K_0 and $\tilde{K}_0$. We transform each integral by an homographic transformation $\tau = a\tau' + b/c\tau' + d$ which maps the integration bounds into ± 1 and one of the remaining roots into infinity.

Each integral is then transformed into

$$\int_{-1}^{+1} \frac{d\tau}{[P(\tau)]^{1/2}},$$

where $P(\tau)$ is a polynomial of third degree completely symmetrical in z_i (in fact, this is a reduced form of an elliptic integral). Now

$$\int_{\tau^-(z_1)}^{\tau^+(z_1)} \frac{d\tau}{[K\tilde{K}]^{1/2}} = \int_{\tau^-(z_3)}^{\tau^+(z_3)} \frac{d\tau}{[K_0\tilde{K}_0]^{1/2}}$$

because one goes from one to the other by a permutation $z_1 \leftrightarrow z_3$.

In the general case the demonstration reduces to the preceding one, if we admit the following representation for $A_2(z_2)$:

$$A(z_2) = \int \frac{1}{z_2 - z_2^0} G(z_2^0) dz_2^0.$$

From the complete symmetry of

$$\int_{\tau^-(z_1)}^{\tau^+(z_1)} \frac{d\tau}{[K\tilde{K}]^{1/2}} = \int_{-1}^{+1} \frac{d\tau}{[P(\tau)]^{1/2}}$$

with respect to the z_i, it follows by analytic continuation in z that the relation (2.12) is still valid and thus the corresponding spectral parts cancel.

In our case, $A_1(z_1)$, $A_2(z_2)$, $A_3(z_3)$ are simple propagators and the demonstration of (10) and (13) is a particular case of the general one. The fact that for $A^{[5,67]}$ we take only the regular part is imposed by the reality argument for the $A_i(z_i)$. (This can be thought of as if the M mass grew from a normal value to its actual one.)

(2) The equality (19) is a consequence of the following relations:

$$\frac{1}{[\tilde{K}]^{1/2}} \ln \frac{V_d - [\tilde{K}'']^{1/2}}{V_d + [\tilde{K}'']^{1/2}} = \int_{s_{67}^{(1)}}^{s_{67}^{(2)}} \frac{ds_{67}}{(s_{67} - m^2)[\tilde{K}(s_{67}, t_{26}, s_{57})]^{1/2}}.$$

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Meson and Baryon Resonances in Relativistic $SU(6)$

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The relativistic $SU(6)$ supermultiplets which can be constructed from two quarks and two antiquarks, three quarks, or four quarks and one antiquark are examined as possible candiates for the classification of meson and baryon resonances. The multiplets can be labeled either by the $SU(12)$ formalism or by the $U(6)\times U(6)$ group of Dashen and Gell-Mann; the two groups lead to equivalent classifications. Those static $SU(6)$ multiplets within a given $SU(12)$ multiplet which represent physical particles (i.e., satisfy Bargmann-Wigner equations) are just the states having the maximal eigenvalue of γ_0 in the $SU(12)$ multiplet. These constitute a representation of the Dashen–Gell-Mann $U(6)\times U(6)$ subgroup of $U(12)$ which commutes with γ_0. Applications to specific resonances are discussed.

It has been noted that $SU(6)$ symmetry[1] and its relativistic modifications[2] severely restrict the classification of resonances which can decay without symmetry breaking into two mesons or into a single meson and a baryon. The analysis of this classification is facilitated by the use of the W-spin and B-spin subgroups of $SU(12)$.[3]

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[1] F. Gürsey and L. A. Radicati, Phys. Rev. Letters **13**, 173 (1964); A. Pais, Phys. Rev. Letters **13**, 175 (1964); B. Sakita, Phys. Rev. **136**, B1756 (1964).

[2] M. A. B. Bég and A. Pais, Phys. Rev. Letters **14**, 267 (1965); R. Delbourgo, A. Salam, and J. Strathdee, Proc. Roy. Soc. (London) **A284**, 146 (1965). B. Sakita and K. C. Wali, Phys. Rev. Letters **14**, 404 (1965); Phys. Rev. **139**, B1355 (1965); K. Bardakci, J. M. Cornwall, P. G. O. Freund, and B. W. Lee, Phys. Rev. Letters **13**, 698 (1964); **14**, 48 (1965).

[3] H. J. Lipkin and S. Meshkov, Phys. Rev. Letters, **14**, 670 (1965).

