PHYSICAL REVIEW VOLUME 140, NUMBER 1B 11 OCTOBER 1965

Particle Correlation Arising from Isospin Pairing in Light Nuclei*

A. GOSWAMI AND L. S. KISSLINGER
Western Reserve University, Cleveland, Ohio
and
Brookhaven National Laboratory, Upton, New York
(Received 21 May 1965)

We consider a new kind of particle correlation in light nuclei consisting of neutron-proton pairs in $T=0$ states, which we call isospin pairing. The importance of such a correlation stems from the fact that the *n-p* $T=0$ force is dominant for $A\leq 30$, especially for $N=Z$ nuclei. It is shown that such an isospin correlation is coherent and, among other things, can provide the necessary decrease in the ground-state band moment of inertia of $N=Z$ even-even nuclei. Detailed properties of the variational wave function are discussed. The possibility of understanding the deformation of light nuclei as a result of competition between $T=0$ and $T=1$ forces is pointed out.

I. INTRODUCTION

THERE is now strong evidence that there is a marked difference between the heavy nuclei, i.e., nuclei in which there is a considerable neutron excess, and light nuclei with regard to their stability against deformations. For the heavy nuclei, study of the low-lying states has led to an analysis of the two-body interaction in terms of a short-range pairing component,[1] an interaction which in two-particle states has nonvanishing matrix elements only for total angular momentum $J=0$, and a long-range component, usually taken as the quadrupole-quadrupole interaction.[2,3]

The pairing force favors spherical symmetry, and resists the tendency of the long-range component to produce deformation. Near closed shells the pairing force dominates, which explains the occurrence of large regions of nuclei with spherical symmetry. Only when there are a large number of particles outside of closed shells does the quadrupole force dominate for these heavy nuclei and a deformation occur. The stability against deformation for heavy nuclei in this picture depends upon the competition of pairing and long-range forces.[2] Using this picture it has thus been possible to predict, among other things, the transition from the vibrational region to the well-known region of deformation in the rare earths.[3–5] We would like to emphasize that in this picture the particles are paired in $J=0$, $T=1$ states. Also, the quadrupole force is usually treated by an approximation (called the quasiparticle random-phase approximation)[4,5] such that only $T=1$ operators are taken into consideration, since protons are always coupled to protons and neutrons to neutrons.

In the light nuclei one does not see extended regions of spherical nuclei, but there seems to be an extremely strong tendency towards deformation. Even in the doubly closed nuclei O^{16} and Ca^{40} the first excited states seem to correspond to deformed nuclei.[6] In the qualitative picture which we consider here, the strong tendency of light nuclei towards deformation is essentially to be understood in terms of a competition between the $T=0$ and $T=1$ components of the two-body force. We take advantage of the fact that isospin is a very good quantum number in these nuclei and that the $J=0$ pairing force only couples particles in states of isospin $T=1$. Therefore, for $T=0$ states the only way the pairing force can be effective will involve correlation of quadruples. Hereafter we restrict ourselves to the $N=Z$ nuclei with $T=0$ ground states.

It should be noted further that for the light nuclei quite generally, the absence of neutron excess weakens the effectiveness of the $T=1$ force compared with the heavy nuclei. In addition to this, a study of the low-lying states of odd-odd nuclei shows a gradual change of relative strengths of the $T=0$ and $T=1$ residual forces. For nuclei heavier than Ca^{40}, the $T=1$ force predominates (i.e., the ground states are $T=1$ for the $N=Z$ nuclei) while for all lighter nuclei (with the exception of Cl^{34}, where the $T=0$ excited state nevertheless is very near to the $T=1$ ground state) the $T=0$ force is predominant. Particularly for nuclei lighter than $A=30$ (i.e., for nuclei in the first half of the *s-d* shell as well as lighter nuclei), the $T=0$ component is significantly larger than the $T=1$ component. This suggests that for these nuclei the pairing force ($T=1$, $J=0$) will not be as important as the $T=0$ force, and that the entire treatment of nuclei in this region must be different from that of heavy nuclei.

There have been numerous efforts during the last five years to include the neutron-proton correlation

* Supported in part by the National Science Foundation.

