relations among the terms of the nonleptonic current:

$$\sqrt{2}(K^0|\pi^0)=\sqrt{2}(\bar{K}^0|\pi^0)=\sqrt{6}(K^0|\eta)=(K_2{}^0|\pi^0)$$
$$=\sqrt{3}(K_2{}^0|\eta)=-(K^+|\pi^+), \quad (13)$$

and

$$(K_1{}^0|\pi^0)=(K_1{}^0|\eta)=0.$$

Then we have from Eq. (13)

$$\omega(K_1{}^0|\pi^0e^+e^-)=\omega(\eta|K_1{}^0e^+e^-)=0.$$

It also follows from CP invariance and conservation of angular momentum that

$$\omega(K_2{}^0|\pi^0e^+e^-)=\omega(\eta|K_2{}^0e^+e^-)=0.$$

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Regge Poles in Quantum Electrodynamics with Massive Photons

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Starting from the Lagrangian theory of quantum electrodynamics with massive photons, we find that the lowest order radiative corrections to the fourth order "box" diagram of the Compton scattering amplitude contribute terms proportional to $(\ln t)^2$ for large t, violating the Regge behavior if uncanceled. The sixth-order ladder diagram and two others obtained from it by interchanging one of the external photon vertices with a virtual photon vertex are re-evaluated. It is found that the unwanted terms contributed by all these diagrams exactly cancel, proving that in this theory, the fermion lies on a Regge trajectory, up to the sixth-order perturbation.

I. INTRODUCTION

In quantum electrodynamics with massive photons, perturbation theory has been applied up to the fourth order to investigate whether the fermion lies on a Regge trajectory.[1] The diagrams that contribute in the second to the fourth order are shown in Figs. 1 and 2. Their sum gives[1]

$$\mathfrak{M}_{\mu\nu}=-\gamma^2\Gamma_{2\mu}(p\!\!\!/-m)^{-1}\{1+\gamma^2[(s-m^2)I_0(s)$$
$$-(s-mp\!\!\!/)I_1(s)]\ln(-t)/8\pi^2\}\Gamma_{1\nu}, \quad (1)$$

where

$$s=(p_1+k_1)^2,$$
$$t=(p_1-p_2)^2,$$
$$I_n(s)=\int_0^1 x^n dx/[m^2x+\lambda^2(1-x)-sx(1-x)],$$
$$\Gamma_{2\mu}=\gamma_\mu-(\mathbf{k}_1k_{2\mu}/k_1\cdot k_2),$$
$$\Gamma_{1\nu}=\gamma_\nu-(\mathbf{k}_2k_{1\nu}/k_1\cdot k_2),$$

and m, λ, are the masses of the fermion and the photon respectively, and γ the coupling constant.[2] With the use of the external photon gauges $\Gamma_{1\nu}$, $\Gamma_{2\mu}$ instead of γ_ν, γ_μ, all other second and fourth order diagrams do not contribute.

The situation is more complicated in the sixth order, where contributions come from more than one diagram. In particular, when radiative corrections for the vertex parts and the self-energy parts are inserted in Fig. 2, terms proportional to $[\ln(-t)]^2$ are obtained. We show in this paper that the contributions from diagrams 3(a)–(f) in Fig. 3 exactly cancel the extra terms contributed by diagrams 4(a)–(c) in Fig. 4, giving the correct coefficient for $[\ln(-t)]^2$ as required by the Regge behavior. Thus the fermion is proved to lie on a Regge trajectory, up to the sixth order of perturbation, in the Lagrangian theory of quantum electrodynamics with massive photons.

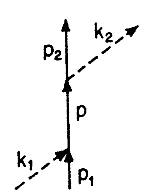

FIG. 1. Second-order Feynman-Dyson diagram.

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† Alfred P. Sloan Foundation Fellow.
[1] M. Gell-Mann, M. L. Goldberger, F. E. Low, E. Marx, and F. Zachariasen, Phys. Rev. 133, B145 (1964).
[2] Our metric is so chosen that $p^2=p_0{}^2-\mathbf{p}^2$.

