Rare $\Delta Q=0$, $\Delta S=1$ Decay Modes of Hyperons and K Mesons*

K. TANAKA
Ohio State University, Columbus, Ohio
(Received 15 June 1965)

The $\Delta Q=0$, $\Delta S=1$ component of the unitary spin current is used in interactions that are both weak and electromagnetic. The effective coupling constant is determined from the experimental decay rate $\Sigma^+ \to p+\gamma$. With the aid of symmetry arguments, the rare decay rates of the modes $B \to B'+\gamma$, $B \to B'+e^++e^-$, and $K \to \pi+e^++e^-$ are computed.

1. INTRODUCTION

THE invariance of the strong interactions under the group SU_3 of unitary spin rotations corresponds to conservation of the eight-component unitary spin current[1]

$$J_i=\mathrm{Tr}\bar{B}\lambda_i B\,,\quad (i=1,-8)\,,$$

where the spatial dependence is suppressed, λ_i are the eight independent traceless 3×3 matrices, and B is the familiar 3×3 traceless matrix representing the eight baryons. Here Tr is the matrix trace over a three-dimensional space that combines isotopic spin and strangeness. The components J_1, J_2, and J_4, J_5 are characterized by the properties $\Delta Q=1$, $\Delta S=0$, and $|\Delta Q|=|\Delta S|=1$, respectively, and are useful in leptonic decays when they are combined with weak lepton currents. The components J_3 and J_8 are characterized by the property $\Delta Q=\Delta S=0$ and are useful in electromagnetic interactions. Then the only components of J_i which have not found their way into elementary interactions are J_6 and J_7 which are characterized by $\Delta Q=0$, $|\Delta S|=1$.

The purpose of this paper is to explore the possibility of using these components in interactions that are both weak and electromagnetic. To be sure, the corresponding Lagrangian must be effective, so we examine the resulting leptonic process $\Sigma^+\to p+\gamma$ and determine the effective coupling constant. The rates of the various rare decay modes that are computed can eventually be compared with experiments.

The modes $B\to B'+\gamma$ and $B\to B'+e^++e^-$ are studied in Sec. 2, and the decay rate of the mode $K\to\pi+e^++e^-$ is estimated in Sec. 3.

2. $B\to B'+\gamma$, $B\to B'+e^++e^-$

The amplitude of the decay mode $B\to B'+\gamma$ has a parity-conserving part and a parity-violating part. In general, both parts can have D- and F-type couplings

$$\begin{aligned}D=&\mathrm{Tr}(\bar{B}BS+\bar{B}SB)\\=&\bar{\Sigma}^+p+\bar{\Xi}^-\Sigma^--(\bar{\Sigma}^0/\sqrt{2}+\bar{\Lambda}/\sqrt{6})n\\&-\bar{\Xi}^0(\Sigma^0/\sqrt{2}+\Lambda/\sqrt{6})\,,\end{aligned}\tag{1}$$

$$\begin{aligned}F=&\mathrm{Tr}(\bar{B}BS-\bar{B}SB)\\=&\bar{\Sigma}^+p-\bar{\Xi}^-\Sigma^--(\bar{\Sigma}^0/\sqrt{2}-3\bar{\Lambda}/\sqrt{6})n\\&+\bar{\Xi}^0(\Sigma^0/\sqrt{2}-3\Lambda/\sqrt{6})\,,\end{aligned}\tag{2}$$

where S is the 3×3 matrix given by $S=(\lambda_6+i\lambda_7)^\dagger/2$. The spatial dependence is suppressed.

The parity-violating part of the interaction of $B\to B'+\gamma$ for D-type coupling (which is CP-invariant) can be written as

$$\begin{aligned}[\bar{\Sigma}^+O_\alpha p-\bar{p}O_\alpha\Sigma^+-(\bar{\Sigma}^0/\sqrt{2}+\bar{\Lambda}/\sqrt{6})O_\alpha n\\+\bar{n}O_\alpha(\Sigma^0/\sqrt{2}+\Lambda/\sqrt{6})-\bar{\Xi}^0O_\alpha(\Sigma^0/\sqrt{2}+\Lambda/\sqrt{6})\\+(\bar{\Sigma}^0/\sqrt{2}+\bar{\Lambda}/\sqrt{6})O_\alpha\Xi^0]A_\alpha\,,\end{aligned}\tag{3}$$

where $O_\alpha=\gamma_5\sigma_{\alpha\beta}\partial_\beta$ and A_α is the electromagnetic field operator. We note that A_α transforms like the singlet member ($U=0$) of the unitary octet and the $\Delta Q=0$, $\Delta S=+1$ current transforms into the $\Delta Q=0$, $\Delta S=-1$ current under the U-spin transformation $\Sigma^+\to p$, $\Xi^0\to n$, $\Xi^-\to\Sigma^-$, $\Sigma^0\to(\Sigma^0+\sqrt{3}\Lambda)/2$, $\Lambda\to(\sqrt{3}\Sigma^0-\Lambda)/2$, and $A_\alpha\to A_\alpha$. As a result, Eq. (3) must be invariant under the U-spin transformation.