1 A. Bohr, B. R. Mottelson, and D. Pines, Phys. Rev. **110**, 936 (1958).

2 B. R. Mottelson, in *The Many-Body Problem* (John Wiley & Sons, Inc., New York, 1959), p. 283.

3 J. P. Elliot, Proc. Roy. Soc. (London) **A245**, 128, 562 (1958). S. T. Beliaev, Kgl. Danske Videnskab. Selskab, Mat. Fys. Medd. **31**, No. 11 (1959).

4 L. S. Kisslinger and R. A. Sorensen, Rev. Mod. Phys. **35**, 853 (1963). For earlier references, see this reference.

5 M. Baranger, Phys. Rev. **120**, 957 (1960); R. Arvieu and M. Veneroni, Compt. Rend. **252**, 670 (1961); T. Marumori, Progr. Theoret. Phys. (Kyoto) **24**, 331 (1960).

6 S. Gorodetzky *et al.*, Phys. Letters **1**, 14 (1962). E. B. Carter, G. E. Mitchel, and R. H. Davis, Phys. Rev. **133**, B1421 (1963); **133**, B1434 (1964). F. Everling, Nucl. Phys. **40**, 670 (1963).

effects. One major problem is connected with the fact that the quadruple correlations must be treated for $J=0$ pairing. This is a very complicated problem.[7–9] On the other hand, if we restrict the force to $J=0$, $T=1$ and look for BCS (Bardeen-Cooper-Schrieffer) solutions for the $N=Z$ nuclei,[10] the ground state corresponds to the shell-model solution (the superconducting solution violates isospin conservation)[11], so that in the usual meaning the pairing energy disappears.[10]

In the present work we completely neglect the $T=1$ component of the force except for its effect on the self-consistent field. One immediate advantage is that we can neglect quadruple correlations entirely if the correlations can be treated in a linear approximation like the BCS theory. Our basic objective is to learn whether there is enough coherency in the $T=0$ part of the n-p interaction to produce an important particle correlation, so that the approximation becomes a realistic one.

A variational method is employed with a variational wave function composed of independent $T=0$ pairs; i.e., this is a BCS treatment with pairing in isospin rather than angular momentum. We refer to this as isospin pairing. The formalism is quite similar to the $J=0$ pairing, but the physical content is entirely different. The quasiparticles corresponding to this isospin pairing are formed by combining protons with neutron holes and vice versa. Thus the BCS wave function consists of terms with various numbers of independent neutron-proton pairs with $T=0$, $J\neq 0$, so that angular momentum is not conserved and the state has the superficial appearance of the sum over both even-even and odd-odd nuclei. The nonconservation of angular momentum is understood with the interpretation that these states are intrinsic states, i.e., the generating wave functions for an entire rotational band from which the rotational states must be projected. One should remind oneself of the derivation of the Hartree-Fock-Bogoliubov[12] equations, where the same type of picture occurs. These states and the corresponding moments of inertia are derived in Sec. II for even-even nuclei. In Sec. III we show the consistency of these wave functions with the fundamental difference between the even-even and odd-odd nuclei seen in experiments. Section IV presents a summary and the conclusion of the paper.

[7] B. Bremont and J. Valatin, Nucl. Phys. **41**, 640 (1963); B. H. Flowers and M. Vajicic, *ibid.* **49**, 586 (1963); for other earlier references and discussions on this, see A. M. Lane, *Nuclear Theory* (William Benjamin, Inc., New York, 1963), Chap. 5.

[8] B. Bayman, in the *Proceedings of Rutherford Jubilee Conference* (Academic Press Inc., New York, 1961), p. 215.

[9] M. K. Pal, in *Proceedings of Low Energy Conference* (Department of Atomic Energy, Government of India, Bombay, 1963).

[10] A. Goswami, thesis, Calcutta University (unpublished), and Nucl. Phys. **60**, 228 (1964).

[11] This energy gap is essentially given by the ground-state expectation value of the $T=1$ pair operators. If the ground state is $T=0$ then the expectation value is zero. This does not imply that in general $T=1$ forces do not act in $T=0$ states, but is simply a statement about the BCS method.

[12] M. Baranger, Phys. Rev. **122**, 992 (1961). M. Baranger and K. Kumar, Nucl. Phys. **62**, 113 (1965).