The decay into two mesons in the $SU(6)$ **35**, or into a meson in the **35** and a baryon in the **56**, is allowed only for those resonances classified in multiplets appearing in the products

$$\mathbf{35}\times\mathbf{35}=\mathbf{1}+\mathbf{35}+\mathbf{35}+\mathbf{189}+\mathbf{280}+\mathbf{280}^*+\mathbf{405}\,, \tag{1a}$$

$$\mathbf{35}\times\mathbf{56}=\mathbf{56}+\mathbf{70}+\mathbf{700}+\mathbf{1134}\,. \tag{1b}$$

Thus, for example, a baryon state classified in the **20** (the original classification of Sakita[1]) cannot decay into a **56** and a **35**. Such a resonance is allowed by $SU(6)$ to decay into a baryon and two mesons, where no pair is in a **35** or **56**. In the $SU(12)$ relativistic extension of $SU(6)$ the **35** and **56** are embedded in the $SU(12)$ representations **143** and **364**, respectively. The $SU(12)$ relations corresponding to Eq. (1) are

$$\begin{aligned}\mathbf{143}\times\mathbf{143}=\mathbf{1}+\mathbf{143}+\mathbf{143}+\mathbf{4212}\\ +\mathbf{5005}+\mathbf{5005}^*+\mathbf{5940}\,,\end{aligned} \tag{2a}$$

$$\mathbf{143}\times\mathbf{364}=\mathbf{364}+\mathbf{572}+\mathbf{16016}+\mathbf{35100}\,. \tag{2b}$$

The limitation of meson-meson and meson-baryon resonances to those $SU(12)$ representations appearing on the right-hand side of Eq. (2) has many consequences including the $SU(6)$ restrictions, Eq. (1). The $SU(6)$ particle content of a given $SU(12)$ representation is determined by the equations of motion which break $SU(12)$ and reduce the number of states which correspond to physical particles. The results are easily obtained from a quark model using W spin and B spin. Both the quark and antiquark have $B=\frac{1}{2}$, $B_3=\frac{1}{2}$ at rest. Thus, for any state considered as a number of quarks and antiquarks at rest, we obtain $B_3=B=N/2$, where N is the total number of quarks and antiquarks in the system. To each $SU(12)$ multiplet there corresponds a value of N. Only those states which have $B_3=B=N/2$ in the decomposition $SU(12)\supset SU(6)_W \times SU(2)_B$ can correspond to states of physical particles at rest. These states constitute a representation of the subgroup $SU(6)\times SU(6)$ of $SU(12)$, whose generators are all those $SU(12)$ generators which commute with B_3.[4]

The decomposition of the $SU(12)$ multiplets that appear in Eq. (2a) according to their $SU(6)\times SU(2)$ content is as follows:

$$\begin{aligned}
\mathbf{143}\ &:\ (35,3)(1,3)(35,1)\,,\\
\mathbf{4212}\ &:\ (189,5)(35,5)(1,5)(280,3)(280^*,3)(189,3)\\
&\quad 2(35,3)(405,1)(189,1)(35,1)(1,1)\,,\\
\mathbf{5005}\ &:\ (280,5)(35,5)(405,3)(280,3)(189,3)2(35,3)\\
&\quad (1,3)(280,1)(280^*,1)(35,1)\,,\\
\mathbf{5005}^*&:\ (280^*,5)(35,5)(405,3)(280^*,3)(189,3)2(35,3)\\
&\quad (1,3)(280^*,1)(280,1)(35,1)\,,\\
\mathbf{5940}\ &:(405,5)(35,5)(1,5)(405,3)(280^*,3)(280,3)\\
&\quad 2(35,3)(405,1)(189,1)(35,1)(1,1)\,.
\end{aligned} \tag{3a}$$

TABLE I. Classification of meson and baryon resonances according to $SU(12)$, $SU(6)\times SU(6)$, and $SU(6)$. Note that the **5005** and **5005*** are placed together since they are not self-adjoint and therefore meson states can appear only in the linear combination **5005+5005*** or **5005−5005***.

$SU(6)$ Rep.	$SU(6)\times SU(6)$	$SU(12)$	$B=B_3$	N	n
	Baryons				
56^+	(56,1)	364	$\frac{3}{2}$	3	0
70^+	(70,1)	572	$\frac{3}{2}$	3	0
700^-, 56^-	(126,6*)	16016	$\frac{5}{2}$	4	1
1134^-, 70^-, 56^-	(210,6*)	35100	$\frac{5}{2}$	4	1
	Mesons				
35^-, 1^-	(6,6*)	143	1	2	1
189^+, 35^+, 1^+	(15,15*)	4212	2	4	2
$280^+ + \overline{280}^+$ $35^+ + 35^+$	(15,21*)+(21,15*)	5005+5005*	2	4	2
405^+, 35^+, 1^+	(21,21*)	5940	2	4	2

