

Inelastic Deuteron Scattering and (d,p) Reactions from Isotopes of Titanium. III. $Ti^{49}(d,p)Ti^{50\dagger}$

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Fifty-six levels in Ti^{50} have been excited by the (d,p) reaction on Ti^{49} at 6-MeV bombarding energy. The differential cross sections for the proton groups were measured at 23 scattering angles. A distorted-wave analysis of the experimental results yielded values of the orbital angular momenta of the transferred neutron and transition strengths for 44 of the observed transitions. A sum-rule analysis indicates that all of the $1f_{7/2}$, $2p_{3/2}$, and $2p_{1/2}$ single-particle strengths and 60% of the $1f_{5/2}$ strength were observed. The present data are compared to other experimental evidence on Ti^{50} , and a level scheme for Ti^{50} is proposed. The spectroscopic data are discussed in terms of the shell model with residual interactions. The results for the transitions to the ground state and three lowest excited states of Ti^{50} are shown to be in disagreement with seniority conservation.

1. INTRODUCTION

THE present paper is a report of the results of an investigation of the $Ti^{49}(d,p)Ti^{50}$ reaction. The 6-MeV deuteron beam from the MIT-ONR electrostatic generator and the multigap spectrograph of Enge and Buechner¹ were used. In previous papers we have reported the results from observations on the $Ti^{50}(d,p)Ti^{51}$ reaction² and on the $Ti^{47}(d,p)Ti^{48}$ reaction³ performed under the same experimental conditions employed in the present experiment.

Fifty-six levels in Ti^{50} were observed, ranging from 0 to 7.66-MeV excitation energy. Forty-four of these transitions showed angular distributions of stripping character. Values of the orbital angular momentum of the transferred neutron (l_n values) and of the transition strengths $(2J_f+1)S_{ij}$ were derived from a distorted-wave (DW) analysis of the observed cross sections.

The experimental results are presented in Sec. 2. In Sec. 3.1 we propose a level scheme for Ti^{50} based on the available experimental data; Sec. 3.2 contains a comparison of the spectroscopic data of the present experiment to spectroscopic information from earlier work. Our results are discussed in Sec. 4, partly in terms of sum-rule limits (Sec. 4.1), and partly in terms of currently used nuclear models^{4,5} (Sec. 4.2).

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¹ H. A. Enge and W. W. Buechner, *Rev. Sci. Instr.* **34**, 155 (1963).

² P. D. Barnes, C. K. Bockelman, O. Hansen, and A. Spurduto, *Phys. Rev.* **136**, B438 (1964) [Paper I on $Ti^{50}(d,p)Ti^{51}$].

³ P. D. Barnes, C. K. Bockelman, O. Hansen, and A. Spurduto, *Phys. Rev.* **138**, B597 (1965). [Paper II on $Ti^{47}(d,p)Ti^{48}$].

⁴ I. Talmi, *Phys. Rev.* **126**, 1096 (1962); A. de-Shalit, in *Selected Topics in Nuclear Theory*, edited by F. Janouch (International

2. RESULTS

The experimental procedure has been described in detail in two previous papers.^{2,3}

Figure 1 presents a $Ti^{49}(d,d)$ angular distribution measured at 6.00-MeV bombarding energy in comparison with an optical-model prediction derived from the $Ti^{48,49,50}$ average optical potential B_4 of Ref. 2. Figure 2

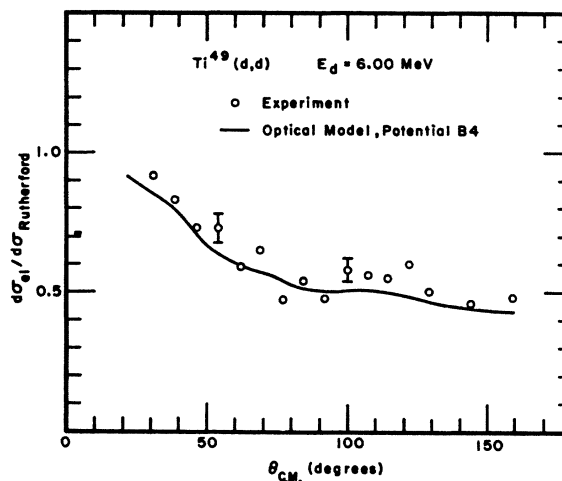


FIG. 1. Angular distributions of 6.0-MeV deuterons elastically scattered from Ti^{49} . The open circles are the experimental cross sections in units of the Rutherford cross section. The vertical bars indicate statistical errors only, and do not include the 24% error in absolute cross section. The solid curve is an optical-model prediction computed from a complex potential (B_4 of Ref. 2) which fits the 6-MeV deuteron scattering from $Ti^{48,49,50}$. Further detail is given in Ref. 2.