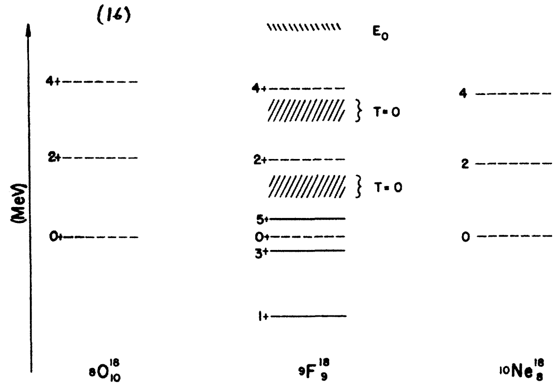

FIG. 1. Experimental energy levels of O^{18}, F^{18}, and Ne^{18}. Only the levels relevant to the discussion in the text are explicitly shown. E_0 is the assumed unperturbed $(d_{5/2})^2$ level.

II. HAMILTONIAN AND EVEN-EVEN SOLUTIONS

A. $J=1$, $T=0$ Specificity Force

In terms of the single-particle creation operators $b_{jk\tau}^\dagger$ for a shell-model particle with spin j and with z components of spin and isospin k and τ (the other quantum numbers are omitted), the Hamiltonian with a general two-body residual force can be written as

$$H=\sum_{jk\tau}\epsilon_{jk\tau}^{(0)}b_{jk\tau}^\dagger b_{jk\tau}-\tfrac{1}{4}\sum_{1234J'T'}G_{J'T'}^{1234}\times[B_{12}^\dagger(J'T')B_{34}(J'T')]_{J=0,T=0}\,,\quad(1)$$

where $B_{12}^\dagger(JT)=[b_1^\dagger b_2^\dagger]_{JT}$ is the tensor operator of rank J and isospin T, formed by vector coupling the particle creation operators in J and T, and the $\epsilon_{jk\tau}^{(0)}$ are the single-particle energies. The $G_{J'T'}^{1234}$ parameters are closely related to the two-particle matrix elements in the J, T states. For example, in the s-d shell, F^{18}, O^{18}, and Ne^{18} isotopes provide this information as in the usual shell-model calculations.[13] For a single j shell the parameters are G_{J1} (J even) and G_{J0} (J odd), and for the $d_{5/2}$ subshell they are determined by the positions of the $T=0$, $J=1$, 3, 5 and $T=1$, $J=0$, 2, 4 levels in F^{18}, and by the $T=1$, $J=0$, 2, 4 levels in O^{18} and Ne^{18}, to the accuracy with which such an extremely simple model can be used. See Fig. 1.

The Hartree-Fock solution, in which representation we wish to study the correlations, defines a basis in which the single-particle creation operators are given by

$$a_{\Lambda k\tau}^\dagger=\sum_j C_{\Lambda k}^j b_{jk\tau}^\dagger\,,\quad(2)$$

with the coefficients $C_{\Lambda k}^j$ determined by the Hartree-Fock calculation, Λ being an additional quantum number introduced to distinguish the different states of the same k, assuming axial symmetry of the Hartree-

[13] I. Talmi, in *Selected Topics in Nuclear Spectroscopy* (North-Holland Publishing Company, Amsterdam, 1964), p. 106.

Fock solutions. The creation operators $a_{\Lambda k\tau}^\dagger$ then correspond to the states in a deformed field with axial symmetry. We do not carry out the Hartree-Fock calculation, but assume that the single-particle states and resulting energies from such a calculation are given. Because of the success of the rotational model[14] one knows that the single-particle states resemble those of Nilsson.[15] However, an important modification is the tendency for the filled levels to be lowered in energy with respect to the unfilled levels in a Hartree-Fock calculation, resulting in an effective gap to the excited intrinsic levels. This has been observed in medium and heavy nuclei,[16,17] where it is not so systematic, since other effects can be as important in renormalizing the shell-model energies within a shell, and in light nuclei[18] where it appears to be an important and systematic effect.

Thus in contrast to the $J=0$ pairing in the heavy nuclei, the energy gap in the even-even nuclei in the region being studied here arises systematically both from renormalization of the single-particle spectrum and from the correlations. For this reason, one should be able to make a reasonable estimate of the magnitude of these correlations without including the consequent renormalization of the Hartree particles, by using the results of a Hartree-Fock calculation with a reasonable two-body interaction. We therefore assume in the numerical calculations that the renormalized single-particle energies ϵ_k are diagonal in the $a_{\Lambda k\tau}^\dagger|0\rangle$ states and we use the numerical values of Kelson and Levinson.[18] However, the particle correlations included in this paper will smear the Fermi surface, reducing the difference between filled and unfilled levels. Therefore if one carries out a complete self-consistent calculation the "single-particle gap" will be reduced and consequently the correlations which we are calculating will be enhanced.