The corresponding reduction for the baryon multiplets, Eq. (2b), is

$$\begin{aligned}
\mathbf{364}\ \ &:\ (56,4)(70,2)\,,\\
\mathbf{572}\ \ &:\ (70,4)(70,2)(56,2)(20,2)\,,\\
\mathbf{16016}&:\ (700,6)(56,6)(1134,4)(700,4)(70,4)(56,4)\\
&\quad (1134,2)(560,2)(70,2)(56,2)\,,\\
\mathbf{35100}&:\ (1134,6)(70,6)(56,6)2(1134,4)(700,4)(560,4)\\
&\quad (540,4)3(70,4)2(56,4)(20,4)2(1134,2)(700,2)\\
&\quad (560,2)2(540,2)3(70,2)2(56,2)(20,2)\,.
\end{aligned} \tag{3b}$$

Using the above criterion, one can read from this decomposition the allowed $SU(6)$ particle multiplets. In addition, we note that the parity of an $SU(12)$ representation is determined under the assumption that the spins of all resonances are given by the $SU(12)$ quantum numbers and there is no angular momentum involved which is external to $SU(12)$. This classification, which we call extreme $SU(12)$ symmetry, is then the same as that obtained in a quark model in which all resonances are composed of quarks and antiquarks in s states. In extreme $SU(12)$ symmetry the parity of an $SU(12)$ representation is determined by the number of antiquarks n that it contains, and equals $(-1)^n$.[5] The $SU(6)$ representations[6] which are appropriate for the classification of meson and baryon resonances are listed in Table I.

In general, a given $SU(12)$ representation contains several different $SU(6)$ representations which can accommodate physical particles. This introduces complications in the W spin and $SU(6)_W$ classification. A physical particle at rest is in an eigenstate of ordinary S spin and is a member of a single multiplet in the

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[4] B_3 is defined as $\gamma_0/2$ both for quarks and antiquarks. B_3 is not conserved in motion. A theory based on the group $U(6)\times U(6)$ of operators which commute with γ_0 has been proposed by R. F. Dashen and M. Gell-Mann, Phys. Letters **17**, 142 (1965). The same classifications of particles are obtained from both broken $U(12)$ and $U(6)\times U(6)$.

[5] A. Salam, J. Strathdee, J. M. Charap and P. T. Matthews, Phys. Letters **15**, 184 (1965).

[6] Note that the decomposition (3) applies to $SU(12)\supset SU(6)_S \times SU(2)_D$ as well as $SU(6)_W\times SU(2)_B$. Since $D_3=B_3=\gamma_0/2$, the choice of allowable physical multiplets is the same in $SU(6)_S$ as in $SU(6)_W$ for particles at rest.

$SU(6)_S$ classification. Since S spin and W spin do not commute, a physical particle at rest might not be in a W-spin eigenstate and might be in a mixture of several states which belong to different $SU(6)_W$ multiplets. For the common meson and baryon multiplets, **143**, **364**, and **572**, this difficulty does not arise. The **364** and **572** each contain only a single $SU(6)$ multiplet and contain only quarks and no antiquarks; thus, their classification under W spin and $SU(6)_W$ is the same as under S spin and $SU(6)_S$ at rest. The **143** contains two $SU(6)$ multiplets, but one is a singlet, which can only mix with a single state in the **35**, namely, the $SU(3)$ singlet. Furthermore, the quark-antiquark system has the peculiar property of W-S flip,[7] in which the singlet and triplet states are simply interchanged, but not mixed. Thus, the $SU(6)$ singlet meson (possibly the X^0) has W spin one, is in the same W-spin triplet as the $SU(3)$ singlet vector-meson states with transverse polarization, and belongs to the meson **35** in $SU(6)_W$. The singlet of $SU(6)_W$ is the longitudinally polarized $SU(3)$ singlet vector-meson state. For all other $SU(12)$ representations listed, some of the particles will be mixtures of states belonging to different $SU(6)_W$ representations.

The information obtained above can now be used for the classification of resonances. We first consider the baryonic resonances. The **364** contains the well-known **56**$^+$. A **70**$^-$ representation has been suggested[8] as a possible location of the γ octet and several other resonances. We see that this **70**$^-$ cannot be a member of **572** because of parity. However, this **70**$^-$ can belong to the **35100**. In such a case its existence would predict the existence of more resonances belonging to the **1134**$^-$ and **56**$^-$ of **35100**. Another possible location of the γ octet is in the **700**$^-$ of **16016**. In this case an additional **56**$^-$ is also predicted. Some candidates for the γ octet are the $Y_1^*(1660)$[9] and the $\Xi^*(1810)$.[10] A spin-$\frac{3}{2}^+$ assignment is also possible, in which case they could be placed in the **70**$^+$ of **572**.