Atomic Energy Agency, Vienna, 1963); A. de-Shalit and I. Talmi, *Nuclear Shell Theory* (Academic Press Inc., New York, 1964).

⁵ J. D. McCullen, B. F. Bayman, and L. Zamick, *Phys. Rev.* **134**, B515 (1964).

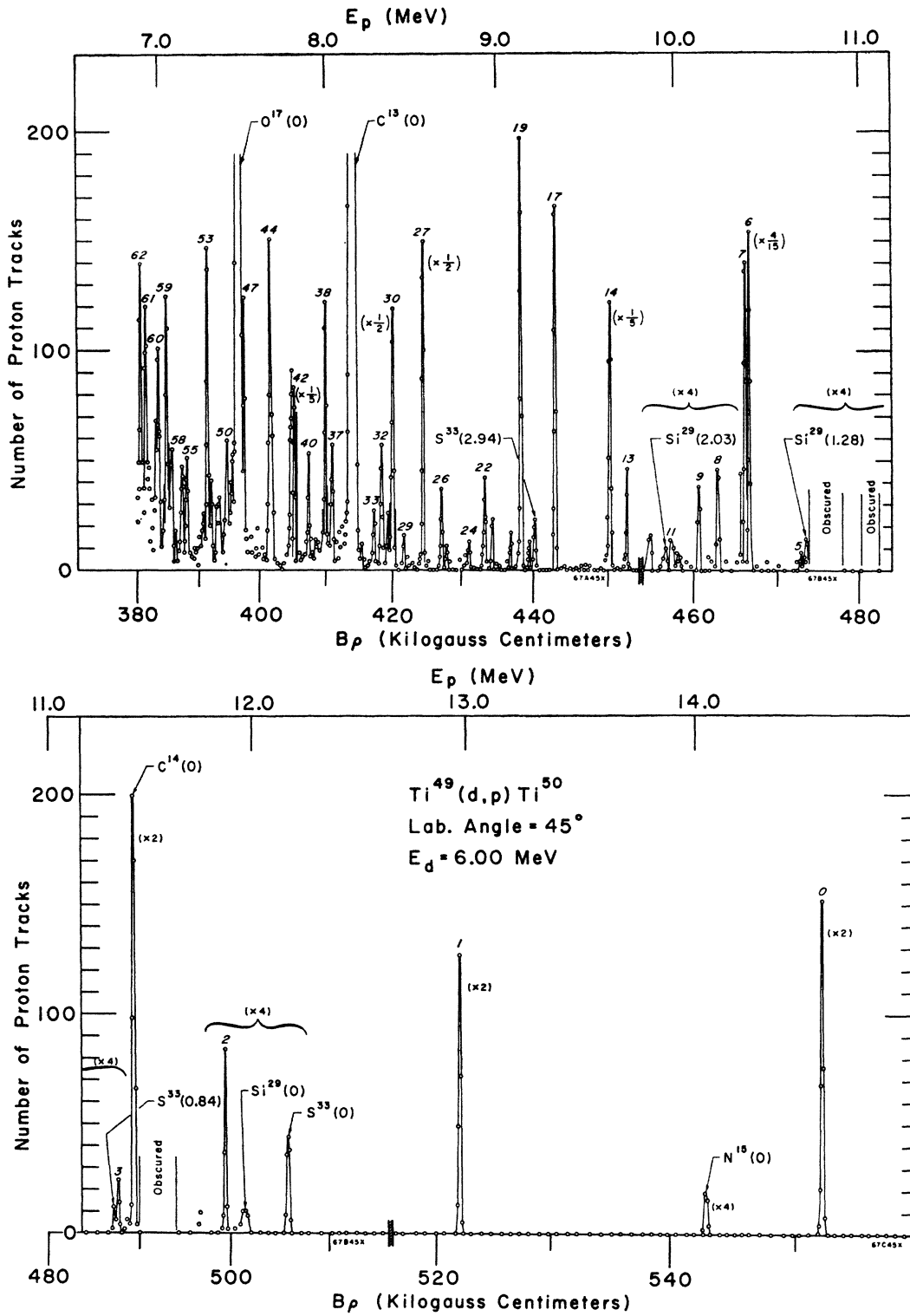


FIG. 2. Measured proton spectrum at laboratory angle 45° . The number of proton tracks in a 0.25-mm strip across the exposed zone is plotted against position along the photographic emulsion. The spectrograph calibration fixes the superimposed scale of magnetic rigidity. Proton groups identified by their kinematic shift as signifying levels in Ti^{50} are labeled with the numbers used to identify these states in Tables I and II. Prominent contaminant groups are also identified.

TABLE I. $\text{Ti}^{49}(d,p)\text{Ti}^{50}$ results. The first column gives the level numbers. Missing numbers indicate that a level known from other experiments (see Table II) was not excited under the present conditions. Column 2 shows the presently obtained excitation energies. The energies marked with an asterisk were measured on the single-gap broad-range spectrograph, and are known to ± 6 keV. Energies for other levels were obtained from these values, combined with Q -value differences measured on the multigap spectrograph, and are accurate to ± 20 keV. The subcolumns of column 3 list the $l_n=0, 1, 2, 3,$ and 4 strengths, respectively, derived from the DW analysis of the measured cross sections. Absolute cross-section errors of $\pm 25\%$ are also assigned to the strengths, except for $l_n=3$ transitions at excitation energies above ≈ 6.5 MeV, where difficulties with impurity groups and background increases the absolute error to $\pm 40\%$. Relative errors are $\pm 10\%$ except for the above mentioned $l_n=3$ transitions where relative errors are $\pm 30\%$. The last column presents the maximum observed cross section for each transition. If this number is given in parentheses, it means that the cross section was measured at an angle other than the one expected to give maximum yield for the l_n values involved, the yield at this angle being obscured.

