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Exchange Electrical Moments for Nuclear Beta and Gamma Transitions*

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The relative importance of exchange electric currents in nuclear beta and gamma transitions is estimated by using their relation to the Van Vleck potential and to the sum rule for nuclear photoabsorption cross section. Though the exchange electric currents are not experimentally observable because of the Siegert theorem, the estimates give us an idea of the difference between a free nucleon and a nucleon inside a nucleus. It is found that the magnitude of the exchange electric current relative to the convection current is more than 40% at the Fermi surface.

I. INTRODUCTION

THERE has been some controversy[1,2] about the question whether there is a significant difference between the conserved-vector-current (CVC) theory[3] and conventional beta theories.[4] By "conventional theory" we mean the bare-nucleon coupling theory. The difference between the conventional and CVC theories stems mainly from the ($\pi\pi e\nu$) coupling with the same vector coupling constant as in nuclear beta decay. The existence of the above coupling has been verified by two kinds of experiments: direct measurement[5] and weak magnetism.[6] The fact that the magnitude of the weak magnetism is just what is expected from CVC theory also implies that it is too small[1] to be influential on the spectra, life times, etc. for the ordinary low-energy beta transitions.

Another consequence of the CVC theory is the appearance of the exchange electric current,[7] which is assumed to vanish in the case of the bare-nucleon coupling theory. For the electromagnetic transitions, the corresponding exchange electric moments have long been known to exist, but the magnitude of their contribution has never been studied carefully, probably because the Siegert theorem implies that the isovector part cannot be experimentally measured.[8,9] In the theory of beta decay, the quantity corresponding to the exchange electric moment had been studied[10,11] long before the CVC theory was proposed. The magnitude, however, turned out to be large according to the semi-empirical estimate due to Ahrens-Feenberg[10] but very small according to Pursey.[11] In this note we try to settle

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[1] S. C. Wu, Rev. Mod. Phys. **36**, 618 (1964).

[2] R. J. Blin-Stoyle, Nucl. Phys. **57**, 232 (1964); C. W. Kim, Nucl. Phys. **49**, 651 (1963); J. Deutsch, *Selected Topics in Nuclear Spectroscopy* (North-Holland Publishing Company, Amsterdam, 1964), p. 323.

[3] R. P. Feynman and M. Gell-Mann, Phys. Rev. **109**, 193 (1958); S. S. Gerschtein and J. B. Zeldovich, Zh. Eksperim. i Teor. Fiz. **29**, 698 (1955) [English transl.: Soviet Phys.—JETP **2**, 576 (1955)].

[4] E. Fermi, Z. Physik **88**, 161 (1934); E. J. Konopinski and G. E. Uhlenbeck, Phys. Rev. **60**, 308 (1941).

[5] A. F. Dunaitsev, V. I. Petrukhin, Yu. D. Prokoshkin, and V. I. Rykalin, Brookhaven National Laboratory Report No. BNL 837 C-39, 1963 (unpublished).

[6] Y. K. Lee, L. W. Mo, and C. S. Wu, Phys. Rev. Letters **10**, 253 (1963).

[7] J. I. Fujita, Phys. Rev. **126**, 202 (1962); Progr. Theoret. Phys. (Kyoto) **28**, 338 (1962); J. Euchler, Z. Phys. **171**, 463 (1963); A. Fujii, J. I. Fujita, and M. Morita, Progr. Theoret. Phys. (Kyoto) **32**, 438 (1964).

[8] A. F. Siegert, Phys. Rev. **52**, 787 (1937); R. G. Sachs, *Nuclear Theory* (Addison-Wesley Publishing Company, Inc., Cambridge, Massachusetts, 1953), p. 243.

[9] Our argument cannot be applied to the isoscalar part. Some quantities associated with the isoscalar exchange current have been considered by Y. Fujii and M. Kawaguchi [Progr. Theoret. Phys. (Kyoto) **26**, 519 (1961)], by M. Kawaguchi and H. Yokomi [Progr. Theoret. Phys. (Kyoto), Suppl. **21**, 71 (1962)], and by D. R. Harrington [Phys. Rev. **133**, B142 (1964)].

[10] T. Ahrens and E. Feenberg, Phys. Rev. **86**, 74 (1952).

[11] D. L. Pursey, Phil. Mag. **42**, 1193 (1951).

this controversy and we confirm the previous assertion[12] that most probably the correct value lies between these two estimates.