II. RADIATIVE CORRECTION TO THE SQUARE DIAGRAM

We first turn our attention to Fig. 3(b). The self-energy part of an electron with momentum q in the lowest order is given as[3]

$$\Sigma(q)=A+B(\mathbf{q}-m)+(\mathbf{q}-m)^2\Sigma_f(q), \tag{2}$$

where A and B are independent of q and are eliminated by renormalization, and where

$$\Sigma_f(q)=(8\pi^2)^{-1}\gamma^2\int_0^1 x(1-x)dx\int_0^1 dz\frac{m(1+x)+(\mathbf{q}+m)(1-x)\{[2m^2xz(1+x)]/[m^2x^2+\lambda^2(1-x)]-1\}}{m^2x^2+\lambda^2(1-x)-x(1-x)z(q^2-m^2)}. \tag{3}$$

Now we have

$$\mathfrak{M}_{\mu\nu}^{(3b)}=i\gamma^4\int\frac{\gamma_\rho(\mathbf{p}_2-\mathbf{l}+m)\Gamma_{2\mu}(\mathbf{p}-\mathbf{l}+m)\Gamma_{1\nu}\Sigma_f(p_1-l)\gamma_\rho}{[(p_2-l)^2-m^2][(p-l)^2-m^2][l^2-\lambda^2]}\frac{d^4l}{(2\pi)^4}. \tag{4}$$

We may substitute Eq. (3) into Eq. (4) and evaluate the resulting expression in the limit $t\to\infty$ and s fixed. This tedious process can be avoided, however, if we note that, for $q\to\infty$, Eq. (3) gives

$$\Sigma_f(q)\to(16\pi^2)^{-1}\gamma^2\ln(q^2-m^2)/(\mathbf{q}-m). \tag{5}$$

If we substitute Eq. (5) into Eq. (4), we find that the integrand differs from that of Fig. 2 only by a factor $(\gamma^2/16\pi^2)\ln[(p_1-l)^2-m^2]$. When $p_1\cdot p_2$ is very large, this factor is approximated by $-(\gamma^2/16\pi^2)\ln\alpha$, where α is the Feynman parameter for the internal electron line of momentum p_1-l in the square diagram. Thus we get

$$\mathfrak{M}_{\mu\nu}^{(3b)}=(4\pi)^{-1}(\gamma^2/4\pi)^3\Gamma_{2\mu}[-(\mathbf{p}+m)I_0(s)+\mathbf{p}I_1(s)]\Gamma_{1\nu}[\ln(-t)]^2. \tag{6}$$

Figure 3(e) is calculated in the same manner and, as expected, gives the same result,

$$\mathfrak{M}_{\mu\nu}^{(3b)}=\mathfrak{M}_{\mu\nu}^{(3e)}. \tag{7}$$

If we insert the self-energy part into the internal electron line of momentum $p-l$ in Fig. 2, the diagram is only of the order $\ln t$ since $\ln[(p-l)^2-m^2]$ is bounded. We may also see this by noticing that $\ln\alpha$, where α is the Feynman parameter for the internal electron line of momentum $p-l$, is not large.

Next we turn our attention to Fig. 3(a) and Fig. 3(c).

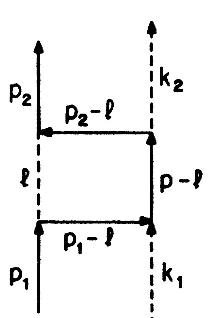

FIG. 2. Fourth-order Feynman-Dyson diagram.