Level No.	E_{ex} (MeV)	$(2J_f+1)S_{ij}^{J_f}$					$(d\sigma/d\Omega)_{\text{max}}$	Level No.	E_{ex} (MeV)	$(2J_f+1)S_{ij}^{J_f}$					$(d\sigma/d\Omega)_{\text{max}}$
		$l=0$	$l=1$	$l=2$	$l=3$	$l=4$				$l=0$	$l=1$	$l=2$	$l=3$	$l=4$	
0	0				11.3		0.21	34	6.325						(0.02)
1	1.555*				3.5		0.27	35	6.392*						0.42
2	2.686*		0.92				0.13	37	6.498*						0.17
3	3.208		0.46				0.01	38	6.536*						0.66
5	3.879		nonstripping				0.01	39	6.592				3.5		0.04
6	4.158		nonstripping				3.2	40	6.636*				2.4		0.10
7	4.184		7.9				3.7	41	6.697						0.02
8	4.322		9.9				0.09	42	6.726*						(1.43)
9	4.422	0.06	0.27				0.08	43	6.744	0.18					0.42
10	4.536		0.15				0.03	44	6.863*						(0.69)
11	4.576		no unique l_n				0.04	45	6.913		1.3		1.9		0.02
13	4.808*		no unique l_n				0.24	46	6.986		nonstripping				0.02
14	4.898*		0.65				4.0	47	7.025		nonstripping				0.40
17	5.203*		10.2				1.14	48	7.049		0.54				0.22
18	5.348		2.7				0.02	49	7.094		0.30				0.16
19	5.395*		nonstripping				1.28	50	7.132			0.36			0.18
20	5.440*		3.1				0.05	51	7.178			0.55			(0.06)
21	5.561*		0.12		0.41		0.17	52	7.229				1.55		0.12
22	5.600*		0.39				0.06	53	7.249*		0.16				0.68
24	5.717*		0.18		0.65		0.04	54	7.280		0.82				0.07
25	5.821		nonstripping				0.03	55	7.387*			0.08		1.3	0.12
26	5.851		0.05		0.83		0.12	56	7.407					6.6	0.18
27	5.956*		0.23		1.2		1.75	57	7.447		0.23				
29	6.079		4.1				(0.07)	58	7.471		no unique l_n				(0.33)
30	6.138*		0.25				0.95	59	7.504		0.43				0.77
31	6.176		0.37		3.8		(0.19)	60	7.550		1.0		1.3		0.27
32	6.210		(0.50)				(0.15)	61	7.631		0.26		2.1		0.60
33	6.250*		0.07		1.3		0.06	62	7.663		0.79		7.3 ^a		0.40