Since the electromagnetic (isovector part) and CVC theories are quite analogous, both cases are treated simultaneously in this note. In Sec. II the essence of previous works is recapitulated. In Sec. III the origin of the exchange electric current is discussed on the basis of the one-boson-exchange model of the nuclear potential.[13] In Sec. IV the relationship between the transition operators and the Van Vleck potential[14] (or the effective mass in the nuclear many-body problem) is studied. From the simple-model calculation in Sec. IV and also the sum rule due to Levinger and Bethe[15] for the nuclear photoabsorption, we are led to the conclusion that the magnitude of the exchange electric current is 40% or more of that of the convection current. The origin of disagreement between Pursey's estimate and ours is also discussed.

II. REVIEW OF EXCHANGE ELECTRIC MOMENTS

In this section, the so-called Siegert theorem[8] and its extension[7] to weak interactions are briefly described.

In the case of electromagnetic transitions, the conservation law of charge,

$$\operatorname{div}\mathbf{J}(\mathbf{x})+[H_N,iJ_0(\mathbf{x})]=0\,, \qquad (1)$$

leads us to the identity

$$\begin{aligned}\langle f|\int \mathbf{J}(\mathbf{x})\cdot \operatorname{grad}U(\mathbf{x})d^3x|i\rangle &=\langle f|\int [H_N,iJ_0(\mathbf{x})]U(\mathbf{x})d^3x|i\rangle \\ &=i(E_f-E_i)\langle f|\int J_0(\mathbf{x})U(\mathbf{x})d^3x|i\rangle, \qquad (2)\end{aligned}$$

where the electromagnetic vector potential is assumed to have the form $\mathbf{A}(\mathbf{x})=\nabla U(\mathbf{x})$, since we are mainly interested in the electric transitions in the following. In Eq. (2) the quantities E_i and E_f represent the masses of the initial and final nuclei, respectively. The nuclear Hamiltonian is given by $H_N=T+V$, where T and V are the kinetic and potential energies, respectively. Now, if we assume that the nuclear potential V commutes with $J_0(\mathbf{x})$, then Eq. (2) is clearly consistent with the assumption of "additivity" for the charge and current densities,

$$J_0(\mathbf{x})=\sum_{i=1}^{A}\tfrac{1}{2}(1+\tau_3^{(i)})e\rho(\mathbf{x}-\mathbf{x}_i)\,, \qquad (3a)$$

and

$$\mathbf{J}(\mathbf{x})=\sum_{i=1}^{A}\tfrac{1}{2}(1+\tau_3^{(i)})e(\mathbf{p}_i/M)\rho(\mathbf{x}-\mathbf{x}_i)\,, \qquad (3b)$$

where $\rho(\mathbf{x})$ represents the Fourier transform of the Hofstadter form factor for a proton. However, in actual fact the nuclear potential V does not commute with $J_0(\mathbf{x})$ because of the presence of exchange forces, so that we must abandon at least one of Eqs. (3). The Siegert theorem asserts that it is still a good approximation to retain Eq. (3a). This will be shown explicitly in the next section. Therefore, the extent of failure of "additivity" is given by the magnitude of exchange electric current, $\int[V,J_0(\mathbf{x})]U(\mathbf{x})d^3x$.

For the weak interactions[7,12] it has been shown that the vector part of the weak current $J_\mu^{(V)\pm}(\mathbf{x})$ should satisfy the continuity relation similar to Eq. (1),

$$\operatorname{div}\mathbf{J}^{(V)\pm}(\mathbf{x})+[H_N-V_c,\,iJ_0^{(V)\pm}(\mathbf{x})]=0\,, \qquad (4)$$

provided that validity of the CVC theory is assumed and a possible small contribution from radiative corrections is neglected. In Eq. (4), V_c represents the charge-dependent parts of H_N such as the Coulomb potential and neutron-proton mass difference. Hereafter, let us rewrite H_N as $T+V+V_c$, the nuclear potential V being assumed to be charge-independent. For the lepton fields of electric type which are given by

$$L(\mathbf{x})=\operatorname{grad}U(\mathbf{x})\,, \qquad (5)$$

the following identity is useful:

$$\begin{aligned}\langle f|\int \mathbf{J}^{(V)\pm}(\mathbf{x})\cdot\operatorname{grad}U(\mathbf{x})d^3x|i\rangle &=\langle f|\int [H_N-V_c,\,iJ_0^{(V)\pm}(\mathbf{x})]U(\mathbf{x})d^3x|i\rangle \\ &=i(E_f-E_i)\langle f|\int J_0^{(V)\pm}(\mathbf{x})U(\mathbf{x})d^3x|i\rangle \\ &\quad-\langle f|\int [V_c,iJ_0^{(V)\pm}(\mathbf{x})]U(\mathbf{x})d^3x|i\rangle. \qquad (6)\end{aligned}$$