The vertex part is written as[3]

$$\Lambda_\rho(q_2,q_1)=L\gamma_\rho+\Lambda_{\rho f}(q_2,q_1), \tag{8}$$

where q_2 and q_1 are the outgoing and incoming electron momenta respectively, and where L is independent of q_1 and q_2 and is removed by renormalization. The function $\Lambda_{\rho f}(q_2,q_1)$ is quite complicated. We note, however, that it is invariant under the transformation $q_1\leftrightarrow q_2$ plus reversing the order of the gamma matrices. Now, as $q_1^2\to\infty$, with q_2^2, $(q_1-q_2)^2$ fixed, we get

$$\begin{aligned}\Lambda_{\rho f}(q_2,q_1)\to&-(16\pi^2)^{-1}\gamma^2\ln(q_1^2-m^2)\gamma_\rho\\&+(8\pi^2)^{-1}\gamma^2m(q_{1\rho}-q_{2\rho})[\ln(q_1^2-m^2)]^2/(q_1^2-m^2)\\&-(4\pi^2)^{-1}\gamma^2(\mathbf{q}_1-\mathbf{q}_2)\gamma_\rho(\mathbf{q}_2-\tfrac{1}{2}\mathbf{q}_1)\\&\times\ln(q_1^2-m^2)/(q_1^2-m^2).\end{aligned} \tag{9}$$

A similar expression is easily obtained when $q_2^2\to\infty$,

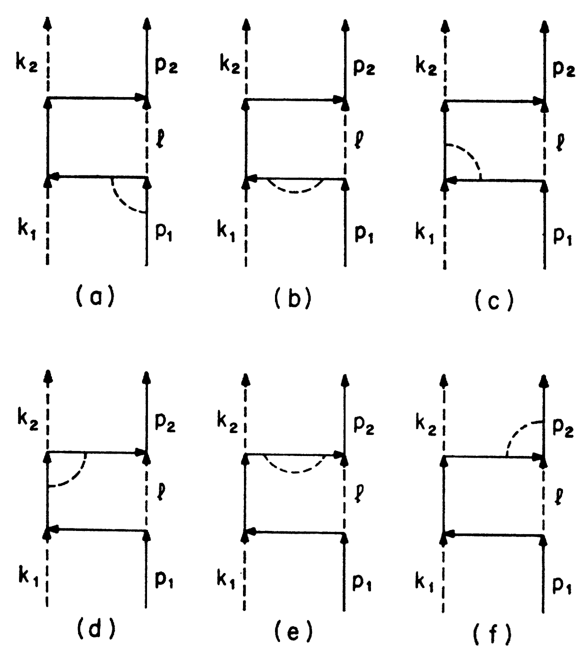

FIG. 3. Radiative corrections to the fourth-order Feynman-Dyson diagram.

[3] See, for instance, J. M. Jauch and F. Rohrlich, *The Theory of Photons and Electrons* (Addison-Wesley Publishing Company, Inc., Reading, Massachusetts, 1959), Secs. 9-4 and 9-6.

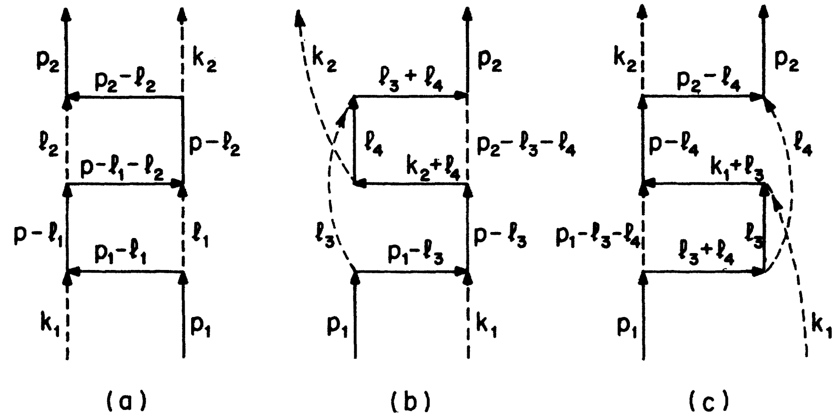

FIG. 4. Sixth-order ladder diagrams.

with q_1^2, $(q_1-q_2)^2$ fixed, from the invariance property mentioned above.

Substituting Eq. (9) into the square diagram, we obtain after some algebra

$$\mathfrak{M}_{\mu\nu}^{(3a)}=-\mathfrak{M}_{\mu\nu}^{(3b)}, \tag{10a}$$

$$\mathfrak{M}_{\mu\nu}^{(3c)}=\mathfrak{M}_{\mu\nu}^{(3b)}. \tag{10b}$$

The fact that the external photon gauge Γ is used should be remembered in deriving Eq. (10b).