^a This distribution could alternatively be fitted with $l_n=2+4$.

displays a $\text{Ti}^{49}(d,p)\text{Ti}^{50}$ spectrum observed at a laboratory angle of 45° .

The excitation energies, l_n values, and transition strengths obtained in the present experiment are listed in Table I. The table also gives the maximum observed cross section for each transition. The complete data may be obtained from the authors.

The l_n values and transition strengths were derived from DW predictions by means of the procedures described in Refs. 2 and 3. Excitation energies were determined from Q -value differences obtained from the multigap-spectrograph measurements together with absolute Q -value determinations for a number of transitions measured in the MIT single-gap spectrograph. The specific levels for which the absolute Q values were obtained are indicated with an asterisk in Table I. The $\text{Ti}^{49}(d,p)\text{Ti}^{50}$ ground-state Q value was measured to be 8.733 ± 0.006 MeV, based on an energy standard for Po^{210} α particles of $5.3042 + 0.0016$ MeV. The measurements are in good agreement with the results of Ref. 6.

3. COMPARISON WITH OTHER EXPERIMENTS

3.1. Level Scheme

The experimental data⁶⁻¹⁵ available on the first 42 levels of Ti^{50} are collected in Table II. No information, except that presently obtained, is known to us beyond level 42. The identification of levels obtained in one experiment with levels obtained in another experiment is always consistent with all the information available, but should generally be taken rather as a proposal than a certain fact (cf., the caption for Table II).

⁶ O. Hansen, Nucl. Phys. **28**, 140 (1961); J. H. Bjerregaard, P. F. Dahl, O. Hansen, and G. Sidenius, *ibid.* **51**, 641 (1964).

⁷ S. Hinds and R. Middleton (private communication).

⁸ D. R. Koehler and W. L. Alford, Nucl. Phys. **41**, 520 (1963).

⁹ G. Chilosi, P. Cuzzacrea, G. B. Vingiani, R. A. Ricci, and H. Morinaga, Nuovo Cimento **27**, 86 (1963).

¹⁰ B. Zeidman (private communication).

¹¹ G. F. Pieper, Phys. Rev. **88**, 1299 (1952).

¹² J. L. Yntema, Phys. Rev. **131**, 811 (1963).

¹³ K. Ilakovac, L. G. Kuo, M. Petravić, I. Slaus, P. Tomas, and G. R. Satchler, Phys. Rev. **128**, 2739 (1962).

¹⁴ H. O. Funsten, N. R. Roberson, and E. Rost, Phys. Rev. **134**, B117 (1964).

¹⁵ W. S. Gray, R. A. Kenefick, and J. J. Kraushaar, Nucl. Phys. **67**, 565 (1965).

3.2. Spectroscopic Evidence

The $Ti^{49}(d, p)Ti^{50}$ reaction has been investigated by Yntema¹² at a bombarding energy of 21 MeV. The spectroscopic results of this experiment are compared to our data in Table III. The agreement of the two sets of $l_n=1$ strengths indeed is very good, whereas the $l_n=3$ strengths for levels (0) and (1) given by Yntema are a factor of ≈ 2 lower than the presently measured values. A similar discrepancy was found in the $Ti^{47}(d, p)-Ti^{48}$ case and might indicate that the intrinsic $l_n=1$ to $l_n=3$ cross-section ratio, as predicted by the DW theory, is incorrect for one (or both) sets of experimental conditions. Yntema finds a small $l_n=3$ admixture in the transition to level (2); the presently obtained angular distribution is pure $l_n=1$. For definiteness, we shall in the further discussions use the present data on the Ti^{50} states (0), (1), and (2).

Proton pickup reactions on V^{51} (Refs. 10, 13) excite levels (0), (1), (2), and (3) with $l_p=3$ transition strengths which are consistent with a description of these states as members of a $(f_{7/2})^2$ proton configuration.