In this paper we have further restricted ourselves to a single j shell for the simplicity in getting numerical solutions. Henceforth we shall therefore consider specifically a single j shell in order to calculate the correlation energy. The entire s-d shell must be used for the calculation of the moment of inertia, and some justification for the use of the parameters determined by the single j shell is given later.

For a single j shell, the states $a_{\Lambda k\tau}^\dagger|0\rangle$ become just $b_{jk\tau}^\dagger|0\rangle$ (the label j can be omitted). We look for BCS solutions but with quasiparticles conserving the z component of isospin and angular momentum[19]

$$\alpha_{k\tau}^\dagger = U_{k\tau}a_{k\tau}^\dagger - V_{k\tau}a_{-k-\tau}\,,\qquad U_{k\tau}^2+V_{k\tau}^2=1\,. \tag{3}$$

In the usual way[20] one can write down the auxiliary Hamiltonian $H'=H-\lambda N$, where N us the number operator and λ the chemical potential, and $H'=H_0+H_{11}+H_{20}+H_{\rm int}$, in which H_0 is independent of quasiparticle operators. H_{11} is of the form of a single-quasiparticle energy operator, H_{20} creates or destroys two quasiparticles, and $H_{\rm int}$ involves four quasiparticle operators (in normal form). So long as there exists an energy gap between the quasiparticle vacuum and the two-quasiparticle states one can proceed in the usual manner: neglect $H_{\rm int}$ and determine the transformation (3) such that H_{20} vanishes. The H_{20} part of the $T=0$ component of the auxiliary Hamiltonian (assuming that the self-consistent field can be approximated by replacing $\epsilon_{jk\tau}^{(0)}$ by $\epsilon_{jk\tau}$ as described above) is easily seen to be given by

$$H_{20}=\tfrac{1}{2}\sum_{k\tau}\Bigg\{(\epsilon_{k\tau}-\lambda)2U_{k\tau}V_{k\tau}(-1)^{1/2-\tau} -\sum_J(-1)^J\frac{2}{(2J+1)^{1/2}}G_{0J}\sum_{k'>0}C_{k'-k'0}{}^{jjJ} \times U_{k'\tau}V_{k'\tau}C_{k-k0}{}^{jjJ}(U_{k\tau}^2-V_{k\tau}^2)\Bigg\}(-1)^{1/2-\tau} \times(\alpha_{-k-\tau}^\dagger\alpha_{k\tau}^\dagger+\alpha_{k\tau}\alpha_{-k-\tau})\,, \tag{4}$$

the C's being Clebsch-Gordan coefficients with the Condon-Shortley[21] phase. The solutions are found by setting $H_{20}=0$ and taking the average number of particles in the ground state $|\Psi_0\rangle$ to be the mass number A for the isotope in question, $\langle\Psi_0|N|\Psi_0\rangle=A$, or alternatively, by calculating the commutators of b and $b^\dagger$ with H and linearizing with respect to b and $b^\dagger$.

In this subsection we consider the special case in which only the $J=1$ component of the force is used (the $J=1$, $T=0$ force is of course the most important one), i.e., take all the $G_{0J}=0$ except for G_{01}. Let us define

$$\Delta=(G_{01}/\sqrt{3}j(j+1))\sum_{k>0}U_kV_k(-1)^{j-k}. \tag{5}$$

The gap equation, obtained by setting $H_{20}=0$, is

$$1=\frac{G_{01}(2j+1)}{3\sqrt{3}j(j+1)}\sum_{k>0}\frac{k^2}{[(\epsilon_k-\lambda)^2+k^2\Delta^2]^{1/2}}. \tag{6}$$

[14] A. Bohr and B. R. Mottelson, Kgl. Danske Videnskab. Selskab, Mat. Fys. Medd. 27, No. 16 (1953).

[15] S. G. Nilsson, Kgl. Danske Videnskab. Selskab, Mat. Fys. Medd. 29, No. 16 (1955); B. R. Mottelson and S. G. Nilsson, *ibid.* 1, No. 8 (1959).