Similarly, we have

$$\mathfrak{M}_{\mu\nu}^{(3d)}=-\mathfrak{M}_{\mu\nu}^{(3e)}=-\mathfrak{M}_{\mu\nu}^{(3f)}. \tag{11}$$

From Eqs. (7), (10), and (11) we obtain the total contribution of radiative corrections to the fourth-order square diagram as

$$\mathfrak{M}_{\mu\nu}^{(3a)}+\mathfrak{M}_{\mu\nu}^{(3b)}+\mathfrak{M}_{\mu\nu}^{(3c)}+\mathfrak{M}_{\mu\nu}^{(3d)}+\mathfrak{M}_{\mu\nu}^{(3e)}+\mathfrak{M}_{\mu\nu}^{(3f)} = (2\pi)^{-1}(\gamma^2/4\pi)^3\Gamma_{2\mu}[-(p\!\!\!/+m)I_0(s) + p\!\!\!/ I_1(s)]\Gamma_{1\nu}[\ln(-t)]^2. \tag{12}$$

III. DISCUSSION

The calculation in the above section is straightforward and in fact quite simple. On the other hand, if Regge asymptotic behavior holds, the sum of the sixth-order diagrams should give, in consistency with (1), a total contribution to order $[\ln(-t)]^2$

$$-\gamma^2\Gamma_{2\mu}(p\!\!\!/-m)^{-1}\{\tfrac{1}{2}[(s-m^2)I_0(s)-(s-mp\!\!\!/)I_1(s)]^2\times[\gamma^2\ln(-t)/8\pi^2]^2\}\Gamma_{1\nu}. \tag{13}$$

Thus the term of Eq. (12) must be canceled by the contribution from some other diagrams. What diagrams can these be? In view of the fact that Eq. (12) involves the functions $I_0(s)$ and $I_1(s)$, which are connected with the fourth-order square diagram, these diagrams must have a two-particle intermediate state in the s channel which are the only diagrams that can be reduced to the fourth-order square diagram when some of the vertices are fused together. Furthermore, the contribution of these diagrams must not be independent of t. These two requirements leave us with only the three diagrams of Fig. 4.

These diagrams have been considered in detail by Polkinghorne.[4] He found that they, taken together, give just Eq. (13). Therefore, if his computation is correct, the asymptotic behavior is not of the Regge form. In the next section, we re-evaluate the contribution of the diagrams of Fig. 4, and our result is in disagreement with that of Polkinghorne. Instead, we find that the contribution Eq. (12) is canceled and we indeed obtain the desired answer Eq. (13).

IV. CONTRIBUTION FROM DIAGRAM a OF FIGURE 4

In this section we consider diagram (a) of Fig. 4. We have

$$\mathfrak{M}_{\mu\nu}^{(4a)}=\gamma^6\int\frac{\gamma_\rho(p\!\!\!/_2-l\!\!\!/_2+m)\Gamma_{2\mu}(p\!\!\!/-l\!\!\!/_2+m)\gamma_\sigma(p\!\!\!/-l\!\!\!/_1-l\!\!\!/_2+m)\gamma_\rho(p\!\!\!/-l\!\!\!/_1+m)\Gamma_{1\nu}(p\!\!\!/_1-l\!\!\!/_1+m)\gamma_\sigma}{[(p_2-l_2)^2-m^2][(p-l_2)^2-m^2][(p-l_1-l_2)^2-m^2][(p-l_1)^2-m^2][(p_1-l_1)^2-m^2][l_1^2-\lambda^2][l_2^2-\lambda^2]}\times\frac{d^4l_1}{(2\pi)^4}\frac{d^4l_2}{(2\pi)^4}. \tag{14}$$

We note that if the numerator in Eq. (14) is set to unity, the right side of Eq. (14) is of the order[1,5] $[\ln(-t)]^2/t$.

[4] J. C. Polkinghorne, J. Math. Phys. **5**, 1491 (1964).