Analogously to the electromagnetic case, we may assume the "additivity" only for the weak charge density, $J_0^{(V)\pm}(\mathbf{x})$ (not for the weak current density),

$$J_0^{(V)\pm}(\mathbf{x})=g^{(V)}\sum_{i=1}^{A}\tau_\pm^{(i)}\rho(\mathbf{x}-\mathbf{x}_i)\,, \qquad (7)$$

and the nuclear-potential term $\int[V,iJ_0^{(V)\pm}(\mathbf{x})]U(\mathbf{x})d^3x$ stands for the extent of deviation from "additivity." For practical purposes, Eq. (6) is especially useful[7] when we adopt the so-called Ahrens-Feenberg approxima-

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[12] J. I. Fujita, Brookhaven National Laboratory Report No. BNL 837 C-39, 1963 (unpublished), p. 340.

[13] N. Hoshizaki, S. Otsuki, W. Watari, and M. Yonezawa, Progr. Theoret. Phys. (Kyoto) **27**, 1199 (1962); S. Sawada, T. Ueda, W. Watari, and M. Yonezawa, *ibid.* **28**, 991 (1962); **32**, 380 (1964): A. Scotti and D. Y. Wong, Phys. Rev. Letters **10**, 142 (1963).

[14] H. A. Bethe and R. F. Bacher, Rev. Mod. Phys. **8**, 82 (1936).

[15] J. S. Levinger and H. A. Bethe, Phys. Rev. **78**, 115 (1950).

tion[10] or, equivalently, the isomultiplet approximation for the term including V_c.[12] However, we do not consider this point further in this paper.

III. ORIGIN OF EXCHANGE ELECTRIC CURRENT

In order to clarify physical meaning of the exchange currents, let us first examine the one-pion exchange process shown in (a) or (a′) of Fig. 1. The matrix element can be written in terms of the function given by

$$\mathcal{V}_{\text{exch}}^{(\pi)}(x_1,x_2)_\alpha = -i(2\pi)^{-4}g_{\pi N}^2(\boldsymbol{\tau}^{(1)}\times\boldsymbol{\tau}^{(2)})_\alpha\int d^4q\int d^4\kappa \times\gamma_5^{(1)}\Delta_F(\tfrac{1}{2}\kappa-q)\Gamma_\mu(\tfrac{1}{2}\kappa-q,\,\tfrac{1}{2}\kappa+q) \times\Delta_F(\tfrac{1}{2}\kappa+q)\gamma_5^{(2)}a_\mu(\kappa)e^{-iq(x_1-x_2)}e^{-i\kappa(x_1+x_2)/2}, \quad (8a)$$

where the electromagnetic vector potential $A_\mu(x)$ [or the lepton fields $L_\mu(x)$] is given in the form

$$A_\mu(x)\ [\text{or } L_\mu(x)]=\int a_\mu(\kappa)e^{-i\kappa x}d^4\kappa. \quad (8b)$$

In the case of a pion being exchanged we have

$$\Gamma_\mu(\tfrac{1}{2}\kappa-q,\,\tfrac{1}{2}\kappa+q)=2Cq_\mu F_\pi(\kappa^2), \quad (8c)$$

where C stands for the unit charge e or the weak coupling constant $g^{(V)}$, and $F_\pi(\kappa^2)$ is the pion electromagnetic form factor. If the momentum transfer κ is sufficiently small, namely $|\kappa^2|\ll\mu^2$, we have the following relations:

$$\Delta_F(\tfrac{1}{2}\kappa-q)\approx\Delta_F(\tfrac{1}{2}\kappa+q)\approx(q^2+\mu^2)^{-1}, \quad (9a)$$

and

$$\Delta_F(\tfrac{1}{2}\kappa-q)\Gamma_\mu(\tfrac{1}{2}\kappa-q,\,\tfrac{1}{2}\kappa+q)\Delta_F(\tfrac{1}{2}\kappa+q) \approx-2C(\partial/\partial q_\mu)\Delta_F(q), \quad (9b)$$

according to the Ward identity.[16] Inserting Eq. (9) into Eq. (8a) and carrying out the partial integration in q we obtain

$$\mathcal{V}_{\text{exch}}^{(\pi)}(x_1,x_2)_\alpha = -2(2\pi)^{-4}g_{\pi N}^2(\boldsymbol{\tau}^{(1)}\times\boldsymbol{\tau}^{(2)})_\alpha\int d^4q \times\gamma_5^{(1)}(q^2+\mu^2)^{-1}\gamma_5^{(2)}e^{-iq\cdot(x_1-x_2)}(x_1-x_2)_\mu \times\int d^4\kappa a_\mu(\kappa)e^{-i\kappa(x_1+x_2)/2}. \quad (10a)$$