Extreme $SU(12)$ predicts that all baryonic resonances that belong to the **700** or **1134** have negative parity. There are some cases when resonances must belong to these representations. These are all particles belonging to the $SU(3)$ multiplets **35**, **27**, and **10*** and all particles whose spin is $\frac{5}{2}$. An additional prediction is that no meson-baryon resonance can have spin greater than $\frac{5}{2}$. This means that all $Y=2$ or $T=\frac{5}{2}$ resonances have negative parity. No $Y=2$ resonance has yet been reported. The $T=\frac{5}{2}$ $p\pi^+\pi^+$ enhancement reported[11] at 1600 may belong to a **35** of $SU(3)$.[12] Its decay into $N^{*++}\pi^+$ is likely. In some cases the above predictions do not hold: The $N_{1/2}^*$ (1688) decays mainly into πN and has spin and possible parity $\frac{5}{2}^+$.[13] The $Y_0^*(1815)$ decays mainly into $\bar{K}N$ and has spin $\frac{5}{2}$ and its parity is supposedly positive.[13] The $N_{3/2}^*(1920)$ is known to have spin $\frac{7}{2}$.[13] Spins higher than $\frac{5}{2}$ have also been suggested for $N_{1/2}^*(2190)$ and $N_{3/2}^*(2360)$.[13]

Extreme $SU(12)$ predicts that only the meson-meson resonances which belong to the **143** have negative parity. This representation accommodates the known **35** and a singlet, possibly the X^0. Other meson-meson resonances, in particular those having spin 2, or those which belong to $SU(3)$ representations other than **1** or **8**, have positive parity. The maximum allowed spin is 2. Classifications of the existing resonances into $SU(6)$ and $SU(12)$ multiplets have been suggested.[14] No obvious contradictions to extreme $SU(12)$ predictions can be deduced from the present experimental data.

A possible remedy for the troubles of extreme $SU(12)$ would be the introduction of orbital angular momentum for the quark systems. Such an approach was favored in the days before the relativistic extension of $SU(6)$ was suggested. In that case the total angular momentum of a resonance in its rest system is the vector sum of the spin, which is given by $SU(12)$, and the orbital angular momentum L which is external to $SU(12)$ and transforms like a singlet under it. One then obtains "super-supermultiplets" with an additional multiplicity factor of $2L+1$ over the $SU(6)$ multiplicity. The parity and total-angular-momentum assignments will thus be modified. The introduction of an orbital angular momentum will also modify other predictions of $SU(12)$, such as the incorrect predictions of polarizations in meson-baryon scattering processes.

Another possible treatment for decays forbidden by extreme $SU(12)$ which are nevertheless observed, is the introduction of symmetry breaking by a spurion. However, the properties of the spurion should be chosen to insure that a selection rule forbidding the decay does not remain even after the symmetry breaking.[15]

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[7] D. Horn, M. Kugler, H. J. Lipkin, S. Meshkov, J. C. Carter, and J. J. Coyne, Phys. Rev. Letters **14**, 717 (1965).

[8] I. P. Gyuk and S. F. Tuan, Phys. Rev. Letters **14**, 121 (1965).

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[10] So far, spin and parity are undetermined, see R. Armenteros, in *Proceedings of the International Conference on High Energy Physics, Dubna, 1964* (Moscow, 1965); G. A. Smith, James S. Lindsey, Janice Button-Shafer, and Joseph J. Murray, Phys. Rev. Letters **14**, 25 (1965).

[11] G. Goldhaber, S. Goldhaber, T. O'Halloran, B. C. Shen, in *Proceedings of the International Conference on High Energy Physics, Dubna, 1964* (Moscow, 1965); G. Alexander, O. Benari, B. Haber, U. Karshon, N. Kidron, B. Reuter, A. Shapira, E. Simopoulou, and G. Yekutieli (private communication). (The decay into $N^{*++}\pi^+$ is possible in this work); G. Goldhaber, *Proceedings of the Second Coral Gables Conference, 1965* (W. H. Freeman and Company, San Francisco, California, 1965).

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[14] R. Delbourgo, M. A. Rashid, and J. Strathdee, Phys. Rev. Letters **14**, 719 (1965).

[15] See footnote 6 in H. J. Lipkin, Phys. Rev. Letters **14**, 513 (1965).