[5] P. G. Federbush and M. T. Grisaru, Ann. Phys. (N. Y.) **22**, 263 (1963).

The numerator in Eq. (14) can be written as

$$[2p_{2\rho}-\gamma_\rho l_2]\Gamma_{2\mu}(p-l_2+m)\gamma_\sigma(p-l_1-l_2+m) \times\gamma_\rho(p-l_1+m)\Gamma_{1\nu}[2p_{1\sigma}-l_1\gamma_\sigma]. \quad (15)$$

To evaluate Eq. (15), we move p_2 to the left and p_1 to the right until they operate on the proper spinors to give m. To exhibit the process of evaluation, we shall do one of the terms in Eq. (15) in detail.

We consider the product of $2p_{2\rho}$ and $2p_{1\sigma}$ in Eq. (15), and first move p_1 to the right:

$$\begin{aligned}4\Gamma_{2\mu}(p-l_2+m)p_1(p-l_1-l_2+m)p_2(p-l_1+m)\Gamma_{1\nu}\\ =8[p_1\cdot(p-l_1-l_2)]\Gamma_{2\mu}(p-l_2+m)p_2(p-l_1+m)\Gamma_{1\nu}\\ -8(p_1\cdot p_2)\Gamma_{2\mu}(p-l_2+m)(p-l_1-l_2-m)\\ \times(p-l_1+m)\Gamma_{1\nu}\\ +8[p_1\cdot(p-l_1)]\Gamma_{2\mu}(p-l_2+m)(p-l_1-l_2-m)p_2\Gamma_{1\nu}\\ -4\Gamma_{2\mu}(p-l_2+m)(p-l_1-l_2-m)p_2(p-l_1-m)\\ \times[\{p_1,\Gamma_{1\nu}\}_+-\Gamma_{1\nu}m]. \quad (16)\end{aligned}$$

We shall retain only those terms in $\mathfrak{M}_{\mu\nu}^{(4a)}$ which give at least $[\ln(-t)]^2$ for large $|t|$. Since the denominator alone is of the order $[\ln(-t)]^2/t$, the numerator has to contribute a factor t. We shall show that the last term in Eq. (16) can be neglected. To see this we note that $\{p_1,\Gamma_{1\nu}\}_+$ and $m\Gamma_{1\nu}$ are both finite as $|t|\to\infty$,[1] and to get a factor t we have to form products of p_2 with l_1 or l_2, with the displacement parts of l_1 and l_2 contributing the factor p_1. Now the coefficients of p_1 in the displacements of l_1 and l_2 are proportional to Feynman parameters which are small in the region of integration giving dominant contribution at large t. It is important to observe that this does not necessarily imply that terms involving $(p_2\cdot l_1)$ and $(p_2\cdot l_2)$ are small, since some other factors which have enhancement effects may be present. To avoid getting into uninteresting mathematical details, we here state without proof that terms like $(p_1\cdot l_2)l_1^2$, $(p_1\cdot l_2)l_2^2$, $(p_2\cdot l_1)l_1^2$, $(p_2\cdot l_1)l_2^2$ are of the order $[\ln(-t)]^2$ and must be kept, while terms like $(p_1\cdot l_1)l_1^2$, $(p_1\cdot l_1)l_2^2$, $(p_2\cdot l_2)l_1^2$, $(p_2\cdot l_2)l_2^2$ are of the order unity, and can be discarded. Terms like $(p_1\cdot l_2)(l_1\cdot l_2)$, $(p_2\cdot l_1)(l_1\cdot l_2)$ are even as large as t. However, the contributing part of $(l_1\cdot l_2)$ comes from the products of p_1 in the displacement of l_1 with p_2 in the displacement of l_2. Since p_1 is always moved to the right and p_2 to the left, terms like $(p_1\cdot l_2)\, l_2\, l_1$ can be discarded, while the existence of terms like $(p_1\cdot l_2)\, l_1\, l_2$, if uncanceled, would violate the Regge behavior. Now we may examine the last term of Eq. (16). We move the factor p_2 to the left, and it is easily seen that all terms obtained are small and can be discarded. Also, since the factors $(p_1\cdot p)$ and $(p_2\cdot p)$ are finite as $t\to\infty$ with s fixed, all terms in Eq. (16) involving these factors can be seen to be small.