If we introduce the function

$$\mathcal{V}^{(\pi)}(x_1,x_2)=(2\pi)^{-4}g_{\pi N}^2(\boldsymbol{\tau}^{(1)}\cdot\boldsymbol{\tau}^{(2)}) \times\int d^4q\,\gamma_5^{(1)}\frac{1}{q^2+\mu^2}\gamma_5^{(2)}e^{-iq(x_1-x_2)}, \quad (10b)$$

[16] J. C. Ward, Phys. Rev. **78**, 182 (1950).

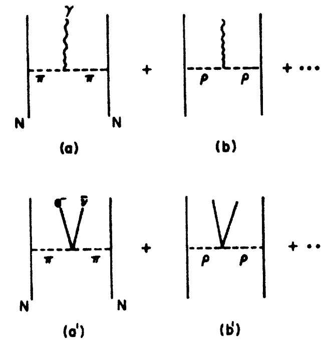

FIG. 1. Feynman diagrams of the exchanged electromagnetic and beta transitions.

corresponding to the simple one-pion-exchange process, we can put Eq. (10a) into the following form:

$$\mathcal{V}_{\text{exch}}^{(\pi)}(x_1,x_2)_\alpha = i[\mathcal{V}^{(\pi)}(x_1,x_2),\,\tau_\alpha^{(1)}x_{1\mu}+\tau_\alpha^{(2)}x_{2\mu}]A_\mu(\tfrac{1}{2}(x_1+x_2)). \quad (10c)$$

It should be noted that in the "static limit" $t_1=t_2$, the term associated with the fourth component A_4 in Eq. (10c) vanishes, to justify the "additivity" for the exchange charge density.

For the electric transitions, we have

$$\mathbf{A}(\mathbf{x})=\boldsymbol{\nabla}U(\mathbf{x}),\quad A_4(\mathbf{x})=0. \quad (11a)$$

In the simplest case for the $E1$ transition, we may put $U(\mathbf{x})$ simply proportional to z, to get the equation

$$\mathbf{x}_i\cdot\mathbf{A}(\tfrac{1}{2}(\mathbf{x}_1+\mathbf{x}_2))=U(\mathbf{x}_i),\quad(i=1,2), \quad (11b)$$

thus we have the important expression for the static exchange electric moment $V_{\text{exch}}^{(\pi)}(\mathbf{x}_1,\mathbf{x}_2)_\alpha$ given by

$$V_{\text{exch}}^{(\pi)}(\mathbf{x}_1,\mathbf{x}_2)_\alpha = i[V^{(\pi)}(\mathbf{x}_1,\mathbf{x}_2),\,\tau_\alpha^{(1)}U(\mathbf{x}_1)+\tau_\alpha^{(2)}U(\mathbf{x}_2)], \quad (12)$$

where $V^{(\pi)}(\mathbf{x}_1,\mathbf{x}_2)$ is the static one-pion-exchange potential obtained by putting $t_1=t_2$ in Eq. (10b).

For the higher multipoles El, we still have Eq. (11a). According to the mean value theorem, we have

$$U(\mathbf{x}_1)-U(\mathbf{x}_2)=(\mathbf{x}_1-\mathbf{x}_2)\cdot(\boldsymbol{\nabla}U(\mathbf{x}))_{\mathbf{x}=\mathbf{x}_0}, \quad (13a)$$

where $\mathbf{x}_0$ denotes an appropriate position between $\mathbf{x}_1$ and $\mathbf{x}_2$. If the variation of $U(\mathbf{x})$ within the range of nuclear potential is small, we have

$$(\mathbf{x}_1-\mathbf{x}_2)\cdot\mathbf{A}(\tfrac{1}{2}(\mathbf{x}_1+\mathbf{x}_2))\approx U(\mathbf{x}_1)-U(\mathbf{x}_2), \quad (13b)$$

which is the approximate relation corresponding to Eq. (11b). It is, then, clear that Eq. (12) holds approximately also for higher El.