[16] B. Cohen, Phys. Rev. 130, 227 (1963); B. Cohen *et al.*, Rev. Mod. Phys. 35, 322 (1963).

[17] N. Freed, thesis, Western Reserve University (unpublished); B. E. Chi, N. Freed, L. S. Kisslinger, and T. Terasawa (to be published).

[18] I. Kelson and C. A. Levinson, Phys. Rev. 136, B269 (1964).

[19] Here we use the phase $V_{k\tau}=(-)^{1/2-\tau}V_{k\tau}'$, where $V_{k,\tau}'=V_{k,-\tau}'$. Note also that for a single j shell, $V_{k\tau}=(-)^{1/2-\tau}(-)^{j-k}(k/|k|)\times|V_{k\tau}|$. The transformation is canonical. Symmetry with respect to k, $-k$ is assumed in the solutions.

[20] N. N. Bogoliubov, Nuovo Cimento 7, 794 (1958); J. G. Valatin, Nuovo Cimento 7, 843 (1958).

[21] E. U. Condon and G. C. H. Shortley, *Theory of Atomic Spectra* (The Macmillan Company, New York, 1935).

Equation (6) is easily derived from Eq. (5) by taking only $J=1$, using $C_{k0k}{}^{j1j}=k/[j(j+1)]^{1/2}$, and setting

$$2U_{k\tau}V_{k\tau}(-1)^{1/2-\tau}=(-1)^{j-k}k\Delta/E_k, \quad (7a)$$

$$U_{k\tau}{}^2-V_{k\tau}{}^2=(\epsilon_k-\lambda)/E_k, \quad (7b)$$

$$E_k=[(\epsilon_k-\lambda)^2+k^2\Delta^2]^{1/2}. \quad (7c)$$

The probabilities of occupation, $V_{k\tau}{}^2$, and nonoccupation, $U_{k\tau}{}^2$, are obtained from

$$2V_{k\tau}{}^2=1-(\epsilon_\lambda-\lambda)/E_k,$$
$$2U_{k\tau}{}^2=1+(\epsilon_\lambda-\lambda)/E_k, \quad (8)$$

with the phases of $V_{k\tau}$ suitably chosen. The parameters λ and Δ are determined from the gap Eq. (6) and the number equation $\langle\Psi_0|N|\Psi_0\rangle=A$

$$j+\tfrac{1}{2}-\tfrac{1}{2}(A-A_0)=\textstyle\sum_k (\epsilon_k-\lambda)/E_k, \quad (9)$$

where A_0 is the number of protons plus neutrons in the filled levels (not included in the calculation).

Equations (6) and (9) are of familiar form and the ground state is the quasiparticle vacuum $\alpha_{k\tau}|\Psi_0\rangle=0$. However, the pairing has been done in k and τ so that the ground state admixes many angular momenta, and is interpreted as an intrinsic state for the ground-state band with $K=0$ and $T=0$. It should be stressed that the violation of angular-momentum conservation is not an extra assumption, but rather an essential part of the picture. Since the main component of the force leading to spherical symmetry, the $J=0$, $T=1$ component, is not present, the system will be deformed and the calculation is being carried out in the intrinsic space. The rotational levels can be projected out in the usual way.

In the numerical calculation for Mg^{24} the force strength G_{01} is fitted to F^{18} (see Fig. 1) assuming that the 1+ ground state is lowered in energy by the residual force by about 6 MeV. The energy levels are taken as $(\epsilon_{1/2},\epsilon_{3/2},\epsilon_{5/2})=(0,3.8,8.7)$. With these parameters the gap parameter $\Delta=0.655$ and $\lambda=5.648$.

The moments of inertia can be calculated by the cranking method with the Hartree-Fock solutions and energies ϵ_u and ϵ_o. With uncorrelated states the moment of inertia is given by[22]

$$\mathscr{I}=2\hbar^2\sum_{\substack{u=\text{unoccupied}\\ o=\text{occupied}}}\frac{|\langle o|j_x|u\rangle|^2}{\epsilon_u-\epsilon_o}. \quad (10)$$

The tendency for the occupied states to be reduced in energy by the residual interaction increases the energy difference between occupied and unoccupied levels, and from Eq. (10) one can see that this single-particle gap will reduce the moment of inertia. Kelson and Levinson have found the reduction compared to Nilsson levels in this region to be about a factor of two. However, even neglecting corrections to the cranking model[23] and neglecting the contributions from the core, both of which increase the moment of inertia, they find that $\hbar^2/2\mathscr{I}=0.17$, compared with the experimental value of 0.22.