The first term in Eq. (16) can be evaluated to be

$$\begin{aligned}-16[p_1\cdot(l_1+l_2)][p_2\cdot(p-l_2)]\Gamma_{2\mu}(p-l_1+m)\Gamma_{1\nu}+8[p_1\cdot(l_1+l_2)][\{\Gamma_{2\mu},p_2\}_+-m\Gamma_{2\mu}](p-l_2-m)(p-l_1+m)\Gamma_{1\nu}\\ \approx 8[p_1\cdot(l_1+l_2)]l_2^2\Gamma_{2\mu}(p-l_1+m)\Gamma_{1\nu}-8[p_1\cdot(l_1+l_2)][(p_2-l_2)^2-m^2]\Gamma_{2\mu}(p-l_1+m)\Gamma_{1\nu}. \quad (17)\end{aligned}$$

The factor $[(p_2-l_2)^2-m^2]$ in the second term of the right side of Eq. (17) can be used to cancel the same factor of the denominator in Eq. (14), and the expression left is independent of p_2 and hence of t. Thus the second term in Eq. (17) is of order unity, and Eq. (17), or the first term of Eq. (16) is approximately

$$8(p_1\cdot l_2)l_2^2\Gamma_{2\mu}(p-l_1+m)\Gamma_{1\nu}. \quad (18)$$

Similarly, the third term of Eq. (16) is approximately

$$8(p_2\cdot l_1)l_1^2\Gamma_{2\mu}(p-l_2+m)\Gamma_{1\nu}. \quad (19)$$

The second term in Eq. (16) is the term retained and discussed in Ref. 1 and everything in it has to be kept. From Eqs. (16), (18), and (19) we obtain

$$\begin{aligned}4\Gamma_{2\mu}(p-l_2+m)p_1(p-l_1-l_2+m)p_2(p-l_1+m)\Gamma_{1\nu}\\ \approx 8(p_1\cdot l_2)l_2^2\Gamma_{2\mu}(p-l_1+m)\Gamma_{1\nu}\\ +8(p_2\cdot l_1)l_1^2\Gamma_{2\mu}(p-l_2+m)\Gamma_{1\nu}\\ -8(p_1\cdot p_2)[(p-l_1)^2-m^2]\Gamma_{2\mu}(p-l_2+m)\Gamma_{1\nu}\\ -8(p_1\cdot p_2)[(p-l_2)^2-m^2]\Gamma_{2\mu}(p-l_1+m)\Gamma_{1\nu}\\ +8(p_1\cdot p_2)\Gamma_{2\mu}(p-l_2+m)(p-m)\\ \times(p-l_1+m)\Gamma_{1\nu}. \quad (20)\end{aligned}$$

As was pointed out in Ref. 1, the last term in Eq. (20) gives the exact amount required by the Regge behavior, and the third and fourth terms in Eq. (20) are of the order $[\ln(-t)]^3$. We shall see later that they are exactly canceled by corresponding terms in diagrams (b) and (c) of Fig. 4.

We now turn to other terms left in Eq. (15). By similar manipulations we may get

$$\begin{aligned}-\gamma_\rho l_2\Gamma_{2\mu}(p-l_2+m)p_1(p-l_1-l_2+m)\gamma_\rho(p-l_1+m)\Gamma_{1\nu}\\ \approx -8(p_1\cdot l_2)l_1^2(\Gamma_{2\mu}l_2+l_2\Gamma_{2\mu})\Gamma_{1\nu}\\ +8m(p_1\cdot l_2)l_1^2\Gamma_{2\mu}\Gamma_{1\nu}\\ +8(p_1\cdot l_2)l_1 p\Gamma_{2\mu}l_1\Gamma_{1\nu}\\ +8(p_1\cdot l_2)(\Gamma_{2\mu}l_2+l_2\Gamma_{2\mu})l_2(p-l_1+m)\Gamma_{1\nu}, \quad (21)\end{aligned}$$