Since Eq. (12) is based on the Ward identity Eq. (9b), the above discussion can be straightforwardly extended to the case where the nuclear potential is given by a superposition of the one-boson-exchange processes,

$$V=V^{(\pi)}+V^{(\rho)}+\cdots. \quad (14a)$$

Thus, corresponding to the sum of the diagrams in

Fig. 1, the static exchanged electric current is derived,

$$V_{\rm exch}(\mathbf{x}_1,\mathbf{x}_2)_\alpha=i[V(\mathbf{x}_1,\mathbf{x}_2),\sum_{i=1}^{2}\tau_\alpha^{(i)}U(\mathbf{x}_1)]. \quad (14b)$$

This is just the relation which we wanted to prove in the nonrelativistic limit.

Our next task is to estimate the magnitude of the commutator $[V,\sum_{i=1}^{A}\boldsymbol{\tau}^{(i)}z_i]$ in a nucleus. Here we would like to point out that a closely related subject has been already investigated thoroughly; the sum rule for the nuclear photoabsorption cross section of the electric dipole type. The best known form is due to Levinger-Bethe,[15]

$$\int\sigma(W)dW=\frac{2\pi^2\alpha}{M}\frac{NZ}{A}(1+0.8x). \quad (15a)$$

For the case $N=Z$ and the Serber force $x=\frac{1}{2}$, we have the value

$$0.015A(1+0.4)\ \text{MeV}. \quad (15b)$$

One can easily see that the commutator $[V,\sum\tau_3^{(i)}z_i]$ with which we are concerned just gives the second term of the right-hand side of Eq. (15a) or Eq. (15b). Since this term depends on the assumed nuclear model, several authors[17] have demonstrated that the coefficient might possibly be bigger than 0.8. However, Eq. (15) seems to be the best since it is consistent with experimental knowledge[18] (see Fig. 2 in Ref. 18) and also with the dispersion relation argument.[19] The Levinger-Bethe sum rule apparently suggests that the magnitude of the contribution of exchange electric currents is 40% of that of the convection currents. This statement is more carefully studied in the next section.

IV. EFFECTIVE SINGLE-BODY EXCHANGE ELECTRIC MOMENTS

The exchange electric moment $i[V,\sum_i\boldsymbol{\tau}^{(i)}\mathbf{r}_i]$ is a two-body operator. If we average one of the two coordinates over a closed Fermi sphere, the corresponding effective single-body form can be obtained. Let us assume that the nuclear potential has the form

$$V=-\sum_{(i,j)}V(r_{ij})\{1-x+xP_{ij}^M\}, \quad (16)$$

where P^M stands for the Majorana space-exchange operator. In Eq. (16) we omitted the spin-orbit interactions, because their contribution is already known to be small[11] and sensitively depends on the individual states. From Eq. (16) we obtain

$$i[V,\sum_i\boldsymbol{\tau}^{(i)}\mathbf{r}_i]=ix\sum_{(ij)}V(r_{ij})(\boldsymbol{\tau}^{(i)}-\boldsymbol{\tau}^{(j)})(\mathbf{r}_i-\mathbf{r}_j)P_{ij}^M. \quad (17)$$

The effective single-body operator $\mathbf{M}_{\rm eff}^{(1)}$ is defined by the expression,

$$\langle f|i[V,\sum\boldsymbol{\tau}^{(i)}\mathbf{r}_i]|i\rangle\equiv\langle f|\mathbf{M}_{\rm eff}^{(1)}|i\rangle. \quad (18)$$

Let us assume that the initial and final states are expressed by the Slater determinants. Since the exchange term may be neglected[14] in the first approximation, the effective operator $\mathbf{M}_{\rm eff}^{(1)}$ is given by

$$\mathbf{M}_{\rm eff}^{(1)}=\sum_i\boldsymbol{\tau}^{(i)}\mathbf{u}^{(i)}, \quad (19a)$$

where

$$\mathbf{u}^{(1)}\phi_1(\mathbf{r}_1)=(A-1)x\times\langle(\phi_2(\mathbf{r}_2),V(r_{12})(\mathbf{r}_1-\mathbf{r}_2)P_{12}^M\phi_1(\mathbf{r}_1)\phi_2(\mathbf{r}_2))\rangle_{\text{state }2}. \quad (19b)$$