The pairing correlations reduce the moment of inertia.[24] This can be seen from the cranking formula with pairing correlations,[25] which is

$$\mathscr{I}=2\hbar^2\sum_{k'>0;\text{all }k}\frac{|\langle k|j_x|k'\rangle|^2}{E_k+E_{k'}}(U_k|V_{k'}|-U_{k'}|V_k|)^2. \quad (11)$$

Calculations in the heavy deformed nuclei have demonstrated[25] that the effects of the usual $J=0$ pairing are large and that the results are in good agreement with experimental data.

In order to compare the results for the moment of inertia of the $J=1$, $T=0$ calculation with the results of Kelson and Levinson to test the effects of this new correlation, one must include all the states which they have used.[26] Here we shall present the justifications for applying our single-j-shell results to the case of several j shells. Firstly, the modification of the self-consistent field by the correlations is not expected to be large (especially for high-lying levels). Secondly, from the Kelson-Levinson results for the energies and wave functions for Mg^{24} one sees that the three K levels which are predominantly $d_{5/2}$ are among the lowest lying levels. The only other low-lying level is predominantly $s_{1/2}$ and thus is not mixed with $d_{5/2}$ levels by the present correlation with the $J=1$ force above, except for the implicit interaction included in the self-consistent field. (This is not so for a general force, but the mixing should still be small.) One can therefore get a fairly accurate estimate of the quasiparticle energies of the other levels (not included in our calculation) by using the values of λ and Δ calculated from the $d_{5/2}$ subshell. The probability of occupation of all the levels can thus be obtained from Eq. (7). Since the force strengths were obtained by considering only the $d_{5/2}$ levels, the results in the calculation with several j levels will be more or less unchanged to the extent that only a renormalization is involved. Taking $\Delta=0.655$ and $\lambda=5.648$, using the Hartree-Fock energies and wave functions of Ref. 18 and calculating the U's by Eq. (7) one finds that $\hbar^2/2\mathscr{I}=0.223$. Thus in this simple calculation the moment of inertia is reduced by the correct amount.

The self-consistent Hartree-Fock-Bogoliubov-type calculation will change the result in two ways: The single-particle gap will be reduced if the particle correlations are included as discussed above, tending to increase the moment of inertia, while the correlations

[22] D. R. Inglis, Phys. Rev. **96**, 1059 (1954); **97**, 1701 (1955).
[23] D. J. Thouless and J. G. Valatin, Nucl. Phys. **31**, 211 (1962).
[24] A. Bohr and B. R. Mottelson, Kgl. Danske Videnskab. Selskab, Mat. Fys. Medd. **30**, No. 1 (1955).
[25] J. J. Griffin and M. Rich, Phys. Rev. Letters **3**, 1342 (1959) and Phys. Rev. **118**, 850 (1960). S. G. Nilsson and O. Prior, Kgl. Danske Videnskab. Selskab, Mat. Fys. Medd. **32**, No. 16 (1961).
[26] We use the energies and wave functions given in Table II, p. B273 of Ref. 18. Mg^{24} corresponds to the case $\theta_{1/2}=1$, $\theta_{3/2}=1$ in that table.

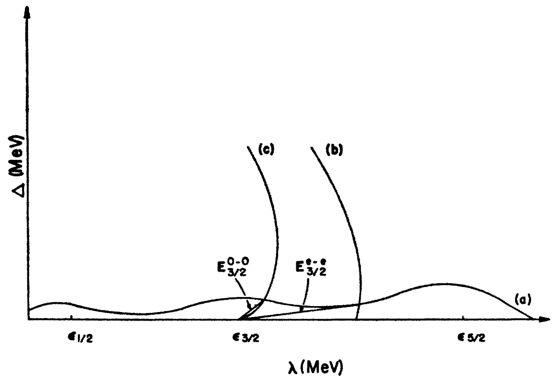

FIG. 2. Solutions to the gap and number equations for even-even and for odd-odd nuclei. Curve (a) is a schematic plot of Δ versus λ from the gap equation (6). Curves (b) and (c) are schematic plots of Δ versus λ from the number equation (9) for even-even and odd-odd nuclei, respectively. The smallest quasiparticle energies in the even-even and odd-odd nuclei are $E^{e\text{-}e}$ and $E^{o\text{-}o}$ corresponding to Mg^{24} and Na^{22}, respectively.

included in this work will be enhanced, reducing the moment of inertia. Only by carrying out the complete calculation can one be sure of the final value, but the new energy gap which appears in this work is important and corresponds to a correlation which must be included. Detailed Hartree-Fock-Bogoliubov–type calculations with isospin correlation are now in progress.

