A in the strong and electromagnetic interactions. Although definitely $e_1\neq0$, the observed equality $s_1/s_2=e_1/e_2$ indicates relative A failure in the electromagnetic interaction to be exactly comparable with that in strong interactions.

[7] S. Coleman and S. L. Glashow, Phys. Rev. Letters **6**, 423 (1961).

2. NON-LEPTONIC HYPERON DECAY

Assume the $\Delta I=\frac{1}{2}$ rule throughout. Then six interaction forms exist for pionic decay of hyperons:

$$B_6\{\sqrt{2}\bar{p}(\epsilon_6)\Lambda\pi^- -\bar{n}(\epsilon_6)\Lambda\pi^0\}+\text{H.c.} \quad (6)$$

$$B_{6'}\{\sqrt{2}\bar{p}(\epsilon_6')[\Sigma^+\pi^0-\Sigma^0\pi^-] -\bar{n}(\epsilon_6')[\Sigma^-\pi^- -\Sigma^+\pi^+]\}+\text{H.c.} \quad (6')$$

$$B_{6''}\{\bar{n}(\epsilon_6'')[\Sigma^+\pi^+ +\Sigma^0\pi^0+\Sigma^-\pi^-]\}+\text{H.c.} \quad (6'')$$

$$B_7\{\sqrt{2}\bar{\Lambda}(\epsilon_7)\Xi^-\pi^- +\bar{\Lambda}(\epsilon_7)\Xi^0\pi^0\}+\text{H.c.} \quad (7)$$

$$B_{7'}\{\sqrt{2}[\bar{\Sigma}^0\pi^- -\bar{\Sigma}^-\pi^0](\epsilon_7')\Xi' +[\bar{\Sigma}^-\pi^+ -\Sigma^+\pi^-](\epsilon_7')\Xi^0\}+\text{H.c.} \quad (7')$$

$$B_{7''}\{[\bar{\Sigma}^+\pi^- +\bar{\Sigma}^0\pi^0+\bar{\Sigma}^-\pi^+](\epsilon_7'')\Xi^0\}+\text{H.c.}, \quad (7'')$$

where $\bar{p}(\epsilon)\Lambda\pi^-=[\bar{\Psi}_p(1+\epsilon\gamma_5)\psi_\Lambda]\phi_-^*$, etc. Here we must use the scalar form of the interaction, even if it is only effective or derived. In the current-current form the vector contribution arises entirely from the A failure expressed by Δ; its use would defeat the purpose of looking for A invariance in VWI.

The 12 constants B and ϵ in the above equations reduce to six under Eqs. (1) and (3):

$$\begin{matrix} B_6=B_7=B, & B_6'=B_7'=B', & B_6''=B_7''=B'', \\ \epsilon_6=-\epsilon_7=\epsilon, & \epsilon_6'=\epsilon_7'=\epsilon', & \epsilon_6''=-\epsilon_7''=\epsilon''. \end{matrix} \quad (8)$$

Experimentally one finds that

$$|B(\Xi^-\to\Lambda\pi^-)|\approx|B(\Lambda\to p\pi^-)|$$

and that $\alpha_\Xi\approx-\alpha_\Lambda$ for these two decays. This verifies to first order one part of Eq. (8) and hence supports Eq. (1). Unfortunately, the only other comparison possible is between $\Xi^0\to\Lambda\pi^0$ and $\Lambda\to n\pi^0$; under the $\Delta I=\frac{1}{2}$ assumption this yields no new information and can never be so accurately determined as the decays involving π^- emission.

To reduce further the number of independent constants in Eq. (8), we may introduce an additional postulate having no direct connection with A or isotopic spin, namely, symmetry under the substitution

$$\begin{bmatrix} p \\ n \\ \Xi^0 \\ \Xi^- \end{bmatrix} \leftrightarrow \begin{bmatrix} \Sigma^+ \\ (\Sigma^0-\Lambda)/\sqrt{2} \\ (\Sigma^0+\Lambda)/\sqrt{2} \\ -\Sigma^- \end{bmatrix}. \quad (9)$$

To give this a name, we denote it as "W invariance." Applied to Eqs. (6)–(8), it yields the additional relations

$$B''=B'=B, \quad \epsilon''=\epsilon. \quad (10)$$

Equation (10) is in good agreement with observed Σ decay; $B''=B'$ is a necessary condition for the absence of polarization in charged-pion decay of Σ^+ and Σ^-.

It would be desirable to fix ϵ' by means of another postulate along the lines of the previous two. One such possibility is the following: Consider the baryon forms $(\bar{\Sigma}^+ n)$ and $(\bar{\Xi}^0\Sigma^-)$, which are unique in requiring $\Delta I_z = +\frac{3}{2}$. They are related by the operation AP (or A); we might expect their relative signature to be opposite to that for comparable terms with $\Delta I_z=\frac{1}{2}$, by a sign alternation rule similar to that between Σ and Λ. Thus, one assumes

$$(\bar{\Sigma}^+ n)/(\bar{\Xi}^0\Sigma^-) = -(\bar{n}\Sigma^-)/(\bar{\Sigma}^+\Xi^0). \tag{11}$$

The appropriate terms in Eqs. (6) and (7) are a sum of the primed and double-primed coordinates; with the previous symmetries in Eqs. (8) and (10), the unequivocal consequence of Eq. (11) is

$$\epsilon' = -\epsilon \tag{12}$$

as observed.