For simplicity, the one-particle wave functions are assumed to have the form

$$\phi_i(\mathbf{r})=(\sqrt{\Omega})^{-1}e^{i\mathbf{p}_i\mathbf{r}}, \quad (20)$$

where $\Omega=(\frac{4}{3})\pi r_0^3A$ represents the nuclear volume. Then we obtain

$$\mathbf{u}^{(1)}=\frac{(A-1)}{\Omega\Omega_p}x(-i\nabla_{p1})\int d^3r\int d^3p_2\ V(r)e^{-i(\mathbf{p}_1-\mathbf{p}_2)\cdot\mathbf{r}}, \quad (21)$$

where $\Omega_p=\frac{4}{3}\pi p_F^3A$, p_F being the Fermi momentum. In calculating Eq. (21) we may carry out the integration with respect to $\mathbf{r}$ over the infinite volume if the range of potential $V(r)$ is much smaller than the nuclear radius. Now Eq. (21) can be expressed in a simple form in terms of the so-called Van Vleck potential[14] $\Phi(p_1)$ as follows:

$$\mathbf{u}^{(1)}=-\nabla_{p1}\Phi(p_1), \quad (22a)$$

where

$$\Phi(p_1)=\frac{(A-1)}{\Omega\Omega_p}x\int_{|p_2|<p_F}d^3p_2\int d^3r\ V(r)e^{i(\mathbf{p}_2-\mathbf{p}_1)\cdot\mathbf{r}}. \quad (22b)$$

It is well known that the existence of Majorana exchange forces leads to the velocity-dependent average potential, namely the Van Vleck potential, and it is one of the most important causes of the difference between the mass of an isolated nucleon and the effective mass of a nucleon in a nucleus.

For the purpose of illustration, let us calculate Eq. (22) by assuming the square-well potential, from which the original sum rule[15] in Eq. (15) was derived, namely

$$V(r)=V_0,\quad r<b$$
$$=0,\quad r>b, \quad (23a)$$

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[17] K. Okamoto, Phys. Rev. **116**, 428 (1959); J. W. Clark, Can. J. Phys. **39**, 385 (1961); K. Okamoto and K. Hasegawa, Progr. Theoret. Phys. (Kyoto) **28**, 137 (1962); T. P. Wang and J. W. Clark, Bull. Am. Phys. Soc. **10**, 71 (1965).

[18] J. H. Carver and D. C. Peaslee, Phys. Rev. **120**, 2155 (1960).

[19] M. Gell-Mann, M. L. Goldberger, and W. E. Thirring, Phys. Rev. **95**, 1612 (1954).

where

$$\begin{aligned} &\text{triplet range} \quad b=1.8\text{ F}, \\ &\qquad\text{depth} \quad V_0=1.50(\tfrac{1}{4}\pi^2)(1/Mb^2); \\ &\text{singlet range} \quad b=2.5\text{ F}, \\ &\qquad\text{depth} \quad V_0=0.95(\tfrac{1}{4}\pi^2)(1/Mb^2). \end{aligned} \tag{23b}$$

In this model Eq. (21) can be easily integrated analytically and we obtain

$$\Phi(p_1)=(A-1)\frac{(4\pi)^2V_0}{\Omega\Omega_p}\left[\frac{\sin p_1b\,\sin p_Fb}{p_1b}-\int_0^b\frac{\sin p_Fr\,\cos p_1r}{r}dr\right], \tag{24a}$$

and

$$\mathbf{u}^{(1)}=(A-1)\frac{(4\pi)^2V_0}{\Omega\Omega_p}\mathbf{p}_1\left[\frac{\sin p_1b\,\sin p_Fb}{p_F{}^3b}-\frac{\sin p_Fb\,\cos p_1b}{p_1{}^2}-\frac{1}{2p_1}\left\{\frac{\sin(p_1-p_F)b}{p_1-p_F}-\frac{\sin(p_1+p_F)b}{p_1+p_F}\right\}\right]. \tag{24b}$$

The numerical results are obtained by inserting Eq. (23) into Eq. (24). In Fig. 2 the ratio R of the effective exchange electric current $\mathbf{u}^{(1)}$ to the convection current $\mathbf{p}_1/M$ is shown for the cases of $r_0=1.2$, 1.37, and 1.5 and $x=\frac{1}{2}$. In Ref. 15, the values $r_0=1.37$ and 1.5 were adopted and the factors $(1+0.91x)$ and $(1+0.80x)$ were obtained in the sum rule. The latter value is usually quoted in references. Figure 2 shows that the nucleons having the momenta approximately equal to p_F are responsible for the above factors. Therefore we can conclude that the ratio of the exchange to convection currents is 40% or more insofar as the momenta close to p_F play an important part.