and

$$\begin{aligned}-2\Gamma_{2\mu}(p-l_2+m)\gamma_\sigma(p-l_1-l_2+m)p_2(p-l_1+m)\Gamma_{1\nu}l_1\gamma_\sigma\\ \approx -8(p_2\cdot l_1)l_2^2\Gamma_{2\mu}(\Gamma_{1\nu}l_1+l_1\Gamma_{1\nu})\\ +8m(p_2\cdot l_1)l_2^2\Gamma_{2\mu}\Gamma_{1\nu}\\ +8(p_2\cdot l_1)\Gamma_{2\mu}l_2\Gamma_{1\nu}p l_2\\ +8(p_2\cdot l_1)\Gamma_{2\mu}(p-l_2+m)l_1(\Gamma_{1\nu}l_1+l_1\Gamma_{1\nu}). \quad (22)\end{aligned}$$

With

$$\gamma_\rho l_2\Gamma_{2\mu}(p-l_2+m)\gamma_\sigma(p-l_1-l_2+m) \times\gamma_\rho(p-l_1+m)\Gamma_{1\nu}l_1\gamma_\sigma\approx 0,$$

the numerator for $\mathfrak{M}_{\mu\nu}^{(4a)}$ is equal to the sum of Eqs. (20), (21), (22).

V. CONTRIBUTION FROM DIAGRAMS (b) AND (c) OF FIGURE 4

We now turn to diagram (b) of Fig. 4. We have

$$\mathfrak{M}_{\mu\nu}^{(4b)}=\gamma^6\int\frac{d^4l_3}{(2\pi)^4}\frac{d^4l_4}{(2\pi)^4}$$
$$\times\frac{\gamma_\rho(l_3+l_4+m)\gamma_\sigma(l_4+m)\Gamma_{2\mu}(k_2+l_4+m)\gamma_\rho(p-l_3+m)\Gamma_{1\nu}(p_1-l_3+m)\gamma_\sigma}{[(l_3+l_4)^2-m^2][l_4^2-m^2][(k_2+l_4)^2-m^2][(p-l_3)^2-m^2][(p_1-l_3)^2-m^2][l_3^2-\lambda^2][(p_2-l_3-l_4)^2-\lambda^2]}.\quad(23)$$

We first state that when the numerator in Eq. (23) is set to unity, the right side of Eq. (23) is of the order $1/t$. Now the numerator is a product of five momenta which can form at most two dot products. To obtain something of the order of $[\ln(-t)]^2$, one of them has to produce a factor t and the other has to give an enhancement effect of $[\ln(-t)]^2$. We next observe that if the internal electron line with momentum l_4 in Fig. 4(b) is contracted, the diagram is identical to Fig. 4(a) with the internal electron line with momentum $p-l_2$ contracted. Therefore, the terms in the numerator of Eq. (23) which have a factor $(l_4^2-m^2)$ can be identified with some corresponding terms in Eq. (14). This fact simplifies the calculation greatly. Otherwise, since terms as large as $[\ln(-t)]^3$ are present in Eq. (14), one has to make sure that not only the dominant terms but also the next order terms are all evaluated correctly.

The numerator in Eq. (23) can be written as

$$\begin{aligned}&2\gamma_\rho(l_3+l_4+m)p_1(l_4+m)\Gamma_{2\mu}(p-p_2+l_4+m)\\&\quad\times\gamma_\rho(p-l_3+m)\Gamma_{1\nu}-\gamma_\rho(l_3+l_4+m)\gamma_\sigma(l_4+m)\\&\qquad\times\Gamma_{2\mu}(p-p_2+l_4+m)\gamma_\rho(p-l_3+m)\Gamma_{1\nu}l_3\gamma_\sigma.\end{aligned}\quad(24)$$