The nonleptonic decay term has now been reduced to just two independent constants, B and ϵ. Although we have tacitly assumed ϵ to be real, nothing about its magnitude has been specified. The postulate of W invariance does not seem to fit very naturally into the framework of SU_3; its efficacy for very weak interactions suggests that they may obey a more complicated charge symmetry.

Precise comparison of data on $\Lambda \to p\pi^-$ and $\Xi^- \to \Lambda\pi^-$ would allow some estimate of A mixture in the physical baryon octet. Write the effective interaction in nonrelativistic form:

$$\begin{aligned} &\bar{p}\{S_\Lambda + P_\Lambda(\boldsymbol{\sigma}\cdot\mathbf{q}/q)\}\Lambda\phi_-^*, \\ &\bar{\Lambda}\{S_\Xi - P_\Xi(\boldsymbol{\sigma}\cdot\mathbf{q}/q)\}\Xi^-\phi_-^*. \end{aligned} \tag{13}$$

Here q is the final momentum and the deviation from A symmetry is expressed by the parameter a in the formulas

$$S_\Lambda = (1-a)S, \quad P_\Lambda = (1+a)P, \quad S_\Xi = (1+a)S, \quad P_\Xi = (1-a)P. \tag{14}$$

The relative assignment of the $(1\pm a)$ terms is unique. Other arrangements would represent: A failure in the VWI instead of in the baryons, if the first two factors were $(1\pm a)$ and the second two $(1\mp a)$; or no failure at all if the S factors were both $(1\pm a)$ and the P factors both $(1\mp a)$. The numerical check on Eq. (14) is that

$$\alpha_\Lambda(\Gamma/q)_\Lambda = -\alpha_\Xi(\Gamma/q)_\Xi. \tag{15}$$

With current data the left-hand side of Eq. (15) is 1.1 ± 0.3 times the right-hand side. Now taking $\alpha_\Lambda = 0.66$, $\alpha_\Xi = -0.4\pm0.1$ we obtain $a = (14\pm5)\%$. This is exactly the amplitude of A mixture anticipated from the crude estimate Δ/M found in boson decays.

3. LEPTONIC DECAY

In leptonic decay processes one might at least hope that Eq. (1) would hold for the hadrons; and this seems to be true in the known cases $\pi_{\mu2}$, $K_{\mu2}$, π_{e3}, and K_{e3}. For leptonic decay of baryons the current-current form of interaction is necessary; to reduce the baryon term to a scalar form, write it as $\{\bar{\psi}(\gamma_\mu L_\mu)(1+\epsilon\gamma_5)\psi\}$, where L_μ is an A-invariant polar vector. Then the $AP=+1$ assumption for baryon terms in conjunction with the $\Delta I=\frac{1}{2}$ rule yields the interaction form

$$\begin{aligned} &\{\bar{\Lambda}\gamma_\mu(1+\epsilon\gamma_5)\Xi^- - \bar{p}\gamma_\mu(1+\epsilon\gamma_5)\Lambda\}l_\mu^- + \text{H.c.} \\ &\quad \pm\{\bar{\Sigma}^0\gamma_\mu(1+\epsilon\gamma_5)\Xi^- - \sqrt{2}\bar{\Sigma}^+\gamma_\mu(1+\epsilon\gamma_5)\Xi^0 \\ &\quad + \bar{p}\gamma_\mu(1+\epsilon\gamma_5)\Sigma^0 + \sqrt{2}\bar{n}\gamma_\mu(1+\epsilon\gamma_5)\Sigma^-\}l_\mu^- + \text{H.c.} \end{aligned} \tag{16}$$

In this case the relative signs of heavy-particle polarization are the same instead of opposite as for $\Xi \to \Lambda\pi$ and $\Lambda \to p\pi$; this results from using the $V-A$ form of interaction. One also expects that $\epsilon^2 = 1$ for at least the bare baryons, with corrections in practice of order 20% as observed in n-p decay. Equation (16) differs from the prediction of an SU_3 theory of leptonic decays.[8] Given 20% corrections to the axial-vector terms, the situation is not yet clear experimentally; but all terms in Eq. (16) are in principle accessible to experiment.

This same approach for $\Delta S=0$ leptonic decays would imply equality in amplitude and $V-A$ relative sign for $n \to p$ and $\Xi^- \to \Xi^0$ decays; for leptonic decays of $Y=0$ baryons the AP rule tells only that the relative signs for the various terms are those of isotopic spin generators: viz., $(\bar{\Sigma}^0\Sigma^- - \bar{\Sigma}^+\Sigma^0)$ and $(\bar{\Lambda}\Sigma^- + \bar{\Sigma}^+\Lambda)$, which makes them first-class currents in Weinberg's sense.[9]

If the VWI interaction is mediated by a vector meson W, then the preceding discussion indicates that $(\partial_\mu W_\mu)$ satisfies Eq. (1). Thus, if W_μ is a polar vector it has $A=+1$ like the ρ; if a pseudovector, it should have $A=-1$.

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[8] N. Cabibbo, Phys. Rev. Letters **10**, 531 (1963).

[9] S. Weinberg, Phys. Rev. **112**, 1375 (1958). Indeed, Eq. (1) simply generalizes to $\Delta S \neq 0$ the statement that all weak interactions are first-class.