B. δ Force

Calculations have also been carried out with a δ force, i.e., a Hamiltonian of the form (1) in which two of the energy splittings in F^{18}, O^{18}, and N^{18} are used to fit one of the $T=0$ and one of the $T=1$ force parameters, while the ratios are determined by the δ force. Only the $T=0$ force is included in the calculation, for which the force parameter ratios are found to be $|G_{01}|:|G_{03}|:|G_{05}| = 10.4:8.9:17.5$, fitting the ground state in F^{18} as before. The calculation is carried out in precisely the same manner as with the $J=1$ force, the change from (4), (5), (6), and (7) being that the functional dependence of the gap parameter on the state is now no longer known. Thus Eq. (7c) defining the quasiparticle energies is replaced by

$$E_k[(\epsilon_k-\lambda)^2+\Delta_k^2]^{1/2}, \quad (7c')$$

and the gap equation now reads

$$\Delta_k=\sum_J \frac{(-1)^J}{(2J+1)^{1/2}} G_{0J} C_{k,-k,0}{}^{jjJ} \sum_{k'} C_{k',-k',0}{}^{jjJ} \frac{\Delta_{k'}}{E_{k'}}, \quad (12)$$

where $C_{k,-k,0}{}^{jjJ}$ denotes a Clebsch-Gordan coefficient. The solution of this equation with Eq. (8) gives $\lambda=6.07$, $\Delta_{5/2}=1.75$, $\Delta_{3/2}=-1.51$, and $\Delta_{1/2}=0.01$. The moment of inertia calculated from this solution in the same manner as described in Sec. II.A is $\hbar^2/2\mathscr{I}=0.26$. In view of the effects which tend to increase the moment of inertia from the present estimate (as already discussed), the value of $\hbar^2/2\mathscr{I}$ found here seems very satisfactory.

The results are not qualitatively different from the $T=0$, $J=1$ force, which further confirms that the correlations found in this work are important qualitatively and not very dependent on the exact nature of the force.

III. ODD-ODD NUCLEI

Since the BCS solution for isospin pairing does not conserve the number of $T=0$ pairs, the wave function for an even-even nucleus contains components with an odd number of neutrons and protons and vice versa. Superficially it would seem that the states of actual even-even and odd-odd nuclei are being mixed, which would be bad, since empirically the energy spectra are quite different. In discussing this problem we shall use Fig. 2, which shows a schematic plot of Δ versus λ as given by the gap equation [curve (a)], and plots of Δ versus λ from Eq. (9) for even-even [curve (b)] and odd-odd [curve (c)] nuclei. The intersection of curves (a) and (b) gives the (λ,Δ) solution for an even-even nucleus, and the intersection of curves (a) and (c) gives it for an odd-odd nucleus.

The essential point is that the gap between the ground states and the excited intrinsic states in the even-even nuclei arise both from the $T=0$ force and from the single-particle gap in the Hartree solutions, and moreover, the gap parameters Δ_k are much smaller than those which one would expect in this mass region from a simple continuation of the results for heavy nuclei. This leads to solutions for the even-even nuclei in which the Fermi level λ is approximately half way between the almost-filled Nilsson state and the almost-empty Nilsson state ($k=\frac{3}{2}$ and $k=\frac{5}{2}$ for Mg^{24}), so that one maintains very large quasiparticle excitation energies E in spite of a small gap parameter Δ_k. Thus the ground state of the even-even nucleus is the quasiparticle vacuum, with the excited two-quasiparticle states well removed.