Let us first examine the first term in Eq. (24). The momentum factor $p-l_3$ in the extreme right position cannot be used to form dot products which give either a factor of t or enhancement effects, since the coefficient of p_2 in the displacement of l_3 involves products of two small Feynman parameters, while p_1 in the displacement of l_3 can be moved to the right to become m, without encountering any of the momentum factors. Thus both dot products have to come from the other four-momentum factors, and we may neglect m in those propagators. Making use of the identity

$$\gamma_\rho ABCDE\gamma_\rho=-2EDCBA,$$

we obtain for the first term of Eq. (24)

$$\begin{aligned}&-4(p-p_2+l_4)\Gamma_{2\mu}l_4p_1(l_3+l_4)(p-l_3+m)\Gamma_{1\nu}\\&\qquad\approx8(p_1\cdot l_4)l_4^2\Gamma_{2\mu}(p-l_3+m)\Gamma_{1\nu}.\end{aligned}\quad(25)$$

Similarly, we can obtain the second term in Eq. (24) as

$$\begin{aligned}&-8(l_3\cdot l_4)l_4^2\Gamma_{2\mu}(p-l_3+m)\Gamma_{1\nu}\\&+8(l_3\cdot l_4)l_4^2\Gamma_{2\mu}\Gamma_{1\nu}l_3.\end{aligned}\quad(26)$$

The numerator of Eq. (23) is equal to the sum of Eqs. (25) and (26). Since all terms of Eqs. (25) and (26) have the factor l_4^2, which can be replaced by $l_4^2-m^2$ as their difference is small, by previous discussion all terms in Eq. (23) can be identified with corresponding terms in Fig. 4(a). Comparing the two contracted diagrams, we see that there is the correspondence

$$l_4=p_2-l_1-l_2,$$
$$l_3=l_1.$$

Thus Eq. (23) is equal to the numerator

$$\begin{aligned}&8(p_1\cdot p_2)[(p-l_2)^2-m^2]\Gamma_{2\mu}(p-l_1+m)\Gamma_{1\nu}\\&\quad-8(p_1\cdot l_2)l_2^2\Gamma_{2\mu}(p-l_1+m)\Gamma_{1\nu}\\&\quad-8(p_2\cdot l_1)l_2^2\Gamma_{2\mu}(p-l_1+m)\Gamma_{1\nu}\\&\qquad+8(p_2\cdot l_1)l_2^2\Gamma_{2\mu}\Gamma_{1\nu}l_1,\end{aligned}\quad(27)$$

evaluated with the denominator of Fig. 4(a).

The contribution of Fig. 4(c) can be obtained from that of 4(b) by the substitution

$$p_2\leftrightarrow p_1,\ l_1\leftrightarrow l_2,\ \Gamma_{2\mu}\leftrightarrow\Gamma_{1\nu}$$

and with the order of the γ matrices reversed.

Adding up the contribution from Figs. 4(a), 4(b), and 4(c) we get as the numerator

$$\begin{aligned}&8(p_1\cdot p_2)\Gamma_{2\mu}(p-l_2+m)(p-m)(p-l_1+m)\Gamma_{1\nu}\\&\quad+8(p_2\cdot l_1)\Gamma_{2\mu}(p-l_2+m)l_1(l_1\Gamma_{1\nu}+\Gamma_{1\nu}l_1)\\&\quad+8(p_1\cdot l_2)(l_2\Gamma_{2\mu}+\Gamma_{2\mu}l_2)l_2(p-l_1+m)\Gamma_{1\nu}.\end{aligned}\quad(28)$$

The first term in Eq. (28) gives exactly Eq. (13), while the last two terms in Eq. (25) can be evaluated to give exactly the negative of Eq. (12). Therefore, the sum of Figs. 3 and 4 gives the correct term required by the Regge behavior. In obtaining Eq. (28), the fact that

$$q_1^2\approx-q_{10}^2\approx\tfrac{1}{2}q_1'^2$$

has been utilized, where q_{10} is the displacement of q_1 and

$$q_1'=q_1-q_{10}.$$

In conclusion, the confirmation of Regge behavior at the sixth order is a nontrivial check which lends credence to its validity to all order. That a contribution from radiative corrections exactly cancels the unwanted terms in ladder Fig. 4 was not anticipated, and the meaning of the cancellation is not entirely clear.

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