When the U-spin transformation is carried out on Eq. (3) and also on the corresponding parity-violating part of the interaction for F-type coupling, both interactions change sign and consequently vanish.[2] In other words, the parity-violating part of the interaction of $B\to B'+\gamma$ vanishes provided it is CP-invariant and constructed from the $\Delta Q=0$, $|\Delta S|=1$ component of the unitary spin current.

The parity-conserving part of the interaction is of the form

$$\begin{aligned}(dD+fF)A+\mathrm{H.c.}=\{&\bar{\Sigma}^+p+(d-f)\bar{\Xi}^-\Sigma^--\bar{\Sigma}^0n/\sqrt{2}\\&-(d-3f)\bar{\Lambda}n/\sqrt{2}-(d-f)\bar{\Xi}^0\Sigma^0/\sqrt{2}\\&-(d+3f)\bar{\Xi}^0\Lambda/\sqrt{6}\}A+\mathrm{H.c.}\,,\end{aligned}\tag{4}$$

where D and F are given in Eqs. (1) and (2) and the factors d and f ($d+f=1$) indicate the ratio of D- to

* Research supported in part by the U. S. Atomic Energy Commission.

[1] M. Gell-Mann, Phys. Rev. **125**, 1067 (1962).

[2] The fact that the parity-violating parts of $\Sigma^+\to p+\gamma$ and $\Xi^-\to\Sigma^-+\gamma$ vanish is noted under more general conditions by Y. Hara, Phys. Rev. Letters **12**, 378 (1964).

TABLE I. The decay rates and branching ratios of $B \to B'+\gamma$.

Mode	Decay rate (10^6 sec^{-1})	Branching ratio
$\Sigma^+ \to p\gamma$	23.3	1.8×10^{-3}
$\Xi^- \to \Sigma^-\gamma$	0.3	5.5×10^{-6}
$\Xi^0 \to \Sigma^0\gamma$	0.2	6.8×10^{-6}
$\Xi^0 \to \Lambda\gamma$	7.8	2.4×10^{-3}
$\Sigma^0 \to n\gamma$	11.9	$<1.2\times10^{-7}$
$\Lambda \to n\gamma$	0.3	8.3×10^{-6}

F-type coupling.[3] The matrix elements of the various $B \to B'+\gamma$ modes are related to that of the mode $\Sigma^+ \to p+\gamma$ by Eq. (4). For the numerical work, the experimental value $d/f=1.7$ that was found for the axial vector part of the $\Delta S=\Delta Q=1$ current is used.[4]

The parity-conserving part of the amplitude of $B(p) \to B'(q)+\gamma(k)$ can be written from invariance arguments as

$$\mathfrak{M}=eGf\bar{u}(q)[(i\gamma_\mu k^2+k_\mu\Delta)F_1 +i\sigma_{\mu\nu}k_\nu MF_2]u(p)\epsilon_\mu/(2|k|^{1/2}), \quad (5)$$

where $\hbar=c=1$, $2M=m(B)+m(B')$, $\Delta=m(B)-m(B')$, $G=10^{-5}m^{-2}(p)$, ϵ_μ is the photon polarization, and f is a dimensionless constant that characterizes the reduction in strength of the $|\Delta S|=1$, $\Delta Q=0$ decay modes due to the strong interactions. The electric and magnetic form factors F_1 and F_2 are assumed to be of order unity. It turns out that f is a few orders of magnitude less than unity.

The decay rate of $\Sigma^+ \to p+\gamma$ with the aid of the amplitude Eq. (5) is[5]

$$\omega_\gamma=4\alpha G^2f^2M^5[\Delta/m(B)]^3, \quad B=\Sigma, \quad (6)$$

where $|F_2|^2=1$ is assumed, and $\alpha=e^2/4\pi=1/137$. From the experimental decay rate[6] $\omega_\gamma=2.33\times10^7$ sec^{-1}, we obtain $f^2=3.17\times10^{-4}$.

The decay rates of the mode $B \to B'+\gamma$ are obtained from Eqs. (4) to (6) and are given in Table I together with their respective branching ratios. The matrix elements are related by symmetry arguments but the kinematic corrections are taken into account. One observes from Table I that the mode that is likely to be observed is $\Xi^0 \to \Lambda+\gamma$, although perhaps it is difficult experimentally.