Neutron pickup reactions^{16, 17} on Ti^{50} indicate a p -neutron admixture in the Ti^{50} ground state of ≈ 0.4 particles; similarly, for the Ti^{49} ground state an admixture of ≈ 0.4 p particles is observed.¹⁶ Recent proton pickup data¹⁸ on Ti shows that the Ti isotopes have almost pure $(f_{7/2})^2$ proton configurations; the $(p_{3/2})^2$ admixtures are probably $\lesssim 0.2$ particles.

4. DISCUSSION

4.1. Strength Functions

The (d, p) transition strengths of Table I are plotted against excitation energy in Fig. 3. Certain characteristic trends observed for the Ti^{50} strength functions are similar to those of Ti^{48} (see Ref. 3): (a) the $l_n=3$ strength divides into two groups which are interpreted as corresponding to $1f_{7/2}$ (low excitation energies) and $1f_{5/2}$ (high-excitation energies) neutron transfers, respectively; (b) the $l_n=1$ strength function does not show such a grouping; (c) the $l_n=1$ strength is distributed over a range of ≈ 5 -MeV excitation energy.

The mean $f_{7/2}$ excitation energy is somewhat lower in the Ti^{50} case than in the Ti^{48} case, reflecting the higher degree of filling of the $1f_{7/2}$ shell in Ti^{50} .

The p strength above 7-MeV excitation is quite low, which may indicate that most of the available p strength has been observed. No indications that the $f_{5/2}$ strength has been used up are seen from Fig. 3.

The onset of $l_n=4, 2,$ and 0 groups observed at the highest excitation energies explored presumably signals the beginning of the $1g_{9/2}, 2d_{5/2},$ and $3s_{1/2}$ strength functions.

TABLE II. Experimental data on the first 42 levels of Ti^{50} . (For levels 43–62, see Table I.) This table is assembled from the data available on the Ti^{50} levels below the highest excitation observed in the present experiment, 7.663 MeV. Level numbers are assigned in column 1 in order of increasing excitation energy. The evidence for levels higher than level 42 is derived only from the present experiment (see Table I). In column 2 are listed values of the excitation energy. For levels observed in the present experiment the values of Table I are listed; for other levels the listed values are those which seem best to the present authors. The known spins and parities are given in column 3. The last column indicates the modes of excitation which have been employed in exciting the level in question. The identification of a particular level seen in one experiment with a level observed in another experiment is often uncertain. The criteria used for the identifications in the present table are: (1) the energies of levels from different experiments must coincide within the errors quoted, and (2) the data on a given level cannot be obviously contradictory, e.g., a level assigned negative parity in one experiment cannot be identified with a state showing l_n -odd stripping in the present experiment [an exception to this rule is level (17)]. In the case of levels (1) and (2) the first criterion is violated by the wide spread of energies quoted in the various experiments [e.g., from 1.4 to 1.570 MeV for level (1)].

Level No.	E_{ex} (MeV)	J^π	Decay	Level reported from				
				(p, p')	(n, d)	(d, p)	(d, He^2)	(t, p)
0	0	0^+	a, b		c	d, e, f, g	h	k
1	1.555	2^+	a, b	d, i, j	c	d, e, f, g	h	
2	2.686	(4^+)	a, b	i, j	c	d, f, g	h	
3	3.208	(6^+)	a, b		c	g	h	
4	3.771					d		
5	3.879	0^+				g		k
6	4.158	$+$				d, e, f, g		
7	4.184	$+$		i, j		d, e, f, g		
8	4.322	$+$		j		d, g		
9	4.422	3^-		i, j		d, g		
10	4.536					g		
11	4.576					g		
12	4.738			j				
13	4.808	$+$		j		d, g		
14	4.898	$+$				d, e, f, g		
15	4.960			j				
16	5.106					d		
17 ¹	5.203	$+$		j		d, f, g		
18	5.348			j		d, g		
19	5.395	$+$		j		d, e, f, g		
20	5.440	$+$		j		g		
21	5.561	$+$		j		g		
22	5.600	$+$		j		g		
23	5.638	0^+		j				k
24	5.717			j		g		
25	5.821	$+$		j		g		
26	5.851	$+$		j		g		
27	5.956	$+$				e, f, g		
28	6.045	0^+						k
29	6.079	$+$		j		g		
30	6.138	$+$				g		
31	6.176	$+$		(j)		g		
32	6.210	$-$		(j)		g		
33	6.250	$+$		(j)		g		
34	6.325					g		
35	6.392	$+$				g		
36	6.459			j				
37	6.498	$+$		j		g		
38	6.536	$+$		j		g		
39	6.592					g		
40	6.636	$+$				g		
41	6.697					g		
42	6.726	$+$		j		g		

^a See Ref. 8.