It is straightforward to extend the above argument on $E1$ to higher multipoles El. Instead of Eq. (17) we use

$$\begin{aligned}[V,\textstyle\sum_i \boldsymbol{\tau}^{(i)}(x_i+iy_i)^l] &= x\sum_{(i,j)} V(r_{ij})(\boldsymbol{\tau}^{(i)}-\boldsymbol{\tau}^{(i)}) \\ &\quad\times\{(x_i+iy_i)^l-(x_j+iy_j)^l\}P_{ij}{}^M \qquad (25a)\\ &\approx x\sum_{(i,j)} V(r_{ij})(\boldsymbol{\tau}^{(i)}-\boldsymbol{\tau}^{(j)})\{(x_i-x_j)+i(y_i-y_j)\} \\ &\quad\times l(x_i+iy_i)^{l-1}P_{ij}{}^M. \qquad (25b)\end{aligned}$$

At the step from Eq. (25a) to Eq. (25b), the fact needed was that $V(r)$ is of sufficiently short range, as noted in Sec. III. Then the rest of the calculation goes through precisely in the same way as in the case of $E1$. Thus the ratio R in Fig. 2 can be applied to general higher multipoles of El type.

V. CONCLUSIONS AND DISCUSSIONS

In Sec. III we proved that the electromagnetic and weak exchange electric currents as shown in Fig. 1 can

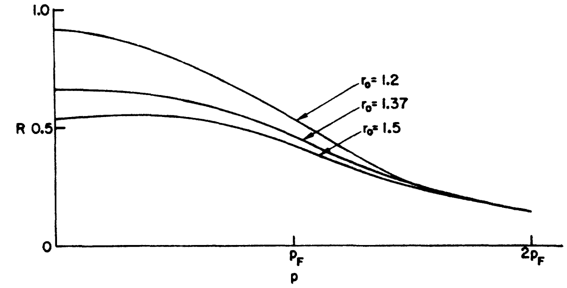

FIG. 2. The ratio R of "effective exchange electric current" to the convection current which was calculated in Sec. IV by assuming the square-well potential and the Fermi-gas model with a sphere of radius $r_0A^{1/3}$ with $r_0=1.2$, 1.37, and 1.5 F. In the figure, the Serber force $x=\frac{1}{2}$ is assumed.

be written in a compact form $i[V,\sum_i \boldsymbol{\tau}^{(i)}U_l(\mathbf{x}_i)]$ for general electric l-pole (El) transitions, in the nonrelativistic limit. If we use only the simplest diagrams in Fig. 1, the one-pion exchange, we obtain the exchange electric current $i[V^{(\pi)},\sum_i \boldsymbol{\tau}^{(i)}U_l(\mathbf{x}_i)]$ which clearly differs greatly from the total commutator[20] $i[V,\sum_i \boldsymbol{\tau}^{(i)}U_l(\mathbf{x}_i)]$.

A simple estimate of the above commutator can be obtained by looking at the Levinger-Bethe sum rule; the latter suggests that the $E1$ transition amplitude in a nucleus is enhanced by about 40%. In order to ascertain this point, we derived in Sec. IV the expression for the effective single-body operator from the two-body operator $[V,\sum_i \boldsymbol{\tau}^{(i)}\mathbf{r}_i]$. The ratio R of the exchange to convection currents turned out to be consistent with the Levinger-Bethe sum rule at the Fermi surface $p\approx p_F$.

The whole content of this note can be more clearly seen by introducing the idea of "effective mass" M^*. As is well known, an average potential inside a nucleus becomes velocity-dependent in the presence of exchange forces; the so-called Van Vleck potential is given by

$$V\approx\Phi(p)=-V_0+\tfrac{1}{2}V''\mathbf{p}^2. \tag{26a}$$

Then the effective mass M^* is defined as

$$\mathbf{p}^2/2M^*=\mathbf{p}^2/2M+\tfrac{1}{2}V''\mathbf{p}^2. \tag{26b}$$

The calculation in Sec. IV for the $E1$ transition amplitude is symbolically expressed as follows:

$$\frac{1}{M}\int\mathbf{p}+\int[V,i\mathbf{r}]\approx\frac{1}{M}\int\mathbf{p}+V''\int\mathbf{p}=\frac{1}{M^*}\int\mathbf{p}. \tag{27a}$$

The ratio R of the exchange to convection currents is now given by

$$R=MV''=(M/M^*)-1. \tag{27b}$$

Figure 2 shows that $R\gtrsim0.4$ at $p\approx p_F$ for $r_0<1.5$. On the other hand, the sum rule for the photoabsorption cross