The J_6+iJ_7 current ($\Delta Q=0$, $|\Delta S|=1$) and the $J_3+J_8/\sqrt{3}$ current ($|\Delta Q|=|\Delta S|=0$) are both components of the unitary spin current so that the following equality should hold in the present approach where the Lagrangian is treated as effective,

$$\omega(B|B'e^+e^-)/\omega_\gamma(B|B'\gamma)=\omega(\Sigma^0|\Lambda e^+e^-)/\omega(\Sigma^0|\Lambda\gamma). \quad (7)$$

The amplitude of $B(p) \to B'(q)+e^-(r)+e^+(s)$ is obtained from Eq. (5) by the substitution

$$\epsilon_\mu/(2|k|)^{1/2} \to e\bar{u}(r)\gamma_\mu v(s)/k^2 \quad (k=r+s).$$

We combine Eqs. (5), (6), and (7) and obtain the ratio[7] of $R=\omega(B|B'e^+e^-)/\omega(B/B'\gamma)$;

$$R=\frac{\alpha}{3\pi}\int_{4m_e^2/\Delta^2}^{1} dx\left[(1-x)^3\left(1-\frac{4m_e^2}{\Delta^2 x}\right)\right]^{1/2} \times\left[\frac{8}{3x}-\mathrm{Re}\frac{(F_1F_2^*)}{|F_2|^2}4\frac{\Delta^2}{M^2}+\left|\frac{F_1}{F_2}\right|^2\frac{4\Delta^2}{3M^2}\right], \quad (8)$$

where $x=-k^2/\Delta^2$, and m_e is the mass of the electron. If $|F_1|\approx|F_2|$, then the second and third terms on the right-hand side of Eq. (8) are negligible because of the factor Δ^2/M^2 compared with the first term, so that

$$R\approx\frac{2\alpha}{3\pi}\left[\ln\left(\frac{\Delta}{m_e}\right)-\frac{4}{3}\right]. \quad (9)$$

The corresponding decay rates for Σ^+ and Ξ^0 are from Table I and Eq. (9),

$$\omega(\Sigma^+|pe^+e^-)=1.76\times10^5\ \text{sec}^{-1}, \quad \omega(\Xi^0|\Lambda e^+e^-)=0.56\times10^5\ \text{sec}^{-1}. \quad (10)$$

3. $K \to \pi+e^++e^-$

The $\Delta Q=0$, $|\Delta S|=1$ component of the unitary spin current is also responsible for the decay mode $K^+(p) \to \pi^+(q)+e^-(r)+e^+(s)$. The amplitude can be written as[8]

$$\mathfrak{M}=2e^2Gf(4\omega_k\omega_\pi)^{-1/2}p_\mu\bar{u}(r)\gamma_\mu v(s), \quad (11)$$

where ω_k and ω_π are the energies of the kaon and pion, respectively.

The decay rate with the aid of the amplitude Eq. (11) is

$$\omega=(\alpha^2G^2f^2m^5k/48\pi)F(m_\pi/m_k), \quad (12)$$

where $F(x)=1-8x^2+8x^6-x^8-24x^4\ln x$. When $F(m_\pi/m_k)=0.6$ and $f^2=3.17\times10^{-4}$ are put into Eq. (12), we find that

$$\omega(K^+|\pi^+e^+e^-)=3.8\times10^2\ \text{sec}^{-1}.$$

The branching ratio is about 10^{-6} which is in agreement with recent estimates.[8,9]

The $\Delta Q=0$, $|\Delta S|=1$ current for pseudoscalar mesons is of the D type in order to insure the usual $|\Delta I|=\frac{1}{2}$ rule. One obtains from Eq. (4) the following

[3] In Eq. (4), one can also take $(dD-fF)A$ which will lead to different relations among the decay modes. The combination in Eq. (4) favors the decay rate of $\Sigma^+ \to p+\gamma$ for the d/f ratio that is considered here.

[4] H. Courant *et al.*, Phys. Rev. **136**, B1791 (1964).

[5] R. E. Behrends, Phys. Rev. **111**, 1691 (1958); G. Calucci and G. Furlan, Nuovo Cimento **21**, 679 (1961).

[6] The value used for $\omega(\Sigma^+ \to p+\gamma)/\omega(\Sigma^+ \to p+\pi^0)=(0.37\pm0.08)10^{-2}$ was reported in a recent publication [M. Bazin *et al.*, Phys. Rev. Letters **14**, 154 (1965)].

[7] G. Feldman and T. Fulton, Nucl. Phys. **8**, 106 (1958); L. E. Evans, Nuovo Cimento **25**, 580 (1962); C. Alff *et al.*, Phys. Rev. **137**, B1105 (1965).

[8] L. B. Okun' and A. P. Rudik, Zh. Eksperim. i Teor. Fiz. **39**, 600 (1960) [English transl.: Soviet Phys.—JETP **12**, 422 (1961)].