^o See Ref. 11.

^b See Ref. 10.

^b See Ref. 9.

[†] See Ref. 12.

[†] See Ref. 14.

^c See Ref. 13.

^{*} Present work.

[‡] See Ref. 15.

^d See Ref. 6.

¹ See Ref. 7 (only $L=0$ states quoted).

² This level is assigned 3^- in Ref. 15, but 3^+ is also a possible assignment. If the negative parity assignment is correct, there are two levels with a separation less than 5 keV in this region.

¹⁶ E. Kashy and T. W. Conlon, Phys. Rev. **135**, B389 (1964).

¹⁷ J. L. Yntema, Phys. Rev. **127**, 1659 (1962).

¹⁸ J. L. Yntema and G. R. Satchler, Phys. Rev. **134**, B976 (1964).

TABLE III. Comparison of the strengths observed in the present work with those derived by Yntema (Ref. 12) at a deuteron energy of 21 MeV. Note that "strength" in the present text is used for the quantity $(2J_f+1)S_{ij}$, contrary to the usage in Ref. 12 where "strength" means the spectroscopic factor itself.

Level No.	$l=1$ strength		$l=3$ strength	
	Ref. 12	Present	Ref. 12	Present
0	5.5 ± 1.4	11.3 ± 2.8
1	1.0 ± 0.2	0.92 ± 0.2	1.9 ± 0.5	3.5 ± 0.9
2	0.45 ± 0.1	0.46 ± 0.1	0.6 ± 0.2	...
6+7	$\approx 20^*$	17.8 ± 4.4

* This number is not given in Ref. 12, but is estimated by us from the data contained in Figs. 6 and 7 of that reference.

The observed strengths of definite (l, j) summed over final states are presented in Table IV together with the theoretical expectations for a $(f_{7/2})^{-1}$ neutron configuration of the target ground state. It appears that the theoretical sum-rule limits for the $f_{7/2}$ and $2p_{3/2} + 2p_{1/2}$ transitions are fulfilled within the experimental errors, whereas only part of the $1f_{5/2}$ strength and very little of the $1g_{9/2}$, $2d_{5/2}$, and $3s_{1/2}$ strengths have been found.

TABLE IV. Sum-rule limits. The strengths of Table I, divided by $(2J_i+1)=8$ and summed over final states are given in the second row for each of the observed l values, which are indicated by the shell-model notations of the first row. The numbers of the second row thus correspond to the summed single-particle strength, in units of neutron holes. The division of the $l_n=3$ strength into $f_{7/2}$ and $f_{5/2}$ is suggested from the strength functions of Fig. 3, as discussed in the text. The theoretical sum-rule limits, assuming a pure $(f_{7/2})^{-1}$ neutron configuration in $\text{Ti}^{49}(0)$, are given in the third row. The sum rules are stated in Ref. 3.

$(2J_f+1)S_{ij}$	$1f_{7/2}$	$2p_{3/2} + 2p_{1/2}$	$1f_{5/2}$	$3s_{1/2}$	$2d_{5/2}$	$1g_{9/2}$
Expt.	1.8 ± 0.5	6.9 ± 1.7	3.7 ± 1.1	0.03	0.2	0.8
Theory	1	6	6	2	6	10

The $1f_{7/2}$ strength of 1.8 ± 0.5 neutron holes observed here would correspond to a neutron pickup strength in $\text{Ti}^{49}(0)$ of 6.2 particles. The number found in the (d, t) experiment of Yntema¹⁷ was 6.5 neutrons, whereas Kashy and Conlon¹⁸ found 4.6 neutrons. As was the case in the $\text{Ti}^{47}(d, p)\text{Ti}^{48}$ reactions,⁸ our number agrees well with the Yntema result, whereas the (p, d) number is lower. However, it should be emphasized that the agreement with Ref. 17 regarding absolute strengths may be accidental.