[20] Our argument in this paper cannot be applied to the magnetic multipoles because, in that case, we have $\mathbf{A}(\mathbf{x})$ or $\mathbf{L}(\mathbf{x})\approx\text{curl}\mathbf{V}(\mathbf{x})$. Most recent review of exchange magnetic moments is given by H. Miyazawa, J. Phys. Soc. Japan (Tokyo) **19**, 1764 (1964).

section is obtained in a similar way:

$$\int \sigma(E1)dW = 2\pi^2\alpha\langle[(\mathbf{p}/M)+[V, i\mathbf{r}]\rangle_{00}$$
$$\approx 2\pi^2\alpha(\tfrac{1}{4}A)[(1/M)+V''] = (\pi^2\alpha A/2M^*). \quad (28)$$

Therefore, if we assume that M^* is a constant independent of momentum, an effective mass M^* appears both in the $E1$ transition amplitude Eq. (27a) and the $E1$ sum rule Eq. (28). In this approximation the Levinger-Bethe sum rule corresponds to $M^*=M/1.4$, and the same M^* applies to the enhancement of $E1$ transition by the factor $(1+R)=1.4$.

This value, $R\cong 0.4$, looks reasonable as can be seen from the following considerations:

(a) The relation $M^*=M/1.4$ is consistent with the effective mass $M^{*\prime}=0.4$–0.6 of the theory of the nuclear matter.[21,22] It is not clear though, whether the quantity $M^{*\prime}$ is equal to M^* in Eq. (26b) because the decrease of effective mass inside a nucleus is due to various causes.[21]

(b) The Levinger-Bethe sum rule, from which the value $R=0.4$ is suggested, is probably a good representation of our experimental knowledge.[18,19] In actual nuclear transitions the nucleons with monenta $p_1\cong p_F$ play the most important part, although the precise average value of p_1 should depend on the individual transition.

(c) The model assumed in Sec. IV is certainly quite unrealistic, but the general features of Fig. 2 seem to be more or less independent of the shape of the nuclear potential. As seen from Eq. (22a), our effective exchange current is only a derivative of the Van Vleck potential, which has been discussed in detail in Ref. 14. Of course it is possible to estimate numerically the commutator $i[V,\sum \boldsymbol{\tau}^{(i)}\mathbf{r}_i]$ for more realistic potential and wave functions, but we preferred not to do so because in practical problems we do not use this quantity at all if we make use of the Siegert theorem[8] or its extension to weak interactions.[7]

It is interesting to note that our value of 40% is about 3 times smaller than that of Ahrens and Feenberg but has the same sign as the latter semiempirical estimate. The reason why the Ahrens-Feenberg value is an overestimate has been discussed by Blin-Stoyle.[2] On the other hand, our estimate is much larger than Pursey's. The origin of disagreement between Pursey's estimate and ours lies in the following facts. Pursey expressed the Majorana exchange operator as $-P^\sigma P^\tau$ (P^σ and P^τ are the spin and isospin exchange operators, respectively), so that the exchange term which Pursey neglected just corresponds to the direct term which we treated in this note, as is clearly seen from the identity

$$-P^\sigma P^\tau(1-P^M P^\sigma P^\tau)=P^M(1-P^M P^\sigma P^\tau).$$

Therefore, Pursey's result should be added to our estimate. It should also be remembered that the state-dependent effect due to spin-orbit forces was omitted from our consideration. Moreover, as mentioned in Sec. IV, there remains some possibility that our value of 40% is an underestimate because we neglected correlations among nucleons.

If we assume that the numerical result, 40%, is exactly correct, we are led to the following equality for the beta transitions:

$$\langle f|[H_N-V_C,\, i^J \textstyle\sum_i r_i{}^J Y_{JM}(\hat{r}_i)\tau_\pm{}^{(i)}]|i\rangle_{\text{CVC}}$$
$$=1.4\{[L(2L+1)]^{1/2}\langle i^{J-1}r^{J-1}\boldsymbol{\alpha}\cdot\mathbf{T}_J{}^{J-1}\rangle\}_{\text{conventional}}, \quad (29a)$$

using here the notation of Ref. 7. As a special case of Eq. (29a), we obtain

$$\left(\int\boldsymbol{\alpha}\right)_{\text{CVC}}=1.4\left(\int\boldsymbol{\alpha}\right)_{\text{conventional}}. \quad (29b)$$

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[21] K. A. Brueckner, *The Many-Body Problem* (John Wiley & Sons, Inc., New York, 1958), p. 47.

[22] The application of $M^{*\prime}$ to the case of $E1$ beta transitions has been independently proposed by R. M. Spector (private communication).