[9] G. L. Glashow and M. Baker, Nuovo Cimento **25**, 857 (1962); M. A. Baqi Bég, Phys. Rev. **132**, 426 (1963).

relations among the terms of the nonleptonic current:

$$\sqrt{2}(K^0|\pi^0)=\sqrt{2}(\bar{K}^0|\pi^0)=\sqrt{6}(K^0|\eta)=(K_2{}^0|\pi^0)$$
$$=\sqrt{3}(K_2{}^0|\eta)=-(K^+|\pi^+)\,, \quad (13)$$

and

$$(K_1{}^0|\pi^0)=(K_1{}^0|\eta)=0\,.$$

Then we have from Eq. (13)

$$\omega(K_1{}^0|\pi^0e^+e^-)=\omega(\eta|K_1{}^0e^+e^-)=0\,.$$

It also follows from CP invariance and conservation of angular momentum that

$$\omega(K_2{}^0|\pi^0e^+e^-)=\omega(\eta|K_2{}^0e^+e^-)=0\,.$$

ACKNOWLEDGMENTS

The author wishes to thank K. Itabashi for a helpful discussion and A. Roberts for his comments that initiated this study.

PHYSICAL REVIEW VOLUME 140, NUMBER 2B 25 OCTOBER 1965

Regge Poles in Quantum Electrodynamics with Massive Photons

HUNG CHENG*
Department of Physics, Harvard University, Cambridge, Massachusetts

AND

TAI TSUN WU†
Gordon McKay Laboratory, Harvard University, Cambridge, Massachusetts

(Received 14 June 1965)

Starting from the Lagrangian theory of quantum electrodynamics with massive photons, we find that the lowest order radiative corrections to the fourth order "box" diagram of the Compton scattering amplitude contribute terms proportional to $(\ln t)^2$ for large t, violating the Regge behavior if uncanceled. The sixth-order ladder diagram and two others obtained from it by interchanging one of the external photon vertices with a virtual photon vertex are re-evaluated. It is found that the unwanted terms contributed by all these diagrams exactly cancel, proving that in this theory, the fermion lies on a Regge trajectory, up to the sixth-order perturbation.

I. INTRODUCTION

IN quantum electrodynamics with massive photons, perturbation theory has been applied up to the fourth order to investigate whether the fermion lies on a Regge trajectory.[1] The diagrams that contribute in the second to the fourth order are shown in Figs. 1 and 2. Their sum gives[1]

$$\mathfrak{M}_{\mu\nu}=-\gamma^2\Gamma_{2\mu}(p\!\!/-m)^{-1}\{1+\gamma^2[(s-m^2)I_0(s)$$
$$-(s-mp\!\!/)I_1(s)]\ln(-t)/8\pi^2\}\Gamma_{1\nu}\,, \quad (1)$$

where

$$s=(p_1+k_1)^2\,,$$
$$t=(p_1-p_2)^2\,,$$
$$I_n(s)=\int_0^1 x^n dx/[m^2x+\lambda^2(1-x)-sx(1-x)]\,,$$
$$\Gamma_{2\mu}=\gamma_\mu-(\mathbf{k}_1 k_{2\mu}/k_1\cdot k_2)\,,$$
$$\Gamma_{1\nu}=\gamma_\nu-(\mathbf{k}_2 k_{1\nu}/k_1\cdot k_2)\,,$$

and m, λ, are the masses of the fermion and the photon respectively, and γ the coupling constant.[2] With the use of the external photon gauges $\Gamma_{1\nu}$, $\Gamma_{2\mu}$ instead of γ_ν, γ_μ, all other second and fourth order diagrams do not contribute.

The situation is more complicated in the sixth order, where contributions come from more than one diagram. In particular, when radiative corrections for the vertex parts and the self-energy parts are inserted in Fig. 2, terms proportional to $[\ln(-t)]^2$ are obtained. We show in this paper that the contributions from diagrams 3(a)–(f) in Fig. 3 exactly cancel the extra terms contributed by diagrams 4(a)–(c) in Fig. 4, giving the correct coefficient for $[\ln(-t)]^2$ as required by the Regge behavior. Thus the fermion is proved to lie on a Regge trajectory, up to the sixth order of perturbation, in the Lagrangian theory of quantum electrodynamics with massive photons.

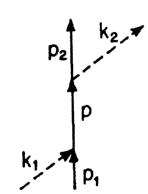

FIG. 1. Second-order Feynman-Dyson diagram.

* Work supported in part by the U. S. Air Force Office of Scientific Research under contract number AF 49(638)-1380.
† Alfred P. Sloan Foundation Fellow.

[1] M. Gell-Mann, M. L. Goldberger, F. E. Low, E. Marx, and F. Zachariasen, Phys. Rev. **133**, B145 (1964).

[2] Our metric is so chosen that $p^2=p_0{}^2-\mathbf{p}^2$.