The presently observed $2p$ strength of 6.9 ± 1.7 holes is consistent with the admixture of approximately 0.4 p particles in the Ti^{49} ground state as seen by Kashy and Conlon.¹⁶

4.2. Comparison to Current Nuclear Models

The Ti^{50} states (0), (1), (2), and (3) seem well described in terms of a proton $(f_{7/2})^2$ configuration. It also appears to be reasonable in a first approximation to describe the Ti^{49} ground state in terms of two $f_{7/2}$ protons and one $f_{7/2}$ neutron hole, ignoring the slight $p_{3/2}$ neutron admixture.

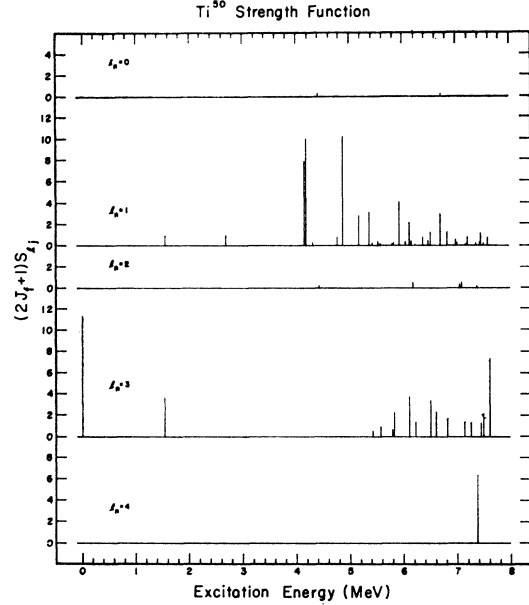


FIG. 3. The strengths $(2J_f+1)S_{ij}$ listed in Table I are plotted as a function of excitation energy for the observed values of orbital angular momentum of the transferred neutron, l_n .

The simplest $\text{Ti}^{49}(0)$ ground-state configuration is one in which the protons couple their spins to zero, i.e.,

$$|\text{Ti}^{49}(0)\rangle = [\pi(f_{7/2})^2_0 \nu(f_{7/2})^{-1}]_{7/2}. \quad (1)$$

Here π indicates protons, and ν indicates neutrons, while the square brackets symbolize vector coupling to a spin of $\frac{7}{2}$.

The only Ti^{50} state excited by a (d, p) reaction is $\text{Ti}^{50}(0)$ in this model. As experiment shows that $\text{Ti}^{50}(1)$ is also excited by a $1f_{7/2}$ transition, excited-proton configurations must be included in $\text{Ti}^{49}(0)$.

In place of the wave function (1), one may substitute

$$|\text{Ti}^{49}(0)\rangle = a_0[\pi(f_{7/2})^2_0 \nu(f_{7/2})^{-1}]_{7/2} + a_2[\pi(f_{7/2})^2_2 \nu(f_{7/2})^{-1}]_{7/2} + a_4[\pi(f_{7/2})^2_4 \nu(f_{7/2})^{-1}]_{7/2} + a_6[\pi(f_{7/2})^2_6 \nu(f_{7/2})^{-1}]_{7/2}. \quad (2)$$

The coefficients a_J are the amplitudes for the occurrence of a proton state of spin J in the Ti^{49} ground state. Wave functions of this type have been used in the $1f_{7/2}$ shell by de-Shalit and Talmi¹⁹ and by McCullen *et al.*⁵

In the work of de-Shalit and Talmi the coefficients a_J were chosen so that the wave functions contain a definite seniority ν as well as a definite isospin T and reduced isospin t . Choosing for the $\text{Ti}^{49}(0)$ wave function $\nu=1$, $T=\frac{5}{2}$ and $t=\frac{1}{2}$ one finds for the (d, p) transition to $\text{Ti}^{50}(0)$ a strength of 7.71, i.e., a strength of 0.3 remains for the transitions to the 2+, 4+, and 6+ $\nu=2$ states of Ti^{50} . According to de-Shalit and Talmi¹⁹ the coefficients

¹⁹ A. de-Shalit and I. Talmi, *Nuclear Shell Theory* (Academic Press Inc., New York, 1964), Chaps. 34 and 35.