The picture for odd-odd nuclei is quite different, for in that case the Fermi level is always quite close to one of these well separated levels, say K_0. Then the quasiparticle vacuum $|\Psi_0\rangle$ and the two-quasiparticle states $\alpha_{K_0}{}^\dagger\alpha_{\bar{K}_0}{}^\dagger|\Psi_0\rangle$ are almost degenerate in energy. Thus the odd-odd terms that are added into the wave function of the even-even nucleus have no resemblance at all to the corresponding terms from the actual odd-odd nuclei; and these odd-odd terms have in fact the essential properties involved in the treatment of the even-even nucleus. Thus the even-even and odd-odd contamination is not a serious problem.

Of course, since there is no gap between the quasiparticle vacuum and the excited states for the real odd-odd nuclei, one must include the quasiparticle interaction terms in order to say anything about the ordering of the states arising from the quasiparticle vacuum and the various states of good angular momentum which arise from the $\alpha_{K_0}{}^\dagger \alpha_{K_0}{}^\dagger |\Psi_0\rangle$ two-quasiparticle states. However, the interesting regularity which one observes in the odd-odd isotopes F^{18}, Na^{22}, and Al^{26}, namely that the ground states are 1+, 3+, and 5+, respectively, would correspond to that expected from a simple coupling rule, since K_0 corresponds to $\frac{1}{2}$, $\frac{3}{2}$, and $\frac{5}{2}$, respectively, in those three isotopes. Detailed calculations for these odd-odd isotopes are being carried out.

IV. SUMMARY AND CONCLUSIONS

In this paper the existence of a new kind of particle correlation in light nuclei has been suggested by the calculation of a finite gap due to the $T=0$ force. The correlation consists of pairing of a neutron and a proton in isospin zero state (and consequently nonzero angular-momentum state) which we have called isospin pairing in contrast to ordinary pairing in the angular momentum zero state.

In the present work, only $N=Z$ nuclei with $T=0$ ground states have been considered, so that $T=1$ independent pairs cannot exist in the ground state. Consequently, the $T=1$ force (in particular the $J=0$ pairing force) can be effective only through quadruple correlation. Furthermore, from a survey of experimental level spectrum data we have pointed out that the $T=0$ force is stronger than $T=1$ force in light nuclei, especially in nuclei of $A\leq 30$. We have therefore neglected the $T=1$ force altogether.

The $T=0$ force is treated taking only the isospin pairing correlation into account. The basic objective has been to learn if there is enough coherence in the $T=0$ part of the n-p interaction to produce an important particle correlation. Therefore, the further approximation of a single j shell has been used and the effect of the self-consistent field is approximately taken care of by taking the single-particle energies determined by a Hartree-Fock calculation. Both of these approximations in fact tend to hinder the correlation effects we are searching for. Firstly, as shown by Kelson and Levinson, the Hartree-Fock calculation already produces an energy gap between the occupied and unoccupied levels, so that taking their results effectively minimizes the gap due to isospin pairing. Secondly, by taking a single j shell ($d_{5/2}$ subshell for Mg^{24} calculation) the degeneracy is decreased, which further tends to lower the gap. However, the calculations become very simple with these two assumptions and the results also should be more convincing.

The mathematical method for the simple case we have treated is very similar to the BCS method with angular-momentum pairing. With a Bogoliubov transformation conserving the z component of angular momentum and isospin (assuming axial symmetry) one finds a finite energy gap both for a constant $J=1$, $T=0$ specificity force and for a δ function $T=0$ force. We believe that this calculation conclusively demonstrates the existence of isospin correlation.

To compare our results with the Hartree-Fock calculation[18] we have computed the moment of inertia of the ground-state band of Mg^{24}. This calculation is crude because for this one needs to know the quasiparticle energies and occupation probabilities of the other k levels which have not been included in the present calculation, but with a reasonable approximation we obtain an improvement in the moment of inertia, bringing the calculated moment of inertia into agreement with the experimental result.

In conclusion, we would like to remark that a complete self-consistent calculation of the Hartree-Fock-Bogoliubov type is necessary to demonstrate how the deformations arise simultaneously with the isospin pairing. Such calculations are now in progress. In the treatment of the heavier nuclei the $T=1$ part of the force will begin to play a more and more important role. It is our conjecture that in contrast to the heavy nuclei, where it is the competition between short-range and long-range parts of the $T=1$ component of the force which determines the tendency toward deformation, in the light nuclei it is the competition between the $T=0$ and $T=1$ components of the force which determines the tendency toward deformation or spherical symmetry.