of fractional parentage for the transitions to the $v=2$ states are equal, i.e., the strengths are proportional to $(2J_f+1)$. This is contrary to experiment; the $2+$ state was observed, but neither the $4+$ nor the $6+$ states were found to have any $l_n=3$ strength. Further, the ratio of the transition strengths to $Ti^{50}(0)$ and to $Ti^{50}(1)$ is experimentally far from the predicted ratio of $\approx 150:1$. It is concluded that wave functions which preserve a definite seniority are in conflict with the present experiment; similar disagreement in predictions for pickup reactions in the Ti isotopes has been noted earlier by Yntema.¹⁷

The wave functions of McCullen *et al.*⁵ do not have a definite seniority but still maintain T as a good quantum number. These wave functions yield for the transitions to $Ti^{50}(0)$ and $Ti^{50}(1)$ a strength ratio of 5 to 1 compared to the experimental ratio of 3.2 ± 0.8 to 1. The theoretical strengths for $l_n=3$ transitions to the $4+$ and $6+$ states of Ti^{50} are negligibly small, in agreement with experiment.

It is concluded that the model of McCullen *et al.*⁵ for $Ti^{49}(0)$, together with an $(f_{7/2})^2$ proton configuration for the final states, gives an essentially correct description of the $f_{7/2}$ part of the (d, p) transitions to states (0), (1), (2), and (3) of Ti^{50} . The assumption of pure $f_{7/2}$ configurations made in Ref. 5, however, does not hold true for the neutron part of the configurations, since a certain amount of p neutron strength is observed experimentally in $Ti^{49}(0)$, $Ti^{50}(1)$, and $Ti^{50}(2)$.

Among the higher lying states the two closely spaced levels at 4.158 and 4.184 MeV are remarkable in that they carry an appreciable part of the available p strength. According to the above discussion the $\pi(f_{7/2})^2_0$ part of the $Ti^{49}(0)$ wave function is dominant; thus it is reasonable to assume that the principal configuration responsible to the 4.158- and 4.184-MeV states is the $(f_{7/2})^{-1}(p_{3/2})^1$ neutron configuration. Four states of spins 2, 3, 4, and 5, respectively, and of positive parity should originate from such a configuration. The recent (t, p) work of Hinds and Middleton⁷ shows that the 4.158-MeV level probably has spin 4 and the 4.184-MeV state has spin 2. Candidates for the spin 3 and 5 states may be found among the remaining strong $l=1$ transitions. A characteristic feature of the $3+$ state of the $(f_{7/2})^{-1}(p_{3/2})^1$ configuration, would be that this state could be excited in direct (d, d) and (p, p) with $L=2$

and spin flip, but it would not be seen in direct (α, α'). It is too early to carry the discussion of the higher excited states further, but the combination of evidence from (d, p), (t, p) and inelastic scattering reactions seems to be a possible spectroscopic tool for such states.

The strong $L=0$ states excited in the (t, p) reaction, when observed in (d, p), are all of nonstripping character, indicating that they do not belong to the $f_{7/2}$ neutron configuration.

State (9) at 4.42 MeV has $l_n=0+2$ character though its strength is quite small. It also appears isolated from other negative parity states by 2 MeV (see, e.g., Fig. 3). Energetically state (9) coincides with the 3- state strongly excited in inelastic proton scattering (see, e.g., Table II). If the states excited in the two experiments indeed are the same, the present results suggest an interesting confirmation of the microscopic character of vibrational states suggested by Mottelson.²⁰ In the microscopic picture a vibrational state is assumed to be built on a superposition of two-quasiparticle shell-model states. In the present case, the octupole vibration would involve states as $f_{7/2}-s_{1/2}$, $f_{7/2}-d_{5/2}$, $f_{7/2}-g_{9/2}$, etc. besides configurations with $f_{5/2}$, $p_{3/2}$, and $p_{1/2}$ particles. The (p, p') process excites many of these modes, the (d, p) transition, however, excites only such modes that involve the $f_{7/2}$ (and $p_{3/2}$) neutrons of $Ti^{49}(0)$.

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²⁰ B. R. Mottelson, in *Proceedings of the International Conference on Nuclear Structure, Kingston, 1960* (University of Toronto Press, Toronto, 1960), p. 525.